Introduction to Supersymmetry 
& 
Implications from the LHC Data

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5 de Dezembro de 2012
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- SUSY Algebra and Representations
- MSSM
  - Particle Content
  - Superpotential
  - Masses
  - Couplings
- Bounds: LEP, Tevatron, · · ·
  - Higgs
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- Conclusions
Books

- *Supersymmetry and Supergravity*, Julius Wess and Jonathan Bagger. \((-, +, +, +)\)
- *Supersymmetric Gauge Field Theory and String Theory*, David Bailin and Alexander Love. \((+, −, −, −)\)
- *Supersymmetry in Particle Physics*, Ian Aitchison. \((+, −, −, −)\)

Other Texts

- *The search for supersymmetry: Probing physics beyond the standard model*, H. E. Haber and G. L. Kane, Phys. Rep. 117 (1985) 75. \((+, −, −, −)\)
- *BUSSTEPP Lectures on Supersymmetry*, José M. Figueroa-O’Farrill, hep-ph/0109172. \((-, +, +, +)\)
- *The Minimal Supersymmetric Standard Model*, Jorge C. Romão, (see my homepage). \((+, −, −, −)\)
Although there is not yet direct experimental evidence for supersymmetry (SUSY), there are many theoretical arguments indicating that SUSY might be of relevance for physics around the 1 TeV scale.

The most commonly invoked theoretical arguments for SUSY are:

- Interrelates matter fields (leptons and quarks) with force fields (gauge and/or Higgs bosons).
- As local SUSY implies gravity (supergravity) it could provide a way to unify gravity with the other interactions.
- As SUSY and supergravity have fewer divergences than conventional field theories, the hope is that it could provide a consistent (finite) quantum gravity theory.
- SUSY can help to understand the mass problem, in particular solve the naturalness problem (and in some models even the hierarchy problem) if SUSY particles have masses \( \leq \mathcal{O}(1\text{TeV}) \).
As the SM is not asymptotically free, at some energy scale \( \Lambda \), the interactions must become strong indicating the existence of new physics. Candidates for this scale: \( M_X \approx O(10^{16} \text{ GeV}) \) in GUT’s or the Planck scale \( M_P \approx O(10^{19} \text{ GeV}) \).

The only consistent way to give masses to the gauge bosons and fermions is through the Higgs mechanism involving at least one spin zero Higgs boson.

Although the Higgs boson mass is not fixed by the theory, a value much bigger than \( < H^0 > \approx G_F^{-1/2} \approx 250 \text{ GeV} \) would imply that the Higgs sector would be strongly coupled making it difficult to understand why we are seeing an apparently successful perturbation theory at low energies.

The one loop radiative corrections to the Higgs boson mass

\[
\delta m_H^2 = O\left( \frac{\lambda^2}{16\pi^2} \right) \Lambda^2 + \ldots
\]

would be too large if \( \Lambda \) is identified with \( \Lambda_{\text{GUT}} \) or \( \Lambda_{\text{Planck}} \).
SUSY cures this problem in the following way. If SUSY were exact, radiative corrections to the scalar masses squared would be absent because the contribution of fermion loops exactly cancels against the boson loops.

Therefore if SUSY is broken, as it must, we should have

$$\delta m_H^2 = \mathcal{O}\left(\frac{\lambda}{16\pi^2}\right) (m_F^2 - m_B^2) \ln \frac{\Lambda}{m_B} + \cdots$$

We conclude that

SUSY provides a solution for the the naturalness problem if the masses of the superpartners are around $\mathcal{O}(1 \text{ TeV})$. This is the main reason behind all the phenomenological interest in SUSY.
The Poincare Algebra

The Poincaré group is made up of the Lorentz group plus the translations. We denote by $J_{\mu\nu}$ the generators of the Lorentz group and by $P_\mu$ the generators of the translations. The algebra is defined by,

\[
[J_{\mu\nu}, J_{\rho\sigma}] = i (g_{\nu\rho}J_{\mu\sigma} - g_{\nu\sigma}J_{\mu\rho} - g_{\mu\rho}J_{\nu\sigma} + g_{\mu\sigma}J_{\nu\rho})
\]

\[
[P_\alpha, J_{\mu\nu}] = i (g_{\mu\alpha}P_\nu - g_{\nu\alpha}P_\mu)
\]

\[
[P_\mu, P_\nu] = 0
\]

One can show that

\[
[P^2, J_{\mu\nu}] = [P^2, P_\mu] = 0
\]

\[
[W^2, J_{\mu\nu}] = [W^2, P_\mu] = [W^2, P^2] = 0
\]

where

\[
W_\mu = -\frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma
\]

is the Pauli-Lubanski vector operator.
The SUSY generators carry Spin 1/2 and obey the following algebra

\[
\begin{align*}
\{Q_\alpha, Q_\beta\} &= 0 \\
\{\overline{Q}_\dot{\alpha}, \overline{Q}_\dot{\beta}\} &= 0 \\
\{Q_\alpha, \overline{Q}_\dot{\beta}\} &= 2 (\sigma^\mu)_{\alpha \dot{\beta}} P_\mu
\end{align*}
\]

where

\[
\sigma^\mu \equiv (1, \sigma^i) \quad \text{and} \quad \overline{\sigma}^\mu \equiv (1, -\sigma^i)
\]

and \(\alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2\) (Weyl 2–component spinor notation).
The commutation relations with the generators of the Poincaré group

\[
[P^\mu, Q_\alpha] = 0 \quad [J^{\mu\nu}, Q_\alpha] = -i (\sigma^{\mu\nu})_\alpha^\beta Q_\beta
\]

One can easily derive that the two invariants of the Poincaré group,

\[
P^2 = P_\alpha P^\alpha \quad W^2 = W_\alpha W^\alpha \quad W_\mu = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma
\]

\[
P^2 \ket{m, s} = m^2 \ket{m, s} \quad W^2 \ket{m, s} = -m^2 s (s + 1) \ket{m, s}
\]

where \( W^\mu \) is the Pauli–Lubanski vector operator, are no longer invariants of the Super Poincaré group:

\[
[Q_\alpha, P^2] = 0 \quad [Q_\alpha, W^2] \neq 0
\]

Irreducible multiplets will have particles of the same mass but different spin.
Simple Results from the Algebra

Number of Bosons = Number of Fermions

$$Q_\alpha |B> = |F> \quad (-1)^{N_F} |B> = |B>$$

$$Q_\alpha |F> = |B> \quad (-1)^{N_F} |F> = -|F>$$

where $$(-1)^{N_F}$$ is the fermion number of a given state. Then we obtain

$$Q_\alpha (-1)^{N_F} = -(-1)^{N_F} Q_\alpha$$

Using this relation we can show that

$$Tr \left[ (-1)^{N_F} \{ Q_\alpha, \overline{Q}_{\dot{\alpha}} \} \right] = Tr \left[ (-1)^{N_F} Q_\alpha \overline{Q}_{\dot{\alpha}} + (-1)^{N_F} \overline{Q}_{\dot{\alpha}} Q_\alpha \right]$$

$$= Tr \left[ -Q_\alpha (-1)^{N_F} \overline{Q}_{\dot{\alpha}} + Q_\alpha (-1)^{N_F} \overline{Q}_{\dot{\alpha}} \right] = 0$$

But we also have

$$Tr \left[ (-1)^{N_F} \{ Q_\alpha, \overline{Q}_{\dot{\alpha}} \} \right] = Tr \left[ (-1)^{N_F} 2 \sigma^\mu_{\alpha\dot{\alpha}} P_\mu \right]$$

$$Tr \left[ (-1)^{N_F} \right] = \# \text{Bosons} - \# \text{Fermions} = 0$$
In the rest frame

\[ \{ Q_\alpha, Q_\dot{\alpha} \} = 2m \delta_{\alpha \dot{\alpha}} \]

This algebra is similar to the algebra of the spin 1/2 creation and annihilation operators. Choose \( |\Omega\rangle \) such that

\[ Q_1 |\Omega\rangle = Q_2 |\Omega\rangle = 0 \]

Then we have 4 states

If \( J_3 |\Omega\rangle = j_3 |\Omega\rangle \)

Two bosons and two fermions states separated by one half unit of spin.
SUSY Representations: Massless case

If \( m = 0 \) then we can choose \( P^\mu = (E, 0, 0, E) \). In this frame

\[
\{Q_\alpha, \overline{Q}_\dot{\alpha}\} = M_{\alpha\dot{\alpha}}
\]

where the matrix \( M \) takes the form

\[
M = \begin{pmatrix}
0 & 0 \\
0 & 4E
\end{pmatrix}
\]

Then \( \{Q_2, \overline{Q}_2\} = 4E \) all others vanish.

We have then just **two** states \(|\Omega\rangle; \overline{Q}_2 |\Omega\rangle\)

If \( J_3 |\Omega\rangle = \lambda |\Omega\rangle \)

<table>
<thead>
<tr>
<th>State</th>
<th>( J_3 )</th>
<th>Eigenvalue</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>\Omega\rangle)</td>
<td></td>
</tr>
<tr>
<td>( \overline{Q}_2</td>
<td>\Omega\rangle)</td>
<td></td>
</tr>
</tbody>
</table>

Two states, one fermion, one boson separated by one half unit of spin.
Superfields for Massless Particles

- **Chiral Superfields:** Spin $0 + \text{Spin } \frac{1}{2}$

  $\Phi = \Phi(\phi, \chi_L)$

  - $\phi$ Complex Scalar: 2 d.o.f
  - $\chi_L$ Chiral Fermion: 2 d.o.f (on-shell)

- **Vector Superfields:** Spin $\frac{1}{2} + \text{Spin } 1$

  $V = V(\lambda, W^\mu)$

  - $\lambda$ Chiral Fermion: 2 d.o.f
  - $W^\mu$ Massless Vector: 2 d.o.f (on-shell)

**Superpartners:**
- $\phi$ superpartner of the fermion: sfermion
- $\lambda$ superpartner of gauge field: gaugino
Gauge Fields

We want to have gauge fields for the gauge group
\( G = SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \). Therefore we will need three vector superfields (or vector supermultiplets) \( \hat{V}_i \) with the following components:

\[
\begin{align*}
\hat{V}_1 & \equiv (\lambda', W_1^\mu) \quad \rightarrow \quad U_Y(1) \\
\hat{V}_2 & \equiv (\lambda^a, W_2^{\mu a}) \quad \rightarrow \quad SU_L(2), \quad a = 1, 2, 3 \\
\hat{V}_3 & \equiv (\tilde{g}^b, W_3^{\mu b}) \quad \rightarrow \quad SU_c(3), \quad b = 1, \ldots, 8
\end{align*}
\]

where \( W_i^\mu \) are the gauge fields and \( \lambda', \lambda \) and \( \tilde{g} \) are the \( U_Y(1) \) and \( SU_L(2) \) gauginos and the gluino, respectively.
## Leptons

As each chiral multiplet only describes one helicity state, we will need two chiral multiplets for each charged lepton (We will assume that the neutrinos do not have mass).

<table>
<thead>
<tr>
<th>Supermultiplet</th>
<th>$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{L}_i \equiv (\tilde{L}, L)_i$</td>
<td>$(1, 2, -\frac{1}{2})$</td>
</tr>
<tr>
<td>$\hat{R}_i \equiv (\tilde{\ell}_R, \ell_L^c)_i$</td>
<td>$(1, 1, 1)$</td>
</tr>
</tbody>
</table>

Each helicity state corresponds to a complex scalar and we have that $\hat{L}_i$ is a doublet of $SU_L(2)$

$$\tilde{L}_i = \begin{pmatrix} \tilde{\nu}_{Li} \\ \tilde{\ell}_{Li} \end{pmatrix}; \quad L_i = \begin{pmatrix} \nu_{Li} \\ \ell_{Li} \end{pmatrix}$$
Quarks

The quark supermultiplets are given in the Table. The supermultiplet $\tilde{Q}_i$ is also a doublet of $SU_L(2)$, that is

<table>
<thead>
<tr>
<th>Supermultiplet</th>
<th>$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{Q}_i \equiv (\tilde{Q}, Q)_i$</td>
<td>$(3, 2, \frac{1}{6})$</td>
</tr>
<tr>
<td>$\tilde{D}_i \equiv (\tilde{d}_R, d^c_L)_i$</td>
<td>$(3, 1, \frac{1}{3})$</td>
</tr>
<tr>
<td>$\tilde{U}_i \equiv (\tilde{u}_R, u^c_L)_i$</td>
<td>$(3, 1, -\frac{2}{3})$</td>
</tr>
</tbody>
</table>

$\tilde{Q}_i = \begin{pmatrix} \tilde{u}_{Li} \\ \tilde{d}_{Li} \end{pmatrix}$ ; $Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$
Higgs Bosons

Finally the Higgs sector. In the MSSM we need at least two Higgs doublets. This is in contrast with the SM where only one Higgs doublet is enough to give masses to all the particles. The reason can be explained in two ways. Either the need to cancel the anomalies, or the fact that, due to the analyticity of the superpotential, we have to have two Higgs doublets of opposite hypercharges to give masses to the up and down type of quarks.

<table>
<thead>
<tr>
<th>Supermultiplet</th>
<th>$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{H}_1 \equiv (H_1, \tilde{H}_1)$</td>
<td>$(1, 2, -\frac{1}{2})$</td>
</tr>
<tr>
<td>$\hat{H}_2 \equiv (H_2, \tilde{H}_2)$</td>
<td>$(1, 2, +\frac{1}{2})$</td>
</tr>
</tbody>
</table>
Most discussions of SUSY phenomenology assume R-Parity conservation where,

\[ R_P = (-1)^{2J + 3B + L} \]

This is the case of the MSSM. It implies:

- SUSY particles are pair produced.
- Every SUSY particle decays into another SUSY particle.
- There is a LSP that it is stable (\[ \cancel{E} \] signature).

But this is just an ad hoc assumption without a deep justification. We will see later what are the consequences of non conservation of R-Parity.
Like in any gauge theory we have

\[
\mathcal{L}_{\text{kin}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i \bar{\lambda}^a \sigma^\mu D_\mu \lambda^a + (D_\mu \phi)^\dagger D^\mu \phi + i \bar{\chi} \sigma^\mu D_\mu \chi
\]

where the field strength \( F_{\mu\nu}^a \) is given by

\[
F_{\mu\nu}^a = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - gf^{abc} W^b_\mu W^c_\nu
\]

and \( f^{abc} \) are the structure constants of the gauge group \( G \). The covariant derivative is

\[
D_\mu = \partial_\mu + ig W^a_\mu T^a
\]

One should note that \( \chi \) and \( \lambda \) are left handed chiral spinors.
For a non Abelian gauge group $G$ we have the usual self–interactions, (cubic and quartic), of the gauge bosons with themselves. But we have a new interaction of the gauge bosons with the gauginos. In two component spinor notation it reads

$$\mathcal{L}_{\lambda \lambda W} = ig f_{abc} \lambda^a \sigma^\mu \overline{\lambda}^b W^c_{\mu} + h.c.$$

where $f_{abc}$ are the structure constants of the gauge group $G$ and the matrices $\sigma^\mu$ are the equivalent of the $\gamma$ matrices in two component language.
Interactions of the Gauge and Matter Multiplets

In the usual non Abelian gauge theories we have the interactions of the gauge bosons with the fermions and scalars of the theory. In the supersymmetric case we also have interactions of the gauginos with the fermions and scalars of the chiral matter multiplet. The general form, in two component spinor notation is,

\[ \mathcal{L}_\Phi W = -g T^a_{ij} W^a_\mu \left( \bar{\chi}_i \sigma^\mu \chi_j + i \phi_i^* \partial_\mu \phi_j \right) + g^2 \left( T^a T^b \right)_{ij} W^a_\mu W^{\mu b} \phi_i^* \phi_j + ig \sqrt{2} T^a_{ij} \left( \lambda^a \chi_j \phi_i^* - \bar{\lambda}^a \bar{\chi}_i \phi_j \right) \]

where the new interactions of the gauginos with the fermions and scalars are given in the last term.
Self Interactions of the Matter Multiplet

These correspond in non supersymmetric gauge theories both to the Yukawa interactions and to the scalar potential. In supersymmetric gauge theories we have less freedom to construct these terms. The first step is to construct the superpotential \( W \). This must be a gauge invariant polynomial function of the scalar components of the chiral multiplet \( \Phi_i \), that is \( \phi_i \). It does not depend on \( \phi_i^* \).

In order to have renormalizable theories the degree of the polynomial must be at most three.

The Yukawa interactions are

\[
L_{Yukawa} = -\frac{1}{2} \left[ \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \chi_i \chi_j + \left( \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \right)^* \bar{\chi_i} \bar{\chi}_j \right]
\]

and the scalar potential is

\[
V_{scalar} = \frac{1}{2} D^a D^a + F_i F_i^*
\]

where

\[
F_i = \frac{\partial W}{\partial \phi_i}, \quad D^a = g \phi_i^* T^a_{ij} \phi_j
\]
The Superpotential and SUSY Breaking Lagrangian

The MSSM Lagrangian is specified by the R–parity conserving superpotential \( W \)

\[
W = \varepsilon_{ab} \left[ h_U^{ij} \hat{Q}_i^a \hat{U}_j^b \hat{H}_2^b + h_D^{ij} \hat{Q}_i^b \hat{D}_j^a \hat{H}_1^a + h_E^{ij} \hat{L}_i^a \hat{R}_j^b \hat{H}_1^a - \mu \hat{H}_1^a \hat{H}_2^b \right]
\]

where \( i, j = 1, 2, 3 \) are generation indices, \( a, b = 1, 2 \) are \( SU(2) \) indices, and \( \varepsilon \) is a completely antisymmetric \( 2 \times 2 \) matrix, with \( \varepsilon_{12} = 1 \). The coupling matrices \( h_U, h_D \) and \( h_E \) will give rise to the usual Yukawa interactions needed to give masses to the leptons and quarks.

If it were not for the need to break SUSY the number of parameters involved would be less than in the SM.
The most general SUSY soft breaking is

\[
- \mathcal{L}_{SB} = M_{ij}^Q \tilde{Q}^a_i \tilde{Q}^a_j + M_{ij}^U \tilde{U}^i_i \tilde{U}^a_j + M_{ij}^D \tilde{D}^i_i \tilde{D}^a_j + M_{ij}^L \tilde{L}^a_i \tilde{L}^a_j + M_{ij}^R \tilde{R}^a_i \tilde{R}^a_j
+ m_{H_1}^2 H_1^a H_1^a + m_{H_2}^2 H_2^a H_2^a - \left[ \frac{1}{2} M_s \lambda_s \lambda_s + \frac{1}{2} M \lambda \lambda + \frac{1}{2} M' \lambda' \lambda' + h.c. \right]
+ \varepsilon_{ab} \left[ A^i U^b U^j H_2^a + A^i D^b D^j H_1^a + A^i E^b E^j R_i H_1^a - B \mu H_1^a H_2^b \right]
\]

**Parameter Counting**

<table>
<thead>
<tr>
<th>Theory</th>
<th>Gauge Sector</th>
<th>Fermion Sector</th>
<th>Higgs Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>SM</td>
<td>(e, g, \alpha_s)</td>
<td>(h_U, h_D, h_E)</td>
<td>(\mu^2, \lambda)</td>
</tr>
<tr>
<td>MSSM</td>
<td>(e, g, \alpha_s)</td>
<td>(h_U, h_D, h_E)</td>
<td>(\mu)</td>
</tr>
<tr>
<td>Broken MSSM</td>
<td>(e, g, \alpha_s)</td>
<td>(h_U, h_D, h_E)</td>
<td>(\mu, M_1, M_2, M_3, A_U, A_D, A_E, B, m_{H_2}^2, m_{H_1}^2, m_Q^2, m_U^2, m_D^2, m_L^2, m_R^2)</td>
</tr>
</tbody>
</table>
The number of independent parameters can be reduced if we impose some further constraints. The most popular is the MSSM coupled to $N = 1$ Supergravity (mSUGRA).

$$A_t = A_b = A_\tau \equiv A,$$

$$m^2_{H_1} = m^2_{H_2} = M_L^2 = M_R^2 = m_0^2, M_Q^2 = M_U^2 = M_D^2 = m_0^2,$$

$$M_3 = M_2 = M_1 = M_{1/2}$$

### Parameter Counting

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Conditions</th>
<th>Free Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_t, h_b, h_\tau, v_1, v_2$</td>
<td>$m_W, m_t, m_b, m_\tau$</td>
<td>$\tan \beta = v_2/v_1$</td>
</tr>
<tr>
<td>$A, B, m_0, M_{1/2}, \mu$</td>
<td>$t_i = 0, i = 1, 2$</td>
<td>$A, m_0, M_{1/2}, \text{sign}(\mu)$</td>
</tr>
</tbody>
</table>
| Total = 10 | Total = 6 | Total = 4"1"

It is remarkable that with so few parameters we can get the correct values for the parameters, in particular $m^2_{H_2} < 0$. For this to happen the top Yukawa coupling has to be large which we know to be true.
The charged gauginos mix with the charged higgsinos giving the so–called charginos. In a basis where $\psi^+ = (-i\lambda^+, \tilde{H}_2^+)$ and $\psi^- = (-i\lambda^-, \tilde{H}_1^-)$, the chargino mass terms in the Lagrangian are

$$L_m = -\frac{1}{2}(\psi^+, \psi^-) \begin{pmatrix} 0 & M^T_C \\ M_C & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + h.c.$$ 

where the chargino mass matrix is given by

$$M_C = \begin{bmatrix} M_2 & \frac{1}{\sqrt{2}}gv_2 \\ \frac{1}{\sqrt{2}}gv_1 & \mu \end{bmatrix}$$

and $M_2$ is the $SU(2)$ gaugino soft mass. We can write this as

$$M_C = \begin{bmatrix} M_2 & \sqrt{2}m_W \sin \beta \\ \sqrt{2}m_W \cos \beta & \mu \end{bmatrix}$$
The Neutralino Mass Matrix

The neutral gauginos mix with the neutral higgsinos giving the so-called neutralinos. In the basis $\psi^0 T = (-i\lambda', -i\lambda^3, H_1^1, \tilde{H}_2^2)$ the neutral fermions mass terms in the Lagrangian are given by

$$\mathcal{L}_m = -\frac{1}{2}(\psi^0)^T M_N \psi^0 + h.c.$$ 

where the neutralino mass matrix is

$$M_N = \begin{bmatrix}
M_1 & 0 & -\frac{1}{2}g'v_1 & \frac{1}{2}g'v_2 \\
0 & M_2 & \frac{1}{2}gv_1 & -\frac{1}{2}gv_2 \\
-\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & -\mu \\
\frac{1}{2}g'v_2 & -\frac{1}{2}gv_2 & -\mu & 0
\end{bmatrix}$$

and $M_1, M_2$ are the gaugino soft mass.
Neutral Higgs Mass Matrices

\[
M_{S^0}^2 = \begin{pmatrix} \tan \beta & -1 \\ -1 & \cot \beta \end{pmatrix} B_\mu + \begin{pmatrix} \cot \beta & -1 \\ -1 & \tan \beta \end{pmatrix} \frac{1}{2} m_Z^2 \sin^2 \beta
\]

with masses

\[
m_{h,H}^2 = \frac{1}{2} \left[ m_A^2 + m_Z^2 + \sqrt{(m_A^2 + m_Z^2)^2 - 4 m_A^2 m_Z^2 \cos 2\beta} \right]
\]

\[
M_{P^0}^2 = \begin{pmatrix} \tan \beta & -1 \\ -1 & \cot \beta \end{pmatrix} B_\mu
\]

with mass

\[
m_A^2 = \frac{B_\mu}{\sin 2\beta}
\]

Sum Rule

\[
m_h^2 + m_H^2 = m_A^2 + m_Z^2
\]

\[
m_h < m_A < m_H \\
m_h < m_Z < m_H
\]
Higgs Boson Mass: Standard Model

In the Standard Model

\[ \mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\lambda}{4} \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 \]

where

\[ \Phi = \begin{pmatrix} \varphi^+ \\ v + H + i\varphi_Z \\ \sqrt{2} \end{pmatrix} \]

Therefore

\[ \mathcal{L}_{\text{Higgs}} = \partial_\mu H \partial^\mu H - \frac{\lambda v^2}{4} H^2 \]

or

\[ M_H = \sqrt{\frac{\lambda}{2}} v \quad \Rightarrow \quad v = 246 \text{ GeV fixed but } \lambda \text{ free.} \]

\[ M_H \text{ free.} \]
Higgs Boson Mass: Radiative corrections

As the top mass is very large there are important radiative corrections to the Higgs boson mass. The most important are:

\[
m_h^2 = m_h^{(0)^2} + \frac{3g^2}{16\pi^2m_W^2}\frac{m_t^4}{\sin^2\beta} \ln \left( \frac{\tilde{m}_t^2 \tilde{m}_{t^2}^2}{m_t^4} \right)
\]
Example of Spectra

**SPS1a**

$m_0 = 100\text{GeV}, m_{1/2} = 250\text{GeV}$

$A_0 = \ -100\text{GeV}, \tan \beta = 10, \mu > 0$

**SPS1b**

$m_0 = 200\text{GeV}, m_{1/2} = 400\text{GeV}$

$A_0 = 0, \tan \beta = 30, \mu > 0$
### Couplings in the MSSM

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Name</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gauge Self-Interaction</strong></td>
<td>VVV, VVVVV</td>
<td><strong>4-Point Coupling</strong></td>
<td>VVff</td>
</tr>
<tr>
<td>3-Point Gauge Coupling</td>
<td>Vff, Vff̃, ṼX̃X, VH̃H, VGH, VGG</td>
<td>3-Point Higgs Coupling</td>
<td>HHVV, HGVV, GGVV</td>
</tr>
<tr>
<td>3-Point Higgs Coupling</td>
<td>Hff, Hff̃, H̃X̃X, HVV</td>
<td>3-Point Goldstone Coupling</td>
<td>fff, fff̃, fff̃f̃f̃f̃f̃</td>
</tr>
<tr>
<td>Other 3-Point</td>
<td>fff̃</td>
<td>Goldstone-Higgs Interaction</td>
<td>HHG, HGG, HHHG, HHGG, HGGG, GGGG</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Ghost</td>
<td>fff̃</td>
</tr>
</tbody>
</table>

*Note: The table lists the couplings and interactions in the MSSM framework.*
Examples of New Couplings: Conserving R-Parity

Summary
Motivation
SUSY Algebra
MSSM
Couplings
• couplings
• New vertices
• Unification
LEP Results
LHC Results
Conclusions

Rule: Change any two lines into the superpartners.
Gauge Couplings Unification

**Summary**

**Motivation**

**SUSY Algebra**

**MSSM**

**Couplings**
- couplings
- New vertices
- Unification

**LEP Results**

**LHC Results**

**Conclusions**

**Standard Model**

\[
\frac{1}{\alpha_i} = \frac{g_i^2}{4\pi} \begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} \rightarrow \begin{pmatrix} U_Y(1) \\ SU_L(2) \\ SU_c(3) \end{pmatrix}
\]

**Supersymmetry**

**Electroweak Theory**

**Quantum Chromodynamics**
Higgs Boson

- High $Q^2$ except $m_t$
- $68\%$ CL
- $m_t$ (Tevatron)
- Excluded

$M_H = 114^{+69}_{-45} \text{ GeV}$

$M_H < 260 \text{ GeV} @ 95\% CL$

$M_H > 114.4 \text{ GeV}$
Summary

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LEP Results

- Higgs
- Charginos ...
- Neutralinos
- Dark Matter

LHC Results

Conclusions

Limits on Charginos, Sneutrinos & Sleptons

\[ \sqrt{s} = 183-208 \text{ GeV} \quad \text{ADLO} \]

\[ \tan \beta = 2 \quad \mu = -200 \text{ GeV} \]

Excluded at 95\% C.L.

\[ M_{\chi} (\text{GeV}) \]

\[ M_{\tilde{l}} (\text{GeV}/c^2) \]

Limits on \( \chi^\pm, \tilde{l}^\pm \approx 100 \text{ GeV} \).
Limits on Neutralinos

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Limits on $\chi^0 \geq 50$ GeV. More model dependent.
Neutralino as Dark Matter

Blue region: Neutralino consistent with dark matter.
The Large Hadron Collider (LHC)

<table>
<thead>
<tr>
<th>Beams</th>
<th>Energy</th>
<th>Luminosity</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEP</td>
<td>e+ e-</td>
<td>200 GeV</td>
</tr>
<tr>
<td>LHC</td>
<td>p p</td>
<td>14 TeV</td>
</tr>
<tr>
<td></td>
<td>Pb Pb</td>
<td>1312 TeV</td>
</tr>
</tbody>
</table>

From LEP to LHC
Collisions at LHC

Proton-Proton (2835 x 2835 bunches)

- Protons/bunch: $10^{11}$
- Beam energy: 7 TeV ($7 \times 10^{12}$ eV)
- Luminosity: $10^{34}$ cm$^{-2}$ s$^{-1}$

- Crossing rate: 40 MHz
- Collisions: $10^7 - 10^9$ Hz

Selection of 1 in 10,000,000,000,000
Higgs to 2 photons ($M_H < 140$ GeV)

$H^0 \rightarrow \gamma\gamma$ is the most promising channel if $M_H$ is in the range 80 – 140 GeV. The high performance PbWO$_4$ crystal electromagnetic calorimeter in CMS has been optimized for this search. The $\gamma\gamma$ mass resolution at $M_{\gamma\gamma} \sim 100$ GeV is better than 1%, resulting in a S/B of $\approx 1/20$.
Higgs to 4 leptons ($140 < M_H < 700$ GeV)

In the $M_H$ range $130 - 700$ GeV the most promising channel is $H^0 \rightarrow ZZ^* \rightarrow 2\ell^+ 2\ell^-$ or $H^0 \rightarrow ZZ \rightarrow 2\ell^+ 2\ell^-$. The detection relies on the excellent performance of the muon chambers, the tracker and the electromagnetic calorimeter. For $M_H \leq 170$ GeV a mass resolution of $\sim 1$ GeV should be achieved with the combination of the 4 Tesla magnetic field and the high resolution of the crystal calorimeter.
Gluinos and squarks can be searched for in various channels with leptons + $E_t^{\text{miss}}$ + jets and discovered for masses up to ~ 2.2 TeV. Sleptons can be discovered for masses up to ~ 350 GeV. The region of parameter space $0.15 < \Omega h^2 < 0.4$ — where LSP would be the Cold Dark Matter particle — is contained well within the explorable region.
Many extensions of the SM have been developed over the past decades:
- Supersymmetry
- Extra-Dimensions
- Technicolor(s)
- Little Higgs
- No Higgs
- GUT
- Hidden Valley
- Leptoquarks
- Compositeness
- 4th generation (t', b')
- LRSM, heavy neutrino
- etc...

(for illustration only)

A complex 2D problem
Experimentally, a signature standpoint makes a lot of sense:
- Practical
- Less model-dependent
- Important to cover every possible signature
First Results from the LHC in 2011

1. SUSY: Jets + Missing $E_T$

- Exclude up to $\sim 1$ TeV for $m$(squark) = $m$(gluino)

---

Jorge C. Romão

Introduction to SUSY – 45
Situation in early 2012

Very precise measurement of $M_W = 80.390 \pm 0.016$ GeV, driven mainly by the Tevatron.

Much of the SM Higgs range had been ruled out by 2011 LHC running.

Excess of events in the low mass region seen in ATLAS and CMS

Exclusions of $M_H$:
- LEP < 114 GeV (arXiv:0602042v1)
2. The 4th of July and after

After 48 years of postulat, 30 years of search (and a few heart attacks), the Higgs is discovered at LHC on the 4th of July: Higgstorial day!

Discrete–Lisbon, 03/12/2012

Implicatons of Higgs discovery – A. Djouadi – p.7/23
Results from the LHC in 2012: Higgs

Higgs combined results

Combination of 5 channels: $bb$, $\tau\tau$, $WW$, $ZZ$, $\gamma\gamma$

Significance $6.9\sigma$ versus $7.8\sigma$ expected.
Results from the LHC in 2012

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LHC Results
- LHC Overview
- Basic Physics
- H: 2photons
- H: 4Leptons
- SUSY at LHC
- Signatures
- First Results 2011
- 2012 Results
Conclusions
4. Implications for pMSSM: mass

Main results:
- Large $M_S$ values needed:
  - $M_S \approx 1$ TeV: only maximal mixing
  - $M_S \approx 3$ TeV: only typical mixing.
- Large $\tan\beta$ values favored but $\tan\beta \approx 3$ possible if $M_S \approx 3$ TeV

How light sparticles can be with the constraint $M_h = 126$ GeV?
- 1s/2s gen. $\tilde{q}$ should be heavy...
  But not main player here: the stops:
  $\Rightarrow m_{\tilde{t}_1} \lesssim 500$ GeV still possible!
- $M_1, M_2$ and $\mu$ unconstrained,
- non-univ. $m_{\tilde{f}}$: decouple $\tilde{\ell}$ from $\tilde{q}$
  EW sparticles can be still very light but watch out the new limits.

Discrete–Lisbon, 03/12/2012  Implications of Higgs discovery – A. Djouadi – p.15/23
Results from the LHC in 2012: SUSY

CMSSM interpretation

LHC Preliminary \( L_{int} = 4.98 \text{ fb}^{-1}, \sqrt{s} = 7 \text{ TeV} \)

- Jets + MHT
- SS Dilepton
- OS Dilepton
- 1 Lepton
- Multi-Lepton
- \( \alpha_T \)
- MT2

DISCRETE 2012, CMS overview, J. Varela
Results from the LHC in 2012: SUSY

**Summary**
- MSUGRA/CMSSM: 0 lep + \( E_T \)\(_{\text{miss}} \)
- Pheno model: 0 lep + \( E_T \)\(_{\text{miss}} \)
- Guino med. \( \tilde{\chi} \) : 1 lep + \( E_T \)\(_{\text{miss}} \)
- GMSB (NLSP): 2 lep (OS) + \( E_T \)\(_{\text{miss}} \)
- GG (bino NLSP): 3 lep + \( E_T \)\(_{\text{miss}} \)
- GG (higgsino NLSP): 3 lep + \( E_T \)\(_{\text{miss}} \)
- LSP: 1/2 lep (+ b-jet) + \( E_T \)\(_{\text{miss}} \)
- Resonance: 3 lep + \( E_T \)\(_{\text{miss}} \)
- R-hadrons: low \( E_T \)\(_{\text{miss}} \)
- Pheno model: 0 lep + \( E_T \)\(_{\text{miss}} \)
- Guino med. \( \tilde{\chi} \) : 1 lep + \( E_T \)\(_{\text{miss}} \)
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**Motivation**
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- H: 2 photons
- H: 4 leptons
- SUSY at LHC
- Signatures
- First Results 2011
- 2012 Results

**Conclusions**

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**ATLAS SUSY Searches**

**ATLAS SUSY Searches**

<table>
<thead>
<tr>
<th>Mass scale [TeV]</th>
<th>Limit (fb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.00 TeV</td>
</tr>
<tr>
<td>1.2</td>
<td>1.24 TeV</td>
</tr>
<tr>
<td>1.15</td>
<td>1.15 TeV</td>
</tr>
<tr>
<td>1.1</td>
<td>1.10 TeV</td>
</tr>
<tr>
<td>1.0</td>
<td>1.00 TeV</td>
</tr>
<tr>
<td>0.9</td>
<td>0.90 TeV</td>
</tr>
<tr>
<td>0.8</td>
<td>0.80 TeV</td>
</tr>
<tr>
<td>0.7</td>
<td>0.70 TeV</td>
</tr>
<tr>
<td>0.6</td>
<td>0.60 TeV</td>
</tr>
</tbody>
</table>

---

**LFV**: pp → \( \ell \ell' \nu \tilde{\nu} \)
**Pheno model**: low \( E_T \)\(_{\text{miss}} \)
**GMSB**: stable \( \tilde{\chi}_1 \)

---

**RVP CMS**: 1 lep + 7 \( E_T \)\(_{\text{miss}} \)
**Bilinear RVP CMS**: 1 lep + 7 \( E_T \)\(_{\text{miss}} \)
**WIMP interaction (DS, Dirac, \( \tilde{\chi}_1 \))**

---

**Jorge C. Romão**
Results from the LHC in 2012: SUSY

\[ \chi^+ \chi^0 \] exclusion limits

7 TeV result

New 8 TeV result

\( \sqrt{s} = 7 \text{ TeV}, L_{\text{int}} = 4.98 \text{ fb}^{-1} \)

\( \sqrt{s} = 8 \text{ TeV}, L_{\text{int}} = 9.2 \text{ fb}^{-1} \)

\( m_{\chi_1} = m_{\tilde{\chi}^0_1} > m_{\tilde{\chi}^+_1} \)

\( m_{\chi_2} = m_{\tilde{\chi}^0_2} > m_{\tilde{\chi}^+_2} \)

\( m_{\tilde{t}} = 0.5 m_{\chi_1} + 0.5 m_{\chi_2} \)

\( m_{\tilde{\chi}^+_1} = m_{\tilde{\chi}^0_1} \)

\( m_{\tilde{\chi}^+_2} = m_{\tilde{\chi}^0_2} \)

\[ \chi^+ \chi^0 \] exclusion limits

\( \text{LEP2 slepton limit} \)

\( \text{LEP2 chargino limit} \)

\( 3/\gamma \text{ (} \tilde{\ell}_L, \text{ BF}(\ell\ell) = 0.5) \)

\( 3/\gamma \text{ (} \tilde{\ell}_R, \text{ BF}(\ell\ell) = 1) \)

\( 3/\gamma \text{ (} \tilde{\ell}_R, \text{ BF}(\ell\ell) = 1) \)

\( 3/\gamma \text{ (} \tilde{\ell}_L, \text{ BF}(\ell\ell) = 0.5) \)

\( 3/\gamma \text{ (} \tilde{\ell}_L, \text{ BF}(\ell\ell) = 1) \)

\( 3/\gamma \text{ (} \tilde{\ell}_R, \text{ BF}(\ell\ell) = 1) \)

\( 2/\gamma \) & \( 3/\gamma \) (no \( \tilde{\tau} \), BF(WZ) = 1)

\( 2/\gamma \) & \( 3/\gamma \) (no \( \tilde{\tau} \), BF(WZ) = 1)

DISCRETE 2012, CMS overview, J. Varela

Jorge C. Romão
Although there is not yet direct experimental evidence for supersymmetry (SUSY), there are many theoretical arguments indicating that SUSY might be of relevance for physics around the 1 TeV scale.

We will be waiting for the LHC verdict!
Lots of things to be done by Experimentalists and Theoreticians