TÉCNICO LISBOA

Homework of Teoria do Campo

Curso de Física Tecnológica - 2015/2016

Hand in until 30/5/2016 at 18:00

The problems are in the framework of the Standard Model (SM). The couplings are in the book. The masses and widths can be found in the *Particle Data Group* (PDG) at `http://pdg.lbl.gov/`.

I

Consider the process

$$\nu_e(p_1) + e^+(p_2) \to \nu_\mu(p_3) + \mu^+(p_4)$$

a) Use the program **qgraf** to find the diagram(s) that contribute at tree level.

b) Write the amplitude for the process. Neglect all the fermion masses but not those of the gauge bosons.

c) Using the technique that you prefer: traces, helicity spinors or helicity (chirality) amplitudes, make a plot of the cross section in the CM frame for $\sqrt{s} \in [1, 200]\text{GeV}$. Express the cross section in picobarn (pb).

d) Use `CalcHEP` to evaluate this process and superimpose the points of `CalcHEP` on top of your curve. Do not forget to verify that you are using the same values for the physical constants.

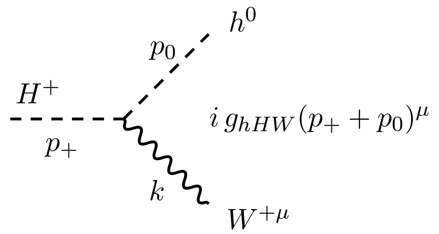
e) When $\sqrt{s} \ll M_W, M_Z$ the cross sections of neutrinos and leptons have the form

$$\sigma = \frac{\lambda}{\pi} G_F^2 s$$

Determine λ and superimpose this curve on you plot. Discuss the validity of the approximation.

II

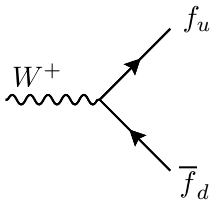
Consider the decay $H^+ \to h^0 + W^+$ in a model where a charged scalar H^+ interacts with the Higgs boson h^0 and with the W with the following Feynman rule,



where g_{hHW} is a dimensionless coupling constant. The momentum p_+ is entering the diagram while k, p_0 are leaving.

a) Evaluate $\Gamma(H^+ \to h^0 + W^+)$ in this model assuming that $m_{H^+} > m_{h^0} + m_W$.

b) Consider now the situation when $m_{H^+} < m_{h^0} + m_W$. In this case we cannot have the above decay, but as the W boson decays into all accessible fermions of the SM, $W^+ \to f_u + \overline{f}_d$



where f_u, f_d are the partners of the doublet that interacts with the W, that is

$$\begin{bmatrix} f_u \\ f_d \end{bmatrix} = \begin{bmatrix} \nu_e \\ e^- \end{bmatrix}, \cdots \begin{bmatrix} u \\ d \end{bmatrix} \cdots$$

we can have the off-shell decay $H^+ \to h^0 + W^* \to h^0 + \sum f_u \overline{f}_d$, where the sum is over all the final states into which the W can decay. Show that we can write,

$$\Gamma(H^+ \to h^0 + W^*) = \frac{1}{\pi} \int d\Delta^2 \, \frac{\Gamma_W \, m_W}{|D(\Delta^2)|^2} \, \Gamma_0(\Delta^2)$$

where

- We summed over all the final states $f_u \overline{f}_d$ and neglected their masses.
- Γ_W, m_W are the total width and the mass of the W, respectively.
- $\Delta = (q_1 + q_2)^2$ is the invariant mass of the fermion pair of momenta q_1 and q_2.
- $D(\Delta^2) = \Delta^2 - m_W^2 + i \, \Gamma_W m_W$, is the denominator of the off-shell W.
- $\Gamma_0(\Delta^2)$ is the width of the process $H^+ \to h^0 + W^+$, calculated in a) but with the substitution $k^2 \to \Delta^2$ instead of $k^2 \to m_W^2$.

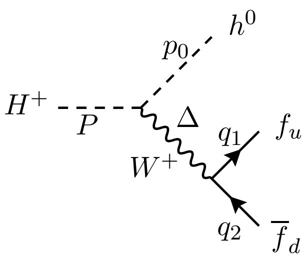
Note: In doing this a useful relation is (see PDG)

$$d\Phi_3(P; p_0, q_1, q_2) = d\Phi_2(P; p_0, \Delta) \; d\Phi_2(\Delta; q_1, q_2) \; (2\pi)^3 \; d\Delta^2$$

where $d\Phi_n(P; p_1, \cdots, p_n)$ is the Lorentz invariant phase space of n particles,

$$d\Phi_n(P; p_1, \cdots, p_n) = \delta^4 \left(P - \sum_i^n p_i \right) \prod_i^n \frac{dp_i}{(2\pi)^3 2E_i}$$

The kinematics is shown in the next figure.



c) Take $m_{h^0} = 125$ GeV, $g_{hHW} = 0.1$. Make a plot of $\Gamma(H^+ \to h^0 + W^+)$ and $\Gamma(H^+ \to h^0 + W^*)$ for $m_{H^+} \in [150, 250]$ GeV. Comment on the result.

NOTES

1. In the web page `http://porthos.ist.utl.pt/CTQFT/` you can find useful examples.