Introduction to Supersymmetry

Jorge C. Romão
Instituto Superior Técnico, Departamento de Física & CFTP
A. Rovisco Pais 1, 1049-001 Lisboa, Portugal

September 21, 2010

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Other Texts

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Motivation

Although there is not yet direct experimental evidence for supersymmetry (SUSY), there are many theoretical arguments indicating that SUSY might be of relevance for physics below the 1 TeV scale.

The most commonly invoked theoretical arguments for SUSY are:

- Interrelates matter fields (leptons and quarks) with force fields (gauge and/or Higgs bosons).
- As local SUSY implies gravity (supergravity) it could provide a way to unify gravity with the other interactions.
- As SUSY and supergravity have fewer divergences than conventional field theories, the hope is that it could provide a consistent (finite) quantum gravity theory.
- SUSY can help to understand the mass problem, in particular solve the naturalness problem (and in some models even the hierarchy problem) if SUSY particles have masses $\leq \mathcal{O}(1\text{TeV})$.

The Naturalness Problem: I

- As the SM is not asymptotically free, at some energy scale Λ, the interactions must become strong indicating the existence of new physics. Candidates for this scale: $M_X \simeq \mathcal{O}(10^{16}$ GeV) in GUT's or the Planck scale $M_P \simeq \mathcal{O}(10^{19}\text{GeV})$.
- The only consistent way to give masses to the gauge bosons and fermions is through the Higgs mechanism involving at least one spin zero Higgs boson.
- Although the Higgs boson mass is not fixed by the theory, a value much bigger than $< H^0 > \simeq G_F^{-1/2} \simeq 250$ GeV would imply that the Higgs sector would be strongly coupled making it difficult to understand why we are seeing an apparently successful perturbation theory at low energies.
- The one loop radiative corrections to the Higgs boson mass

$$\delta m_H^2 = \mathcal{O}\left(\frac{\lambda^2}{16\pi^2}\right)\ \Lambda^2 + \cdots$$

would be too large if Λ is identified with Λ_{GUT} or Λ_{Planck}.

The Naturalness Problem: II

- SUSY cures this problem in the following way. If SUSY were exact, radiative corrections to the scalar masses squared would be absent because the contribution of fermion loops exactly cancels against the boson loops.
- Therefore if SUSY is broken, as it must, we should have [show details]

$$\delta m_H^2 = \mathcal{O}\left(\tfrac{\lambda}{16\pi^2}\right)\ \left(m_F^2 - m_B^2\right) \ln \frac{\Lambda}{m_B} + \cdots$$

- We conclude that

SUSY provides a solution for the the naturalness problem if the masses of the superpartners are below $\mathcal{O}(1\ \text{TeV})$. This is the main reason behind all the phenomenological interest in SUSY.

The Poincare Algebra

The Poincaré group is made up of the Lorentz group plus the translations. We denote by $J_{\mu\nu}$ the generators of the Lorentz group and by P_μ the generators of the translations. The algebra is defined by,

$$[J_{\mu\nu}, J_{\rho\sigma}] = i\,(g_{\nu\rho} J_{\mu\sigma} - g_{\nu\sigma} J_{\mu\rho} - g_{\mu\rho} J_{\nu\sigma} + g_{\mu\sigma} J_{\nu\rho})$$

$$[P_\alpha, J_{\mu\nu}] = i\,(g_{\mu\alpha} P_\nu - g_{\nu\alpha} P_\mu)$$

$$[P_\mu, P_\nu] = 0$$

One can show that [show details]

$$\left[P^2, J_{\mu\nu}\right] = \left[P^2, P_\mu\right] = 0$$

$$\left[W^2, J_{\mu\nu}\right] = \left[W^2, P_\mu\right] = \left[W^2, P^2\right] = 0$$

where

$$W_\mu = -\frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\ J^{\nu\rho}\ P^\sigma$$

is the Pauli-Lubanski vector operator.

SUSY Algebra

The SUSY generators carry Spin 1/2 and obey the following algebra

$$
\begin{aligned}
\{Q_\alpha, Q_\beta\} &= 0 \\
\left\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\right\} &= 0 \\
\left\{Q_\alpha, \overline{Q}_{\dot\beta}\right\} &= 2\left(\sigma^\mu\right)_{\alpha\dot\beta} P_\mu
\end{aligned}
$$

where

$$
\sigma^\mu \equiv (1, \sigma^i) \quad ; \quad \overline{\sigma}^\mu \equiv (1, -\sigma^i)
$$

and $\alpha, \beta, \dot\alpha, \dot\beta = 1, 2$ (Weyl 2–component spinor notation).

SUSY Algebra and the Poincaré Group

The commutation relations with the generators of the Poincaré group

$$[P^\mu, Q_\alpha] = 0 \qquad [J^{\mu\nu}, Q_\alpha] = -i\,(\sigma^{\mu\nu})_\alpha{}^\beta\, Q_\beta$$

One can easily derive that the two invariants of the Poincaré group,

$$P^2 = P_\alpha P^\alpha \qquad W^2 = W_\alpha W^\alpha \qquad W_\mu = -\tfrac{1}{2}\epsilon_{\mu\nu\rho\sigma} J^{\nu\rho} P^\sigma$$

$$P^2 \left|m,s\right\rangle = m^2 \left|m,s\right\rangle \quad W^2 \left|m,s\right\rangle = -m^2 s(s+1) \left|m,s\right\rangle$$

where W^μ is the Pauli–Lubanski vector operator, are no longer invariants of the Super Poincaré group:

$$[Q_\alpha, P^2] = 0 \qquad [Q_\alpha, W^2] \neq 0$$

Irreducible multiplets will have particles of the same mass but different spin.

Simple Results from the Algebra

Number of Bosons = Number of Fermions

$$Q_\alpha|B>=|F> \qquad (-1)^{N_F}|B>=|B>$$
$$Q_\alpha|F>=|B> \qquad (-1)^{N_F}|F>=-|F>$$

where $(-1)^{N_F}$ is the fermion number of a given state. Then we obtain

$$Q_\alpha(-1)^{N_F} = -(-1)^{N_F}Q_\alpha$$

Using this relation we can show that

$$\begin{aligned} Tr\left[(-1)^{N_F}\left\{Q_\alpha,\overline{Q}_{\dot\alpha}\right\}\right] &= Tr\left[(-1)^{N_F}Q_\alpha\overline{Q}_{\dot\alpha}+(-1)^{N_F}\overline{Q}_{\dot\alpha}Q_\alpha\right] \\ &= Tr\left[-Q_\alpha(-1)^{N_F}\overline{Q}_{\dot\alpha}+Q_\alpha(-1)^{N_F}\overline{Q}_{\dot\alpha}\right]=0 \end{aligned}$$

But we also have

$$Tr\left[(-1)^{N_F}\left\{Q_\alpha,\overline{Q}_{\dot\alpha}\right\}\right] = Tr\left[(-1)^{N_F}2\sigma^\mu_{\alpha\dot\alpha}P_\mu\right]$$

$\Rightarrow$

$$Tr\left[(-1)^{N_F}\right] = \#\text{Bosons} - \#\text{Fermions} = 0$$

SUSY Representations: Massive case

In the rest frame $$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = 2\,m\;\delta_{\alpha\dot\alpha}$$

This algebra is similar to the algebra of the spin 1/2 creation and annihilation operators. Choose $|\Omega\rangle$ such that

$$Q_1\,|\Omega\rangle = Q_2\,|\Omega\rangle = 0$$

Then we have 4 states $$|\Omega\rangle\ \ ;\ \ \overline{Q}_1\,|\Omega\rangle\ \ ;\ \ \overline{Q}_2\,|\Omega\rangle\ \ ;\ \ \overline{Q}_1\overline{Q}_2\,|\Omega\rangle$$

If $J_3\,|\Omega\rangle = j_3\,|\Omega\rangle$

State	J_3 Eigenvalue
$\|\Omega\rangle$	j_3
$\overline{Q}_1\,\|\Omega\rangle$	$j_3 + \frac{1}{2}$
$\overline{Q}_2\,\|\Omega\rangle$	$j_3 - \frac{1}{2}$
$\overline{Q}_1\overline{Q}_2\,\|\Omega\rangle$	j_3

Two bosons and two fermions states separated by one half unit of spin.

SUSY Representations: Massless case

If $m = 0$ then we can choose $P^\mu = (E, 0, 0, E)$. In this frame

$$\{Q_\alpha, \overline{Q}_{\dot\alpha}\} = M_{\alpha\dot\alpha}$$

where the matrix M takes the form

$$M = \begin{pmatrix} 0 & 0 \\ 0 & 4E \end{pmatrix}$$

Then $\{Q_2, \overline{Q}_2\} = 4E$ all others vanish.

We have then just **two** states $|\Omega\rangle \;\; ; \;\; \overline{Q}_2 \, |\Omega\rangle$

If $J_3 \, |\Omega\rangle = \lambda \, |\Omega\rangle$

State	J_3 Eigenvalue
$\|\Omega\rangle$	λ
$\overline{Q}_2 \, \|\Omega\rangle$	$\lambda - \frac{1}{2}$

Two states, one fermion one boson separated by one half unit of spin.

Superfields for Massless Particles

[show details]

- Chiral Superfields: Spin 0 + Spin $\frac{1}{2}$

$\Phi = \Phi(\phi, \chi_L)$ → ϕ Complex Scalar: $2\,\mathrm{d.o.f}$; χ_L Chiral Fermion: $2\,\mathrm{d.o.f}$ (on-shell)

- Vector Superfields: Spin $\frac{1}{2}$ + Spin 1

$V = V(\lambda, W^\mu)$ → λ Chiral Fermion: $2\,\mathrm{d.o.f}$; W^μ Massless Vector: $2\,\mathrm{d.o.f}$ (on-shell)

ϕ → superpartner of the fermion: sfermion

λ → superpartner of gauge field: gaugino

Gauge Fields

We want to have gauge fields for the gauge group $G = SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$. Therefore we will need three vector superfields (or vector supermultiplets) $\widehat{V}_i$ with the following components:

$$
\begin{aligned}
&\widehat{V}_1 \equiv (\lambda', W_1^{\mu}) && \rightarrow && U_Y(1) && && \\
&\widehat{V}_2 \equiv (\lambda^a, W_2^{\mu a}) && \rightarrow && SU_L(2) && , && a = 1,2,3 \\
&\widehat{V}_3 \equiv (\widetilde{g}^b, W_3^{\mu b}) && \rightarrow && SU_c(3) && , && b = 1,\dots,8
\end{aligned}
$$

where W_i^{μ} are the gauge fields and λ', λ and $\widetilde{g}$ are the $U_Y(1)$ and $SU_L(2)$ gauginos and the gluino, respectively.

Particle Content: Leptons

Leptons

As each chiral multiplet only describes one helicity state, we will need two chiral multiplets for each charged lepton (We will assume that the neutrinos do not have mass).

Supermultiplet	$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers
$\widehat{L}_i \equiv (\widetilde{L}, L)_i$	$(1, 2, -\frac{1}{2})$
$\widehat{R}_i \equiv (\widetilde{\ell}_R, \ell^c_L)_i$	$(1, 1, 1)$

Each helicity state corresponds to a complex scalar and we have that $\hat{L}_i$ is a doublet of $SU_L(2)$

$$\widetilde{L}_i = \begin{pmatrix} \widetilde{\nu}_{Li} \\ \widetilde{\ell}_{Li} \end{pmatrix} \qquad ; \qquad L_i = \begin{pmatrix} \nu_{Li} \\ \ell_{Li} \end{pmatrix}$$

Particle Content: Quarks

Quarks

The quark supermultiplets are given in the Table. The supermultiplet $\widehat{Q}_i$ is also a doublet of $SU_L(2)$, that is

Supermultiplet	$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers
$\widehat{Q}_i \equiv (\widetilde{Q}, Q)_i$	$(3, 2, \frac{1}{6})$
$\widehat{D}_i \equiv (\widetilde{d}_R, d^c_L)_i$	$(3, 1, \frac{1}{3})$
$\widehat{U}_i \equiv (\widetilde{u}_R, u^c_L)_i$	$(3, 1, -\frac{2}{3})$

$$\widetilde{Q}_i = \begin{pmatrix} \widetilde{u}_{Li} \\ \widetilde{d}_{Li} \end{pmatrix} \qquad ; \qquad Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}$$

Particle Content: Higgs Bosons

Higgs Bosons

Finally the Higgs sector. In the MSSM we need at least two Higgs doublets. This is in contrast with the SM where only one Higgs doublet is enough to give masses to all the particles. The reason can be explained in two ways. Either the need to cancel the anomalies, or the fact that, due to the analyticity of the superpotential, we have to have two Higgs doublets of opposite hypercharges to give masses to the up and down type of quarks.

Supermultiplet	$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers
$\widehat{H}_1 \equiv (H_1, \widetilde{H}_1)$	$(1, 2, -\frac{1}{2})$
$\widehat{H}_2 \equiv (H_2, \widetilde{H}_2)$	$(1, 2, +\frac{1}{2})$

R-Parity

Most discussions of SUSY phenomenology assume R-Parity conservation where,

$$R_P = (-1)^{2J+3B+L}$$

This is the case of the MSSM. It implies:

- SUSY particles are pair produced.
- Every SUSY particle decays into another SUSY particle.
- There is a LSP that it is stable ($\not{E}$ signature).

But this is just an ad hoc assumption without a deep justification. We will see later what are the consequences of non conservation of R-Parity.

Kinetic Terms

Like in any gauge theory we have

$$\mathcal{L}_{kin} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + i\overline{\lambda^a}\overline{\sigma}^\mu D_\mu \lambda^a + (D_\mu \phi)^\dagger D^\mu \phi + i\overline{\chi}\overline{\sigma}^\mu D_\mu \chi$$

where the field strength $F^a_{\mu\nu}$ is given by

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc} W^b_\mu W^c_\nu$$

and f^{abc} are the structure constants of the gauge group G. The covariant derivative is

$$D_\mu = \partial_\mu + i g W^a_\mu T^a$$

One should note that χ and λ are left handed chiral spinors.

Self Interactions of the Gauge Multiplet

For a non Abelian gauge group G we have the usual self–interactions, (cubic and quartic), of the gauge bosons with themselves. But we have a new interaction of the gauge bosons with the gauginos. In two component spinor notation it reads

$$\mathcal{L}_{\lambda\lambda W} = i g f_{abc}\, \lambda^a \sigma^\mu \overline{\lambda}^b\, W_\mu^c + h.c.$$

where f_{abc} are the structure constants of the gauge group G and the matrices σ^μ are the equivalent of the γ matrices in two component language.

Interactions of the Gauge and Matter Multiplets

In the usual non Abelian gauge theories we have the interactions of the gauge bosons with the fermions and scalars of the theory. In the supersymmetric case we also have interactions of the gauginos with the fermions and scalars of the chiral matter multiplet. The general form, in two component spinor notation is,

$$
\begin{aligned}
\mathcal{L}_{\Phi W} &= -g T^a_{ij} W^a_\mu \left(\overline{\chi}_i \overline{\sigma}^\mu \chi_j + i \phi^*_i \overleftrightarrow{\partial}_\mu \phi_j \right) + g^2 \left(T^a T^b \right)_{ij} W^a_\mu W^{\mu b} \, \phi^*_i \phi_j \\
&\quad + i g \sqrt{2} \, T^a_{ij} \left(\lambda^a \chi_j \phi^*_i - \overline{\lambda}^a \overline{\chi}_i \phi_j \right)
\end{aligned}
$$

where the new interactions of the gauginos with the fermions and scalars are given in the last term.

Self Interactions of the Matter Multiplet

These correspond in non supersymmetric gauge theories both to the Yukawa interactions and to the scalar potential. In supersymmetric gauge theories we have less freedom to construct these terms. The first step is to construct the superpotential W. This must be a gauge invariant polynomial function of the *scalar* components of the chiral multiplet Φ_i, that is ϕ_i. It *does not* depend on ϕ_i^*. In order to have renormalizable theories the degree of the polynomial must be at most three.

The Yukawa interactions are

$$\mathcal{L}_{Yukawa} = -\frac{1}{2}\left[\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\,\chi_i\chi_j + \left(\frac{\partial^2 W}{\partial\phi_i\partial\phi_j}\right)^* \,\overline{\chi}_i\overline{\chi}_j\right]$$

and the scalar potential is

$$V_{scalar} = \tfrac{1}{2}D^a D^a + F_i F_i^*$$

where

$$F_i = \frac{\partial W}{\partial \phi_i}, \qquad D^a = g\ \phi_i^* T^a_{ij}\phi_j$$

The Superpotential and SUSY Breaking Lagrangian

The MSSM Lagrangian is specified by the R–parity conserving superpotential W

$$W = \varepsilon_{ab} \left[h_U^{ij} \widehat{Q}_i^a \widehat{U}_j \widehat{H}_2^b + h_D^{ij} \widehat{Q}_i^b \widehat{D}_j \widehat{H}_1^a + h_E^{ij} \widehat{L}_i^b \widehat{R}_j \widehat{H}_1^a - \mu \widehat{H}_1^a \widehat{H}_2^b \right]$$

where $i, j = 1, 2, 3$ are generation indices, $a, b = 1, 2$ are $SU(2)$ indices, and ε is a completely antisymmetric 2×2 matrix, with $\varepsilon_{12} = 1$. The coupling matrices h_U, h_D and h_E will give rise to the usual Yukawa interactions needed to give masses to the leptons and quarks.

If it were not for the need to break SUSY
the number of parameters involved
would be less than in the SM.

SUSY Soft Breaking

The most general SUSY soft breaking is

$$
\begin{aligned}
-\mathcal{L}_{SB} \;=\; & M_Q^{ij2}\widetilde{Q}_i^{a*}\widetilde{Q}_j^a + M_U^{ij2}\widetilde{U}_i\widetilde{U}_j^* + M_D^{ij2}\widetilde{D}_i\widetilde{D}_j^* + M_L^{ij2}\widetilde{L}_i^{a*}\widetilde{L}_j^a + M_R^{ij2}\widetilde{R}_i\widetilde{R}_j^* \\
& + m_{H_1}^2 H_1^{a*}H_1^a + m_{H_2}^2 H_2^{a*}H_2^a - \left[\frac{1}{2}M_s\lambda_s\lambda_s + \frac{1}{2}M\lambda\lambda + \frac{1}{2}M'\lambda'\lambda' + h.c.\right] \\
& + \varepsilon_{ab}\left[A_U^{ij}h_U^{ij}\widetilde{Q}_i^a\widetilde{U}_j H_2^b + A_D^{ij}h_D^{ij}\widetilde{Q}_i^b\widetilde{D}_j H_1^a + A_E^{ij}h_E^{ij}\widetilde{L}_i^b\widetilde{R}_j H_1^a - B\mu H_1^a H_2^b\right]
\end{aligned}
$$

Parameter Counting

Theory	Gauge Sector	Fermion Sector	Higgs Sector
SM	e, g, α_s	h_U, h_D, h_E	μ^2, λ
MSSM	e, g, α_s	h_U, h_D, h_E	μ
Broken MSSM	e, g, α_s	h_U, h_D, h_E	$\mu, M_1, M_2, M_3, A_U, A_D, A_E, B$ $m_{H_2}^2, m_{H_1}^2, m_Q^2, m_U^2, m_D^2, m_L^2, m_R^2$

The Constrained MSSM

The number of independent parameters can be reduced if we impose some further constraints. The most popular is the MSSM coupled to $N=1$ Supergravity (mSUGRA).

$$A_t = A_b = A_\tau \equiv A\,,$$

$$m^2_{H_1} = m^2_{H_2} = M^2_L = M^2_R = m^2_0\,, M^2_Q = M^2_U = M^2_D = m^2_0\,,$$

$$M_3 = M_2 = M_1 = M_{1/2}$$

Parameter Counting

Parameters	Conditions	Free Parameters
$h_t,\ h_b,\ h_\tau,\ v_1,\ v_2$	$m_W,\ m_t,\ m_b,\ m_\tau$	$\tan\beta = v_2/v_1$
$A,\ B,\ m_0,\ M_{1/2},\ \mu$	$t_i = 0,\ i=1,2$	$A,\ m_0,\ M_{1/2},\ \text{sign}(\mu)$
Total = 10	Total = 6	Total = 4+"1"

It is remarkable that with so few parameters we can get the correct values for the parameters, in particular $m^2_{H_2} < 0$. For this to happen the top Yukawa coupling has to be large which we know to be true.

The Chargino Mass Matrices

The charged gauginos mix with the charged higgsinos giving the so–called charginos. In a basis where $\psi^{+T} = (-i\lambda^+, \widetilde{H}_2^+)$ and $\psi^{-T} = (-i\lambda^-, \widetilde{H}_1^-)$, the chargino mass terms in the Lagrangian are

$$\mathcal{L}_m = -\frac{1}{2}(\psi^{+T}, \psi^{-T}) \begin{pmatrix} \mathbf{0} & \boldsymbol{M}_C^T \\ \boldsymbol{M}_C & \mathbf{0} \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + h.c.$$

where the chargino mass matrix is given by

$$\boldsymbol{M}_C = \begin{bmatrix} M_2 & \frac{1}{\sqrt{2}} g v_2 \\ \frac{1}{\sqrt{2}} g v_1 & \mu \end{bmatrix}$$

and M_2 is the $SU(2)$ gaugino soft mass. We can write this as

$$\boldsymbol{M}_C = \begin{bmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{bmatrix}$$

The Neutralino Mass Matrix

The neutral gauginos mix with the neutral higgsinos giving the so–called neutralinos. In the basis $\psi^{0T} = (-i\lambda', -i\lambda^3, \widetilde{H}_1^1, \widetilde{H}_2^2)$ the neutral fermions mass terms in the Lagrangian are given by

$$\mathcal{L}_m = -\frac{1}{2}(\psi^0)^T \boldsymbol{M}_N \psi^0 + h.c.$$

where the neutralino mass matrix is

$$\boldsymbol{M}_N = \begin{bmatrix} M_1 & 0 & -\frac{1}{2}g'v_1 & \frac{1}{2}g'v_2 \\ 0 & M_2 & \frac{1}{2}gv_1 & -\frac{1}{2}gv_2 \\ -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & -\mu \\ \frac{1}{2}g'v_2 & -\frac{1}{2}gv_2 & -\mu & 0 \end{bmatrix}$$

and M_1, M_2 are the gaugino soft mass. [show details]

Neutral Higgs Mass Matrices

$$\boldsymbol{M}^2_{S^0} = \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix} B\mu + \begin{pmatrix} \cot\beta & -1 \\ -1 & \tan\beta \end{pmatrix} \frac{1}{2} m_Z^2 \sin^2\beta$$

with masses

$$m^2_{h,H} = \frac{1}{2}\left[m_A^2 + m_Z^2 \mp \sqrt{(m_A^2+m_Z^2)^2 - 4m_A^2 m_Z^2 \cos 2\beta}\right]$$

$$\boldsymbol{M}^2_{P^0} = \begin{pmatrix} \tan\beta & -1 \\ -1 & \cot\beta \end{pmatrix} B\mu$$ with mass $$m_A^2 = \frac{B\mu}{\sin 2\beta}$$

Sum Rule

$$m_h^2 + m_H^2 = m_A^2 + m_Z^2$$ ⇒ $$m_h < m_A < m_H$$ $$m_h < m_Z < m_H$$

In the Standard Model

$$\mathcal{L}_{\rm Higgs} = (D_\mu \Phi)^\dagger D^\mu \Phi - \frac{\lambda}{4}\left(\Phi^\dagger \Phi - \frac{v^2}{2}\right)^2$$

where

$$\Phi = \begin{pmatrix} \varphi^+ \\ \frac{v+H+i\varphi_Z}{\sqrt{2}} \end{pmatrix}$$

Therefore

$$\mathcal{L}_{\rm Higgs} = \partial_\mu H \partial^\mu H - \frac{\lambda v^2}{4} H^2$$

or

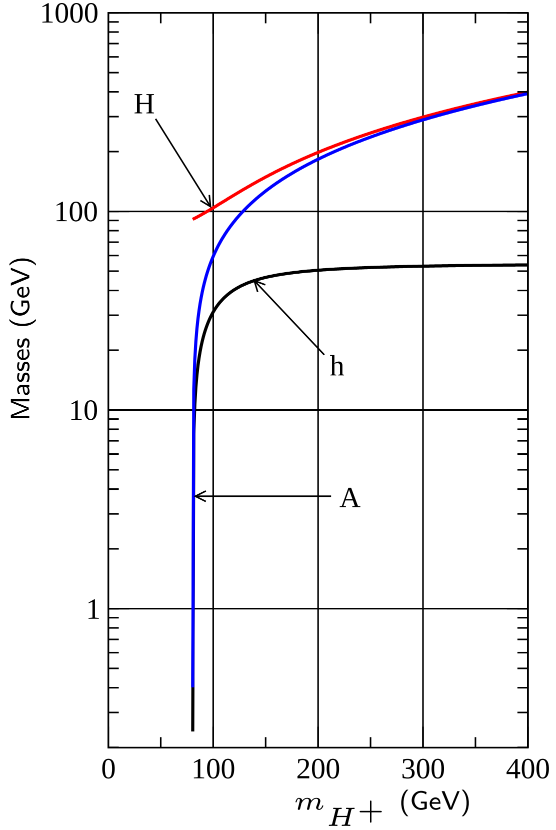
$$M_H = \sqrt{\frac{\lambda}{2}}\, v$$ ⇒ $v = 246$ GeV fixed but λ free. ⇒ M_H free.

Higgs Boson Mass: Radiative corrections

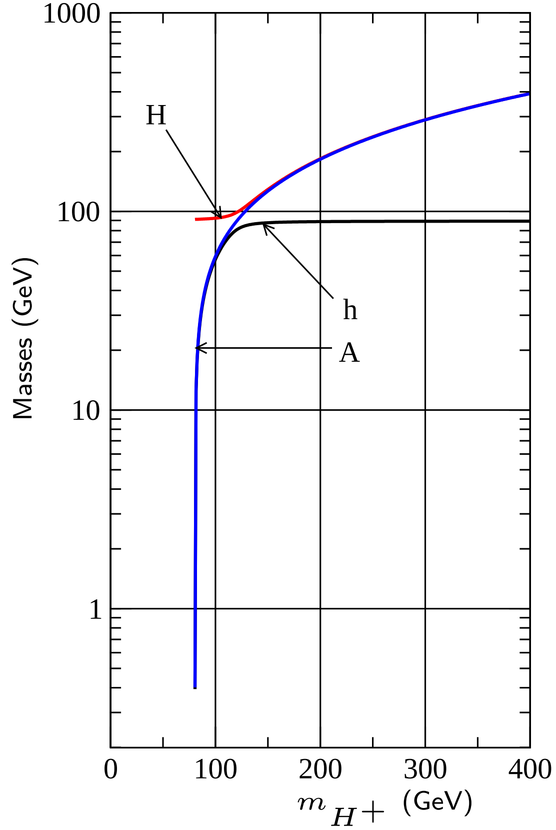
As the top mass is very large there are important radiative corrections to the Higgs boson mass. The most important are:

$$m_h^2 = m_h^{(0)\,2} + \frac{3g^2}{16\pi^2 m_W^2} \frac{m_t^4}{\sin^2\beta} \ln\left(\frac{\tilde{m}_{t_1}^2 \tilde{m}_{t_2}^2}{m_t^4}\right)$$

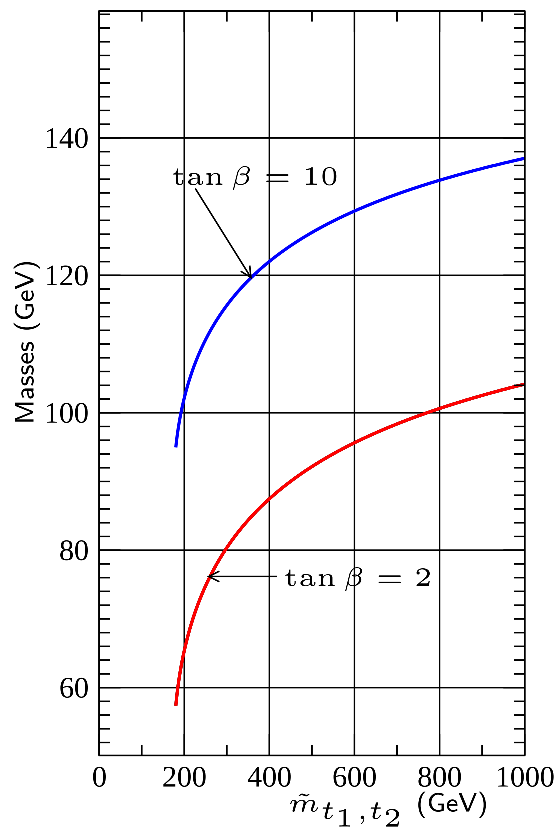
$\tan\beta = 2$



$\tan\beta = 10$

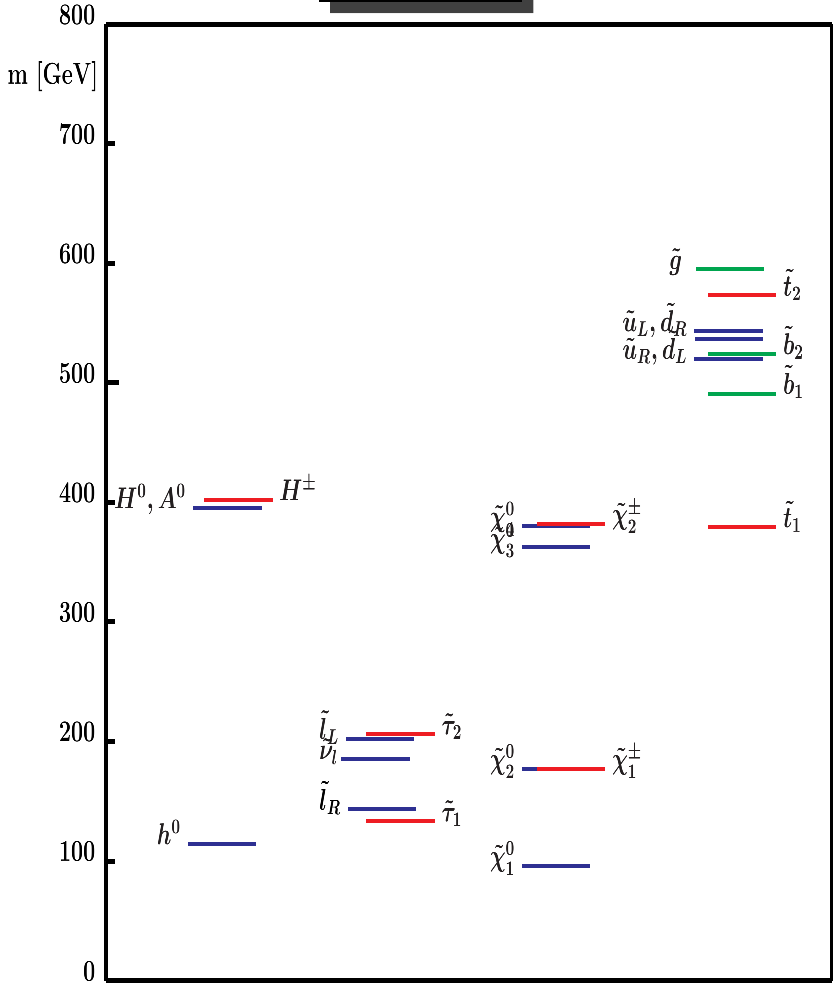


m_h with RC



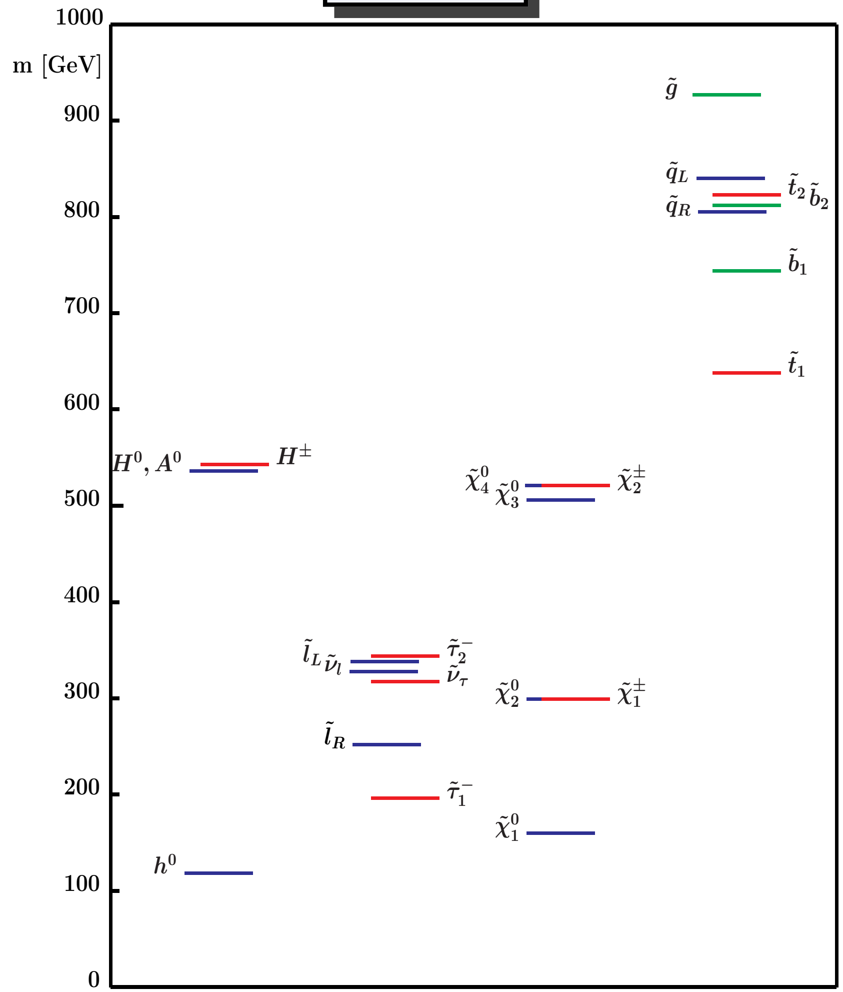
Example of Spectra

SPS1a



$m_0 = 100\text{GeV},\ m_{1/2} = 250\text{GeV}$
$A_0 = -100\text{GeV},\ \tan\beta = 10,\ \mu > 0$

SPS1b



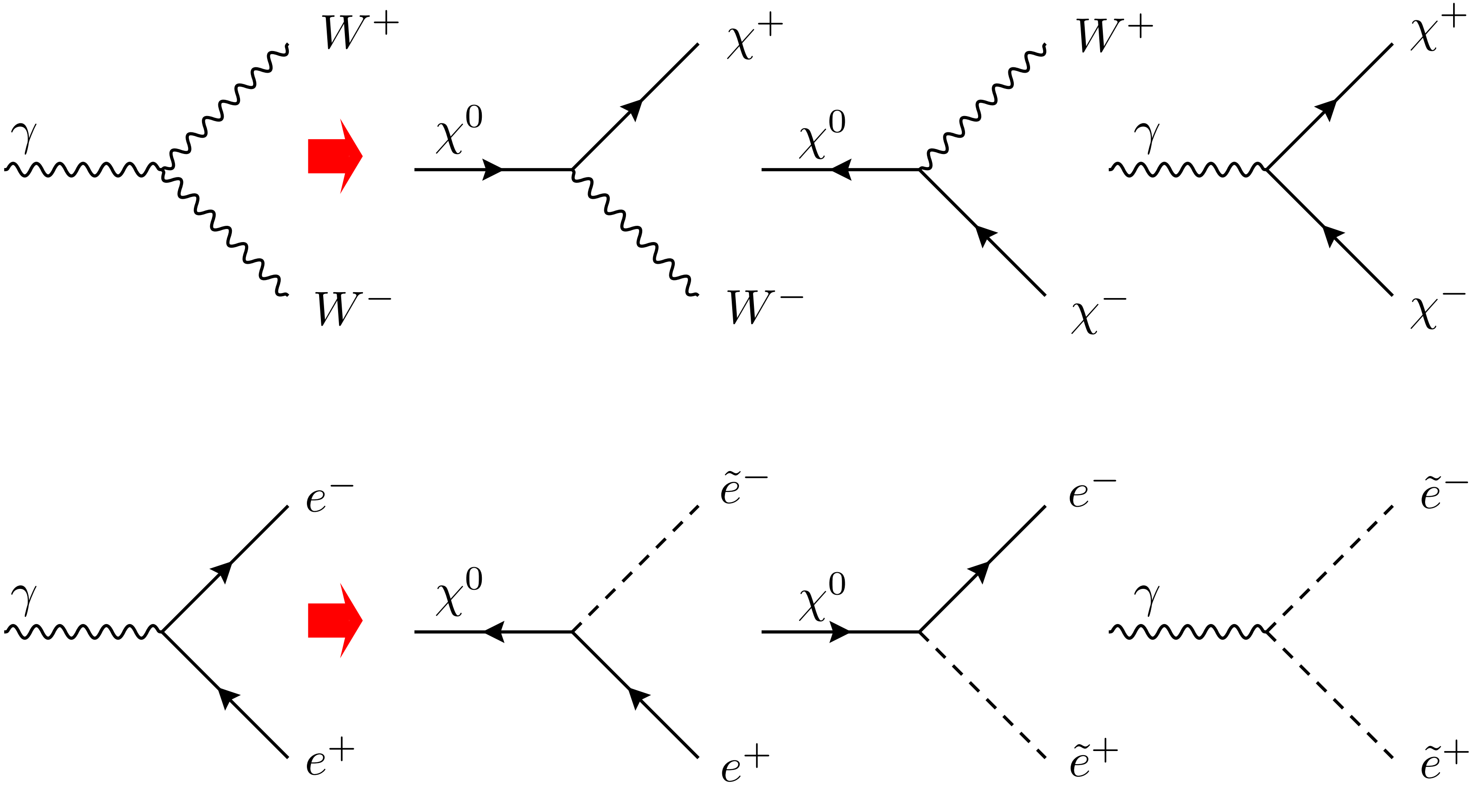
$m_0 = 200\text{GeV},\ m_{1/2} = 400\text{GeV}$
$A_0 = 0,\ \tan\beta = 30,\ \mu > 0$

Couplings in the MSSM

Name	Type
Gauge Self-Interaction	VVV VVVV
3-Point Gauge Coupling	Vff $V\tilde{f}\tilde{f}$ $V\tilde{\chi}\tilde{\chi}$ VHH VGH VGG
3-Point Higgs Coupling	Hff $H\tilde{f}\tilde{f}$ $H\tilde{\chi}\tilde{\chi}$ HVV
3-Point Goldstone Coupling	Gff $G\tilde{f}\tilde{f}$ $G\tilde{\chi}\tilde{\chi}$ GVV
Other 3-Point	$\tilde{f}f\tilde{\chi}$

Name	Type
4-Point Coupling	$VVff$ $HHVV$ $HGVV$ $GGVV$ $\tilde{f}\tilde{f}HH$ $\tilde{f}\tilde{f}GH$ $\tilde{f}\tilde{f}GG$ $\tilde{f}\tilde{f}\tilde{f}\tilde{f}$
Goldstone-Higgs Interaction	HHG HGG $HHHG$ $HHGG$ $HGGG$ $GGGG$
Ghost	$\overline{\omega}\omega$ V $\overline{\omega}\omega$ H $\overline{\omega}\omega$ G

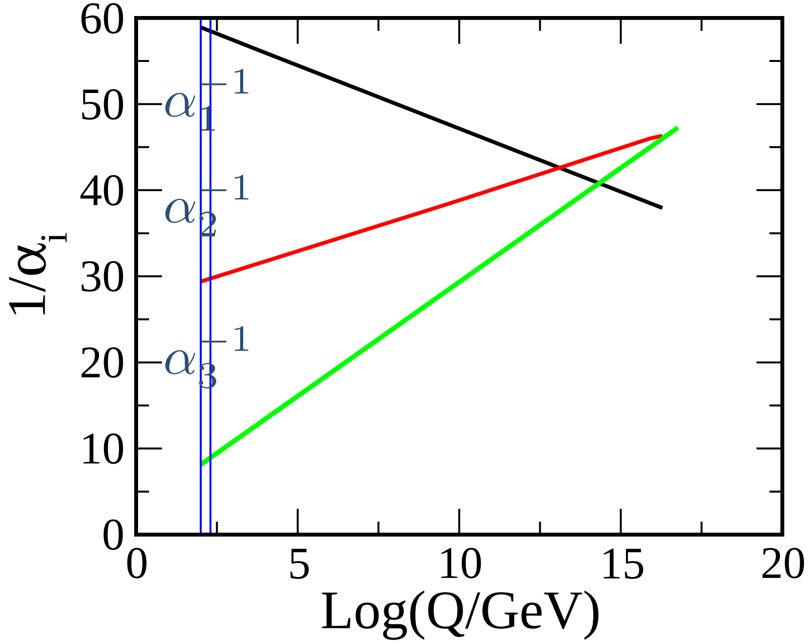
Examples of New Couplings: Conserving R-Parity



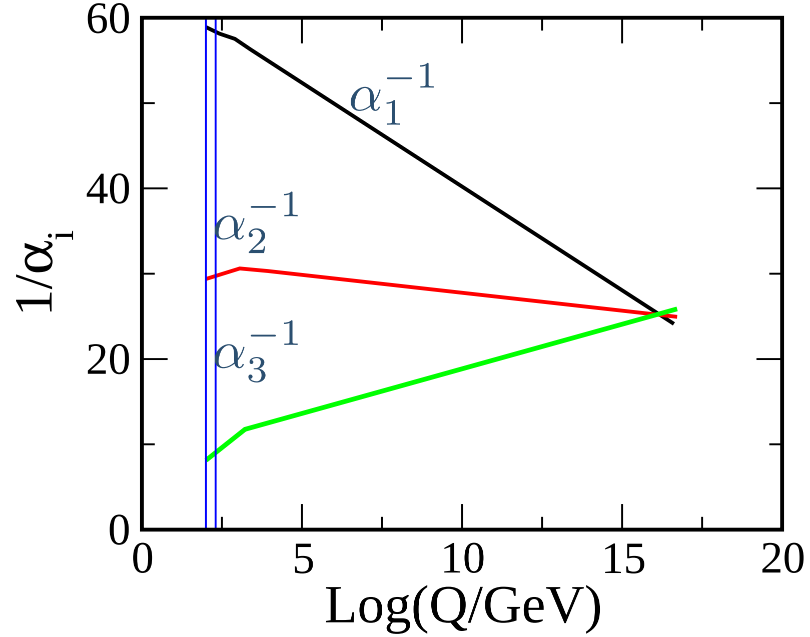
Rule: Change any two lines into the superpartners.

Gauge Couplings Unification

Standard Model



Supersymmetry



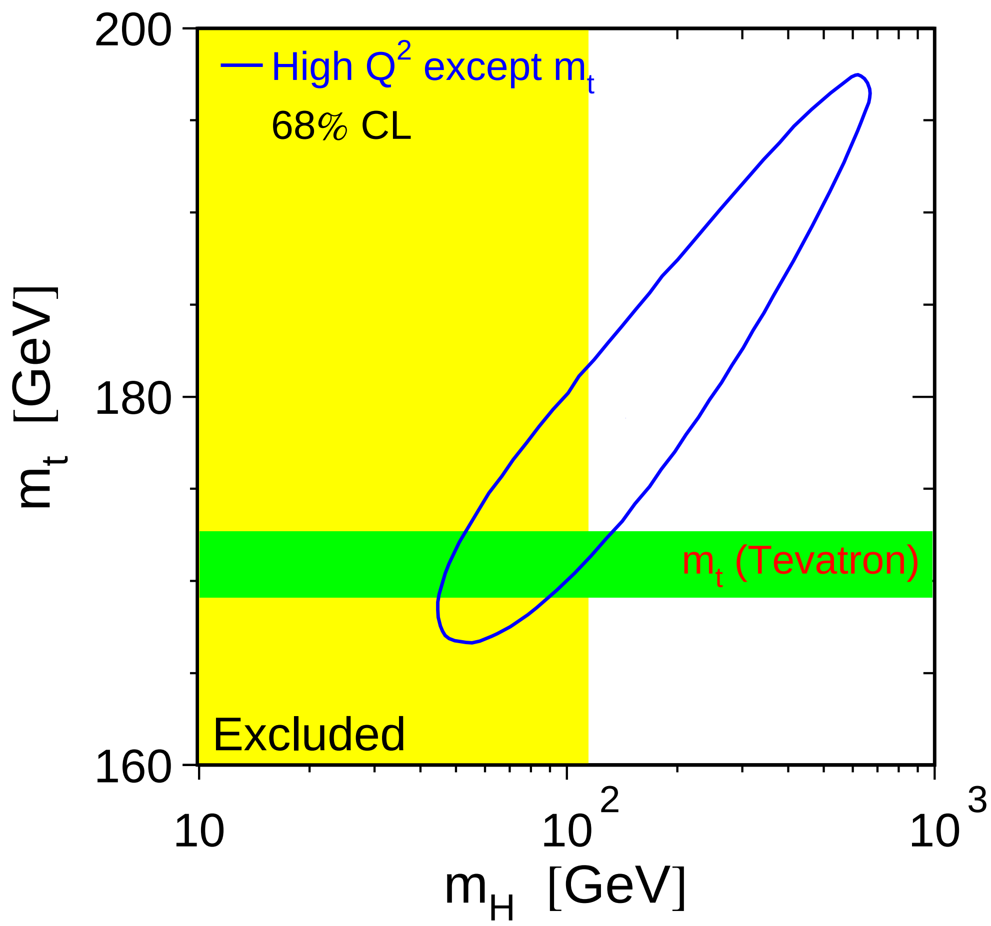
$$\alpha_i = \frac{g_i^2}{4\pi}$$

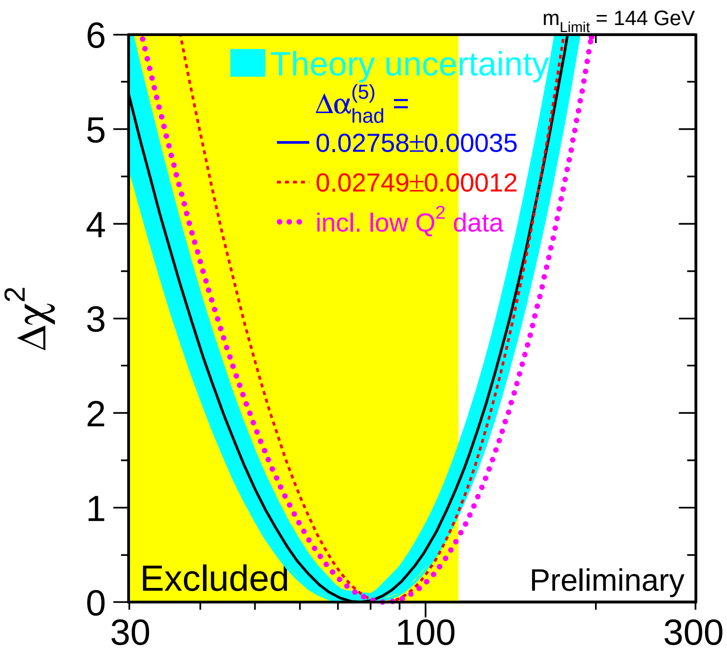
$$\begin{pmatrix} g_1 \\ g_2 \\ g_3 \end{pmatrix} \Rightarrow \begin{pmatrix} U_Y(1) \\ SU_L(2) \\ SU_c(3) \end{pmatrix}$$

Electroweak Theory

Quantum Chromodynamics

Higgs Boson



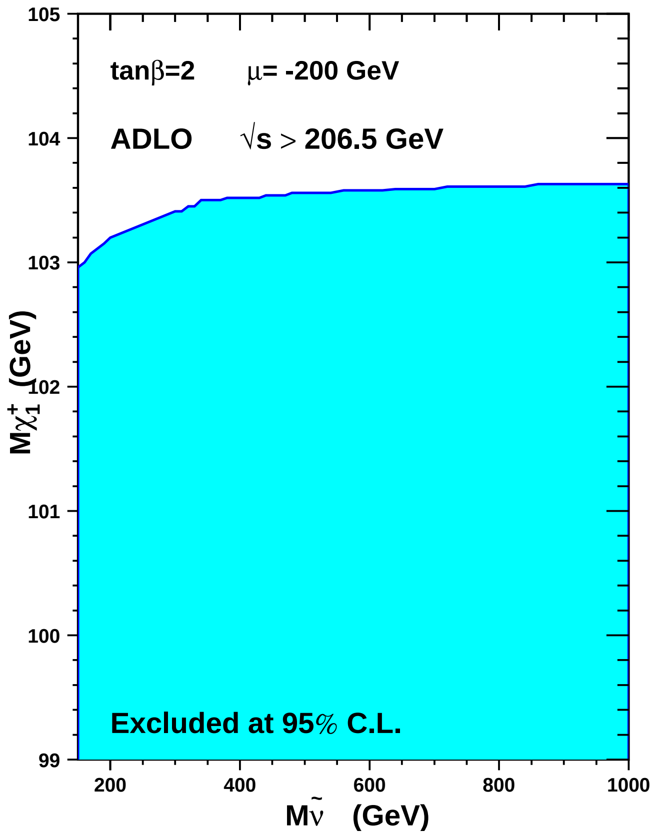


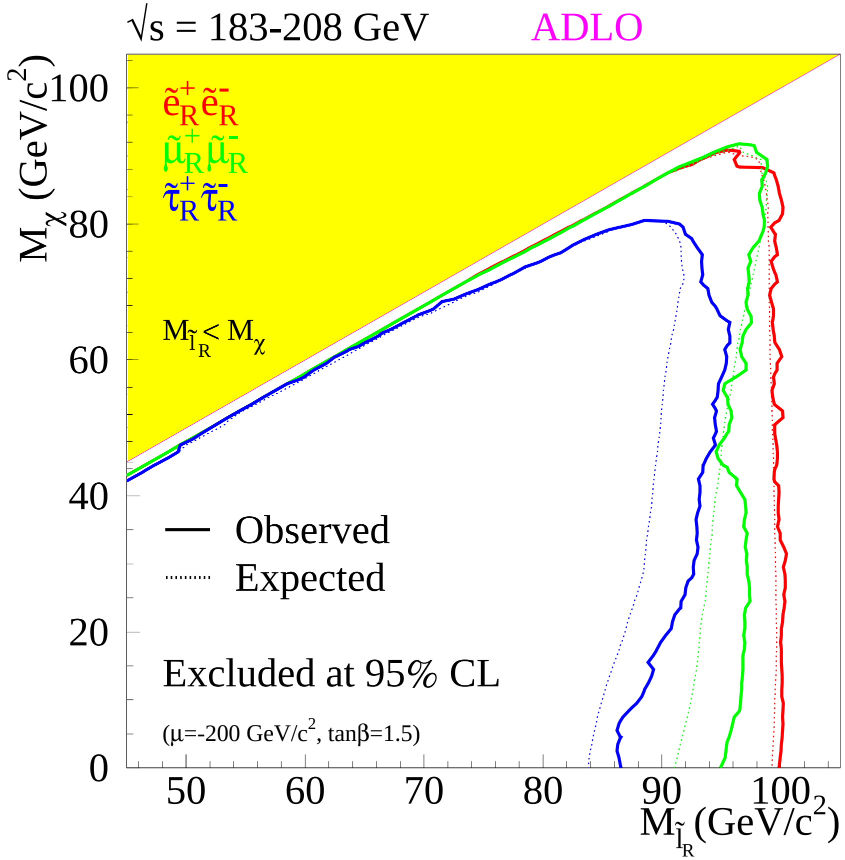
$M_H = 114^{+69}_{-45}$ **GeV**

$M_H < 260$ **GeV @** $95\% CL$

$M_H > 114.4$ **GeV**

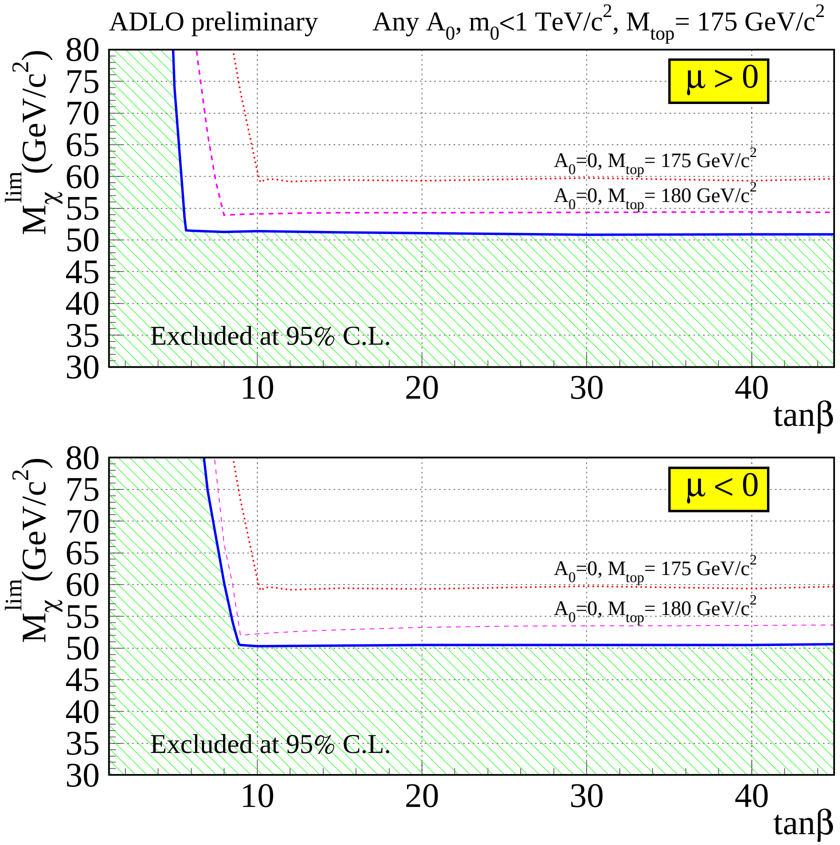
Limits on Charginos, Sneutrinos & Sleptons

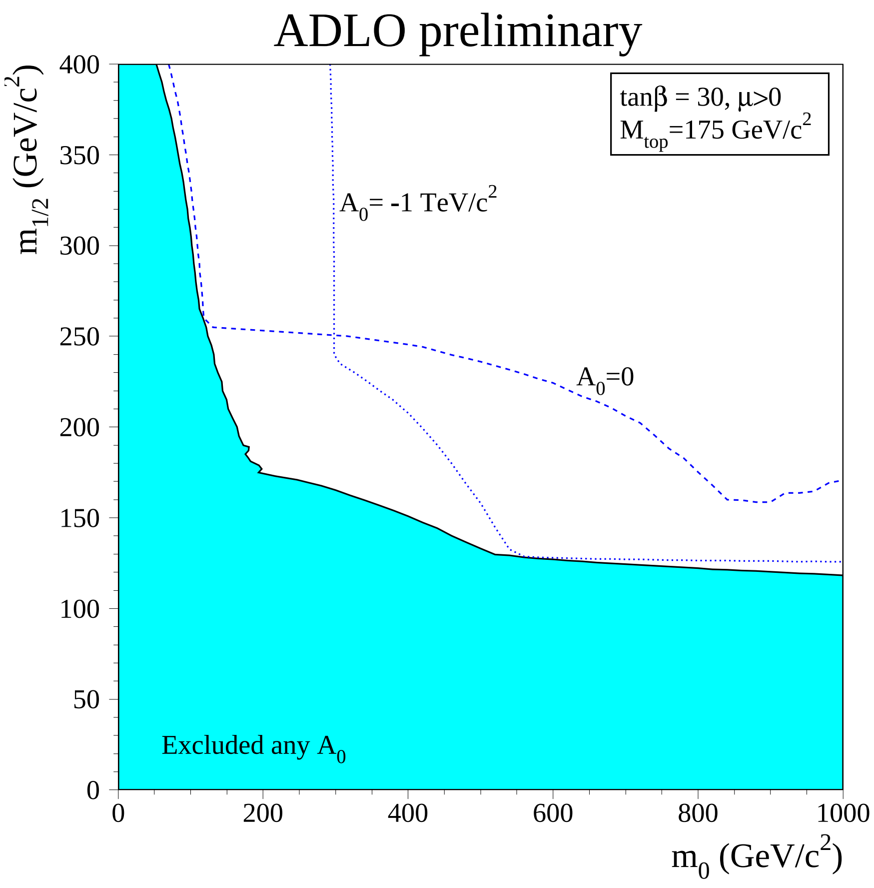




Limits on $\chi^\pm$, $\tilde{l}^\pm \simeq 100$ GeV.

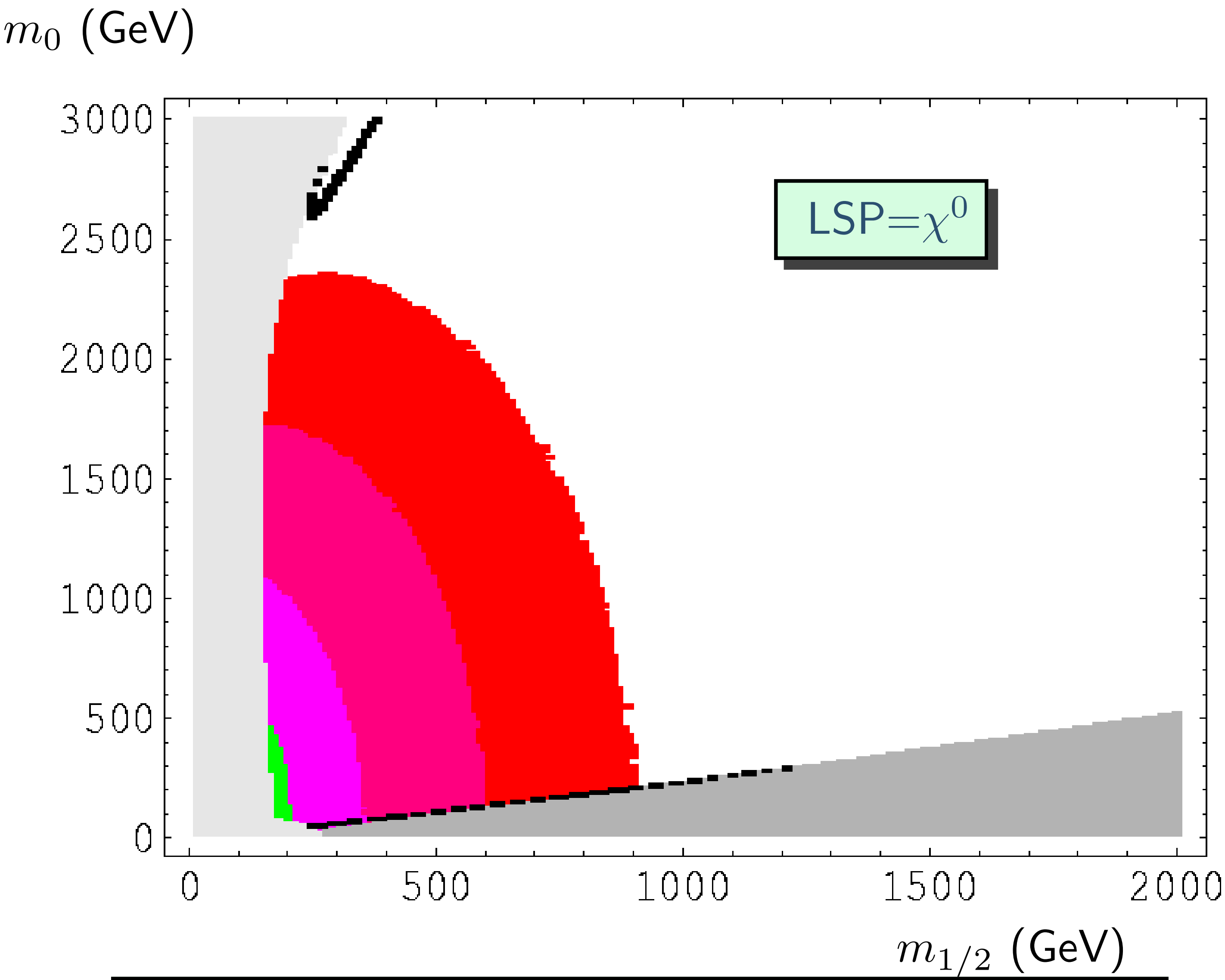
Limits on Neutralinos





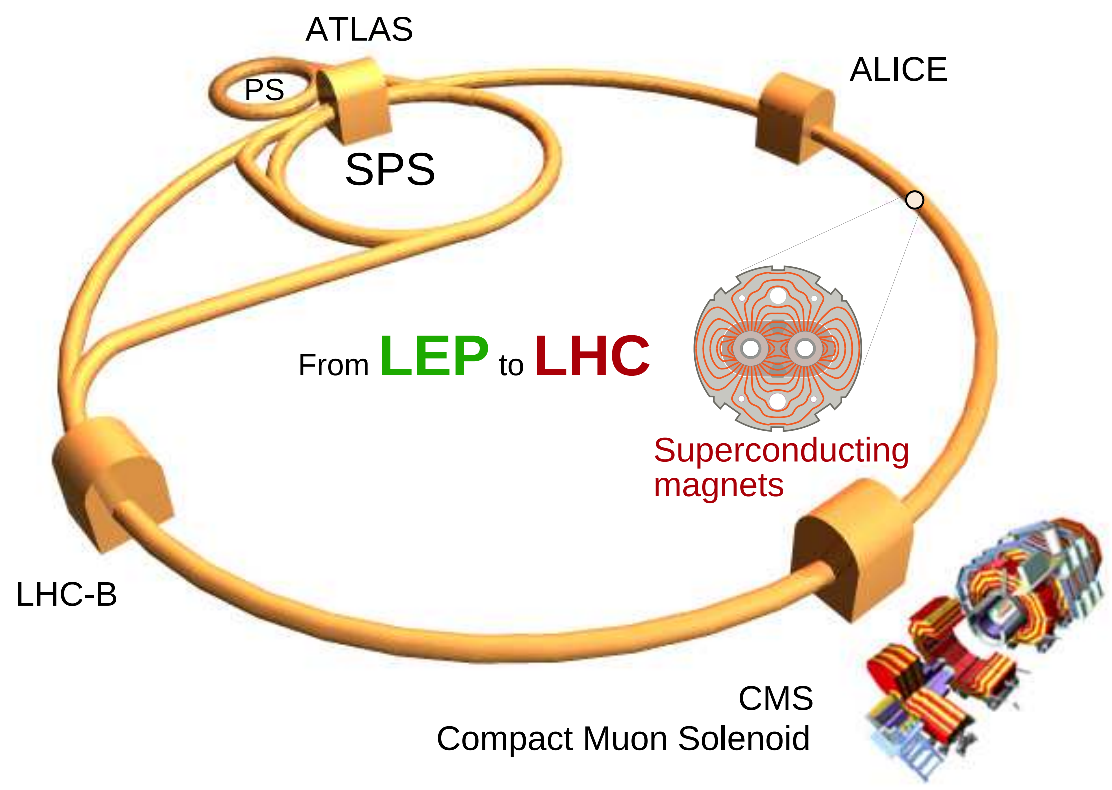
Limits on $\chi^0 \geq 50\ \mathrm{GeV}$. More model dependent.

Neutralino as Dark Matter



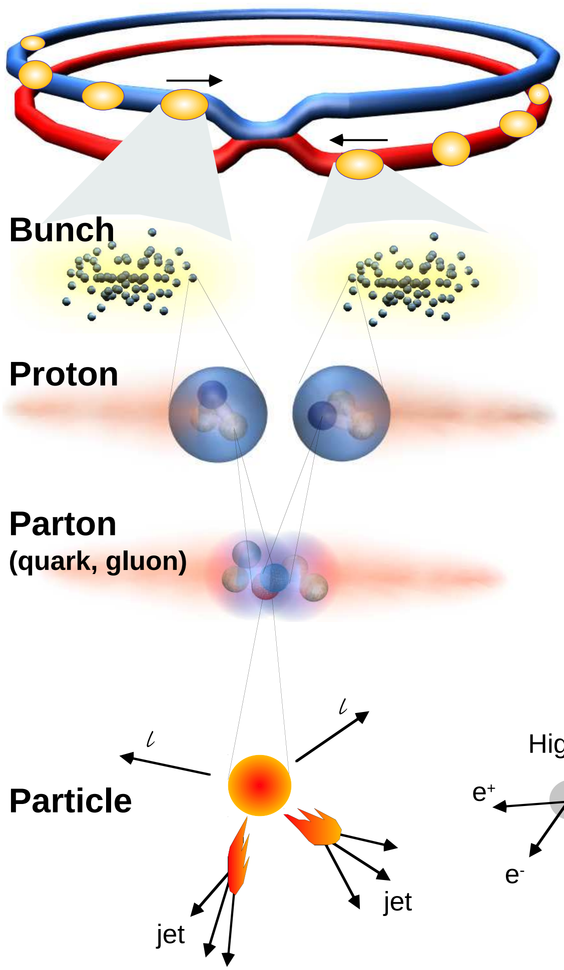
Black lines: Neutralino consistent with dark matter.

The Large Hadron Collider (LHC)

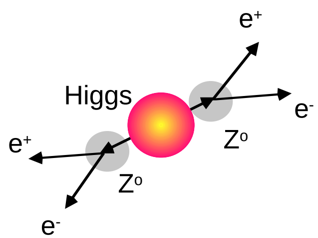


	Beams	Energy		Luminosity
LEP	e+ e-	200	GeV	10^{32} cm^{-2}s^{-1}
LHC	p p	14	TeV	10^{34}
	Pb Pb	1312	TeV	10^{27}

Collisions at LHC



Proton-Proton	(2835 x 2835 bunches)
Protons/bunch	10^{11}
Beam energy	7 TeV (7×10^{12} eV)
Luminosity	10^{34} cm^{-2} s^{-1}
Crossing rate	40 MHz
Collisions ≈	10^7 - 10^9 Hz



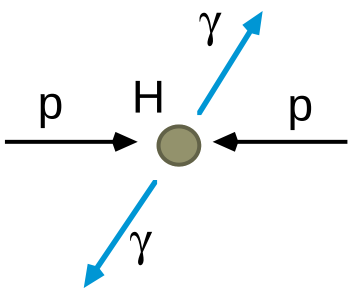
Higgs

SUSY.....

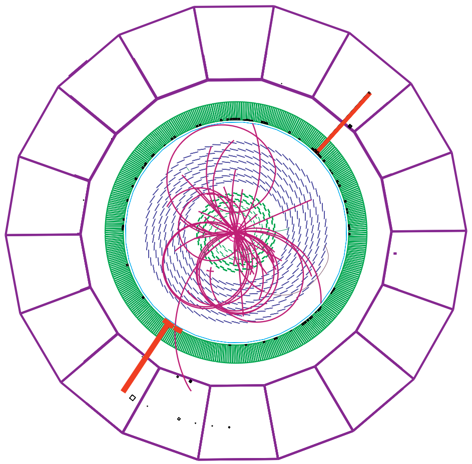
Selection of 1 in 10,000,000,000,000

SM Higgs at LHC: Two photons

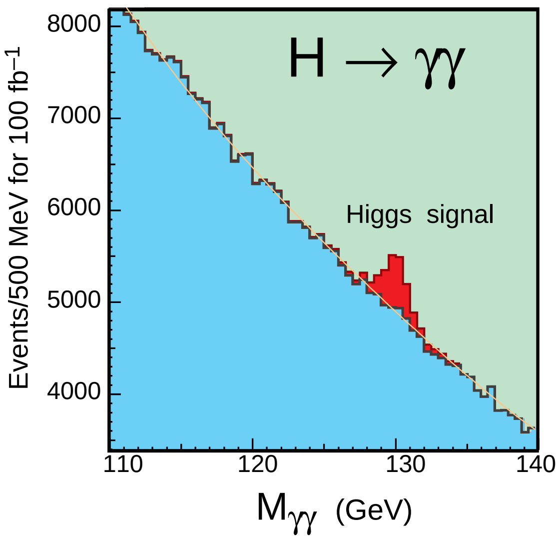
Higgs to 2 photons (M_H < 140 GeV)



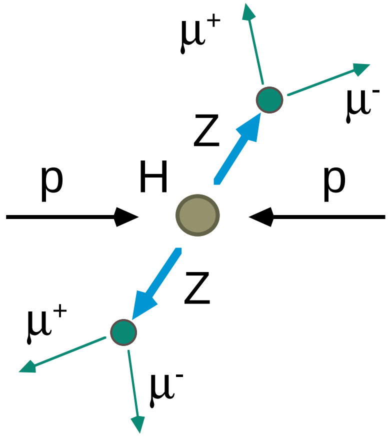
$H^0 \to \gamma\gamma$ is the most promising channel if M_H is in the range 80 – 140 GeV. The high performance $PbWO_4$ crystal electromagnetic calorimeter in CMS has been optimized for this search. The $\gamma\gamma$ mass resolution at $M_{\gamma\gamma}$ ~ 100 GeV is better than 1%, resulting in a S/B of ≈1/20



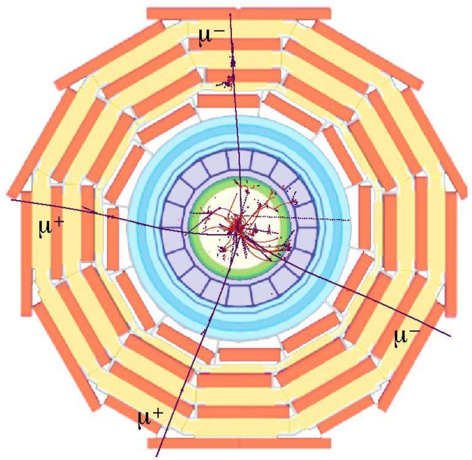
M_{Higgs} = 100 GeV



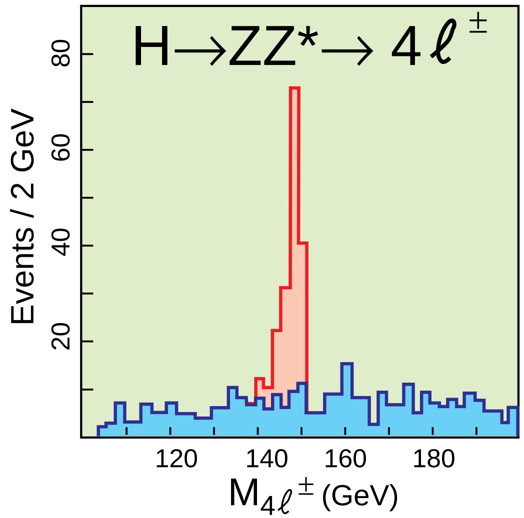
Higgs to 4 leptons (140 < M_H < 700 GeV)



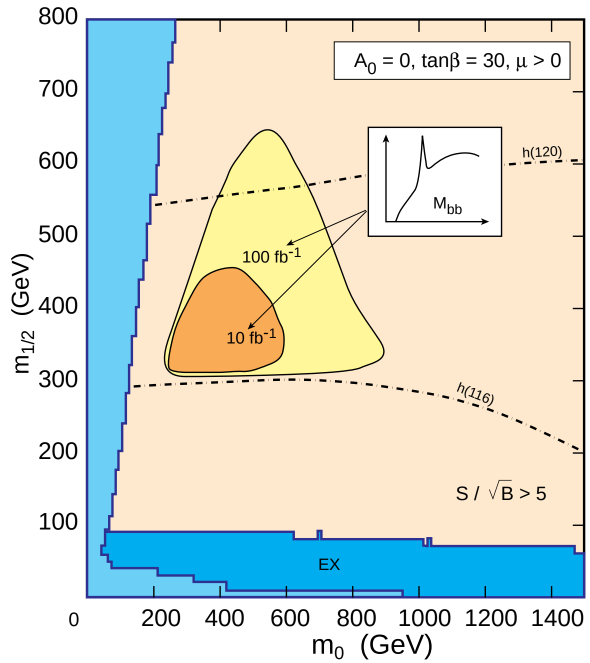
In the M_H range 130 - 700 GeV the most promising channel is $H^0 \to ZZ^* \to 2\ell^+ 2\ell^-$ or $H^0 \to ZZ \to 2\ell^+ 2\ell^-$. The detection relies on the excellent performance of the muon chambers, the tracker and the electromagnetic calorimeter.
For $M_H \leq 170$ GeV a mass resolution of ~1 GeV should be achieved with the combination of the 4 Tesla magnetic field and the high resolution of the crystal calorimeter



M_{Higgs} = 150 GeV

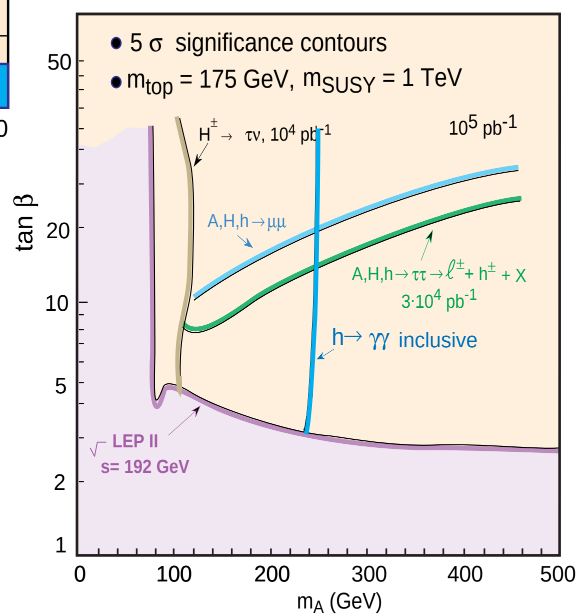


SUSY Higgs: discovery ranges



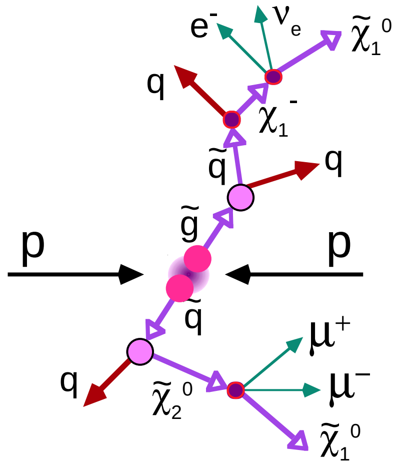
Example of the domain of parameter space of mSUGRA-MSSM where the h^0 can be discovered via its decay in $b\bar{b}$

Higgs bosons in MSSM

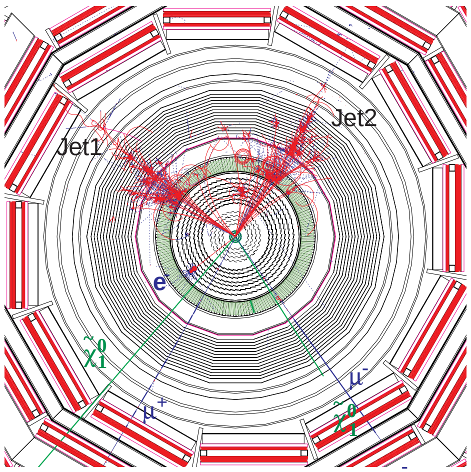


The search for the various MSSM Higgs bosons in different decay modes allows the exploration of most of the parameter region $(\tan\beta, m_A)$

Sparticles

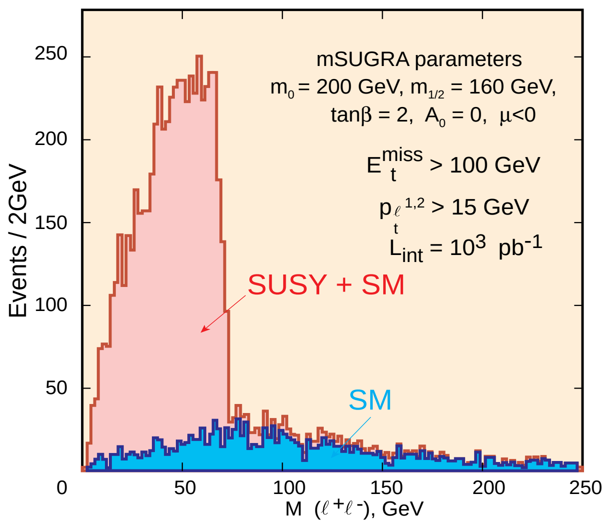


Production of sparticles may reveal itself though some spectacular kinematical spectra, with a pronounced "edge" in the $\ell^+\ell^-$ mass spectrum reflecting $\chi_2^0 \to \ell^+\ell^-\chi_1^0$ production and decay.
An example of such a spectrum in inclusive $\ell^+\ell^- + E_t^{miss}$ and of a 3 $\ell^\pm$ production event are shown below

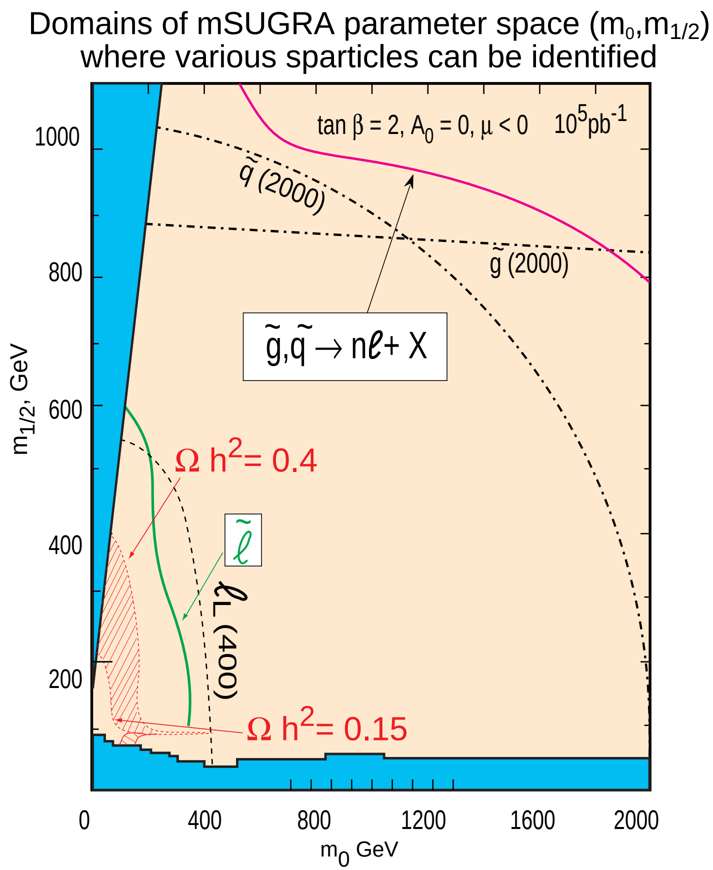


SUSY event with
3 leptons + 2 Jets signature

Inclusive $\ell^+\ell^- + E_t^{miss}$ final states



Sparticles: discovery ranges



Gluinos and squarks can be searched for in various channels with leptons + E_t^{miss} + jets and discovered for masses up to ~ 2.2 TeV. Sleptons can be discovered for masses up to ~ 350 GeV. The region of parameter space $0.15 < \Omega h^2 < 0.4$ — where LSP would be the Cold Dark Matter particle — is contained well within the explorable region

Sparticles cannot escape discovery at the LHC

Conclusions

Although there is not yet direct experimental evidence for supersymmetry (SUSY), there are many theoretical arguments indicating that SUSY might be of relevance for physics below the 1 TeV scale.

We will be waiting for the LHC verdict!

Lots of things to be done by Experimentalists and Theoreticians