

# Hide & Seek with SUSY

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# Summary

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# Problems with the Standard Model

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We believe that, despite its enormous success the Standard Model of strong, weak and electromagnetic interactions has many problems. These are of two kinds

Experimental evidence

- ◆ Neutrino masses and oscillations
- ◆ Dark Matter
- ◆ Dark Energy

Theoretical questions

Here are a few:

- ◆ Why the fermions have those strange hypercharge assignments that ensure, by accident, the cancellation of anomalies?
- ◆ Why these gauge groups. Why they have so different strengths?
- ◆ Why the replication to have 3 families?
- ◆ Why is the electroweak scale much lower than the Planck scale (hierarchy problem)?

# Historical Solution: go to shorter distances

Historically, many problems were solved by going to smaller distances. Here is a couple of examples

## □ Gases

- ◆ Ideal gases  $\Leftrightarrow$  Free point like molecules
- ◆ Deviation from ideal gas  $\Leftrightarrow$  size of molecules  $\Rightarrow$  Van der Waals equation of state

## □ Mendeleev Periodic Table

- ◆ Table explained by bound electrons + spin + Pauli principle
- ◆ Existence of nuclides explained by looking inside the nucleus at protons and neutrons  $\Rightarrow$  smaller distances

Idea: New Physics should appear below the typical scale of the SM

$$v \simeq (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{GeV}, \quad d = \frac{\hbar c}{vc^2} \simeq 0.8 \times 10^{-16} \text{cm}$$

# An Example: Problems with Classical Electromagnetism

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- In classical electromagnetism we have electrons (charges) plus fields  $\vec{E}$ ,  $\vec{B}$
- We can calculate the Coulomb self-energy of the electron. It diverges as  $1/r$  so we have to apply a cutoff, the “size” of the electron  $r_e$

$$\Delta E_{\text{Coulomb}} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e}$$

- Therefore the mass of the electron receives a contribution

$$(m_e c^2)_{\text{observed}} = (m_e c^2)_{\text{bare}} + \Delta E_{\text{Coulomb}}$$

- We know experimentally that  $r_e \leq 10^{-17}$  cm which implies  $\Delta E \geq 10$  GeV

$$0.511 \text{ MeV} = (-9999.489 + 10000.000) \text{ MeV}$$

- To avoid this fine tuned cancellation we would have to say that classical electromagnetism does not hold for distances below  $d_{\text{classical}}$  such that

$$m_e c^2 \simeq \frac{1}{4\pi\epsilon_0} \frac{e^2}{d} \Rightarrow d_{\text{classical}} \simeq 2.8 \times 10^{-13} \text{ cm} \simeq \text{size of nucleus}$$

# Positron: New degrees of freedom and a new symmetry

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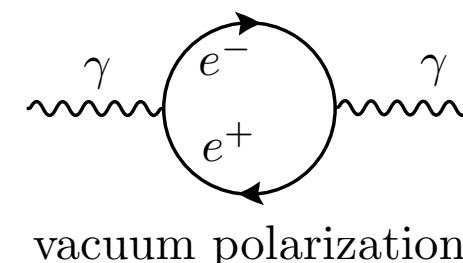
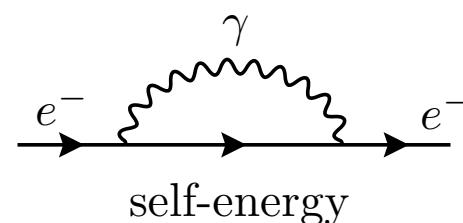
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The problem was solved with QED and the discovery of the positron. We have processes like



- Vacuum fluctuations are allowed if they occur for

$$\Delta t \sim \hbar / \Delta E \sim \hbar / (2m_e c^2)$$

- This introduces a new scale

$$d_{\text{QED}} \sim c \Delta t \sim \hbar c / (2m_e c^2) \simeq 200 \times 10^{-13} \text{ cm}$$

- Therefore classical electromagnetism should indeed break much before what we got previously, that is

$$d_{\text{QED}} \gg d_{\text{classical}}$$

# New contributions to the self energy

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- With the vacuum fluctuations one should also consider a process where one electron sitting in the vacuum annihilates with the positron produced by the vacuum fluctuations, remaining the other electron.
- This gives an additional contribution calculated by Weisskopf which exactly cancels the Coulomb self-energy

$$\Delta E_{\text{pair}} = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r_e} + \frac{3\alpha}{4\pi} m_e c^2 \ln\left(\frac{\hbar}{m_e c r_e}\right)$$

So

$$\Delta E = \Delta E_{\text{Coulomb}} + \Delta E_{\text{pair}} = \frac{3\alpha}{4\pi} m_e c^2 \ln\left(\frac{\hbar}{m_e c r_e}\right)$$

and

$$(m_e c^2)_{\text{observed}} = (m_e c^2)_{\text{bare}} \left[ 1 + \ln\left(\frac{\hbar}{m_e c r_e}\right) \right]$$

- ◆ Proportional to  $(m_e c^2)_{\text{bare}}$
- ◆ Small because of the logarithm. Even for  $r_e = r_{\text{Planck}} = 1.6 \times 10^{-33}$  cm we only get a 9% correction

# Lessons to be taken

- The fact that the correction is proportional to  $m_e$  is a consequence of a new (broken) symmetry with the anti-particle, chiral symmetry.
- In the limit of exact symmetry we have  $m_e = 0$  and the symmetry protects the electron from getting a mass from self-energy corrections
- Doubling the degrees of freedom, and adding a new symmetry leads to the cancellation of divergences. We obtain a good theory for the electron down to very short distances.

This is the paradigm for one of the motivations for supersymmetry as we will now explain in a simple example. We do this before introducing the full structure of supersymmetry

# The Higgs and Supersymmetry

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- In the Standard Model the Higgs potential is

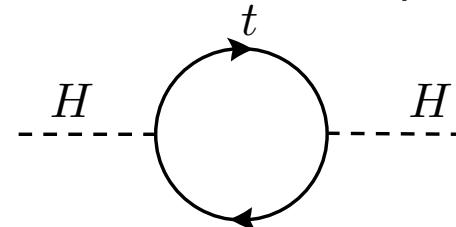
$$V = \mu^2 |H|^2 + \lambda |H|^4$$

The spontaneous breaking of the symmetry occurs when

$$v^2 = 2 \langle H \rangle^2 = -\frac{\mu^2}{\lambda} = (246 \text{ GeV})^2$$

Because unitarity requires  $\lambda \leq 1$  we must have  $-\mu^2 = \mathcal{O}((246 \text{ GeV})^2)$

- However from the top loop



$$\Rightarrow \Delta\mu^2 = -6 \frac{y_t^2 \hbar^2}{4\pi^2 c^2 r_H^2}$$

where  $r_H$  is the “size” of the Higgs.

- An argument along the previous lines would tell us that the SM would not hold for distances below

$$r_H \leq \sqrt{\frac{6 y_t^2 \hbar^2}{4\pi^2 |\mu^2| c^2}} \simeq 10^{-18} \text{ cm}$$

# The Higgs and Supersymmetry ...

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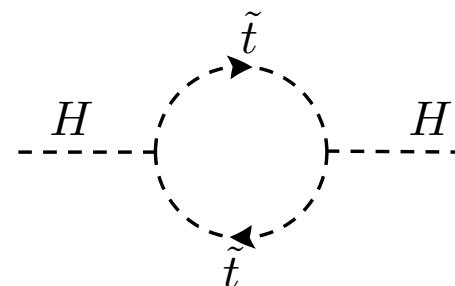
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How should we make the SM consistent down to smaller distances? From the previous argument we should:

- Enlarge the number of degrees of freedom (particles)
- Enlarge the symmetry, possibly a broken one.

Suppose that we have a scalar field, call it stop,  $\tilde{t}$ , that couples to the Higgs in a manner related, because of some symmetry, to the coupling of the Higgs to the top. With this appropriate coupling we obtain from the diagram



$$\Delta\mu_{\text{stop}}^2 = +6 \frac{y_t^2}{4\pi^2} \frac{\hbar^2}{r_H^2 c^2} - 6 \frac{y_t^2}{4\pi^2} (m_{\tilde{t}}^2 - m_t^2) \ln \left( \frac{\hbar^2}{r_H^2 m_t^2 c^2} \right)$$

# The Higgs and Supersymmetry · · ·

Therefore we now have

$$\Delta\mu^2 = -6 \frac{y_t^2}{4\pi^2} (m_{\tilde{t}}^2 - m_t^2) \ln \left( \frac{\hbar^2}{r_H^2 m_t^2 c^2} \right)$$

Comments:

- QFT tell us that loops of fermions and scalars have a relative minus sign, so the new particles to be partners of the top (fermion) should be bosons
- The symmetry must be responsible for the needed relation between the couplings  $H\bar{t}t$  and  $H\tilde{t}^*\tilde{t}$
- Like in the positron analogue, in the limit of exact symmetry,  $\Delta\mu^2 = 0$
- Unlike the positron analogue we do not know  $m_{\tilde{t}}$ . For the argument to hold one must have

$$m_{\tilde{t}} \leq \text{Few TeV}$$

Such a symmetry exists, it is called Supersymmetry (SUSY)

# Other Motivations

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Historically supersymmetry appeared (around 1970) due to a different type of arguments. The most commonly invoked theoretical arguments for SUSY are:

- It evades the Coleman-Mandula theorem, showing that it is possible to enlarge the Poincaré group in a non trivial manner.
- Interrelates matter fields (leptons and quarks) with force fields (gauge and/or Higgs bosons).
- As local SUSY implies gravity (supergravity) it could provide a way to unify gravity with the other interactions.
- As SUSY and supergravity have fewer divergences than conventional field theories, the hope is that it could provide a consistent (finite) quantum gravity theory.
- SUSY can help to understand the mass problem, in particular solve the naturalness problem (and in some models even the hierarchy problem) if SUSY particles have masses  $\leq \mathcal{O}(\text{FewTeV})$ .

# The Poincare Algebra

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The Poincaré group (PG) is made up of the Lorentz group plus the translations. We denote by  $J_{\mu\nu}$  the generators of the Lorentz group and by  $P_\mu$  the generators of the translations. The algebra is defined by,

$$[J_{\mu\nu}, J_{\rho\sigma}] = i(g_{\nu\rho}J_{\mu\sigma} - g_{\nu\sigma}J_{\mu\rho} - g_{\mu\rho}J_{\nu\sigma} + g_{\mu\sigma}J_{\nu\rho})$$

$$[P_\alpha, J_{\mu\nu}] = i(g_{\mu\alpha}P_\nu - g_{\nu\alpha}P_\mu)$$

$$[P_\mu, P_\nu] = 0$$

One can show that

$$[P^2, J_{\mu\nu}] = [P^2, P_\mu] = 0$$

$$[W^2, J_{\mu\nu}] = [W^2, P_\mu] = [W^2, P^2] = 0$$

where

$$W_\mu = -\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma$$

is the Pauli-Lubanski vector operator.

- $P^2, W^2$  are the Casimir operators of PG
- Irreducible reps  $\Rightarrow |m, s\rangle$

Rest frame:

$$W^\mu = (0, m\vec{S}) \Rightarrow W^2 = -m^2 s(s+1)$$

# SUSY Algebra

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The SUSY generators carry Spin 1/2 and obey the following algebra

$$\{Q_\alpha, Q_\beta\} = 0$$

$$\{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(\sigma^\mu)_{\alpha\dot{\beta}} P_\mu$$

where

$$\sigma^\mu \equiv (1, \sigma^i) ; \quad \bar{\sigma}^\mu \equiv (1, -\sigma^i)$$

and  $\alpha, \beta, \dot{\alpha}, \dot{\beta} = 1, 2$  (Weyl 2-component spinor notation).

# SUSY Algebra and the Poincaré Group

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The commutation relations with the generators of the Poincaré group

$$[P^\mu, Q_\alpha] = 0 \quad [J^{\mu\nu}, Q_\alpha] = -i (\sigma^{\mu\nu})_\alpha{}^\beta Q_\beta$$

One can easily derive that the two invariants of the Poincaré group,

$$P^2 = P_\alpha P^\alpha \quad W^2 = W_\alpha W^\alpha \quad W_\mu = -\frac{1}{2} \epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma$$

$$P^2 |m, s\rangle = m^2 |m, s\rangle \quad W^2 |m, s\rangle = -m^2 s(s+1) |m, s\rangle$$

where  $W^\mu$  is the Pauli–Lubanski vector operator, are no longer invariants of the Super Poincaré group:

$$[Q_\alpha, P^2] = 0 \quad [Q_\alpha, W^2] \neq 0$$

Irreducible multiplets will have particles of the same mass but different spin.

# Simple Results from the Algebra

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Number of Bosons = Number of Fermions

$$Q_\alpha |B\rangle = |F\rangle \quad (-1)^{N_F} |B\rangle = |B\rangle$$

$$Q_\alpha |F\rangle = |B\rangle \quad (-1)^{N_F} |F\rangle = -|F\rangle$$

where  $(-1)^{N_F}$  is the fermion number of a given state. Then we obtain

$$Q_\alpha (-1)^{N_F} = -(-1)^{N_F} Q_\alpha$$

Using this relation we can show that

$$\begin{aligned} Tr [(-1)^{N_F} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\}] &= Tr [(-1)^{N_F} Q_\alpha \bar{Q}_{\dot{\alpha}} + (-1)^{N_F} \bar{Q}_{\dot{\alpha}} Q_\alpha] \\ &= Tr [-Q_\alpha (-1)^{N_F} \bar{Q}_{\dot{\alpha}} + Q_\alpha (-1)^{N_F} \bar{Q}_{\dot{\alpha}}] = 0 \end{aligned}$$

But we also have

$$\begin{aligned} Tr [(-1)^{N_F} \{Q_\alpha, \bar{Q}_{\dot{\alpha}}\}] \\ = Tr [(-1)^{N_F} 2\sigma_{\alpha\dot{\alpha}}^\mu P_\mu] \end{aligned}$$



$$Tr [(-1)^{N_F}] = \# \text{Bosons} - \# \text{Fermions} = 0$$

# SUSY Representations: Massive case

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In the rest frame

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = 2m \delta_{\alpha\dot{\alpha}}$$

This algebra is similar to the algebra of the spin 1/2 creation and annihilation operators. Choose  $|\Omega\rangle$  such that

$$Q_1 |\Omega\rangle = Q_2 |\Omega\rangle = 0$$

Then we have 4 states

If  $J_3 |\Omega\rangle = j_3 |\Omega\rangle$



$$|\Omega\rangle ; \bar{Q}_1 |\Omega\rangle ; \bar{Q}_2 |\Omega\rangle ; \bar{Q}_1 \bar{Q}_2 |\Omega\rangle$$

State	$J_3$ Eigenvalue
$ \Omega\rangle$	$j_3$
$\bar{Q}_1  \Omega\rangle$	$j_3 + \frac{1}{2}$
$\bar{Q}_2  \Omega\rangle$	$j_3 - \frac{1}{2}$
$\bar{Q}_1 \bar{Q}_2  \Omega\rangle$	$j_3$

Two bosons and two fermions  
states separated by  
one half unit of spin

# SUSY Representations: Massless case

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If  $m = 0$  then we can choose  $P^\mu = (E, 0, 0, E)$ . In this frame

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = M_{\alpha\dot{\alpha}}$$

where the matrix  $M$  takes the form

$$M = \begin{pmatrix} 0 & 0 \\ 0 & 4E \end{pmatrix}$$

Then  $\{Q_2, \bar{Q}_2\} = 4E$  all others vanish.

We have then just **two** states  $|\Omega\rangle ; \bar{Q}_2 |\Omega\rangle$

If  $J_3 |\Omega\rangle = \lambda |\Omega\rangle$

State	$J_3$ Eigenvalue
$ \Omega\rangle$	$\lambda$
$\bar{Q}_2  \Omega\rangle$	$\lambda - \frac{1}{2}$

Two states, one fermion  
one boson separated by  
one half unit of spin

# Superfields for Massless Particles

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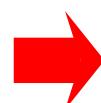
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- Chiral Superfields: Spin 0 + Spin  $\frac{1}{2}$

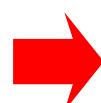
$$\Phi = \Phi(\phi, \chi_L)$$



$\phi$  Complex Scalar: 2 d.o.f  
 $\chi_L$  Chiral Fermion: 2 d.o.f (on-shell)

- Vector Superfields: Spin  $\frac{1}{2}$  + Spin 1

$$V = V(\lambda, W^\mu)$$



$\lambda$  Chiral Fermion: 2 d.o.f  
 $W^\mu$  Massless Vector: 2 d.o.f (on-shell)

$\phi$
$\lambda$

- superpartner of the fermion: sfermion  
 superpartner of gauge field: gaugino

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## □ Gauge Fields

We want to have gauge fields for the gauge group

$G = SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ . Therefore we will need three vector superfields (or vector supermultiplets)  $\hat{V}_i$  with the following components:

$$\hat{V}_1 \equiv (\lambda', W_1^\mu) \rightarrow U_Y(1)$$

$$\hat{V}_2 \equiv (\lambda^a, W_2^{\mu a}) \rightarrow SU_L(2) , \quad a = 1, 2, 3$$

$$\hat{V}_3 \equiv (\tilde{g}^b, W_3^{\mu b}) \rightarrow SU_c(3) , \quad b = 1, \dots, 8$$

where  $W_i^\mu$  are the gauge fields and  $\lambda'$ ,  $\lambda$  and  $\tilde{g}$  are the  $U_Y(1)$  and  $SU_L(2)$  gauginos and the gluino, respectively.

# Particle Content: Leptons

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## □ Leptons

As each chiral multiplet only describes one helicity state, we will need two chiral multiplets for each charged lepton (We will assume that the neutrinos do not have mass).

Supermultiplet	$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers
$\hat{L}_i \equiv (\tilde{L}, L)_i$	$(1, 2, -\frac{1}{2})$
$\hat{R}_i \equiv (\tilde{\ell}_R, \ell_L^c)_i$	$(1, 1, 1)$

Each helicity state corresponds to a complex scalar and we have that  $\hat{L}_i$  is a doublet of  $SU_L(2)$

$$\tilde{L}_i = \begin{pmatrix} \tilde{\nu}_{Li} \\ \tilde{\ell}_{Li} \end{pmatrix} \quad ; \quad L_i = \begin{pmatrix} \nu_{Li} \\ \ell_{Li} \end{pmatrix}$$

# Particle Content: Quarks

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## □ Quarks

The quark supermultiplets are given in the Table. The supermultiplet  $\hat{Q}_i$  is also a doublet of  $SU_L(2)$ , that is

Supermultiplet	$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers
$\hat{Q}_i \equiv (\tilde{Q}, Q)_i$	$(3, 2, \frac{1}{6})$
$\hat{D}_i \equiv (\tilde{d}_R, d_L^c)_i$	$(3, 1, \frac{1}{3})$
$\hat{U}_i \equiv (\tilde{u}_R, u_L^c)_i$	$(3, 1, -\frac{2}{3})$

$$\tilde{Q}_i = \begin{pmatrix} \tilde{u}_{Li} \\ \tilde{d}_{Li} \end{pmatrix} \quad ; \quad Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$$

# Particle Content: Higgs Bosons

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## □ Higgs Bosons

Finally the Higgs sector. In the MSSM we need at least two Higgs doublets. This is in contrast with the SM where only one Higgs doublet is enough to give masses to all the particles. The reason can be explained in two ways. Either the need to cancel the anomalies, or the fact that, due to the analyticity of the superpotential, we have to have two Higgs doublets of opposite hypercharges to give masses to the up and down type of quarks.

Supermultiplet	$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers
$\hat{H}_1 \equiv (H_1, \tilde{H}_1)$	$(1, 2, -\frac{1}{2})$
$\hat{H}_2 \equiv (H_2, \tilde{H}_2)$	$(1, 2, +\frac{1}{2})$

# R-Parity

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Most discussions of SUSY phenomenology assume **R-Parity** conservation where,

$$R_P = (-1)^{2J+3B+L}$$

This is the case of the **MSSM**. It implies:

- SUSY particles are pair produced.
- Every SUSY particle decays into another SUSY particle.
- There is a **LSP** that it is stable ( *E* signature ).

- This is just an **ad hoc** assumption without a deep justification
- However, if it is required we have a candidate for dark matter, the **LSP** that is stable
- This has been a big motivation for the MSSM

# The Superpotential and SUSY Breaking Lagrangian

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The MSSM Lagrangian is specified by the R-parity conserving superpotential  $W$

$$W = \varepsilon_{ab} \left[ h_U^{ij} \hat{Q}_i^a \hat{U}_j \hat{H}_2^b + h_D^{ij} \hat{Q}_i^b \hat{D}_j \hat{H}_1^a + h_E^{ij} \hat{L}_i^b \hat{R}_j \hat{H}_1^a - \mu \hat{H}_1^a \hat{H}_2^b \right]$$

where  $i, j = 1, 2, 3$  are generation indices,  $a, b = 1, 2$  are  $SU(2)$  indices, and  $\varepsilon$  is a completely antisymmetric  $2 \times 2$  matrix, with  $\varepsilon_{12} = 1$ . The coupling matrices  $h_U, h_D$  and  $h_E$  will give rise to the usual Yukawa interactions needed to give masses to the leptons and quarks.

If it were not for the need to break SUSY  
the number of parameters involved  
would be less than in the SM.

# Self Interactions of the Matter Multiplet

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- R-Parity

- Superpotential

- Self Matter Int

- Soft breaking

- CMSSM

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These correspond in non supersymmetric gauge theories both to the Yukawa interactions and to the scalar potential. In supersymmetric gauge theories we have less freedom to construct these terms. The first step is to construct the superpotential  $W$ . This must be a gauge invariant polynomial function of the *scalar* components of the chiral multiplet  $\Phi_i$ , that is  $\phi_i$ . It *does not* depend on  $\phi_i^*$ . In order to have renormalizable theories the degree of the polynomial must be at most three.

The Yukawa interactions are

$$\mathcal{L}_{Yukawa} = -\frac{1}{2} \left[ \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \chi_i \chi_j + \left( \frac{\partial^2 W}{\partial \phi_i \partial \phi_j} \right)^* \bar{\chi}_i \bar{\chi}_j \right]$$

and the scalar potential is

$$V_{scalar} = \frac{1}{2} D^a D^a + F_i F_i^*$$

where

$$F_i = \frac{\partial W}{\partial \phi_i}, \quad D^a = g \phi_i^* T_{ij}^a \phi_j$$

- In  $V_{Higgs}$  we have  $\lambda |\phi|^4 \Rightarrow g^2 |\phi|^4$
- The Higgs mass is not completely free!

# SUSY Soft Breaking

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The most general SUSY soft breaking is

$$\begin{aligned}
 -\mathcal{L}_{SB} = & M_Q^{ij2} \tilde{Q}_i^{a*} \tilde{Q}_j^a + M_U^{ij2} \tilde{U}_i \tilde{U}_j^* + M_D^{ij2} \tilde{D}_i \tilde{D}_j^* + M_L^{ij2} \tilde{L}_i^{a*} \tilde{L}_j^a + M_R^{ij2} \tilde{R}_i \tilde{R}_j^* \\
 & + m_{H_1}^2 H_1^{a*} H_1^a + m_{H_2}^2 H_2^{a*} H_2^a - \left[ \frac{1}{2} M_s \lambda_s \lambda_s + \frac{1}{2} M \lambda \lambda + \frac{1}{2} M' \lambda' \lambda' + h.c. \right] \\
 & + \varepsilon_{ab} \left[ A_U^{ij} h_U^{ij} \tilde{Q}_i^a \tilde{U}_j H_2^b + A_D^{ij} h_D^{ij} \tilde{Q}_i^b \tilde{D}_j H_1^a + A_E^{ij} h_E^{ij} \tilde{L}_i^b \tilde{R}_j H_1^a - B \mu H_1^a H_2^b \right]
 \end{aligned}$$

## Parameter Counting

Theory	Gauge Sector	Fermion Sector	Higgs Sector
SM	$e, g, \alpha_s$	$h_U, h_D, h_E$	$\mu^2, \lambda$
MSSM	$e, g, \alpha_s$	$h_U, h_D, h_E$	$\mu$
Broken MSSM	$e, g, \alpha_s$	$h_U, h_D, h_E$	$\mu, M_1, M_2, M_3, A_U, A_D, A_E, B$ $m_{H_2}^2, m_{H_1}^2, m_Q^2, m_U^2, m_D^2, m_L^2, m_R^2$

# The Constrained MSSM

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The number of independent parameters can be reduced if we impose some further constraints. The most popular is the MSSM coupled to  $N = 1$  Supergravity (mSUGRA).

$$A_t = A_b = A_\tau \equiv A ,$$

$$m_{H_1}^2 = m_{H_2}^2 = M_L^2 = M_R^2 = m_0^2 , M_Q^2 = M_U^2 = M_D^2 = m_0^2 ,$$

$$M_3 = M_2 = M_1 = M_{1/2}$$

## Parameter Counting

Parameters	Conditions	Free Parameters
$y_t, h_b, h_\tau, v_1, v_2$	$m_W, m_t, m_b, m_\tau$	$\tan \beta = v_2/v_1$
$A, B, m_0, M_{1/2}, \mu$	$t_i = 0, i = 1, 2$	$A, m_0, M_{1/2}, \text{sign}(\mu)$
Total = 10	Total = 6	Total = 4 + "1"

It is remarkable that with so few parameters we can get the correct values for the parameters, in particular  $m_{H_2}^2 < 0$ . For this to happen the top Yukawa coupling has to be large which we know to be true.

# Neutral Higgs Mass Matrices

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- Scalar Higgs

- SM Higgs Mass

- Higgs Mass RC

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$$\mathbf{M}_{S^0}^2 = \begin{pmatrix} \tan \beta & -1 \\ -1 & \cot \beta \end{pmatrix} B\mu + \begin{pmatrix} \cot \beta & -1 \\ -1 & \tan \beta \end{pmatrix} \frac{1}{2} m_Z^2 \sin^2 \beta$$

with masses

$$m_{h,H}^2 = \frac{1}{2} \left[ m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos 2\beta} \right]$$

$$\mathbf{M}_{P^0}^2 = \begin{pmatrix} \tan \beta & -1 \\ -1 & \cot \beta \end{pmatrix} B\mu$$

with mass

$$m_A^2 = \frac{B\mu}{\sin 2\beta}$$

**Sum Rule**

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2$$



$$\begin{aligned} m_h < m_A < m_H \\ m_h < m_Z < m_H \end{aligned}$$

# Higgs Boson Mass: Standard Model

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In the Standard Model

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\lambda}{4} \left( \Phi^\dagger \Phi - \frac{v^2}{2} \right)^2$$

where

$$\Phi = \begin{pmatrix} \varphi^+ \\ \frac{v+H+i\varphi_Z}{\sqrt{2}} \end{pmatrix}$$

Therefore

$$\mathcal{L}_{\text{Higgs}} = \partial_\mu H \partial^\mu H - \frac{\lambda v^2}{4} H^2 + \dots$$

or

$$M_H = \sqrt{\frac{\lambda}{2}} v \rightarrow \boxed{v = 246 \text{ GeV fixed but } \lambda \text{ free.}} \rightarrow \boxed{M_H \text{ free.}}$$

# Higgs Boson Mass: Radiative corrections

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- Scalar Higgs

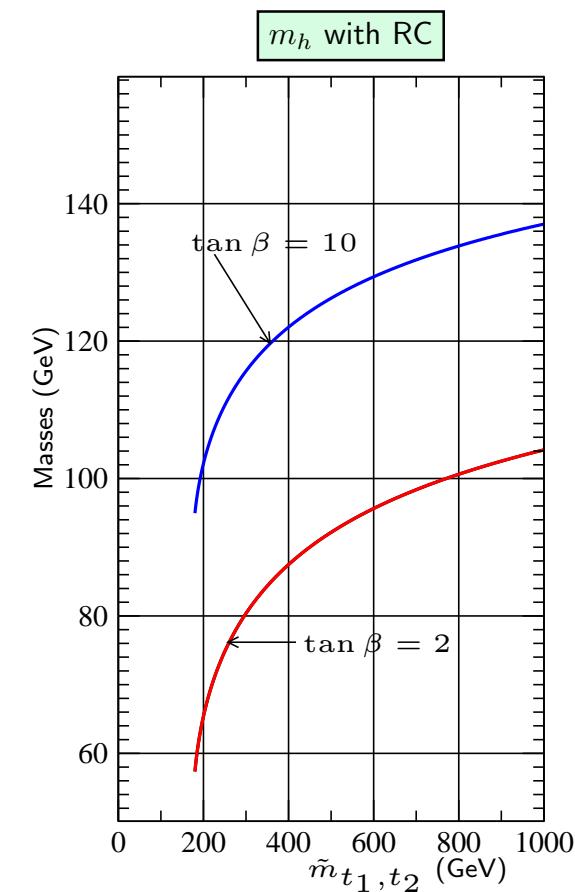
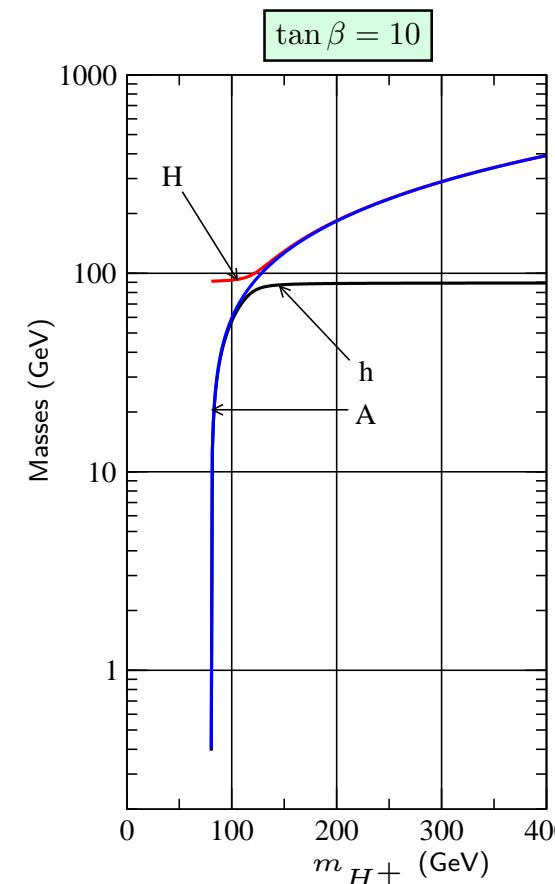
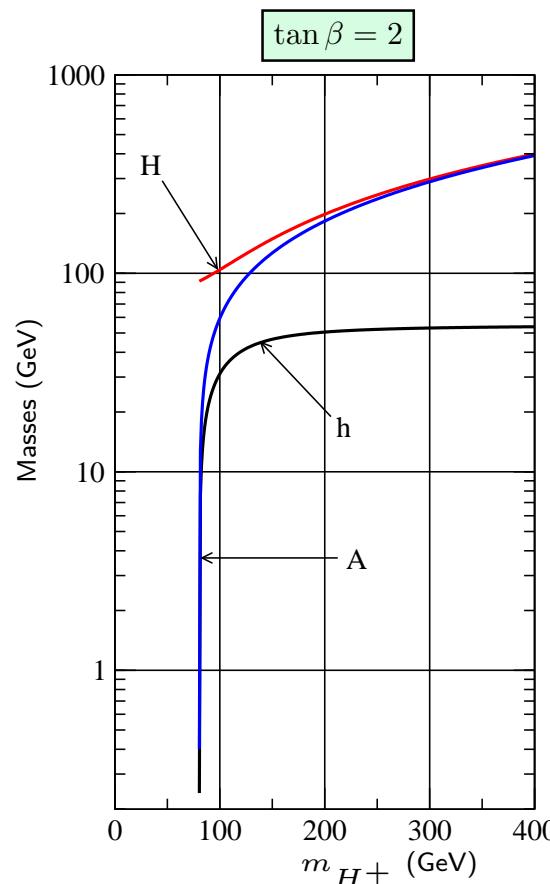
- SM Higgs Mass

- Higgs Mass RC

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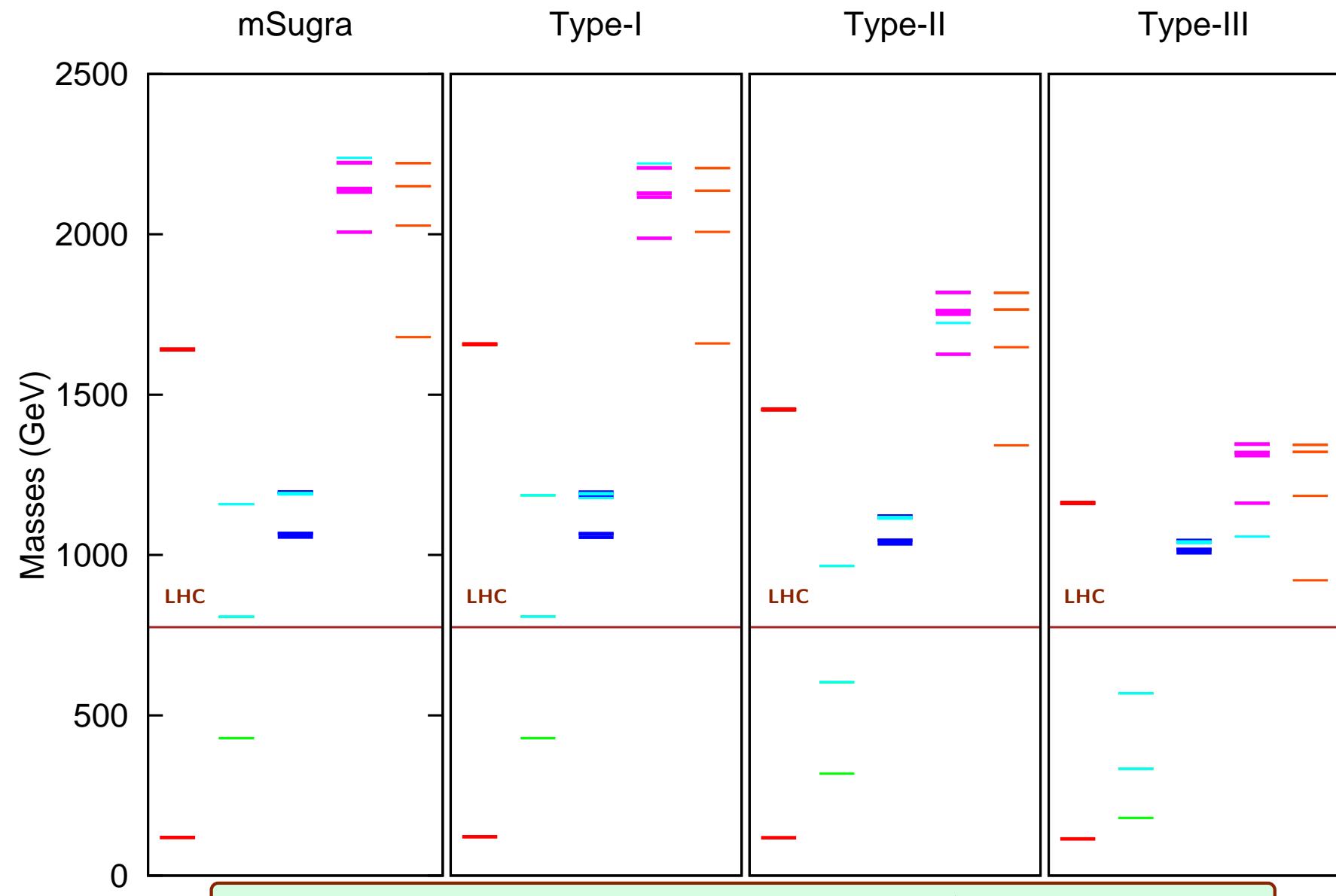
As the top mass is very large there are important radiative corrections to the Higgs boson mass. The most important are:

$$m_h^2 = m_h^{(0)2} + \frac{3g^2}{16\pi^2 m_W^2} \frac{m_t^4}{\sin^2 \beta} \ln \left( \frac{\tilde{m}_{t_1}^2 \tilde{m}_{t_2}^2}{m_t^4} \right)$$



# Example of Spectra: $M_{1/2} = m_0 = 1 \text{ TeV}$ ( $M_{\text{SS}} = 10^{14} \text{ GeV}$ )

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# Couplings in the MSSM

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Name	Type	Name	Type
Gauge Self-Interaction	VVV VVVV	4-Point Coupling	$VVff$ $HHVV$ $HGVV$ $GGVV$ $\tilde{f}\tilde{f}HH$ $\tilde{f}\tilde{f}GH$ $\tilde{f}\tilde{f}GG$ $\tilde{f}\tilde{f}\tilde{f}\tilde{f}$
3-Point Gauge Coupling	$Vff$ $V\tilde{f}\tilde{f}$ $V\tilde{\chi}\tilde{\chi}$ $VHH$ $VGH$ $VGG$	Goldstone-Higgs Interaction	$HHG$ $HGG$ $HHHG$ $HHGG$ $HGGG$ $GGGG$
3-Point Higgs Coupling	$Hff$ $H\tilde{f}\tilde{f}$ $H\tilde{\chi}\tilde{\chi}$ $HVV$	Ghost	$\bar{\omega}\omega V$ $\bar{\omega}\omega H$ $\bar{\omega}\omega G$
3-Point Goldstone Coupling	$Gff$ $G\tilde{f}\tilde{f}$ $G\tilde{\chi}\tilde{\chi}$ $GVV$		
Other 3-Point	$\tilde{f}f\tilde{\chi}$		

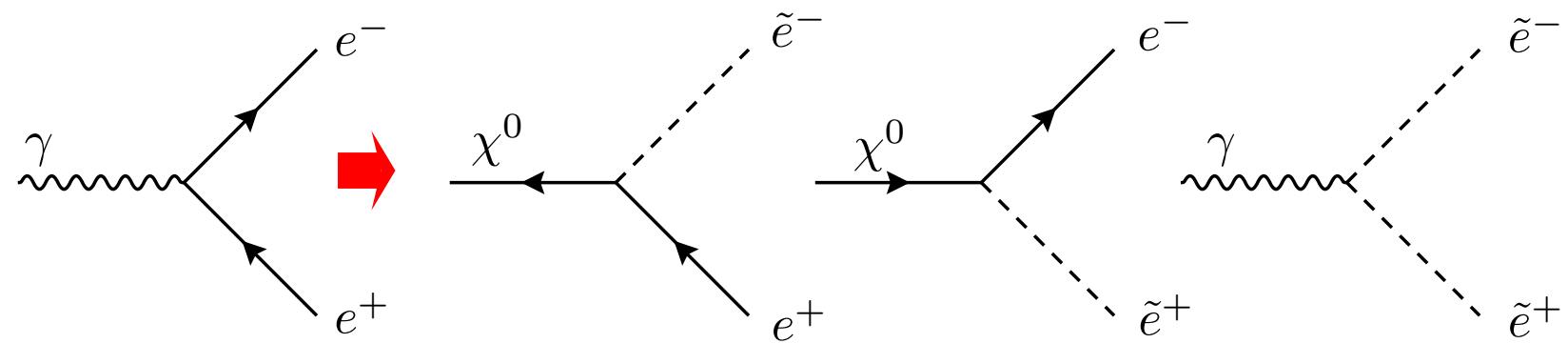
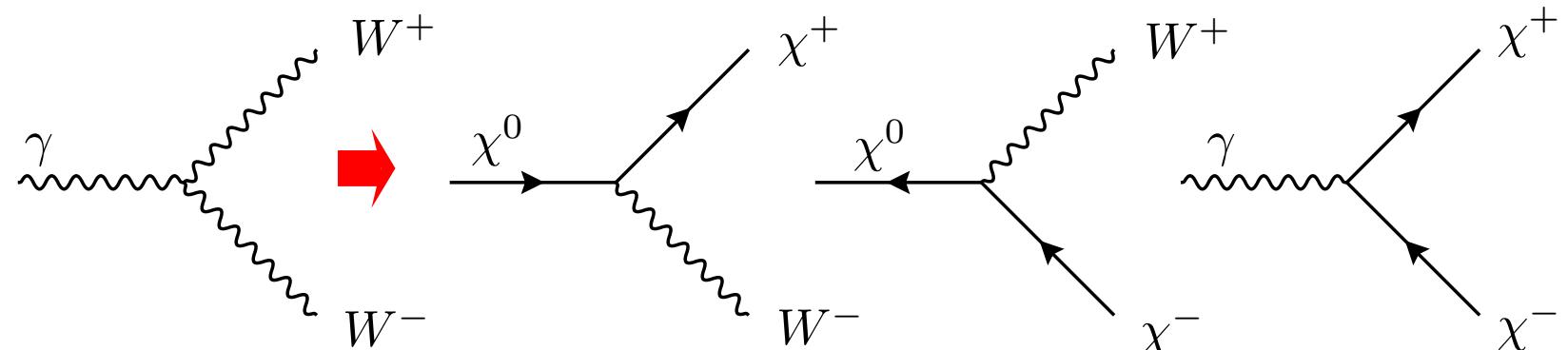
# Examples of New Couplings: Conserving R-Parity

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- New vertices

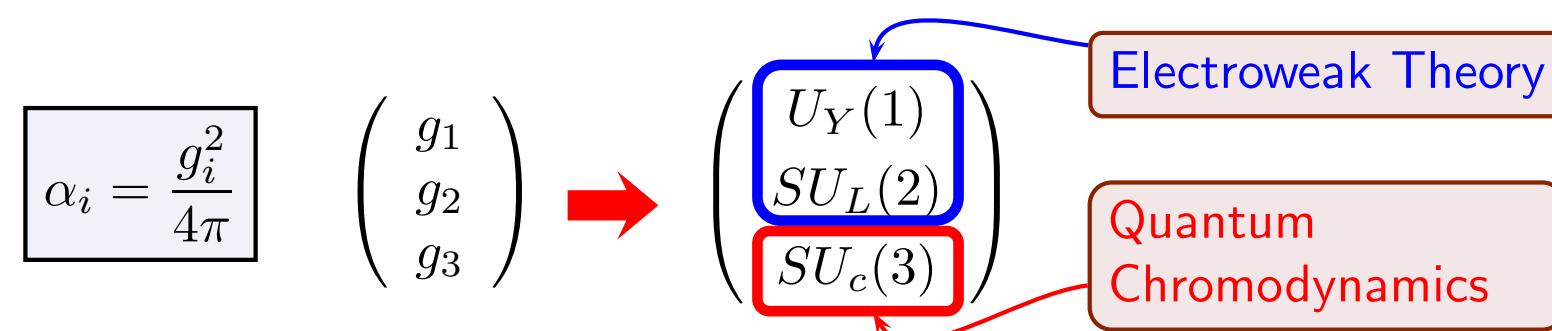
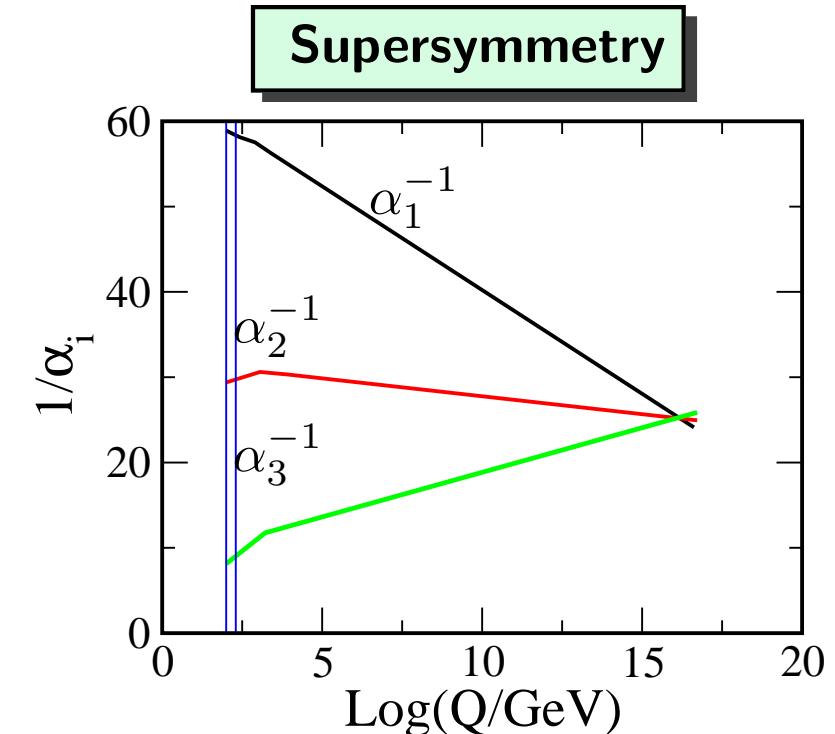
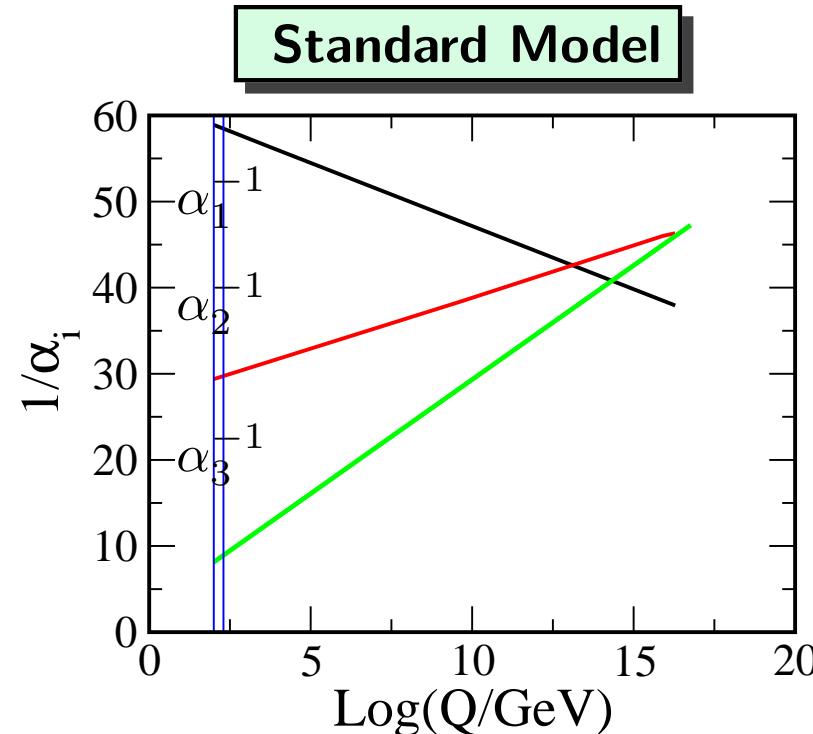
- Unification

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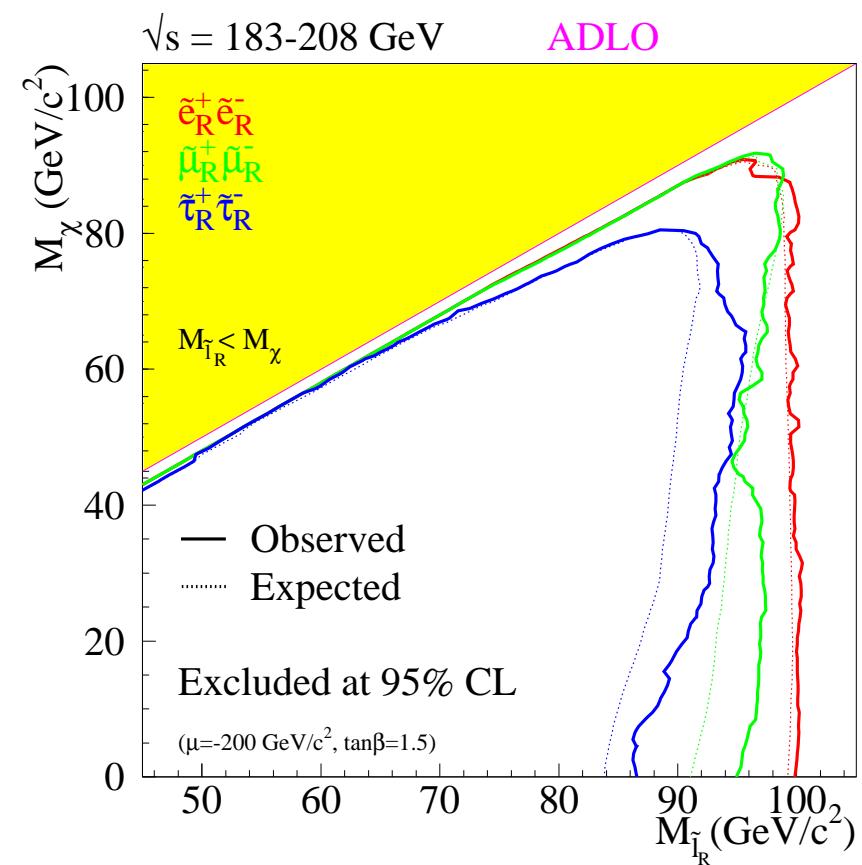
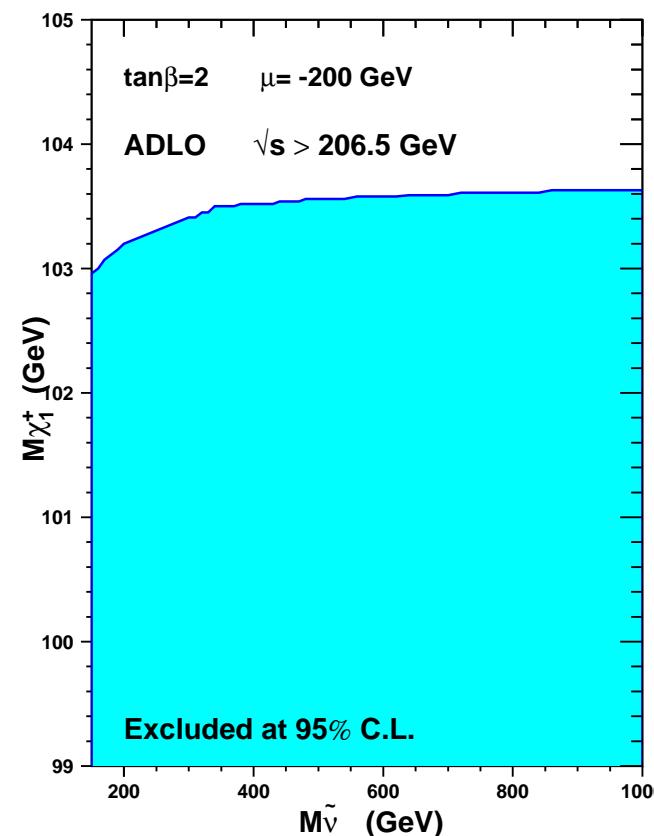
Rule: Change any two lines into the superpartners.

# Gauge Couplings Unification

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# LEP Limits on Charginos, Sneutrinos & Sleptons

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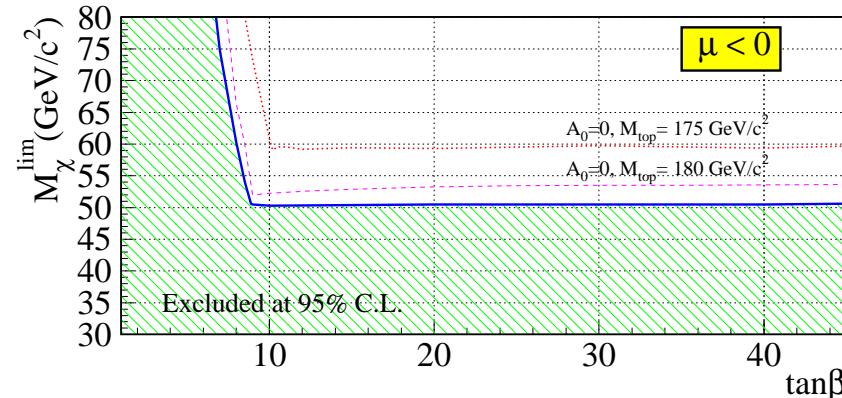
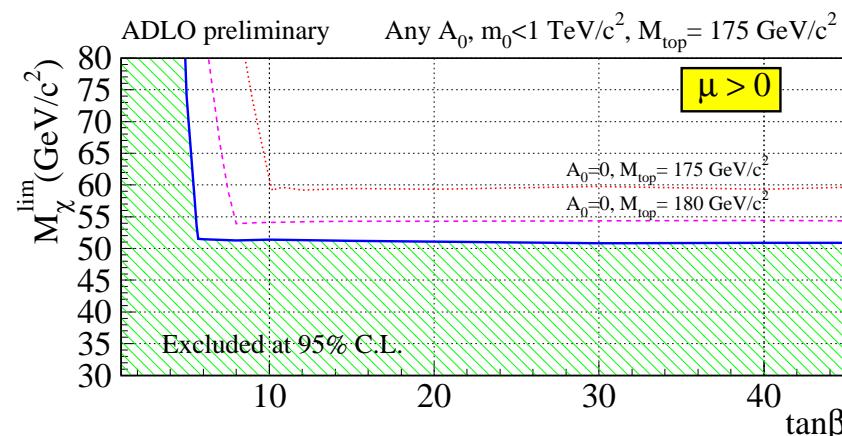
Limits on  $\chi^\pm, \tilde{l}^\pm \simeq 100$  GeV.

# LEP Limits on Neutralinos

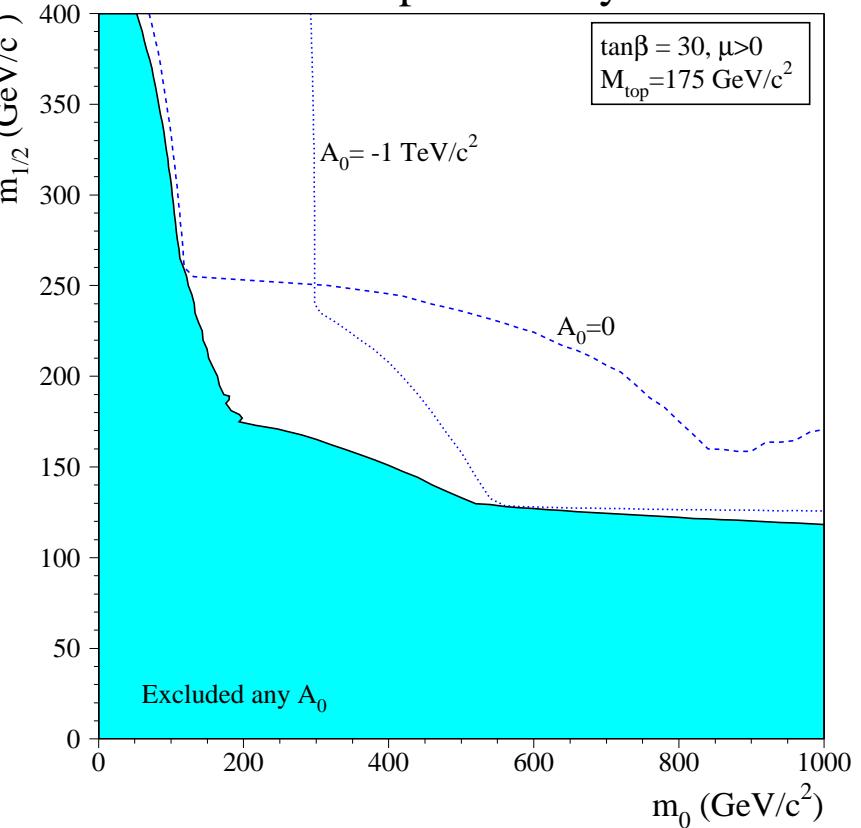
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ADLO preliminary



Limits on  $\chi^0 \geq 50 \text{ GeV}$ . More model dependent.

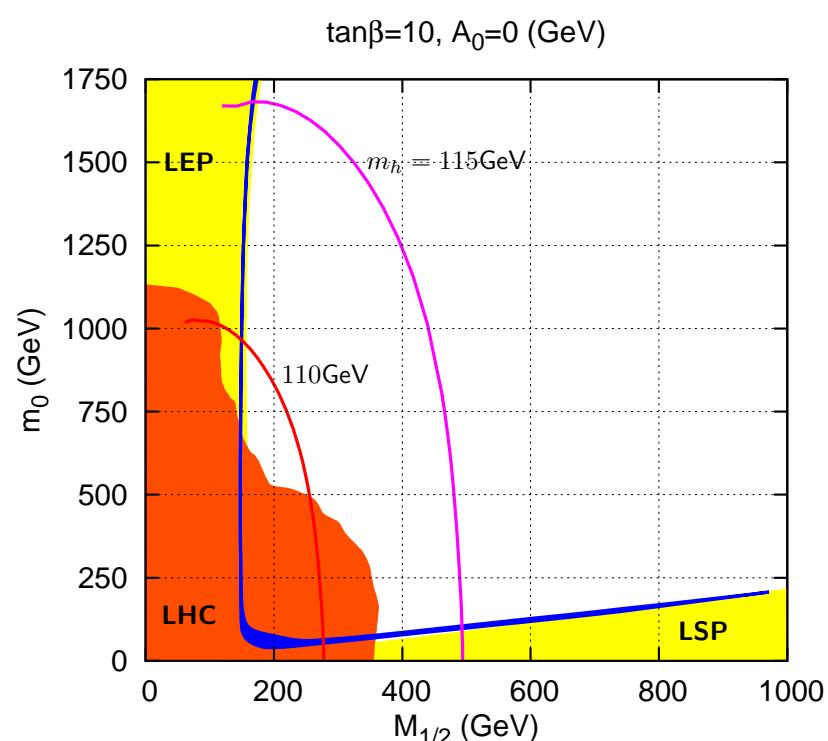
# Neutralino Dark Matter

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In mSugra only four (three) very specific regions can explain the WMAP data ( $3\sigma$ )

$$0.081 \leq \Omega h^2 \leq 0.129$$

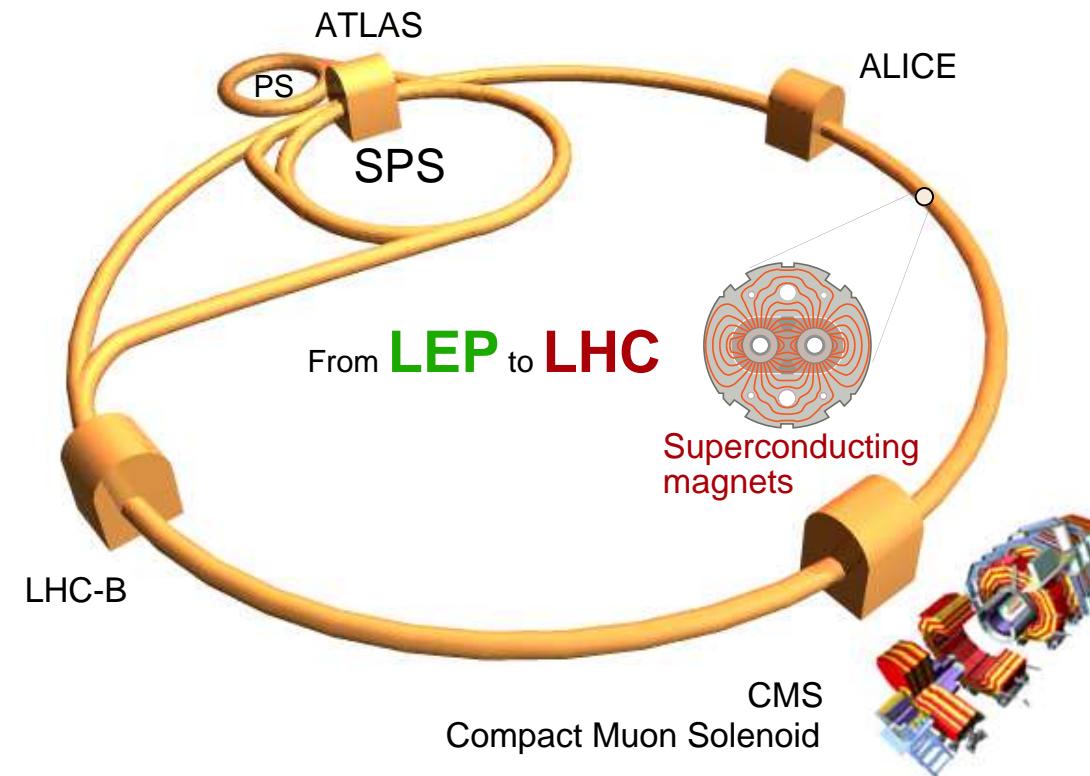
- (The bulk region)
- The co-annihilation line
- The “focus point” line
- The “higgs funnel” region  
(large  $\tan \beta$ )



Pre-LHC Plot  
 $m_h = 125 \text{ GeV}$  outside plot!

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## The Large Hadron Collider (LHC)



	Beams	Energy	Luminosity
LEP	e+ e-	200 GeV	$10^{32} \text{ cm}^{-2}\text{s}^{-1}$
LHC	p p	14 TeV	$10^{34}$
	Pb Pb	1312 TeV	$10^{27}$

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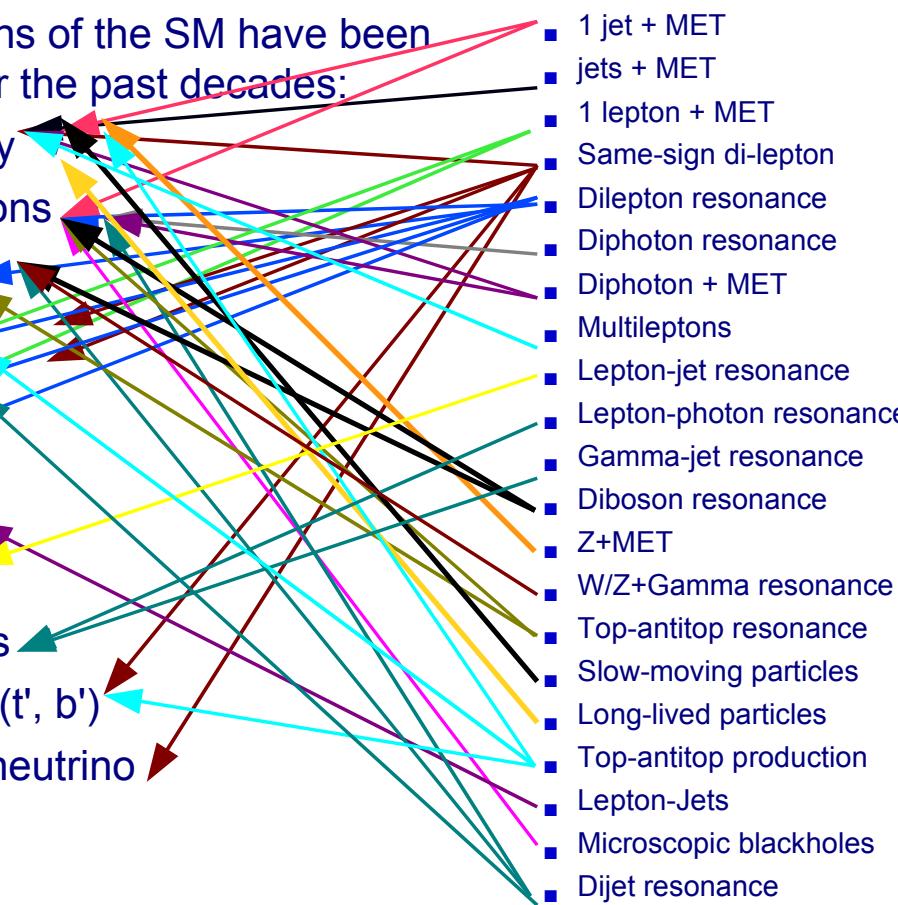
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## A very long list of models x signatures

- Many extensions of the SM have been developed over the past decades:
  - Supersymmetry
  - Extra-Dimensions
  - Technicolor(s)
  - Little Higgs
  - No Higgs
  - GUT
  - Hidden Valley
  - Leptoquarks
  - Compositeness
  - 4<sup>th</sup> generation (t', b')
  - LRSM, heavy neutrino
  - etc...
- 

(for illustration only)

**A complex 2D problem**

Experimentally,  
**a signature standpoint**  
makes a lot of  
sense:

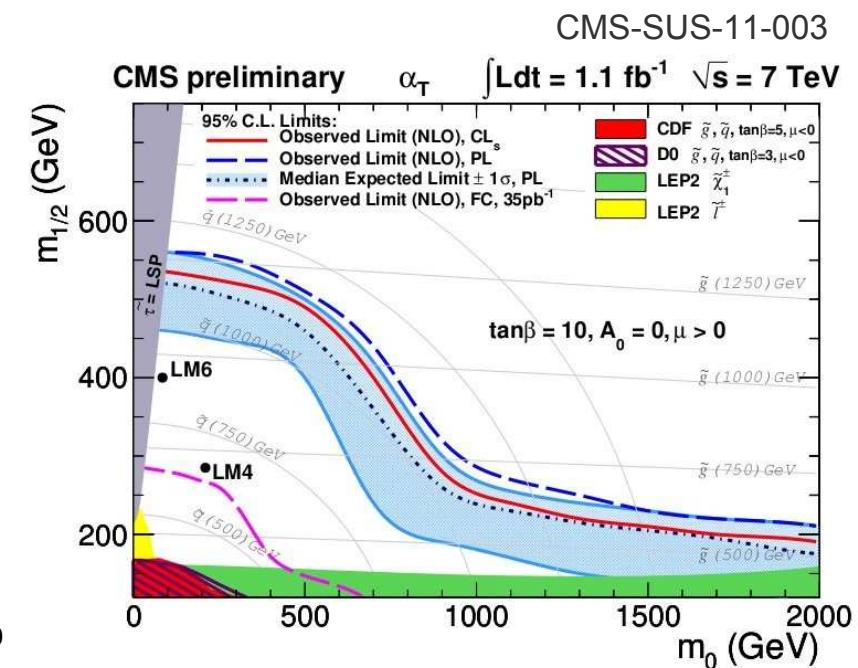
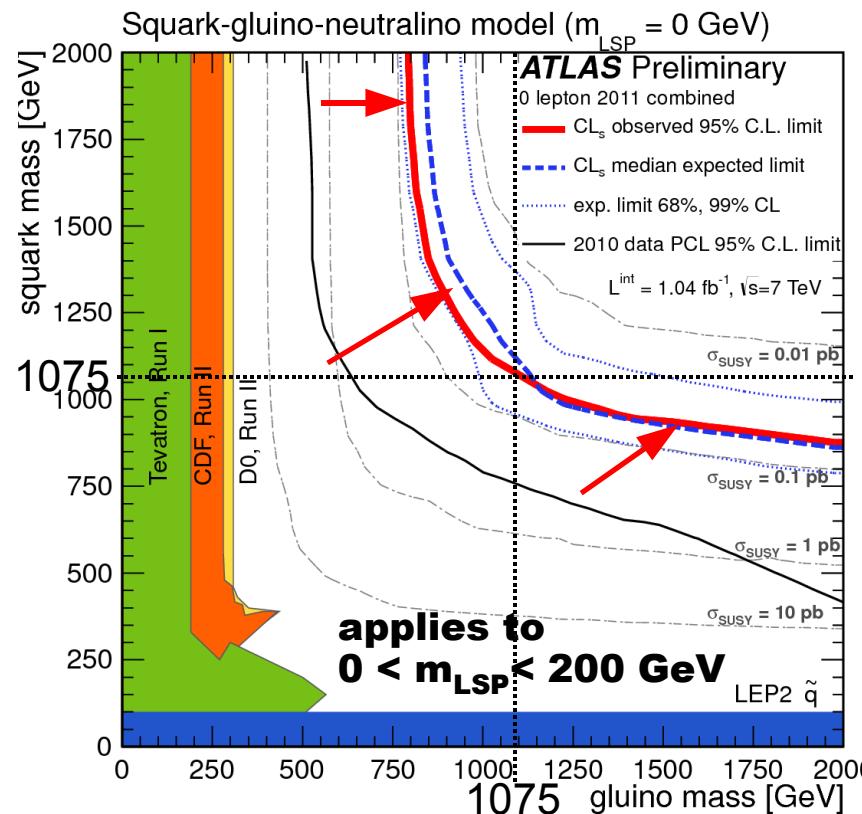
- Practical
- Less model-dependent
- **Important to cover every possible signature**

## 1. SUSY: Jets + Missing $E_T$

$$\tilde{q} \rightarrow q \tilde{\chi}_1^0$$

$$\tilde{g} \rightarrow q \bar{q} \tilde{\chi}_1^0$$

- Exclude up to  $\sim 1$  TeV for  $m(\text{squark}) = m(\text{gluino})$



# Results from the LHC in 2012

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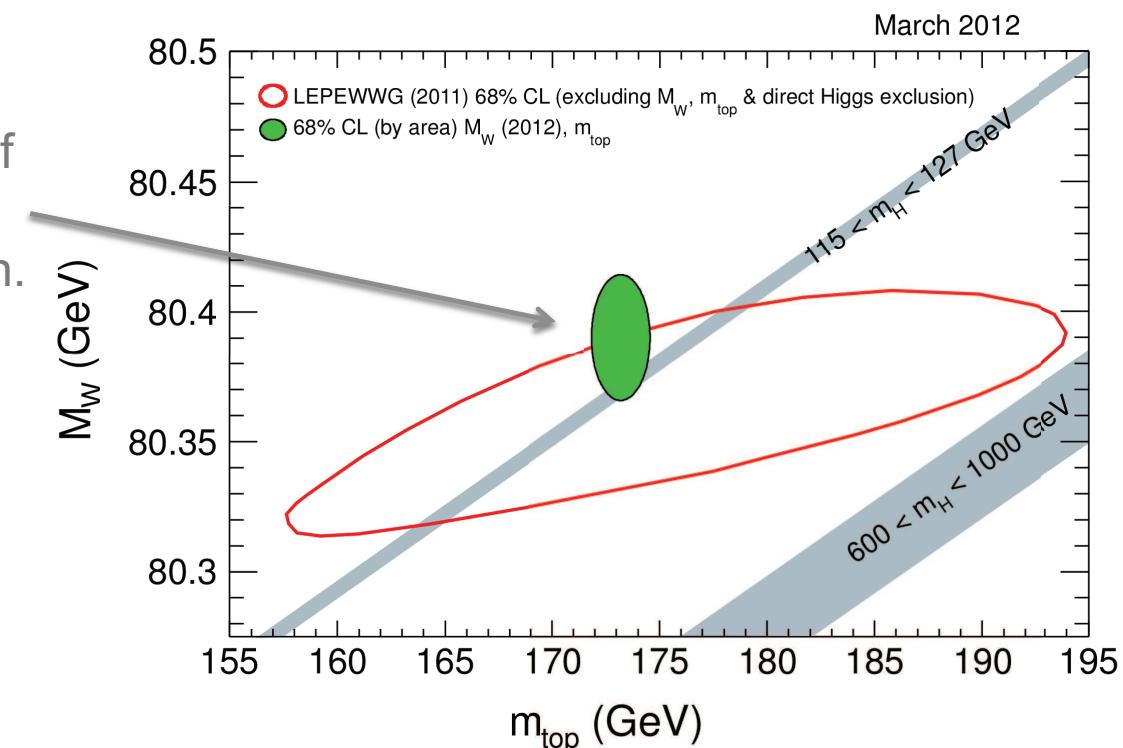

## Situation in early 2012



Very precise measurement of  $M_W = 80.390 \pm 0.016$  GeV, driven mainly by the Tevatron.

Much of the SM Higgs range had been ruled out by 2011 LHC running.

Excess of events in the low mass region seen in ATLAS and CMS



Exclusions of  $M_H$ :

- LEP  $< 114$  GeV (arXiv:0602042v1)
- Tevatron [156, 177] GeV ( arXiv:1107.5518)
- LHC [ $\sim 127$ , 600] GeV arXiv:1202.1408 (ATLAS)  
arXiv:1202.1488 (CMS)

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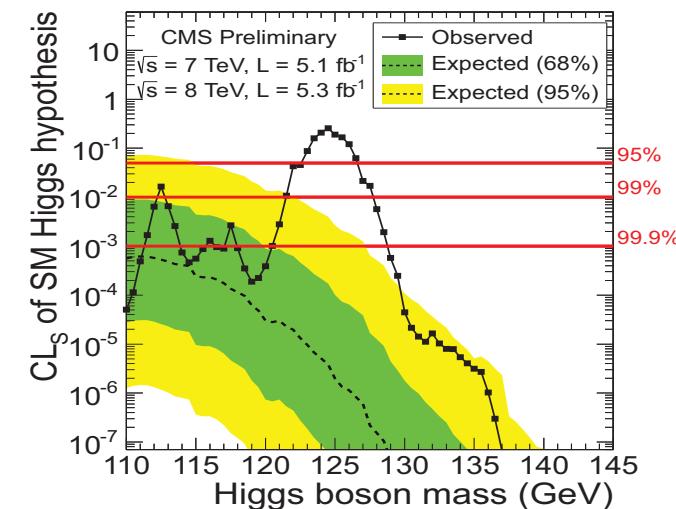
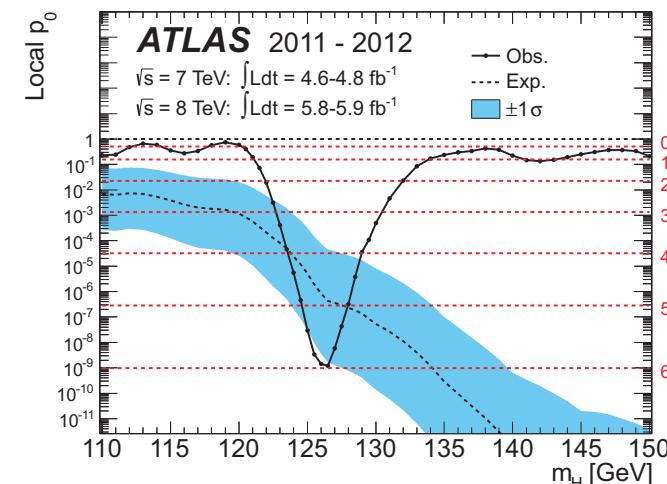
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## 2. The 4th of July and after

**After 48 years of postulat, 30 years of search (and a few heart attacks),  
the Higgs is discovered at LHC on the 4th of July: Higgstорical day!**



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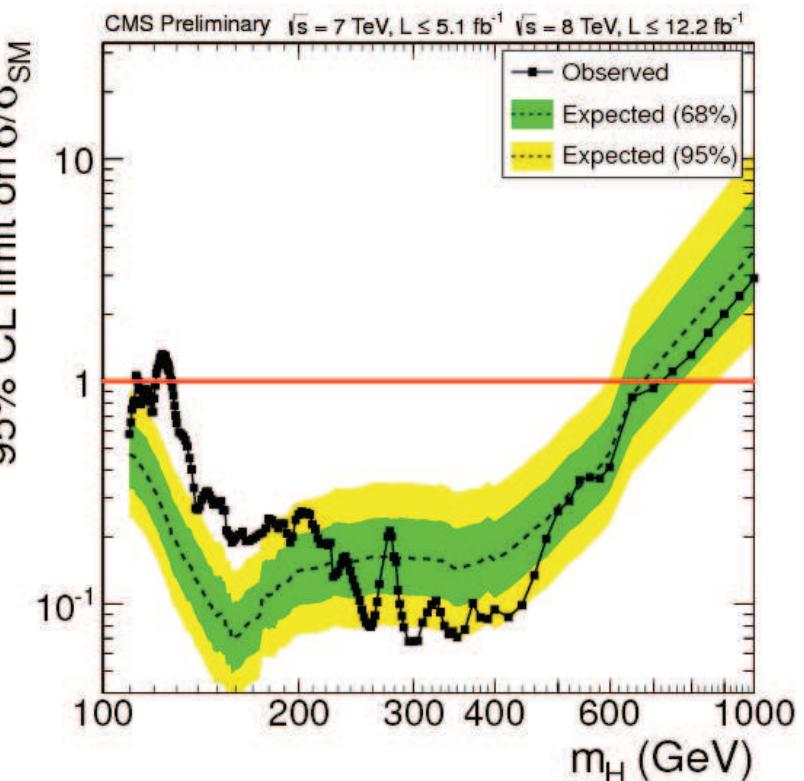
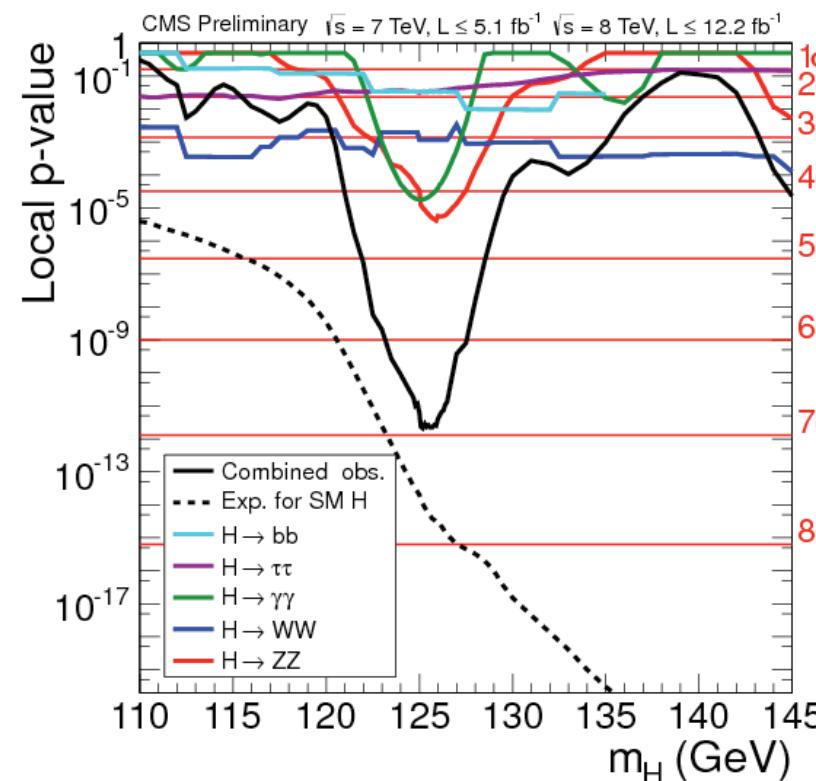
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## Higgs combined results



Combination of 5 channels:  $b\bar{b}$ ,  $\tau\tau$ ,  $WW$ ,  $ZZ$ ,  $\gamma\gamma$

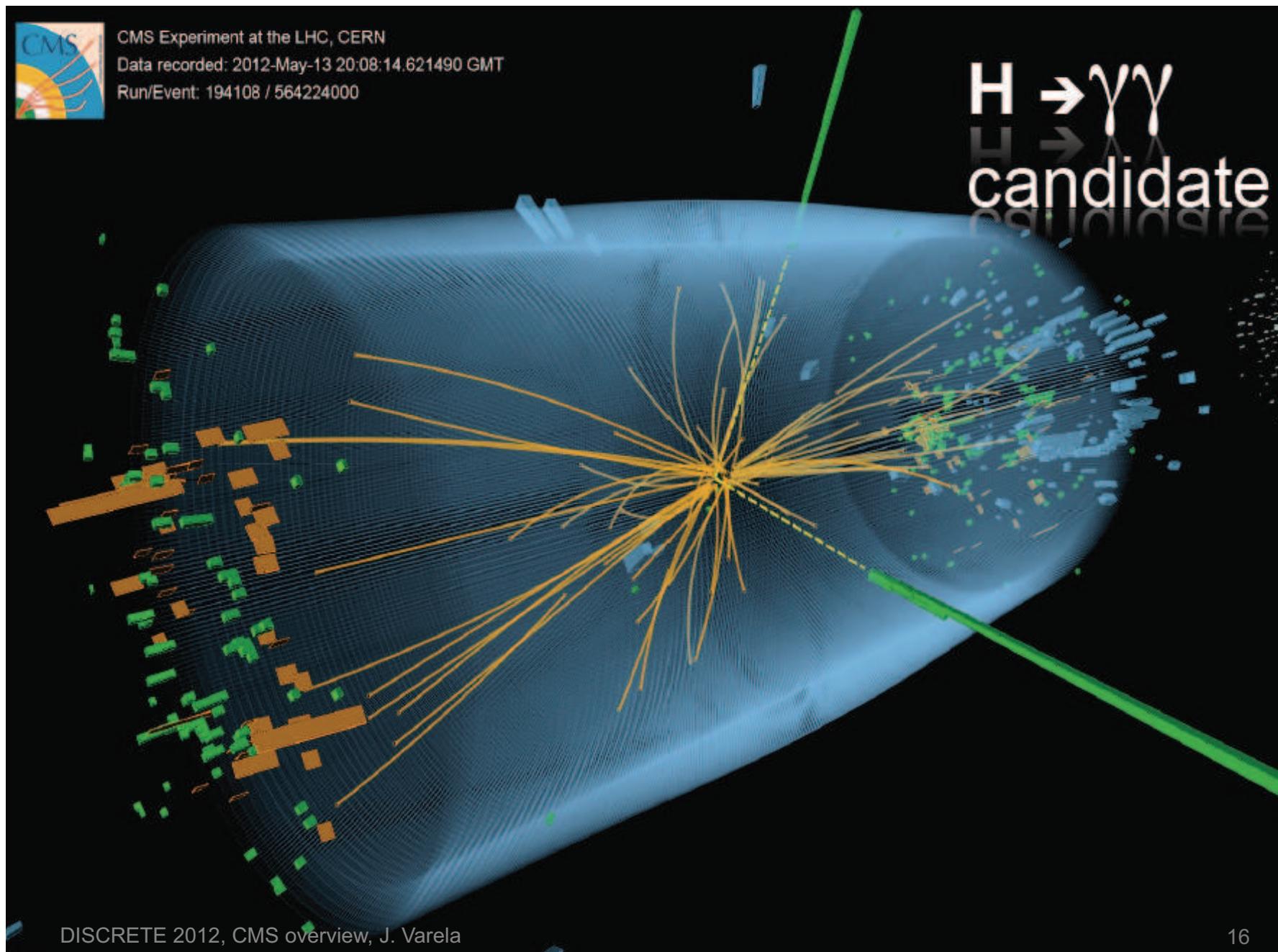


Significance  $6.9\sigma$  versus  $7.8\sigma$  expected.

# Results from the LHC in 2012

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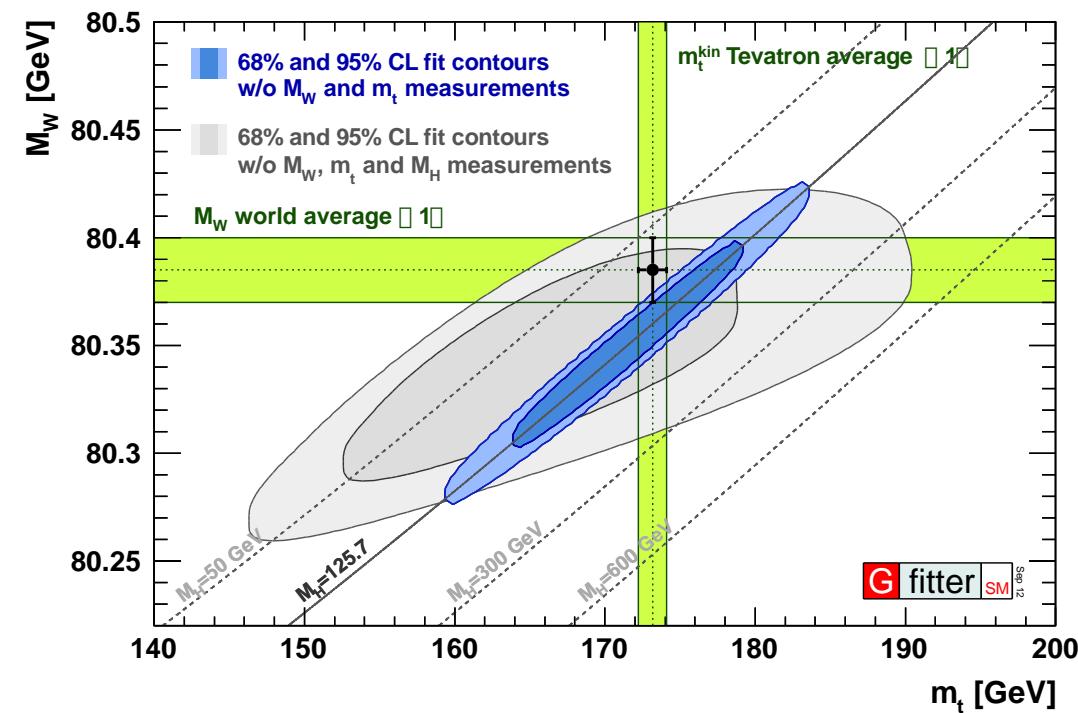
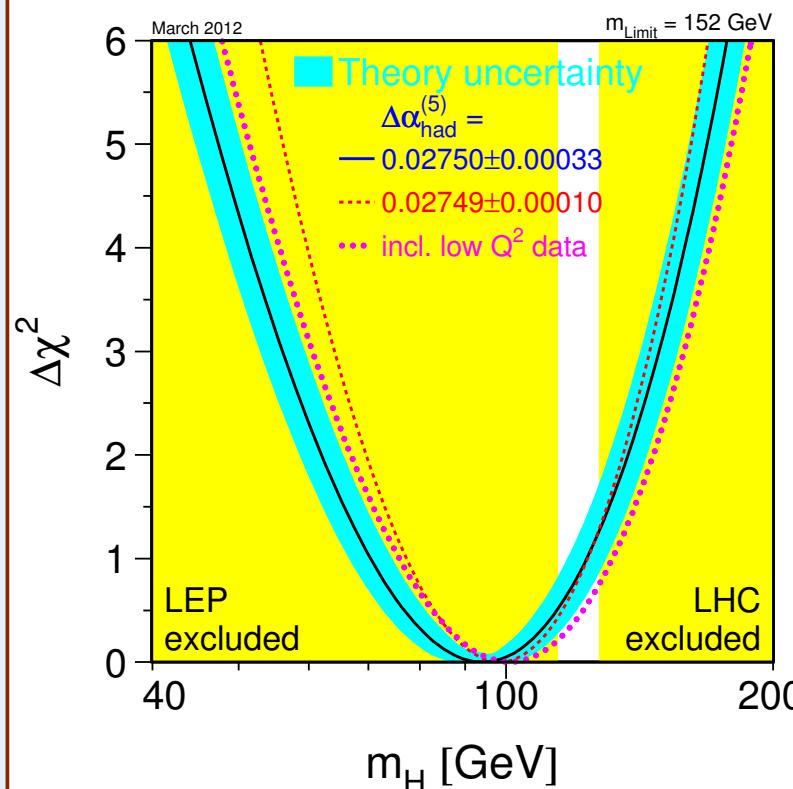
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- Results from direct searches in good agreement with indirect evidence
- The Higgs boson is light!



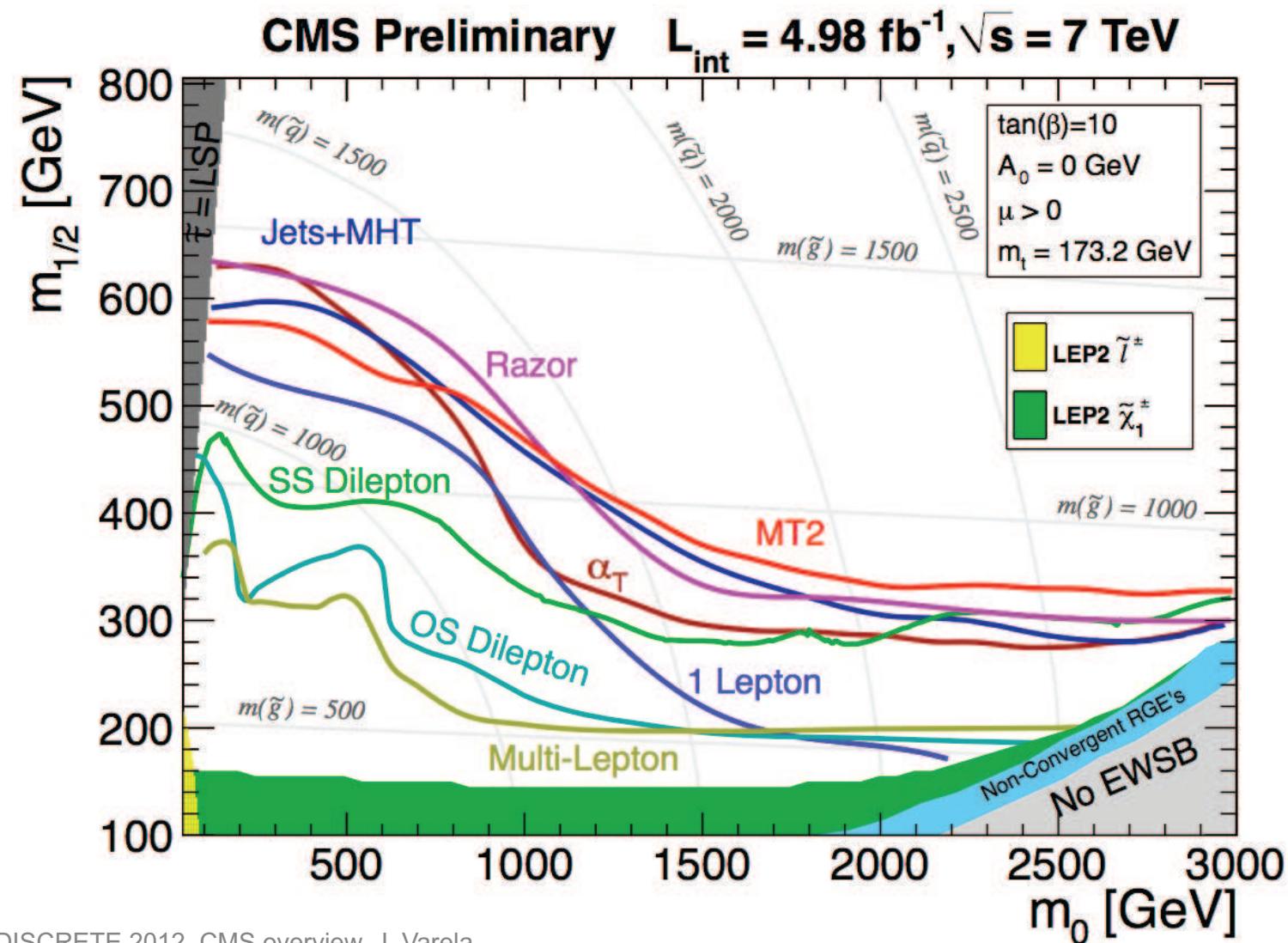
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## CMSSM interpretation



DISCRETE 2012, CMS overview, J. Varela

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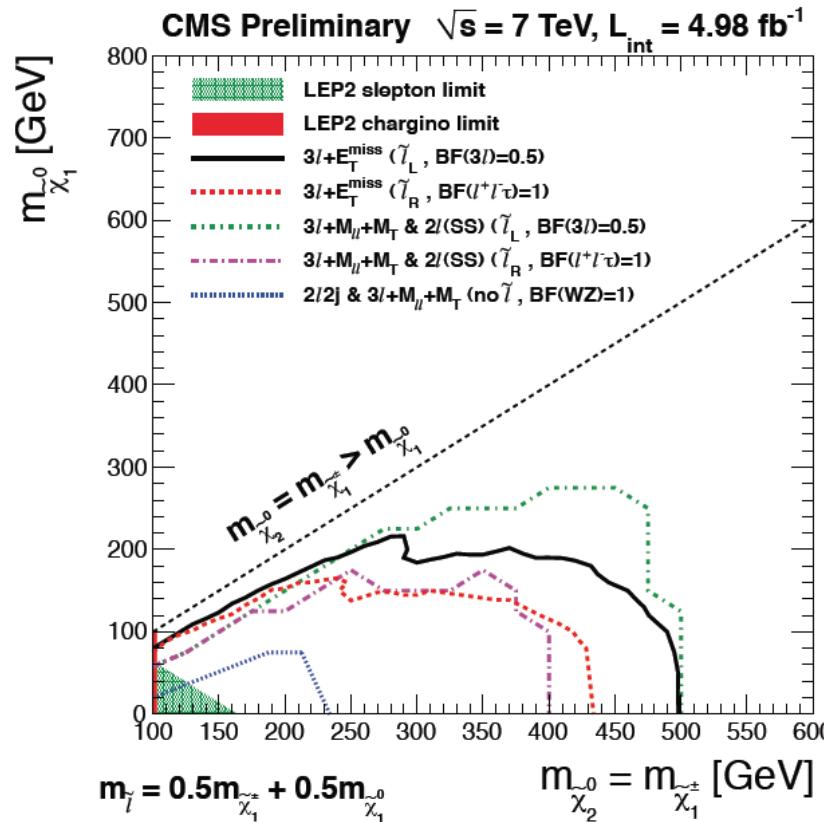
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## $\chi^+\chi^0$ exclusion limits

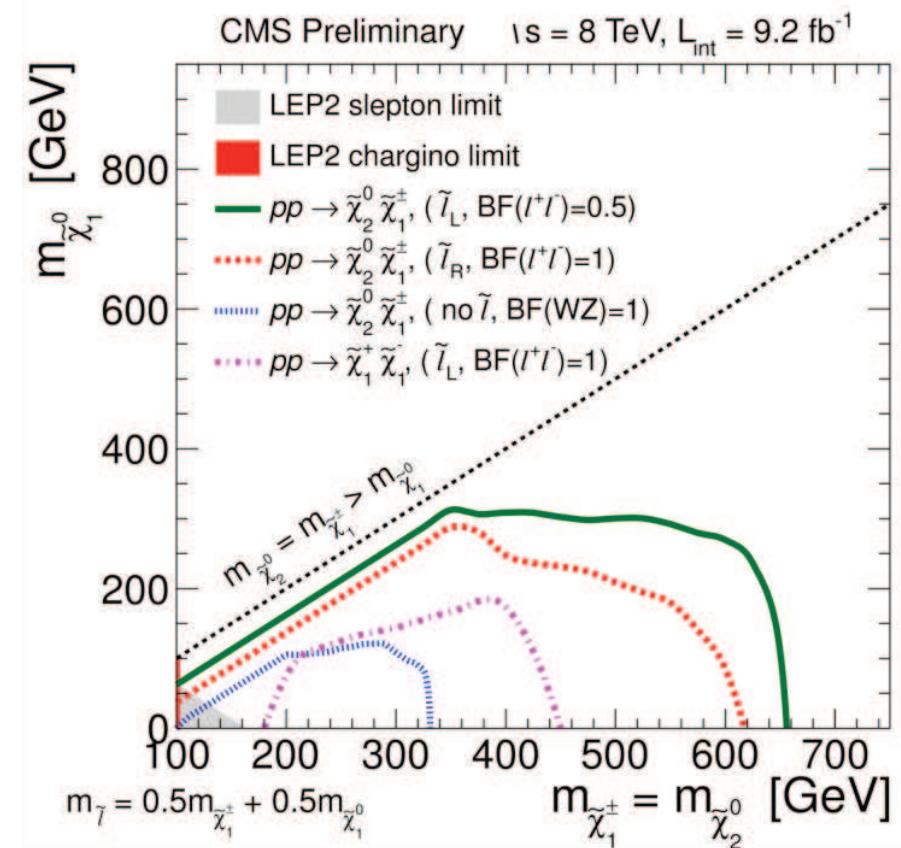


### 7 TeV result



SUS-12-006

### New 8 TeV result



SUS-12-022

# Results from the LHC in 2012: SUSY

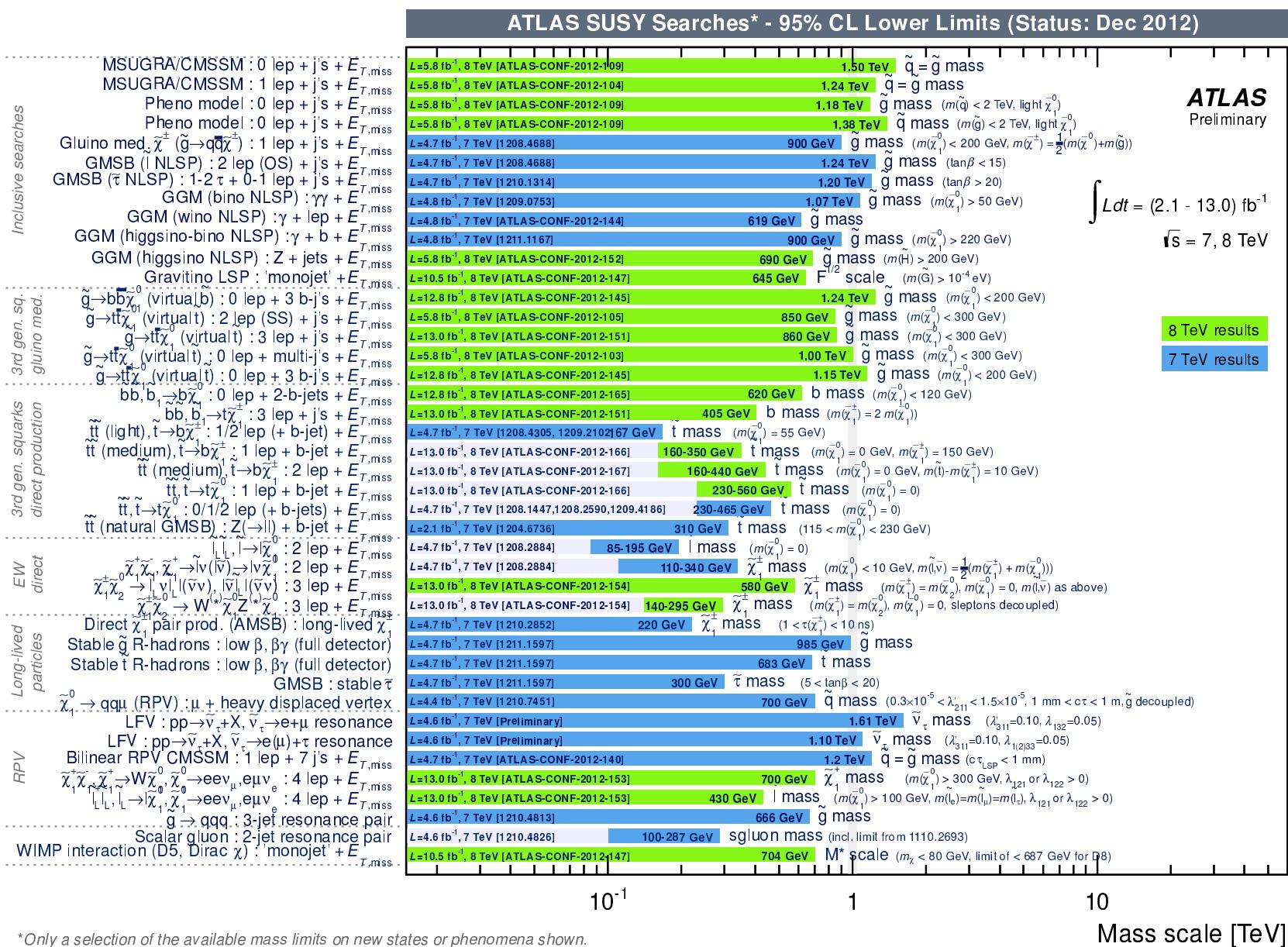
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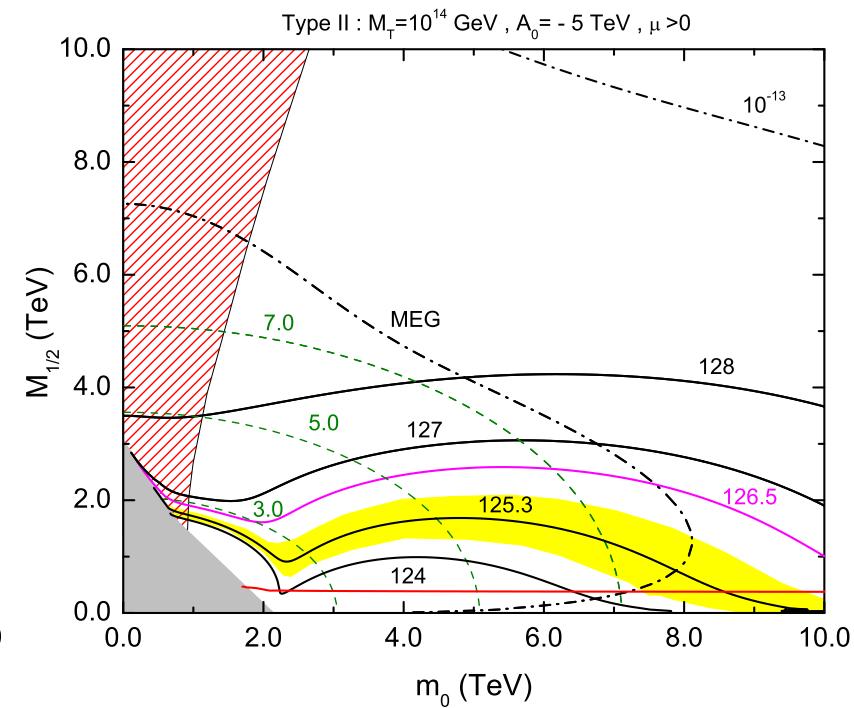
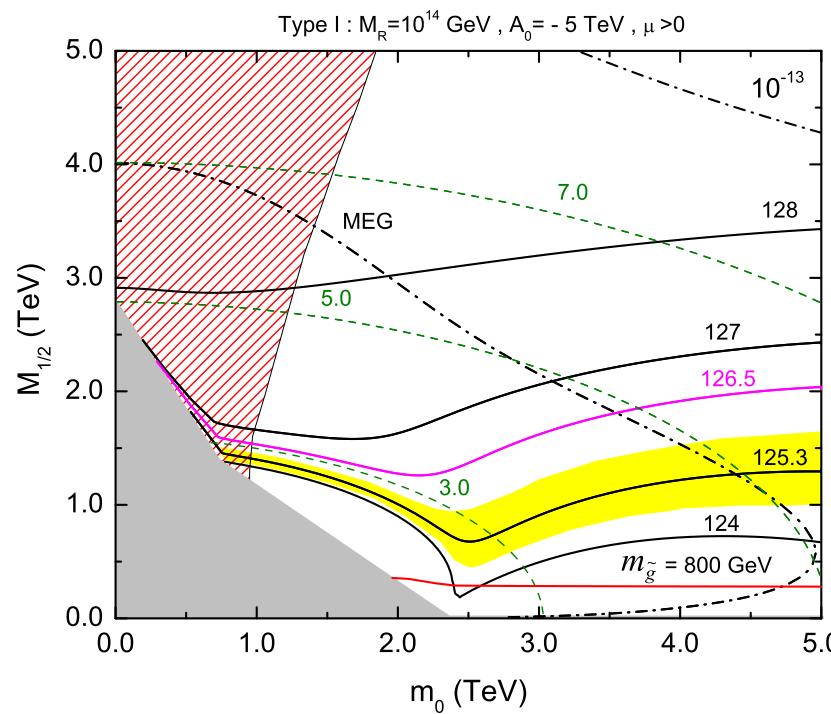
- 2012 Results

[Implications](#)


# Implications for SUSY: 1

- From the experimental evidence
- From internal consistency of SUSY

**Spectra is Heavy  $\mathcal{O}(\text{few TeV})$**



Hirsch, Joaquim and Vicente, JHEP11(2012)109

## Implications for SUSY: 2

Two possible views:

### □ The Pessimist

- ◆ Spectra is too heavy
- ◆ Too much fine tuning
- ◆ Even if true, LHC will not see it. It will be hiding forever!

### □ The Optimist

- ◆ After all, the Higgs boson is light in agreement with indirect searches and with the SUSY predictions
- ◆ Given the Higgs mass around 125 GeV, the spectra is where it should be!
- ◆ With a bit of luck LHC will see it and the seeking will be successful

Conclusion: While SUSY is still hiding we should continue seeking for it at LHC

# Bibliography

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## Books

- ◆ *Supersymmetry and Supergravity*, Julius Wess and Jonathan Bagger.  
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- ◆ *Supersymmetric Gauge Field Theory and String Theory*, David Bailin and Alexander Love.  $(+, -, -, -)$
- ◆ *Supersymmetry in Particle Physics*, Ian Aitchison.  $(+, -, -, -)$

## Other Texts

- ◆ *The search for supersymmetry: Probing physics beyond the standard model*, H. E. Haber and G. L. Kane, Phys. Rep. 117 (1985) 75.  
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- ◆ *A Supersymmetry primer*, Stephen Martin, hep-ph/9709356.  
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- ◆ *BUSSTEPP Lectures on Supersymmetry*, José M. Figueroa-O'Farrill, hep-ph/0109172.  $(-, +, +, +)$
- ◆ *The Minimal Supersymmetric Standard Model*, Jorge C. Romão, (see my homepage).  $(+, -, -, -)$