Example of calculations in the MSSM

1) Neutralino Mass Matrix

Our Higgs doublets are defined

$$H_1 = \begin{pmatrix} \dfrac{v_1 + \sigma_1^0 + i\varphi_1^0}{\sqrt{2}} \\ H_1^- \end{pmatrix} ; \quad H_2 = \begin{pmatrix} H_2^+ \\ \dfrac{v_2 + \sigma_2^0 + i\varphi_2^0}{\sqrt{2}} \end{pmatrix}$$

$$Y = -\frac{1}{2} \qquad\qquad Y = \frac{1}{2}$$

we are going to find the neutralino mass matrix in the basis $\psi^{0T} \equiv (-i\lambda', -i\lambda^3, \tilde{H}_1^1, \tilde{H}_2^2)$, that is

$$\mathcal{L} = -\frac{1}{2} (\psi^0)^T M_N \psi^0 + h.c.$$

the terms contributing come from several sources:

i) Yukawa terms

$$\mathcal{L}_Y = -\frac{1}{2} \frac{\partial W}{\partial\phi_i \partial\phi_j} \chi_i \chi_j + h.c.$$

the relevant terms in W are

$$W = -\mu H_1^1 H_2^2 + \dots$$

So

$$\mathcal{L}^{(1)} = \frac{1}{2}\mu \tilde{H}_1^1 \tilde{H}_2^2 + \frac{1}{2}\mu \tilde{H}_2^2 \tilde{H}_1^1 + h.c.$$

ii) Gauge Matter interaction

The generic term is

$$\mathcal{L}_{\Phi W} = i g \sqrt{2}\, T^a_{ij}\, \lambda^a\, \chi_j\, \phi^*_i + h.c.$$

we have to consider $SU(2)$ and $U_Y(1)$. We have

SU(2):

$$T^a \equiv \frac{\sigma^a}{2}$$

$$\lambda^a \frac{\sigma^a}{2} = \begin{pmatrix} \frac{\lambda^3}{2} & \frac{\lambda^+}{\sqrt{2}} \\ \frac{\lambda^-}{\sqrt{2}} & -\frac{\lambda^3}{2} \end{pmatrix}$$

for H_1 we have

$$\mathcal{L}^{(2)} = i g \sqrt{2} \left(\frac{v_1}{\sqrt{2}} + \cdots \;,\; \right) \begin{pmatrix} \frac{\lambda^3}{2} & \frac{\lambda^+}{\sqrt{2}} \\ \frac{\lambda^-}{\sqrt{2}} & -\frac{\lambda^3}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \tilde{H}_1^1 \\ \tilde{H}_1^- \end{pmatrix}$$

$$+ h.c$$

$$= i g \sqrt{2} \frac{v_1}{\sqrt{2}} \frac{\lambda_3}{2} \tilde{H}_1^1 + h.c + \cdots$$

$$= i \frac{g}{2} v_1 \lambda_3 \tilde{H}_1^1 + h.c. + \cdots$$

In a similar way for H_2

$$\mathcal{L}^{(3)} = -i \frac{g}{2} v_2 \lambda_3 \tilde{H}_2^2 + h.c + \cdots$$

$U_Y(1)$: for H_1 $(Y = -\frac{1}{2})$

$$\mathcal{L}^{(4)} = i\left(-\frac{g'}{2}\right)\sqrt{2}\,\frac{v_1}{\sqrt{2}}\,\lambda'\,\tilde{H}_1^1 + h.c.$$

for H_2 $(Y = +\frac{1}{2})$

$$\mathcal{L}^{(5)} = i\left(\frac{g'}{2}\right)\sqrt{2}\,\frac{v_2}{\sqrt{2}}\,\lambda'\,\tilde{H}_2^2 + h.c.$$

iii) Soft breaking terms

$$\mathcal{L}^{(6)} = \frac{1}{2}M_1\lambda'\lambda' + \frac{1}{2}M_2\lambda^3\lambda^3 + \ldots$$

Finally

$$\mathcal{L}_{\text{Neutralinos}} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + \ldots + \mathcal{L}^{(6)}$$

$$= \frac{1}{2}\mu\,\tilde{H}_1^1\,\tilde{H}_2^2 + \frac{1}{2}\mu\,\tilde{H}_2^2\,\tilde{H}_1^1$$

$$+ i\,\frac{g}{2}\,v_1\,\lambda_3\,\tilde{H}_1^1 - i\,\frac{g}{2}v_2\,\lambda_3\,\tilde{H}_2^2$$

$$- i\,\frac{g'}{2}\,v_1\,\lambda'\,\tilde{H}_1^1 + i\,\frac{g'}{2}\,v_2\,\lambda'\,\tilde{H}_2^2$$

$$+ \frac{1}{2}M_1\,\lambda'\lambda' + \frac{1}{2}M_2\,\lambda^3\lambda^3 + h.c.$$

or

$$M_N = \begin{bmatrix} M_1 & 0 & -\frac{1}{2}g'v_1 & \frac{1}{2}g'v_2 \\ 0 & M_2 & \frac{1}{2}gv_1 & -\frac{1}{2}gv_2 \\ -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & -\mu \\ \frac{1}{2}g'v_2 & -\frac{1}{2}gv_2 & -\mu & 0 \end{bmatrix}$$

2) <u>Lepton Mass Matrices</u>

The relevant terms come from

$$W = (h_E)_{ij} \, L^2_i \, H^1_1 \, R_j$$

giving

$$\mathcal{L}_M = -\frac{v_1}{\sqrt{2}} (h_E)_{ij} \, l_{L_i} \, l^c_{L_j} - \frac{v_1}{\sqrt{2}} (h^*_E)_{ij} \, \overline{l}_{L_i} \, \overline{l^c}_j$$

Define the Dirac 4-spinor

$$l \equiv \begin{pmatrix} \overline{l^c_L} \\ \\ l_L \end{pmatrix}$$

Now using

$$l_i \, l^c_j = l^c_j \, l_i = \overline{l}_j \, P_L \, l_i$$

$$\overline{l}_i \, \overline{l^c}_j = \overline{l}_i \, P_R \, l_j \qquad (1)$$

we get

$$\mathcal{L}_M = -\bar{\ell}\, M_E P_R \ell - \bar{\ell}\, M_E^\dagger P_L \ell$$

$$= -\bar{\ell}_L M_E \ell_R - \bar{\ell}_R M_E^\dagger \ell_L$$

with

$$(M_E)_{ij} = \frac{v_1}{\sqrt{2}} (h_E^*)_{ij}$$

<u>Proof of (1)</u>:

take

$$\Psi_1 = \begin{pmatrix} \bar{\psi}_1 \\ \chi_1 \end{pmatrix} \quad ; \quad \Psi_2 = \begin{pmatrix} \bar{\psi}_2 \\ \chi_2 \end{pmatrix}$$

then

$$\bar{\Psi}_1 \Psi_2 = (\psi_1 , \bar{\chi}_1) \underbrace{\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}}_{\gamma^0} \begin{pmatrix} \bar{\psi}_2 \\ \chi_2 \end{pmatrix}$$

$$= \psi_1 \chi_2 + \bar{\chi}_1 \bar{\psi}_2$$

$$\bar{\Psi}_1 \gamma_5 \Psi_2 = (\psi_1 , \bar{\chi}_1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \bar{\psi}_2 \\ \chi_2 \end{pmatrix}$$

$$= (\psi_1 , \bar{\chi}_1) \begin{pmatrix} -\chi_2 \\ \bar{\psi}_2 \end{pmatrix} = -\psi_1 \chi_2 + \bar{\chi}_1 \bar{\psi}_2$$

thefore:

$$\begin{cases} \bar{\chi}_1 \bar{\psi}_2 = \bar{\Psi}_1 P_R \Psi_2 \\ \chi_2 \psi_1 = \bar{\Psi}_1 P_L \Psi_2 \end{cases}$$

For future reference:

$$\bar{\Psi}_1 \gamma_\mu \Psi_2 = (\psi_1, \bar{\chi}_1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix} \begin{pmatrix} \bar{\psi}_2 \\ \chi_2 \end{pmatrix}$$

$$= (\psi_1, \bar{\chi}_1) \begin{pmatrix} \sigma^\mu & 0 \\ 0 & \bar{\sigma}^\mu \end{pmatrix} \begin{pmatrix} \bar{\psi}_2 \\ \chi_2 \end{pmatrix}$$

$$= \psi_1 \sigma^\mu \bar{\psi}_2 + \bar{\chi}_1 \bar{\sigma}^\mu \chi_2$$

$$\bar{\Psi}_1 \gamma_\mu \gamma_5 \Psi_2 = (\psi_1, \bar{\chi}_1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & \bar{\sigma}^\mu \\ \sigma^\mu & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \bar{\psi}_2 \\ \chi_2 \end{pmatrix}$$

$$= (\psi_1, \bar{\chi}_1) \begin{pmatrix} \sigma^\mu & 0 \\ 0 & -\bar{\sigma}^\mu \end{pmatrix} \begin{pmatrix} \bar{\psi}_2 \\ \chi_2 \end{pmatrix}$$

$$= \psi_1 \sigma^\mu \bar{\psi}_2 - \bar{\chi}_1 \bar{\sigma}^\mu \chi_2$$

therefore:

$$\bar{\chi}_1 \bar{\sigma}^\mu \chi_2 = \bar{\Psi}_1 \gamma_\mu P_L \Psi_2$$

$$\psi_1 \sigma^\mu \bar{\psi}_2 = \bar{\Psi}_1 \gamma_\mu P_R \Psi_2$$

Note also that, in general

$$\psi_1 \sigma^\mu \bar{\psi}_2 = - \bar{\psi}_2 \bar{\sigma}^\mu \psi_1$$

so

$$\bar{\psi}_2 \bar{\sigma}^\mu \psi_1 = - \bar{\Psi}_1 \gamma_\mu P_R \Psi_2 .$$