Prof. Jorge C. Romão (Responsável)

2° Exam: July 4th, 2014 – 18h30

Duration: 2h30

I (3 values)

a) Consider an elastic collision in which a particle with mass m_1 and momentum p_{Lab} collides with a particle with mass m_2 at rest in the Lab frame. Show that the energy loss of the incident particle can be written as

$$\Delta E = \frac{m_2\, p^2_{\text{Lab}}}{s} \left(1 - \cos\theta_{\text{CM}}\right)$$

where s is the square of the energy in the CM and θ_{CM} is the scattering angle in the CM frame, that is, the angle between the outgoing and the incoming particle with mass m_1. **Hint:** The energy loss of particle with mass m_1 is the energy gain of the particle with mass m_2.

b) Consider the equality

$$\gamma^\mu \sigma^{\alpha\beta} \gamma_5 \gamma^\nu\, \sigma_{\alpha\beta} = A g^{\mu\nu} + B g^{\mu\nu}\gamma_5 + C\sigma^{\mu\nu} + D\epsilon^{\mu\nu\alpha\beta}\, \sigma_{\alpha\beta}$$

Determine A, B, C e D.

II (3 values)

Draw the Feynman diagrams for the following processes in the Standard Model:

a) $e^- + \nu_e \to e^- + \nu_e$ b) $e^- + e^+ \to W^+ + W^-$ c) $t + \overline{b} \to H + W^+$

Do not evaluate anything, just draw the diagrams.

III (5 values)

Consider the process $\overline{\nu}_\mu(p_1) + e^-(p_2) \to \overline{\nu}_\mu(p_3) + e^-(p_4)$ in the Standard Model.

a) Draw the diagram(s) that contribute in lowest order.

b) Write the amplitude for the process.

c) If we neglect the lepton masses and consider that the energy in the CM frame, $\sqrt{s}$, is much less that the W and Z boson masses the cross section can be written, just on dimensional grounds as

$$\sigma = \frac{\lambda}{\pi} G_F^2\, s$$

Determine λ. **Note: Use the assignment for the momenta given above**.

IV (5 values)

Consider the process $W^-(p) \to e^-(q_1) + \overline{\nu}_e(q_2) + \gamma(k)$ in the Standard Model (Feynman rules at the end)

a) Write the amplitude for the process

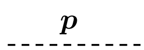
b) Show that the amplitude is gauge invariant, that is, if we write $\mathcal{M} \equiv \epsilon^{\mu *}(k)\, \mathcal{M}_\mu$ where k is the photon 4-momentum, then we must have $k^\mu \mathcal{M}_\mu = 0$. In this problem neglect the lepton masses. Recall that $\epsilon^\alpha(p)p_\alpha = \epsilon^\alpha(k)k_\alpha = 0$ respectively for the Z and the photon.

V (4 values)

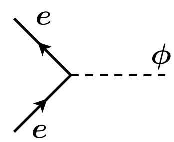
Consider the theory described by the following Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi\; - \frac{1}{2}m_\phi^2\,\phi^2 - \frac{1}{3!}\mu\phi^3 - g\,\overline{\psi}\psi\,\phi$$

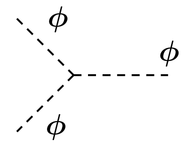
where ϕ is a neutral scalar field and ψ is the electron. The constant g is dimensionless ($\hbar = c = 1$), while μ has dimension of a mass. Besides QED the theory has the following extra propagator and vertices:



$$\frac{i}{p^2 - m_\phi^2}$$



$$-i\,g$$



$$-i\,\mu$$

Now consider the one *loop* corrections in the model described above. In all answers consider only the one-particle irreducible diagrams. Do not evaluate any expression.

a) Draw the diagram(s) for the self-energy of the electron at one *loop*.

b) Draw the diagram(s) for the self-energy of the scalar ϕ at one *loop*.

c) Draw the diagram(s) for corrections to the vertex $\overline{\psi}\psi\phi$ at one *loop*. Discuss the superficial degree of divergence, that is, count the powers of momentum.

d) Draw the diagram(s) for corrections to the vertex ϕ^3 at one *loop*. Discuss the superficial degree of divergence, that is, count the powers of momentum.

e) Is the theory renormalizable? Justify the answer.

Some expressions

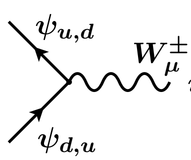
- In the CM frame we have:

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2}\frac{|\vec{p}_{\text{CM}}|}{m^2}\,\overline{|M|}^2, \qquad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2\,s}\frac{|\vec{p}_{3\text{CM}}|}{|\vec{p}_{1\text{CM}}|}\,\overline{|M|}^2$$

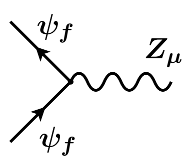
  for a decay, and for a process $p_1 + p_2 \to p_3 + p_4$, respectively.

- $\text{Tr}[\not{a}\not{b}\not{c}\not{d}\gamma_5] = -4i\,\epsilon^{\alpha\beta\gamma\delta}a_\alpha b_\beta c_\gamma d_\delta,\qquad \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu}{}^{\gamma\delta} = -2g^{\alpha\gamma}g^{\beta\delta} + 2g^{\alpha\delta}g^{\beta\gamma}$

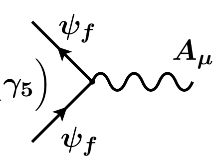
- In the Standard Model $M_W = M_Z\cos\theta_W$, $g_V^f = \frac{1}{2}T_3^f - Q_f\sin^2\theta_W$, $g_A^f = \frac{1}{2}T_3^f$ e $G_F = \sqrt{2}\,g^2/(8M_W^2)$.



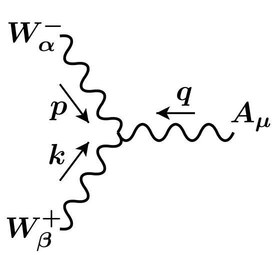
$$i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2}$$



$$i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f - g_A^f\gamma_5\right)$$



$$-ieQ_f\gamma_\mu$$



$$-ie\left[g_{\alpha\beta}(p-k)_\mu + g_{\beta\mu}(k-q)_\alpha + g_{\mu\alpha}(q-p)_\beta\right]$$