TÉCNICO LISBOA

Mestrado em Eng. Física Tecnológica (MEFT)
**Teoria de Campo**
1º semestre de 2013-14

Prof. Jorge C. Romão (Responsável)

# 1º Exam: June 9th, 2014 – 15h00

**Duration: 2h30**

## I (3 values)

a) Consider the collision $\boldsymbol{K^+ + p \to X + p}$, in the laboratory frame, where the proton is at rest. It is known that the initial linear momentum of the $\boldsymbol{K^+}$ is **200 GeV/c** and that the linear momentum of the final proton is in the same direction as the direction as the initial $\boldsymbol{K^+}$ and is **50 GeV/c**. Determine the mass of the $\boldsymbol{X}$ particle. ($\boldsymbol{m_{K^+} = 493}$ **MeV**, $\boldsymbol{m_p = 938}$ **MeV**).

b) Consider the equality

$$\gamma_\alpha \sigma^{\mu\alpha} \gamma_\beta \sigma^{\nu\beta} = A g^{\mu\nu} + B g^{\mu\nu} \gamma_5 + C \sigma^{\mu\nu} + D \epsilon^{\mu\nu\alpha\beta} \sigma_{\alpha\beta}$$

Determine $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ e $\boldsymbol{D}$.

## II (3 valores)

Draw the Feynman diagrams for the following processes in the Standard Model:
a) $e^- + e^+ \to \nu_e + \overline{\nu}_e$ b) $W^- \to e^- + \overline{\nu}_e + \gamma$ c) $H \to t + \overline{b} + W^-$
Do not evaluate anything, just draw the diagrams.

## III (5 values)

Consider the process $\mu^- + e^+ \to \overline{\nu}_e + \nu_\mu$ in the Standard Model

a) Draw the diagram(s) that contribute in lowest order.

b) Write the amplitude for the process.

c) If we neglect the lepton masses and consider that the energy in the CM frame, $\sqrt{s}$, is much less that the $\boldsymbol{W}$ and $\boldsymbol{Z}$ boson masses the cross section can be written as

$$\sigma = \frac{\lambda}{\pi} G_F^2 \, s$$

Determine $\boldsymbol{\lambda}$.

## IV ( 5 values)

Consider the process $Z(p) \to e^-(q_1) + e^+(q_2) + \gamma(k)$ in the Standard Model

a) Write the amplitude for the process

b) Show that the amplitude is gauge invariant, that is, if we write $\mathcal{M} \equiv \epsilon^\mu(k) \, \mathcal{M}_\mu$ where $\boldsymbol{k}$ is the photon 4-momentum, then we must have $k^\mu \mathcal{M}_\mu = 0$.

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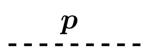
**V** (4 valores)

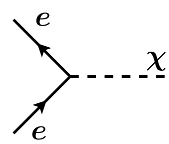
Consider the theory described by the following Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \frac{1}{2}\partial_\mu \chi \, \partial^\mu \chi \; - \frac{1}{2} m_\chi^2 \, \chi^2 - g \, \overline{\psi} \psi \, \chi$$

where $\chi$ is a neutral scalar field (spin 0) and $\psi$ is the electron. The constant $g$ is dimensionless in the sistem of units where $\hbar = c = 1$. Besides QED the theory has the following extra propagator and vertice:




$$\frac{i}{p^2 - m_\chi^2}$$




$$-i\,g$$

Now consider the one *loop* corrections in the model described above. In all answers consider only the one-particle irreducible diagrams. Do not evaluate any expression.

a) Draw the diagram(s) for the self-energy of the electron at one *loop*.

b) Draw the diagram(s) for the self-energy of the scalar $\chi$ at one *loop*.

c) Draw the diagram(s) for corrections to the vertice $\overline{\psi}\psi\chi$ at one *loop*. Discuss the superficial degree of divergence, that is, count the powers of momentum.

d) Draw the diagram(s) for corrections to the vertice $\overline{\psi}\gamma_\mu\psi A^\mu$ at one *loop*. Discuss the superficial degree of divergence, that is, count the powers of momentum.

e) Is the theory renormalizable? Justify the answer.

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**Some expressions**

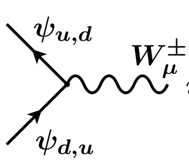
- In the CM frame we have:

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} \frac{|\vec{p}_{\text{CM}}|}{m^2} \overline{|M|}^2, \qquad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2\, s} \frac{|\vec{p}_{3\text{CM}}|}{|\vec{p}_{1\text{CM}}|} \overline{|M|}^2$$

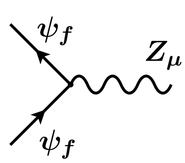
  for a decay, and for a process $p_1 + p_2 \to p_3 + p_4$, respectively.

- $\text{Tr}[\not{a}\not{b}\not{c}\not{d}\gamma_5] = -4i\,\epsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta, \qquad \epsilon^{\mu\nu\alpha\beta}\epsilon_{\mu\nu}{}^{\gamma\delta} = -2g^{\alpha\gamma}g^{\beta\delta} + 2g^{\alpha\delta}g^{\beta\gamma}$

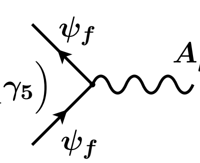
- In the Standard Model $M_W = M_Z \cos\theta_W$, $g_V^f = \frac{1}{2}T_3^f - Q_f \sin^2\theta_W$, $g_A^f = \frac{1}{2}T_3^f$ e $G_F = \sqrt{2}\, g^2/(8M_W^2)$.




$$i\frac{g}{\sqrt{2}}\gamma_\mu \frac{1-\gamma_5}{2}$$




$$i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f - g_A^f\gamma_5\right)$$




$$-ieQ_f\gamma_\mu$$