2$^{\text{nd}}$ Exam: July 6th, 2017 – 18h30

Duration: 2h30

I (3 points)

a) The LEP accelerator was an e^-e^+ collider that was built at CERN before the present LHC. The maximum of energy that was obtained was $\sqrt{s} = 209$ GeV.

Consider the process $e^- + e^+ \to H + \nu_\mu + \overline{\nu}_\mu$. What are the minimum and the maximum of energy of the H (Higgs boson with mass $m_H = 125$ GeV) in the CM frame for that value of the CM energy? If the electron was at rest, what would be the energy of positron beam needed to have the same $\sqrt{s}$?

b) Consider a string of an even number of γ matrices

$$S = \not{a}_1 \not{a}_2 \cdots \not{a}_n$$

where n is even. Show that

$$S = A\,\mathbb{1} + B\gamma_5 + C^{\mu\nu}\sigma_{\mu\nu}$$

where A, B are constants and $C^{\mu\nu}$ is an anti-symmetric tensor. Find an expression to obtain $C^{\mu\nu}$ from S.

II (3 points)

Draw the Feynman Diagrams for the following processes in the Standard Model:

a) $e^+ + e^- \to H + W^+ + W^-$ b) $\nu_e + e^+ \to \mu^+ + \nu_\mu + \gamma$ c) $u + \overline{u} \to W^- + W^+$

Only draw the diagrams, do not calculate or write the amplitudes. Neglect the Higgs couplings to fermions.

III (5 points)

Consider the process $e^-(p_1) + \mu^+(p_2) \to \nu_e(p_3) + \overline{\nu}_\mu(p_4)$ in the Standard Model

a) Draw the diagram(s) that contribute in lowest order in perturbation theory.

b) Write the amplitude for the process.

c) Neglect the fermion masses and consider that the center of mass energy is such that $\sqrt{s} \ll M_Z, M_W$. In these conditions determine the differential cross section $d\sigma/d\Omega$ in the center of mass frame.

d) Show that the total cross section can be written, in this approximation, as

$$\sigma = \frac{\lambda}{\pi} G_F^2\, s$$

Determine λ.

e) Without doing the calculations and using the crossing symmetry, determine $\sum_{\text{spins}} |\mathcal{M}'|^2$ for the process

$$e^-(p_1') + \nu_\mu(p_2') \to \nu_e(p_3') + \mu^-(p_4')\ .$$

Using this result evaluate then the total cross section for this process. What is the corresponding value of λ?

IV (5 points)

Consider the process $e^-(p_1) + \mu^+(p_2) \to \nu_e(p_3) + \overline{\nu}_\mu(p_4) + \gamma(k)$ in the Standard Model. **Neglect** the masses of all the leptons.

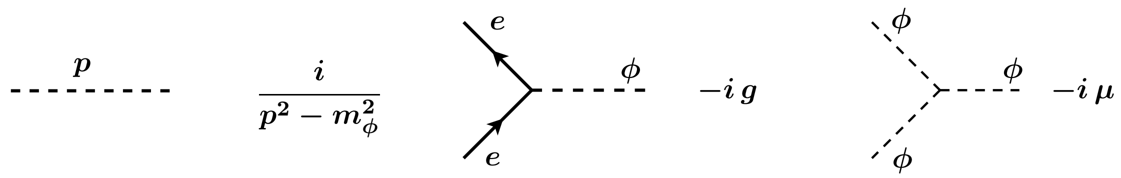
a) Draw the diagrams and write the amplitudes that contribute in lowest order in perturbation theory.

b) Show the gauge invariance of the total amplitude, that is if we write $\mathcal{M} \equiv \epsilon^{*\mu}(k)\,\mathcal{M}_\mu$ where k is the photon 4-momentum, then we should have $k^\mu \mathcal{M}_\mu = 0$. **Note:** All particles are on-shell.

For the following problem consider the theory described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi \; - \frac{1}{2}m_\phi^2\,\phi^2 - g\,\overline{\psi}\psi\,\phi - \frac{\mu}{3!}\phi^3$$

where ϕ is a neutral scalar field (spin 0) and ψ is the electron. The constant g is dimensionless and the constant μ has dimensions of mass (in our system with $\hbar = c = 1$). Besides QED, the propagator and new vertices are:



$\frac{i}{p^2 - m_\phi^2}$ $\qquad -i\,g \qquad -i\,\mu$

V (4 points)

Consider the *one-loop* corrections in this model. In all answers consider only one-particle irreducible diagrams.

a) Draw the diagram(s) that contribute to the self-energy of the field ϕ. Discuss the superficial degree of divergence.

b) Draw the diagram(s) that contribute to the one-loop correction for the vertex $A\psi\psi$. Discuss the superficial degree of divergence.

c) Draw the diagram(s) that contribute to the one-loop correction for the vertex $\phi\phi\phi$. Discuss the superficial degree of divergence.

d) Is the theory renormalizable? Justify carefully your answer.

Useful expressions

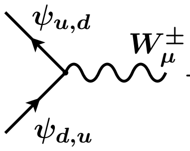
- In the `CM` frame we have:

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2}\frac{|\vec{p}_{\text{CM}}|}{m^2}\,\overline{|M|}^2, \qquad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2\,s}\frac{|\vec{p}_{3\text{CM}}|}{|\vec{p}_{1\text{CM}}|}\,\overline{|M|}^2$$

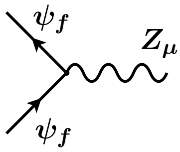
  for a decay and for a $p_1 + p_2 \to p_3 + p_4$ scattering, respectively.

- $\text{Tr}[\not{a}\not{b}\not{c}\not{d}\gamma_5] = -4i\,\epsilon^{\alpha\beta\gamma\delta}a_\alpha b_\beta c_\gamma d_\delta, \qquad \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu}{}^{\gamma\delta} = -2g^{\alpha\gamma}g^{\beta\delta} + 2g^{\alpha\delta}g^{\beta\gamma}$

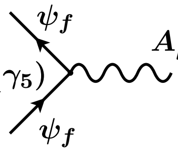
- In the Standard Model $M_W = M_Z \cos\theta_W$, $g_V^f = \frac{1}{2}T_3^f - Q_f \sin^2\theta_W$, $g_A^f = \frac{1}{2}T_3^f$ e $G_F = \sqrt{2}\, g^2/(8M_W^2)$.
- Some constants: $m_Z = 91.19$ GeV, $\Gamma_Z = 2.495$ GeV, $G_F = 1.1664 \times 10^{-5}$ GeV^{-2}, $\sin^2\theta_W = 0.23$, $m_H = 125$ GeV, $\hbar c = 197.327$ MeVfermi.
- Some Feynman rules



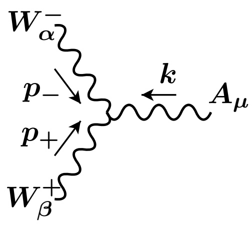
$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2}$



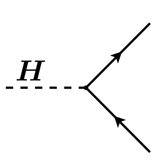
$-i\frac{g}{\cos\theta_W}\gamma_\mu(g_V^f - g_A^f\gamma_5)$



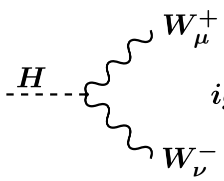
$-ieQ_f\gamma_\mu$



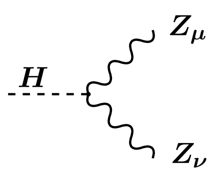
$-ie\,[g_{\alpha\beta}(p_- - p_+)_\mu + g_{\beta\mu}(p_+ - k)_\alpha + g_{\mu\alpha}(k - p_-)_\beta]$



$-i\frac{g}{2}\,\frac{m_f}{M_W}$



$ig\, M_W\, g_{\mu\nu}$



$i\frac{g}{\cos\theta_W}\, M_Z\, g_{\mu\nu}$