CFTP/15-xx

August 10, 2017

Couplings in the C2HDM

Duarte Fontes,* Jorge C. Romão,† and João P. Silva‡
Departamento de Física and CFTP, Instituto Superior Técnico
Universidade de Lisboa, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

Rui Santos§
Centro de Física Teórica e Computacional, Faculdade de Ciências, Universidade de Lisboa, Campo Grande, Edifício C8 P-1749-016 Lisboa, Portugal

Abstract

PACS numbers: 14.60.Pq 12.60.Fr 14.60.St

*Electronic address: duartefontes@tecnico.ulisboa.pt
†Electronic address: jorge.romao@tecnico.ulisboa.pt
‡Electronic address: jpsilva@cftp.ist.utl.pt
§Electronic address: rasantos@fc.ul.pt

I. INTRODUCTION

We collect here the couplings of the neutral and charged Higgs bosons in the C2HDM in the Unitary gauge. The conventions are that all particles and momenta are incoming into the vertex. As for the SM subset we use the notation for the covariant derivatives contained in Romão and Silva [1], with all etas positive.

II. COUPLINGS

A. Couplings of neutral Higgs to Fermions

The couplings of neutral Higgs to fermions can be written in general for all the neutral Higgs bosons of the model h_i in a compact form

$$\mathcal{L} = -\frac{m_f}{v}\,\overline{f}(a_i + ib_i\gamma_5)f\,h_i \tag{1}$$

with the coefficients presented in table I.

	Type I	Type II	Lepton Specific	Flipped
Up	$\frac{R_{i2}}{s_\beta} - ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} - ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} - ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} - ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$
Down	$\frac{R_{i2}}{s_\beta} + ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i1}}{c_\beta} - is_\beta\frac{R_{i3}}{c_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} + ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i1}}{c_\beta} - is_\beta\frac{R_{i3}}{c_\beta}\gamma_5$
Leptons	$\frac{R_{i2}}{s_\beta} + ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$	$\frac{R_{i1}}{c_\beta} - is_\beta\frac{R_{i3}}{c_\beta}\gamma_5$	$\frac{R_{i1}}{c_\beta} - is_\beta\frac{R_{i3}}{c_\beta}\gamma_5$	$\frac{R_{i2}}{s_\beta} + ic_\beta\frac{R_{i3}}{s_\beta}\gamma_5$

Table I: Yukawa couplings of the scalar, h_i, in the form $a_i + ib_i\gamma_5$.

B. Couplings of charged Higgs to Fermions

The couplings of the charged Higgs to fermions can be expressed in the following Lagrangian

$$\mathcal{L} = \frac{\sqrt{2}}{v}\,\overline{\psi}_{d_i}\left[m_{\psi_{d_i}}\eta_L P_L + m_{\psi_{u_i}}\eta_R P_R\right]\psi_{u_i}H^- + \frac{\sqrt{2}}{v}\,\overline{\psi}_{u_i}\left[m_{\psi_{d_i}}\eta_L P_R + m_{\psi_{u_i}}\eta_R P_L\right]\psi_{d_i}H^+ \tag{2}$$

where $(\psi_{u_i}, \psi_{u_i}) = (u_i, d_i), (\nu_i, \ell_i)$, for quarks and leptons in an obvious notation. The couplings $\eta_{L,R}$ are given in table II. In these expressions we neglect the masses of the neutrinos, so in the last line in table II the zeros mean that the corresponding mass in Eq. (2) is zero.

	Type I	Type II	Lepton Specific	Flipped
η_L^q	$-\cot\beta$	$\tan\beta$	$-\cot\beta$	$\tan\beta$
η_R^q	$\cot\beta$	$\cot\beta$	$\cot\beta$	$\cot\beta$
η_L^ℓ	$-\cot\beta$	$\tan\beta$	$\tan\beta$	$-\cot\beta$
η_R^ℓ	0	0	0	0

Table II: Couplings of the charged Higgs boson to fermions.

C. Cubic interactions with Higgs

$$[h_1, h_1, h_1] \quad \text{See Ref. [2]} \tag{3}$$

$$[h_1, h_1, h_2] \quad \text{See Ref. [2]} \tag{4}$$

$$[h_1, h_2, h_2] \quad \text{See Ref. [2]} \tag{5}$$

$$[h_2, h_2, h_2] \quad \text{See Ref. [2]} \tag{6}$$

$$[h_1, h_1, h_3] \quad \text{See Ref. [2]} \tag{7}$$

$$[h_1, h_2, h_3] \quad \text{See Ref. [2]} \tag{8}$$

$$[h_2, h_2, h_3] \quad \text{See Ref. [2]} \tag{9}$$

$$[h_1, h_3, h_3] \quad \text{See Ref. [2]} \tag{10}$$

$$[h_2, h_3, h_3] \quad \text{See Ref. [2]} \tag{11}$$

$$[h_3, h_3, h_3] \quad \text{See Ref. [2]} \tag{12}$$

$$\begin{aligned}[h_1, H^+, H^-] = -i\, v\, [&\mathrm{Im}(\lambda_5) R_{13} \cos(\beta)\sin(\beta) \\ &+ R_{11}\cos(\beta)(\lambda_3\cos(\beta)^2 - (\mathrm{Re}(\lambda_5) - \lambda_1 + \lambda_4)\sin(\beta)^2) \\ &+ R_{12}(-(\mathrm{Re}(\lambda_5) - \lambda_2 + \lambda_4)\cos(\beta)^2\sin(\beta) + \lambda_3\sin(\beta)^3)\Big]\end{aligned} \tag{13}$$

$$\begin{aligned}[h_2, H^+, H^-] = -i\, v\, [&\mathrm{Im}(\lambda_5) R_{23} \cos(\beta)\sin(\beta) \\ &+ R_{21}\cos(\beta)(\lambda_3\cos(\beta)^2 - (\mathrm{Re}(\lambda_5) - \lambda_1 + \lambda_4)\sin(\beta)^2) \\ &+ R_{22}(-(\mathrm{Re}(\lambda_5) - \lambda_2 + \lambda_4)\cos(\beta)^2\sin(\beta) + \lambda_3\sin(\beta)^3)\Big]\end{aligned} \tag{14}$$

$$\begin{aligned}[h_3, H^+, H^-] = -i\, v\, [&\mathrm{Im}(\lambda_5) R_{33} \cos(\beta)\sin(\beta) \\ &+ R_{31}\cos(\beta)(\lambda_3\cos(\beta)^2 - (\mathrm{Re}(\lambda_5) - \lambda_1 + \lambda_4)\sin(\beta)^2)\end{aligned}$$

$$+R_{32}(-(\mathrm{Re}(\lambda_5) - \lambda_2 + \lambda_4)\cos(\beta)^2\sin(\beta) + \lambda_3\sin(\beta)^3)\Big] \tag{15}$$

To make contact with our previous notation [3], we note that

$$[h_i, H^+, H^-] \equiv i\,\lambda_i\,v \equiv i\,g_{h_i H^+ H^-} \tag{16}$$

where the λ_i or $g_{h_i H^+ H^-}$ can be read from Eqs. (13)-(15). The λ_i are in the notation used in Ref.[3] and should not be confused with the parameters in the potential.

D. Cubic interactions with Gauge bosons

One Gauge boson

$$[h_1, H^-, W^{+\mu}] = \frac{g}{2}\,(p_1 - p_-)^\mu\,(R_{13} - i\,R_{12}\cos(\beta) + i\,R_{11}\sin(\beta)) \tag{17}$$

$$[h_2, H^-, W^{+\mu}] = \frac{g}{2}\,(p_2 - p_-)^\mu\,(R_{23} - i\,R_{22}\cos(\beta) + i\,R_{21}\sin(\beta)) \tag{18}$$

$$[h_3, H^-, W^{+\mu}] = \frac{g}{2}\,(p_3 - p_-)^\mu\,(R_{33} - i\,R_{32}\cos(\beta) + i\,R_{31}\sin(\beta)) \tag{19}$$

$$[h_1, H^+, W^-_\mu] = \frac{g}{2}\,(p_1 - p_+)^\mu\,(R_{13} + i\,R_{12}\cos(\beta) - i\,R_{11}\sin(\beta)) \tag{20}$$

$$[h_2, H^+, W^-_\mu] = \frac{g}{2}\,(p_2 - p_+)^\mu\,(R_{23} + i\,R_{22}\cos(\beta) - i\,R_{21}\sin(\beta)) \tag{21}$$

$$[h_3, H^+, W^-_\mu] = \frac{g}{2}\,(p_3 - p_+)^\mu\,(R_{33} + i\,R_{32}\cos(\beta) - i\,R_{31}\sin(\beta)) \tag{22}$$

$$[h_1, h_2, Z^\mu] = -\frac{g}{2c_W}\,(p_1 - p_2)^\mu\,[(R_{13}R_{22} - R_{12}R_{23})\cos(\beta)$$

$$+(-R_{13}R_{21} + R_{11}R_{23})\sin(\beta)] \tag{23}$$

$$= \frac{g}{2c_W}\,(p_1 - p_2)^\mu\,[\cos(\beta)R_{31} + \sin(\beta)R_{32}] \tag{24}$$

$$[h_1, h_3, Z^\mu] = -\frac{g}{2c_W}\,(p_1 - p_3)^\mu\,[(R_{13}R_{32} - R_{12}R_{33})\cos(\beta)$$

$$+(-R_{13}R_{31} + R_{11}R_{33})\sin(\beta)] \tag{25}$$

$$= \frac{g}{2c_W}\,(p_1 - p_2)^\mu\,(-1)\,[\cos(\beta)R_{21} + \sin(\beta)R_{22}] \tag{26}$$

$$[h_2, h_3, Z^\mu] = -\frac{g}{2c_W}\,(p_2 - p_3)^\mu\,[(R_{23}R_{32} - R_{22}R_{33})\cos(\beta)$$

$$+(-R_{23}R_{31} + R_{21}R_{33})\sin(\beta)] \tag{27}$$

$$= \frac{g}{2c_W}\,(p_1 - p_2)^\mu\,[\cos(\beta) R_{11} + \sin(\beta) R_{12}] \tag{28}$$

$$[A^\mu, H^+, H^-] = -i\, e\,(p_+ - p_-)^\mu \tag{29}$$

$$[H^+, H^-, Z^\mu] = -i\,\frac{g}{2c_W}(c_W^2 - s_W^2)\,(p_+ - p_-)^\mu \tag{30}$$

Notice that we can also write a simplified version of the couplings $h_i h_j Z$ as

$$[h_i, h_j, Z^\mu] = \frac{g}{2c_W}\,(p_i - p_j)^\mu\,\epsilon_{ijk}\,[\cos(\beta) R_{k1} + \sin(\beta) R_{k2}] \tag{31}$$

Two Gauge bosons

$$[h_1, W_\mu^+, W_\nu^-] = i\, g M_W\, g_{\mu\nu}\,[R_{11}\cos(\beta) + R_{12}\sin(\beta)] \equiv i\, g M_W\, g_{\mu\nu}\; C_1 \tag{32}$$

$$[h_2, W_\mu^+, W_\nu^-] = i\, g M_W\, g_{\mu\nu}\,[R_{21}\cos(\beta) + R_{22}\sin(\beta)] \equiv i\, g M_W\, g_{\mu\nu}\; C_2 \tag{33}$$

$$[h_3, W_\mu^+, W_\nu^-] = i\, g M_W\, g_{\mu\nu}\,[R_{31}\cos(\beta) + R_{32}\sin(\beta)] \equiv i\, g M_W\, g_{\mu\nu}\; C_3 \tag{34}$$

$$[h_1, Z_\mu, Z_\nu] = i\,\frac{g M_Z}{c_W}\, g_{\mu\nu}\,[R_{11}\cos(\beta) + R_{12}\sin(\beta)] \equiv i\,\frac{g M_Z}{c_W}\, g_{\mu\nu}\; C_1 \tag{35}$$

$$[h_2, Z_\mu, Z_\nu] = i\,\frac{g M_Z}{c_W}\, g_{\mu\nu}\,[R_{21}\cos(\beta) + R_{22}\sin(\beta)] \equiv i\,\frac{g M_Z}{c_W}\, g_{\mu\nu}\; C_2 \tag{36}$$

$$[h_3, Z_\mu, Z_\nu] = i\,\frac{g M_Z}{c_W}\, g_{\mu\nu}\,[R_{31}\cos(\beta) + R_{32}\sin(\beta)] \equiv i\,\frac{g M_Z}{c_W}\, g_{\mu\nu}\; C_3 \tag{37}$$

where in Eqs. (32)-(34) and Eqs. (35)-(37) we used a shortened notation to make contact with our previous conventions [3].

Sometimes it is useful to write some couplings in a shortened version. For instance we can write

$$\begin{aligned}[h_i, H^-, W^{+\,\mu}] =& i\;(p_i - p_-)^\mu\;\frac{g}{2}\,(-i\, R_{i3} - R_{i2}\cos(\beta) + R_{i1}\sin(\beta))\\ \equiv& i\;(p_i - p_-)^\mu\; g_{h_i H^- W^+}\end{aligned} \tag{38}$$

where

$$g_{h_i H^- W^+} \equiv \frac{g}{2}\,(-i\, R_{i3} - R_{i2}\cos(\beta) + R_{i1}\sin(\beta)) \tag{39}$$

In a similar way

$$[h_i, H^+, W^{-\,\mu}] = -i\;(p_i - p_+)^\mu\; g_{h_i H^+ W^-} \tag{40}$$

where

$$g_{h_i H^+ W^-} = g^*_{h_i H^- W^+} \tag{41}$$

E. Cubic Interactions with Goldstones

Here we collect **some** cubic interactions with the Goldstone bosons.

$$
\begin{aligned}
[h_i, G^-, W^{+\,\mu}] &= -i\,\frac{g}{2}(p_i - p_-)^\mu (R_{i1}\cos\beta + R_{i2}\sin\beta) = i\,\frac{g}{2}(p_i - p_-)^\mu\ \mathrm{Im}(V^\dagger V)_{1,i+1} \\
&= -i\,(p_i - p_-)^\mu \frac{1}{2m_W} g_{hWW} \qquad (42)
\end{aligned}
$$

$$
\begin{aligned}
[h_i, G^+, W^{-\,\mu}] &= i\,\frac{g}{2}(p_i - p_+)^\mu (R_{i1}\cos\beta + R_{i2}\sin\beta) = -i\,\frac{g}{2}(p_i - p_+)^\mu\ \mathrm{Im}(V^\dagger V)_{1,i+1} \\
&= i\,(p_i - p_+)^\mu \frac{1}{2m_W} g_{hWW} \qquad (43)
\end{aligned}
$$

$$
\begin{aligned}
[h_i, G^0, Z^\mu] &= -\frac{g}{2c_W}(p_0 - p_i)^\mu (R_{i1}\cos\beta + R_{i2}\sin\beta) \\
&= \frac{g}{2c_W}(p_0 - p_i)^\mu\ \mathrm{Im}(V^\dagger V)_{1,i+1} \qquad (44)
\end{aligned}
$$

$$
[h_i, G^0, G^0] = -i\frac{g}{2}\frac{m_{h_i}^2}{m_W}(R_{i1}\cos\beta + R_{i2}\sin\beta) = i\frac{g}{2}\frac{m_{h_i}^2}{m_W}\ \mathrm{Im}(V^\dagger V)_{1,i+1} \qquad (45)
$$

$$
[h_i, G^+, G^-] = -i\frac{g}{2}\frac{m_{h_i}^2}{m_W}(R_{i1}\cos\beta + R_{i2}\sin\beta) = i\frac{g}{2}\frac{m_{h_i}^2}{m_W}\ \mathrm{Im}(V^\dagger V)_{1,i+1} \qquad (46)
$$

$$
[G^0, H^+, H^-] = 0 \qquad (47)
$$

$$
\begin{aligned}
[G^0, h_i, h_j] &= i\frac{g}{2m_W}(m_{h_i}^2 - m_{h_j}^2)\,[\sin\beta\,(R_{i1}R_{j3} - R_{i3}R_{j1}) + \cos\beta\,(R_{i3}R_{j2} - R_{i2}R_{j3})] \\
&= -i\frac{g}{2m_W}(m_{h_i}^2 - m_{h_j}^2)\ \mathrm{Im}(V^\dagger V)_{i+1,j+1} \qquad (48)
\end{aligned}
$$

$$
\begin{aligned}
[h_i, G^+, H^-] &= -\frac{g}{2}\frac{m_{H^+}^2 - m_{h_i}^2}{m_W}\,(R_{i3} - iR_{i2}\cos\beta + iR_{i1}\sin\beta) \\
&= i\frac{g}{2}\frac{m_{H^+}^2 - m_{h_i}^2}{m_W}\,\left[U^\dagger V\right]_{2,i+1} \\
&= -i\,\frac{m_{H^+}^2 - m_{h_i}^2}{m_W} g_{h_i H^- W^+} \qquad (49)
\end{aligned}
$$

$$
[h_i, G^-, H^+] = -i\,\frac{m_{H^+}^2 - m_{h_i}^2}{m_W} g_{h_i H^+ W^-} \qquad (50)
$$

$$
[W^{\mp\mu}, G^\pm, A^\nu] = i\,e\,m_W g^{\mu\nu} \qquad (51)
$$

$$
[G^+, G^-, A^\mu] = -i\,e\,(p_+ - p_-)^\mu \qquad (52)
$$

$$
[G^0, G^+, W^{-\,\mu}] = \frac{g}{2}\,(p_0 - p_+)^\mu \qquad (53)
$$

$$[G^0, G^-, W^{-\,\mu}] = \frac{g}{2}\,(p_0 - p_-)^\mu \tag{54}$$

$$[G^+, G^-, Z^\mu] = -i\frac{g}{2c_W}\cos 2\theta_W\,(p_+ - p_-)^\mu \tag{55}$$

$$[G^\pm, W^{\mp\mu}, Z^\nu] = -i\,g\,m_Z\sin^2\theta_W g^{\mu\nu} \tag{56}$$

where

$$g_{hWW} = g\,m_W\,(R_{i1}\cos\beta + R_{i2}\sin\beta) = -gm_W\;\mathrm{Im}(V^\dagger V)_{1,i+1} \tag{57}$$

F. Quartic Interactions with Goldstones

Here we collect **some** quartic interactions with the Goldstone bosons.

$$\begin{aligned}[W^{\pm\mu}, G^\mp, h_i, A^\nu] &= i\frac{eg}{2}\,(R_{i1}\cos\beta + R_{i2}\sin\beta)\,g^{\mu\nu} \\ &= i\,\frac{e}{2m_W}\,g_{hWW}\,g^{\mu\nu}\end{aligned} \tag{58}$$

$$\begin{aligned}[G^-, G^+, H^+, H^-] = -i\,\Big[&(\lambda_3+\lambda_4)(\cos^4\beta + \sin^4\beta) \\ &-2\sin^2\beta\cos^4\beta(2\mathrm{Re}\,(\lambda_5) - \lambda_1 - \lambda_2 + \lambda_3 + \lambda_4)\Big]\end{aligned} \tag{59}$$

$$[G^-, G^+, G^-, G^+] = -2i\,\Big[\lambda_1\cos^4\beta + \lambda_2\sin^4\beta + 2\sin^2\beta\cos^2\beta\,(\mathrm{Re}(\lambda_5) + \lambda_3 + \lambda_4)\Big] \tag{60}$$

$$[G^-, G^-, H^+, H^+] = -2i\,\Big[\lambda_5\cos^4\beta + \lambda_5^*\sin^4\beta + \sin^2\beta\cos^4\beta(\lambda_1 + \lambda_2 - 2(\lambda_3+\lambda_4)\Big] \tag{61}$$

G. Quartic interactions with Higgs

$$[h_1, h_1, h_1, h_1] \quad \text{See Ref. [2]} \tag{62}$$

$$[h_1, h_1, h_1, h_2] \quad \text{See Ref. [2]} \tag{63}$$

$$[h_1, h_1, h_2, h_2] \quad \text{See Ref. [2]} \tag{64}$$

$$[h_1, h_2, h_2, h_2] \quad \text{See Ref. [2]} \tag{65}$$

$$[h_2, h_2, h_2, h_2] \quad \text{See Ref. [2]} \tag{66}$$

$$[h_1, h_1, h_1, h_3] \quad \text{See Ref. [2]} \tag{67}$$

$$[h_1, h_1, h_2, h_3] \quad \text{See Ref. [2]} \tag{68}$$

$$[h_1, h_2, h_2, h_3] \quad \text{See Ref. [2]} \tag{69}$$

$$[h_2, h_2, h_2, h_3] \quad \text{See Ref. [2]} \tag{70}$$

$$[h_1, h_1, h_3, h_3] \quad \text{See Ref. [2]} \tag{71}$$

$$[h_1, h_2, h_3, h_3] \quad \text{See Ref. [2]} \tag{72}$$

$$[h_2, h_2, h_3, h_3] \quad \text{See Ref. [2]} \tag{73}$$

$$[h_1, h_3, h_3, h_3] \quad \text{See Ref. [2]} \tag{74}$$

$$[h_2, h_3, h_3, h_3] \quad \text{See Ref. [2]} \tag{75}$$

$$[h_3, h_3, h_3, h_3] \quad \text{See Ref. [2]} \tag{76}$$

$$[h_1, h_1, H^+, H^-] \quad \text{See Ref. [2]} \tag{77}$$

$$[h_1, h_2, H^+, H^-] \quad \text{See Ref. [2]} \tag{78}$$

$$[h_2, h_2, H^+, H^-] \quad \text{See Ref. [2]} \tag{79}$$

$$[h_1, h_3, H^+, H^-] \quad \text{See Ref. [2]} \tag{80}$$

$$[h_2, h_3, H^+, H^-] \quad \text{See Ref. [2]} \tag{81}$$

$$[h_3, h_3, H^+, H^-] \quad \text{See Ref. [2]} \tag{82}$$

$$[H^+, H^+, H^-, H^-] \quad \text{See Ref. [2]} \tag{83}$$

H. Quartic interactions with Gauge bosons

$$[A_\mu, A_\nu, H^+, H^-] = 2\, i\, e^2\, g_{\mu\nu} \tag{84}$$

$$[A_\mu, h_1, H^-, W^+_\nu] = -\frac{eg}{2}\left[R_{13} - i\, R_{12}\cos(\beta) + i\, R_{11}\sin(\beta)\right]\, g_{\mu\nu} \tag{85}$$

$$[A_\mu, h_2, H^-, W^+_\nu] = -\frac{eg}{2}\left[R_{23} - i\, R_{22}\cos(\beta) + i\, R_{21}\sin(\beta)\right]\, g_{\mu\nu} \tag{86}$$

$$[A_\mu, h_3, H^-, W^+_\nu] = -\frac{eg}{2}\left[R_{33} - i\, R_{32}\cos(\beta) + i\, R_{31}\sin(\beta)\right]\, g_{\mu\nu} \tag{87}$$

$$[A_\mu, h_1, H^+, W^-_\nu] = \frac{eg}{2}\left[R_{13} + i\, R_{12}\cos(\beta) - i\, R_{11}\sin(\beta)\right]\, g_{\mu\nu} \tag{88}$$

$$[A_\mu, h_2, H^+, W^-_\nu] = \frac{eg}{2}\left[R_{23} + i\, R_{22}\cos(\beta) - i\, R_{21}\sin(\beta)\right]\, g_{\mu\nu} \tag{89}$$

$$[A_\mu, h_3, H^+, W^-_\nu] = \frac{eg}{2}\left[R_{33} + i\, R_{32}\cos(\beta) - i\, R_{31}\sin(\beta)\right]\, g_{\mu\nu} \tag{90}$$

$$[H^+, H^-, W^+_\mu, W^-_\nu] = i\,\frac{g^2}{2}\, g_{\mu\nu} \tag{91}$$

$$[h_1, h_1, W^+_\mu, W^-_\nu] = i\,\frac{g^2}{2}\,g_{\mu\nu} \tag{92}$$

$$[h_1, h_2, W^+_\mu, W^-_\nu] = 0 \tag{93}$$

$$[h_2, h_2, W^+_\mu, W^-_\nu] = i\,\frac{g^2}{2}\,g_{\mu\nu} \tag{94}$$

$$[h_1, h_3, W^+_\mu, W^-_\nu] = 0 \tag{95}$$

$$[h_2, h_3, W^+_\mu, W^-_\nu] = 0 \tag{96}$$

$$[h_3, h_3, W^+_\mu, W^-_\nu] = i\,\frac{g^2}{2}\,g_{\mu\nu} \tag{97}$$

$$[A_\mu, H^+, H^-, Z_\nu] = i\,\frac{eg}{c_W}(c_W^2 - s_W^2)g_{\mu\nu} \tag{98}$$

$$[h_1, H^-, W^+_\mu, Z_\nu] = \frac{e^2}{2c_W}\left[R_{13} - i\,R_{12}\cos(\beta) + i\,R_{11}\sin(\beta)\right]\,g_{\mu\nu} \tag{99}$$

$$[h_2, H^-, W^+_\mu, Z_\nu] = \frac{e^2}{2c_W}\left[R_{23} - i\,R_{22}\cos(\beta) + i\,R_{21}\sin(\beta)\right]\,g_{\mu\nu} \tag{100}$$

$$[h_3, H^-, W^+_\mu, Z_\nu] = \frac{e^2}{2c_W}\left[R_{33} - i\,R_{32}\cos(\beta) + i\,R_{31}\sin(\beta)\right]\,g_{\mu\nu} \tag{101}$$

$$[h_1, H^+, W^-_\mu, Z_\nu] = -\frac{e^2}{2c_W}\left[R_{13} + i\,R_{12}\cos(\beta) - i\,R_{11}\sin(\beta)\right]\,g_{\mu\nu} \tag{102}$$

$$[h_2, H^+, W^-_\mu, Z_\nu] = -\frac{e^2}{2c_W}\left[R_{23} + i\,R_{22}\cos(\beta) - i\,R_{21}\sin(\beta)\right]\,g_{\mu\nu} \tag{103}$$

$$[h_3, H^+, W^-_\mu, Z_\nu] = -\frac{e^2}{2c_W}\left[R_{33} + i\,R_{32}\cos(\beta) - i\,R_{31}\sin(\beta)\right]\,g_{\mu\nu} \tag{104}$$

$$[H^+, H^-, Z_\mu, Z_\nu] = i\,\frac{g^2}{2c_W^2}(c_W^2 - s_W^2)^2 g_{\mu\nu} \tag{105}$$

$$[h_1, h_1, Z_\mu, Z_\nu] = i\,\frac{g^2}{2c_W^2}g_{\mu\nu} \tag{106}$$

$$[h_1, h_2, Z_\mu, Z_\nu] = 0 \tag{107}$$

$$[h_2, h_2, Z_\mu, Z_\nu] = i\,\frac{g^2}{2c_W^2}g_{\mu\nu} \tag{108}$$

$$[h_1, h_3, Z_\mu, Z_\nu] = 0 \tag{109}$$

$$[h_2, h_3, Z_\mu, Z_\nu] = 0 \tag{110}$$

$$[h_3, h_3, Z_\mu, Z_\nu] = i\, \frac{g^2}{2c_W^2} g_{\mu\nu} \tag{111}$$

We can use Eq. (39) to write,

$$[A_\mu, h_i, H^-, W_\nu^+] = -i\, e\, g_{h_i H^- W^+}\, g_{\mu\nu} \tag{112}$$

$$[A_\mu, h_i, H^+, W_\nu^-] = -i\, e\, g^*_{h_i H^- W^+}\, g_{\mu\nu} \tag{113}$$

III. FORMULAS FOR THE DECAY WIDTHS WITH CHARGED HIGGS

We collect here relevant formulas for the decays of neutral and charged Higgs.

A. Decays of Neutral Higgs

1. Decay of Neutral Higgs into Charged Higgs

We get for the decay $h_i \to H^+ + H^-$,

$$\Gamma(h_i \to H^+ + H^-) = \frac{g^2_{h_i H^+ H^-}}{16\pi m_{h_i}} \sqrt{1 - \frac{4m^2_{H^+}}{m^2_{h_i}}} \tag{114}$$

where $g_{h_i H^+ H^-}$ is given in Eq. (16).

2. Decay of Neutral Higgs into Charged Higgs and W boson

We also have the decay into charged Higgs and W bosons,

$$\Gamma(h_i \to W^+ + H^-) = |g_{h_i H^+ W^-}|^2 \frac{m^3_{h_i}}{16\pi\, m_W^2} \lambda^3(m_W^2, m^2_{H^+}; m^2_{h_i}) \tag{115}$$

where

$$\lambda(x, y; z) = \sqrt{\left(1 - \frac{x}{z} - \frac{y}{z}\right)^2 - 4\, \frac{xy}{z^2}} \tag{116}$$

and the coupling $g_{h_i H^+ W^-}$ is given in Eq. (39). One should note that in the literature there are different definitions of the λ function. Ours it is dimensionless, and it includes the square root. We also divide by the last argument, and the arguments are the squares of the masses.

3. Decay of Neutral Higgs into Charged Higgs and off-shell W boson

We also want the decay $h_i \to W^{*+} + H^-$ with an off-shell W^* boson. This can be done using the method explained in Ref.[4]. In fact we can write the result, for massless decay products of the W^*, in the form,

$$\Gamma(h_i \to W^{*+} H^-) = \frac{1}{\pi} \int d\Delta^2 \frac{\Gamma_W M_W}{|D(\Delta^2|^2} \Gamma_0(\Delta) \tag{117}$$

where

$$\Gamma_0(\Delta) = |g_{h_i H^+ W^-}|^2 \frac{m_{h_i}^3}{16\pi\, m_W^2} \lambda^3(\Delta^2, m_{H^+}^2; m_{h_i}^2) \tag{118}$$

is the on-shell decay for a boson with $k^2 = \Delta^2$, and the denominator is from the off-shell propagator,

$$|D(\Delta^2)|^2 = (\Delta^2 - m_W^2)^2 + m_W^2 \Gamma_W^2 \tag{119}$$

The integral in Eq. (117) in not easy to give in analytic form, but it can be done numerically. For this we introduce the following variables,

$$x_W = \frac{m_W}{m_{h_i}}, \quad x_H = \frac{m_{H^+}}{m_{h_i}}, \quad \delta = \frac{\Gamma_W}{m_{h_i}}, \quad y = \frac{2\, E_{H^-}}{m_{h_i}} \tag{120}$$

where E_{H^-} is the the energy of the recoiling H^-. Using

$$E_{H^-} = \frac{m_{h_i}^2 + m_{H^+}^2 - \Delta^2}{2 m_{h_i}} \tag{121}$$

we finally get

$$\Gamma(h_i \to W^{*+} + H^-) = \frac{\Gamma_W\, m_{h_i}}{16\pi^2 m_W} |g_{h_i H^+ W^-}|^2 R(x_H, x_W, \delta) \tag{122}$$

where the function R is given by

$$R(x_H, x_w, \delta) = \int_{2x_H}^{1+x_H^2} dy\ \frac{\left(y^2 - 4x_H^2\right)^{3/2}}{(1 + x_H^2 - x_W^2 - y)^2 + x_W^2 \delta^2} \tag{123}$$

B. Decay of Charged Higgs

1. Decay of Charged Higgs into Fermions

For the decay into quarks, like $H^+ \to t + \overline{b}$ we have, without radiative corrections,

$$\Gamma(H^+ \to t + \overline{b}) = \frac{3 G_F m_{H^+}^3}{4\pi\sqrt{2}} |V_{tb}|^2 \lambda(m_t^2, m_b^2; m_{H^+}^2) \Big[(1 - x_t - x_b) \left({\eta_R^q}^2 x_t + {\eta_L^q}^2 x_b \right) - 4 x_t x_b\, \eta_L^q \eta_R^q \Big] \tag{124}$$

where

$$x_t = \frac{m_t^2}{m_{H^+}^2}, \quad x_b = \frac{m_b^2}{m_{H^+}^2} \tag{125}$$

and the η's can be read from TableII. For the other families the same formula can be used with obvious modifications. For the decays into charged leptons, we neglect the neutrino masses and obtain

$$\Gamma(H^+ \to \ell^+ + \overline{\nu}_\ell) = \frac{G_F m_{H^+}}{4\pi\sqrt{2}}\, m_\ell^2 \left(1 - \frac{m_\ell^2}{m_{H^+}^2}\right)^2 {\eta_L^\ell}^{\,2} \tag{126}$$

2. Decay of Charged Higgs into Gauge Bosons

We have

$$\Gamma(H^+ \to W^+ + h_i) = |g_{h_i H^+ W^-}|^2 \frac{m_{H^+}^3}{16\pi\, m_W^2} \lambda^3(m_W^2, m_{h_i}^2; m_{H^+}^2) \tag{127}$$

3. Decay of Charged Higgs into off-shell Gauge Bosons

We also want the decay $H^+ \to W^{*+} + h_i$ with an off-shell W^* boson. This again can be done using the method explained in Ref.[4]. In fact we can write the result, for massless decay products of the W^*, in the form,

$$\Gamma(H^+ \to W^{*+} + h_i) = \frac{1}{\pi} \int d\Delta^2 \frac{\Gamma_W M_W}{|D(\Delta^2|^2} \Gamma_0(\Delta) \tag{128}$$

where

$$\Gamma_0(\Delta) = |g_{h_i H^+ W^-}|^2 \frac{m_{H^+}^3}{16\pi\, m_W^2} \lambda^3(\Delta^2, m_{h_i}^2; m_{H^+}^2) \tag{129}$$

is the on-shell decay for a boson with $k^2 = \Delta^2$, and the denominator is from the off-shell propagator,

$$|D(\Delta^2)|^2 = (\Delta^2 - m_W^2)^2 + m_W^2 \Gamma_W^2 \tag{130}$$

We notice that these expressions are just what we had in Eq. (117) and Eq. (118), with the exchange of $h_i \leftrightarrow H^-$. The integral will have the same form if we introduce the variables,

$$x_W = \frac{m_W}{m_{H^+}}, \quad x_h = \frac{m_{h_i}}{m_{H^+}}, \quad \delta = \frac{\Gamma_W}{m_{H^+}}, \quad y = \frac{2\, E_{h_i}}{m_{H^+}} \tag{131}$$

where now E_{h_i} is the the energy of the recoiling h_i. In the same way we get,

$$\Gamma(H^+ \to W^{*+} + h_i) = \frac{\Gamma_W\, m_{H^+}}{16\pi^2 m_W} |g_{h_i H^+ W^-}|^2 R(x_h, x_w, \delta) \tag{132}$$

where the function R function was given before in Eq. (123). The agreement can be seen in Fig. 1

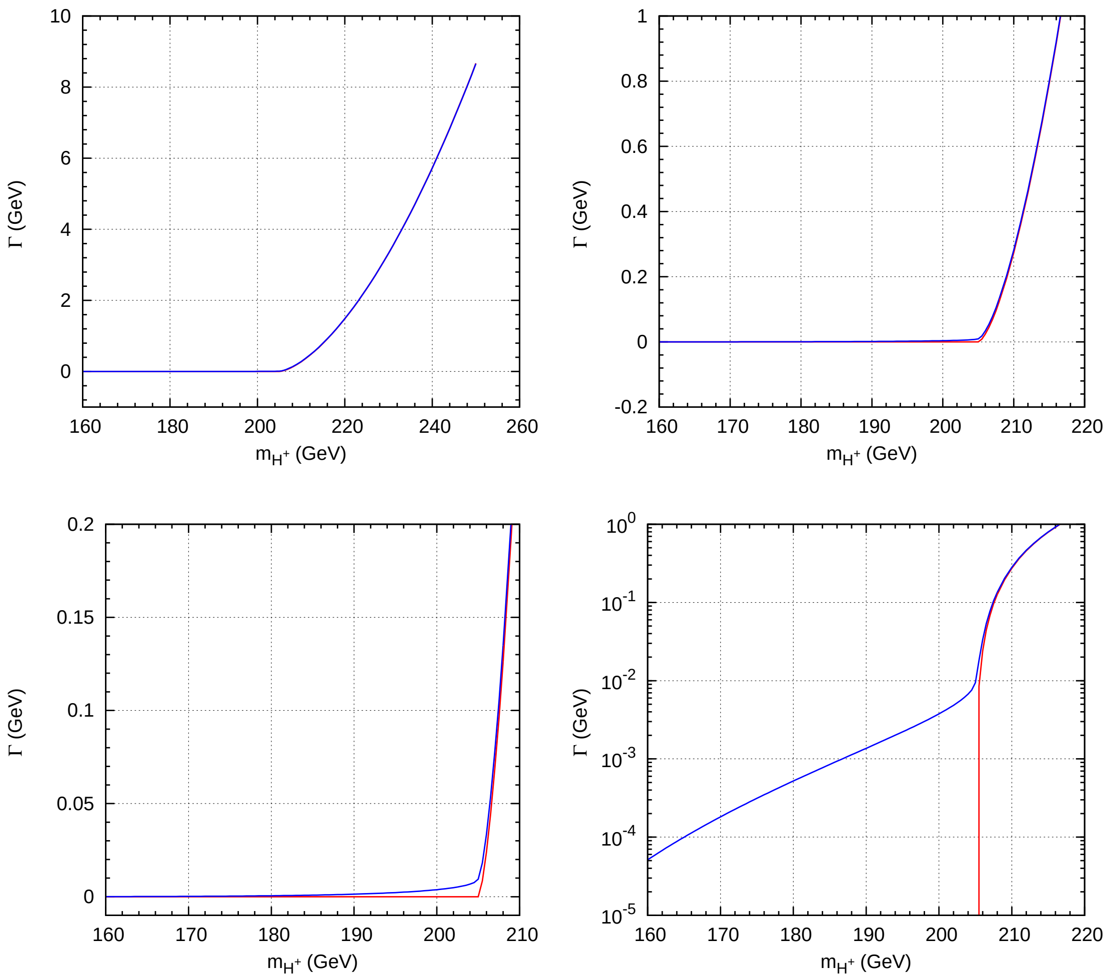

Figure 1: Comparison of the off-shell formula with the on-shell case. In blue we use Eq. (122) and in red Eq. (127). We put the dimensionless coupling $|g_{h_i H^+ W^-}|^2 = 1$.

IV. COMPARISON WITH THE FORMULAS IN HDECAY

As there were some differences for the off-shell decays with W and Z bosons we address the comparison here.

A. Decay $h_i -> H^- + W^{*+}$

We start from our result, Eqs. (122) and (123), which we write here again,

$$\Gamma(h_i \to W^{*+} + H^-) = \frac{\Gamma_W \, m_{h_i}}{16\pi^2 m_W} |g_{h_i H^+ W^-}|^2 R(x_H, x_W, \delta_W) \tag{133}$$

where the function R is given by

$$R(x_H, x_w, \delta) = \int_{2x_H}^{1+x_H^2} dy \, \frac{\left(y^2 - 4x_H^2\right)^{3/2}}{(1 + x_H^2 - x_W^2 - y)^2 + x_W^2 \delta_W^2} \tag{134}$$

To make contact with the expression in HDecay, Eq. (131) of Spira's hep-ph/9705337, we first write our couplings in terms of the definition of HDecay (which we denote with an hat)

$$g_{h_i H^+ W^-} = \frac{g}{2} \hat{g}_{h_i H^+ W^-} \tag{135}$$

and use the theoretical formula for the width of the W,

$$\Gamma_W^{\text{th}} = \frac{3 G_F m_W^3}{2\pi \sqrt{2}} \tag{136}$$

to obtain

$$\begin{aligned} \Gamma(h_i \to W^{*+} + H^-) =& |\hat{g}_{h_i H^+ W^-}|^2 \frac{3 G_F \, g^2 m_W^2 m_{h_i}}{128 \pi^3 \sqrt{2}} R(x_H, x_W, \delta_W) && (137) \\ =& |\hat{g}_{h_i H^+ W^-}|^2 \frac{3 G_F^2 m_W^4 m_{h_i}}{32 \pi^3} R(x_H, x_W, \delta_W) && (138) \\ =& |\hat{g}_{h_i H^+ W^-}|^2 \frac{9 G_F^2 m_W^4 m_{h_i}}{16 \pi^3} \frac{1}{6} R(x_H, x_W, \delta_W) && (139) \end{aligned}$$

which leads to the identification (in the limit $\delta_W \to 0$)

$$G_{H^\pm W^\mp} \equiv \frac{1}{6} R(x_H, x_W, \delta_W \to 0) \tag{140}$$

I have verified numerically that this is correct. Now, why do we still have a small difference when comparing the outputs? The reason is that I have used the experimental width of the W and in the above formula we used the theoretical result. Putting numbers,

$$\Gamma_W^{\text{exp}} = 2.0843 \, \text{GeV}, \quad \Gamma_W^{\text{th}} = 2.04339741 \, \text{GeV}, \quad \frac{\Gamma_W^{\text{exp}}}{\Gamma_W^{\text{th}}} = 1.0200169 \tag{141}$$

In conclusion, my result for the off-shell decays of the W should be higher by that factor. I verified that this is indeed the case.

B. Decay $h_i -> h_j + Z$

1. *The expression*

Our result now reads

$$\Gamma(h_i \to h_j + Z) = \frac{\Gamma_Z \, m_{h_i}}{16 \pi^2 m_Z} |g_{h_i h_j Z}|^2 R(x_{h_j}, x_Z, \delta_Z) \tag{142}$$

where the function R was given before and

$$x_{h_j} = \frac{m_{h_j}}{m_{h_i}}, \quad x_Z = \frac{m_Z}{m_{h_i}}, \quad \delta_Z = \frac{\Gamma_Z}{m_{h_i}} \ . \tag{143}$$

Now we put the couplings in the HDecay notation

$$g_{h_i h_j Z} = \frac{g}{2\, c_W} \hat{g}_{h_i h_j Z} \tag{144}$$

and use the theoretical width of the Z,

$$\Gamma_Z^{\rm th} = \frac{9 G_F m_Z^3}{3\sqrt{2}\pi} \delta'_Z \tag{145}$$

where

$$\delta'_Z = \frac{7}{12} - \frac{10}{9} s_W^2 + \frac{40}{27} s_W^4 \tag{146}$$

Putting everything together we have

$$\begin{aligned}
\Gamma(h_i \to h_j + Z) =& |\hat{g}_{h_i h_j Z}|^2 \ \frac{9 G_F m_Z^2 g^2}{192 \pi^3 \sqrt{2} c_W^2} \delta'_Z \ R(x_{h_j}, x_Z, \delta_Z) && (147)\\
=& |\hat{g}_{h_i h_j Z}|^2 \ \frac{9 G_F^2 m_Z^4}{48 \pi^3} \delta'_Z \ R(x_{h_j}, x_Z, \delta_Z) && (148)\\
=& |\hat{g}_{h_i h_j Z}|^2 \ \frac{9 G_F^2 m_Z^4}{8 \pi^3} \delta'_Z \ \frac{1}{6} R(x_{h_j}, x_Z, \delta_Z) && (149)
\end{aligned}$$

Now if we use the identification of Eq. (140) we agree with Eq. (130) of Spira without the extra $\frac{1}{2}$ factor.

2. *The theoretical and experimental width*

As before our way of doing, using the experimental width will differ slightly from HDecay. Putting numbers we get

$$\Gamma_Z^{\rm exp} = 2.494270\,\text{GeV}, \quad \Gamma_Z^{\rm th} = 2.4415103\,\text{GeV}, \quad \frac{\Gamma_Z^{\rm exp}}{\Gamma_Z^{\rm th}} = 1.02161 \tag{150}$$

3. *Numerical precision*

For Point 3 we agree after we introduce the above corrections: factor of two and the ratio of the experimental width over the theoretical formula. However for Point 2, even after we introduce the previous corrections we are not in complete agreement for the decay $h_3 \to h_2 + Z$. Also if I implement the Spira formula I get a different result from Maggie. I believe that this is due to numerical problems with the formula for HVH (or G_{ij}) for X very close to 1. We are getting a number of order 10^{-16} subtracting numbers close to 1.

In my way of doing things the integral is very well behaved in the whole interval, in fact the function is almost constant. I checked with Mathematica and then I get a result very close to Maggie's, which is off by the factor of two. It turns out that my compiler gets less precise there.

In summary:

- For $h_i - > H^- + W^{*+}$ we agree except for the correction factor for the widths. Note that here the charged Higgs is some GeVs away from h_3, so no precision problems.

- $h_i - > h_j + Z$ I think that the factor 1/2 should not be there. Also we need to correct for the ratio of widths. But for the case where he masses are almost degenerate I get problems with G_{ij} and HVH. For Point 3 where this does not occur precision is not a problem and with the above corrections we agree.

---

[1] J. C. Romao and J. P. Silva, Int. J. Mod. Phys. **A27**, 1230025 (2012), [1209.6213].

[2] D. Fontes, M. Muhlleitner, J. C. Romao, R. Santos and J. P. Silva, (2016), In preparation. Se web page at http://porthos.tecnico.ulisboa.pt/arXiv/C2HDM/.

[3] D. Fontes, J. C. Romao and J. P. Silva, JHEP **12**, 043 (2014), [1408.2534].

[4] J. C. Romao and S. Andringa, Eur. Phys. J. **C7**, 631 (1999), [hep-ph/9807536].