TÉCNICO LISBOA

Homework of Teoria de Campo

Curso de Física Tecnológica - 2016/2017

Hand in until 5/6/2017 at 18:00

The problems are in the framework of the Standard Model (SM). The couplings are in the book. The masses and widths can be found in the *Particle Data Group* (PDG) at `http://pdg.lbl.gov/`.

I

Consider the process

$$e^-(p_1) + e^+(p_2) \to Z(p_3) + Z(p_4)$$

In this question **do not neglect** the electron mass.

a) Use the program **qgraf** to find the diagrams that contribute at tree level and write the amplitudes for each of the diagrams.

b) Consider that the Z bosons are longitudinally polarized, that is, in the frame where the Z moves with velocity $\vec{\beta}$, the longitudinal polarization vector can be written as

$$\varepsilon_L^\mu(k) = (\gamma\beta, \gamma\hat{\beta})$$

Show that this definition satisfies the required properties $\varepsilon_L(k)\cdot\varepsilon_L(k) = -1$ e $\varepsilon_L(k)\cdot k = 0$, where $\vec{\beta} = \vec{k}/E$, $\gamma^{-1} = \sqrt{1-\beta^2}$ and $\hat{\beta} = \vec{\beta}/\beta$.

c) At high energy, $\sqrt{s} \gg M_Z$, unitarity tells us that the total amplitude cannot grow with the energy. However, at high energy we have

$$\overline{v}(p_2)u(p_1) \propto \sqrt{s}, \quad p_i \propto \sqrt{s}, \quad \varepsilon_L^\mu(p_i) \propto \frac{\sqrt{s}}{M_Z}$$

Using these relations show that at high energy we have, for any of the amplitudes, $\mathcal{M}_i$,

$$\mathcal{M}_i \propto s, \text{ or } \propto m_e\sqrt{s}$$

d) By making an appropriate expansion in powers of $1/x$, where $x = s/4M_Z^2$, show explicitly that the bad behavior cancels, for any scattering angle, when we add all the amplitudes. Note that you have to make a consistent expansion in $1/x$ including all the terms, for instance,

$$\begin{aligned}
s =& 4M_Z^2 x,\\
t =& M_Z^2 + m_e^2 - 2M_Z^2 x\left(1 - \sqrt{1-\frac{1}{x}}\sqrt{1-\frac{m_e^2}{M_Z^2}\frac{1}{x}}\cos\theta\right)\\
u =& M_Z^2 + m_e^2 - 2M_Z^2 x\left(1 + \sqrt{1-\frac{1}{x}}\sqrt{1-\frac{m_e^2}{M_Z^2}\frac{1}{x}}\cos\theta\right)\\
\beta_Z =& \sqrt{1-\frac{1}{x}}, \quad \beta_e = \sqrt{1-\frac{m_e^2}{M_Z^2}\frac{1}{x}}
\end{aligned}$$

II

Consider again the process

$$e^-(p_1) + e^+(p_2) \to Z(p_3) + Z(p_4)$$

In this question **neglect** the electron mass.

a) Using the method of your choice determine $\langle |\mathcal{M}|^2 \rangle$ and the differential cross section in the `CM` frame, $d\sigma/d\Omega$.

b) Make a plot of the **total** cross section, $\sigma(e^-e^+ \to ZZ)$, in pb for $\sqrt{s} \in [182.5, 2000]$ GeV.

c) Use the program **CalcHEP** to evaluate the same process and superimpose the points obtained with **CalcHEP** on your previous plot. Make sure that you use the same constants in both cases.

d) In one plot compare the total cross section with the t-channel, u-channel and interference in the smaller interval $\sqrt{s} \in [182.5, 300]$ GeV. Use linear scales.

e) In the same plot compare your results for the total cross section with those of **CalcHEP** in the same smaller interval $\sqrt{s} \in [182.5, 300]$ GeV. Use linear scales.

NOTES

1. In the web page `http://porthos.ist.utl.pt/CTQFT/` you can find useful examples.
2. On any of the questions you can use all the available software, on the condition that you present the codes used.