Mestrado em Eng. Física Tecnológica (MEFT)
Teoria de Campo
2º semestre de 2016-17

Prof. Jorge C. Romão (Responsável)

1st Exam: June 14th, 2017 – 15h00

Duration: 2h30

I (3 points)

a) Consider the process $p+p \to p+p+p+p$ in the Lab frame where one proton is at rest. Determine the minimum energy of the incoming proton beam for the reaction to take place? What is, for this energy, the velocity of the `CM` frame (in the Lab frame)?

b) Consider a string of an odd number of γ matrices

$$S = \not{a}_1 \not{a}_2 \cdots \not{a}_n$$

where n is odd. Show that

$$S = V_\mu \gamma^\mu + A_\mu \gamma^\mu \gamma_5$$

where V_μ, A_μ are 4-vectors.

II (3 points)

Draw the Feynman Diagrams for the following processes in the Standard Model:

a) $e^- + e^+ \to Z + \gamma + \gamma$ b) $e^- + \overline{\nu}_e \to W^- + Z$ c) $H \to b + \overline{b} + Z$

Only draw the diagrams, do not calculate or write the amplitudes. Do not neglect the Higgs couplings to fermions.

III (5 points)

Consider the process $t(k) \to W^+(p_1) + b(p_2)$ in the Standard Model. In this question neglect the mass of the bottom quark.

a) Draw the diagram(s) that contribute in lowest order and write the amplitude.

b) Evaluate the total decay width $\Gamma(t \to W^+ + b)$ as a function of the parameters of the model.

c) Knowing that the polarization vector of a longitudinally polarized W boson, in the frame where it moves with velocity $\vec{\beta}$ is given by $\varepsilon^\mu_L = (\gamma\beta, \gamma\vec{\beta}/\beta)$, determine the fraction of the decays that are longitudinally polarized, that is,

$$R_L \equiv \frac{\Gamma_L}{\Gamma_{\text{Total}}}$$

d) Determine the numerical value of R_L. What would be the result if, for $m_b \simeq 0$ we also had $m_t \gtrsim m_W$? Comment the result.

IV (5 points)

Consider the process $W^-(p_1) \to d(p_3) + \overline{u}(p_4) + \gamma(k)$ in the Standard Model. Neglect the masses of the quarks.

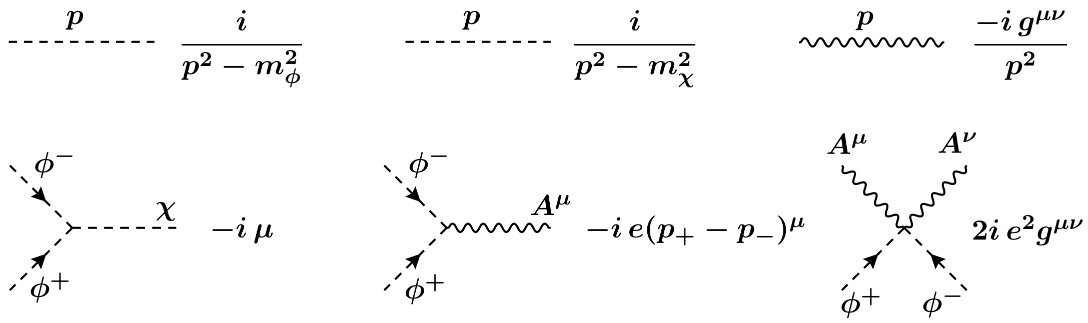
a) Draw the diagrams and write the amplitudes that contribute in lowest order in perturbation theory.

b) Show the gauge invariance of the total amplitude, that is if we write $\mathcal{M} \equiv \epsilon^{*\mu}(k)\, \mathcal{M}_\mu$ where k is the photon 4-momentum, then we should have $k^\mu \mathcal{M}_\mu = 0$. **Note:** All particles are on-shell.

For the following problem consider the theory described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}(\partial_\mu A^\mu)^2 + (\partial_\mu - i\,eA_\mu)\,\phi^-\,(\partial^\mu + i\,eA^\mu)\,\phi^+$$
$$+\frac{1}{2}\partial_\mu\chi\,\partial^\mu\chi - m_\phi^2\,\phi^+\phi^- - \frac{1}{2}m_\chi^2\,\chi^2 - \mu\,\phi^+\phi^-\chi$$

where χ is a neutral spin 0 field, $\phi^\pm$ is a charged spin 0 field and A^μ is the photon. The constant e is the proton charge and μ has the dimensions of mass in our system of units, $\hbar = c = 1$. This theory has the following propagators and vertices.



V (4 points)

Consider the *one-loop* corrections in this model. In all answers consider only one-particle irreducible diagrams.

a) Draw the diagrams that contribute to the vacuum polarization of the photon. Discuss the superficial degree of divergence.

b) Draw the diagrams that contribute to the one-loop correction to the vertex χAA. Discuss the superficial degree of divergence.

c) Draw the diagrams that contribute to the one-loop correction for the vertex $\chi\chi\phi^+\phi^-$. Discuss the superficial degree of divergence.

d) Is the theory renormalizable? Justify carefully your answer.

Useful expressions

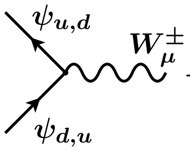
- In the `CM` frame we have:

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2}\,\frac{|\vec{p}_{\rm CM}|}{m^2}\,\overline{|M|}^2, \qquad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2\, s}\,\frac{|\vec{p}_{3\rm CM}|}{|\vec{p}_{1\rm CM}|}\,\overline{|M|}^2$$

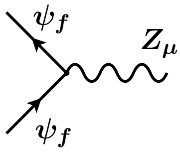
  for a decay and for a $p_1 + p_2 \to p_3 + p_4$ scattering, respectively.

- $\mathrm{Tr}[\not{a}\not{b}\not{c}\not{d}\gamma_5] = -4i\,\epsilon^{\alpha\beta\gamma\delta}a_\alpha b_\beta c_\gamma d_\delta, \qquad \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu}{}^{\gamma\delta} = -2g^{\alpha\gamma}g^{\beta\delta} + 2g^{\alpha\delta}g^{\beta\gamma}$

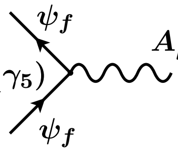
- In the Standard Model $M_W = M_Z \cos\theta_W$, $g_V^f = \frac{1}{2}T_3^f - Q_f \sin^2\theta_W$, $g_A^f = \frac{1}{2}T_3^f$ e $G_F = \sqrt{2}\, g^2/(8M_W^2)$.
- Some constants: $m_Z = 91.19$ GeV, $\Gamma_Z = 2.495$ GeV, $G_F = 1.1664 \times 10^{-5}$ GeV^{-2}, $\sin^2\theta_W = 0.23$, $m_H = 125$ GeV, $\hbar c = 197.327$ MeVfermi.
- Some Feynman rules



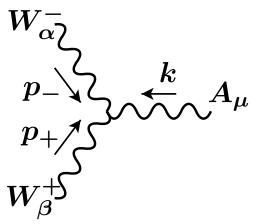
$$-i\frac{g}{\sqrt{2}}\gamma_\mu \frac{1-\gamma_5}{2}$$



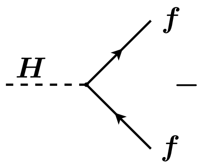
$$-i\frac{g}{\cos\theta_W}\gamma_\mu(g_V^f - g_A^f\gamma_5)$$



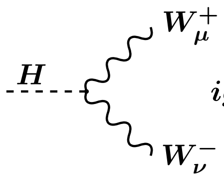
$$-ieQ_f\gamma_\mu$$



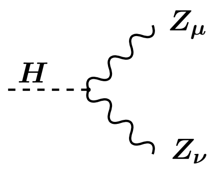
$$-ie\,[g_{\alpha\beta}(p_- - p_+)_\mu + g_{\beta\mu}(p_+ - k)_\alpha + g_{\mu\alpha}(k - p_-)_\beta]$$



$$-i\frac{g}{2}\,\frac{m_f}{M_W}$$



$$ig\,M_W\,g_{\mu\nu}$$



$$i\frac{g}{\cos\theta_W}\,M_Z\,g_{\mu\nu}$$