## $\boxed{\text{I}}$

I.1- ~~FALSO~~ VAL. PR. DE OP. HERMÍTICO SÃO REAIS

I.2 - FALSO NESSE POT. EXISTE PELO MENOS UM ESTADO PAR, MAS PODE NÃO EXISTIR NENHUM OUTRO.

I.3 - VERDADEIRO + NODOS $\Rightarrow$ MAIOR CURVATURA $\Rightarrow \left\langle \frac{d^2}{dx^2} \right\rangle \uparrow \Rightarrow E \uparrow$

I.4 - FALSO O.H. $\Rightarrow [H, P] = 0 \Rightarrow$ PARIDADES NÃO SE MISTURAM NA EVOL. TEMP.
$\Rightarrow \psi$ É SEMPRE PAR e $\frac{7}{2}\hbar\omega = \left(3+\frac{1}{2}\right)\hbar\omega$ É ÍMPAR

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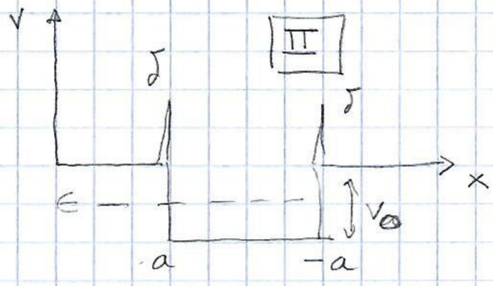
## $\boxed{\text{II}}$




II.1 $|x| > a \qquad \alpha^2 = \frac{2m}{\hbar^2}|E|$

$|x| < a \qquad q^2 = \frac{2m}{\hbar^2}\left(V_0 - |E|\right)$

$$u(x) = \begin{cases} C_1 e^{+\alpha x} & x < -a \\ A\cos(qx) + B\sin(qx) & |x| < a \\ C_2 e^{-\alpha x} & x > a \end{cases}$$

POT. PAR $\Rightarrow$ · SOL PARES OU ÍMPARES
· BASTA VER $x = a$

SOL PARES $\Rightarrow \quad C_1 = C_2 \quad , \quad B = 0$

$$\begin{cases} -\alpha C e^{-\alpha a} - \left[-A q \sin(qa)\right] = \frac{\gamma}{a} C e^{-\alpha a} \\ C e^{-\alpha a} = A\cos(qa) \end{cases}$$

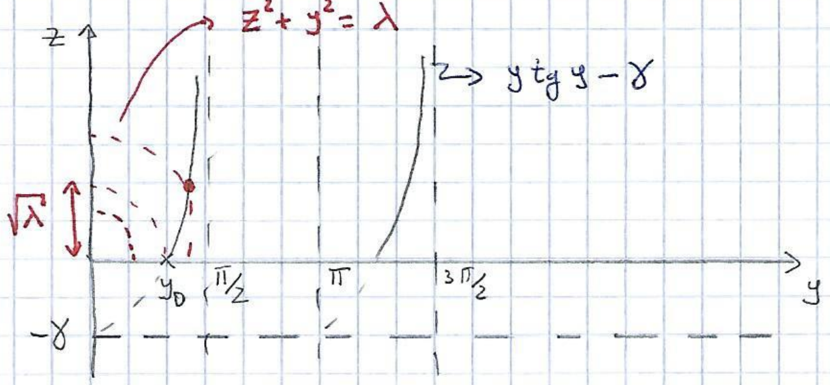
$$q A \sin(qa) = C e^{-\alpha a}\left[\frac{\gamma}{a} + \alpha\right]$$

$$A\cos(qa) = C e^{-\alpha a} \qquad \Rightarrow \boxed{y \operatorname{tg} y = \gamma + z}$$

$$\underbrace{qa}_{y} \operatorname{tg}(\underbrace{qa}_{y}) = \gamma + \underbrace{\alpha a}_{z}$$

II.2 $z = y\, tg\, y - \gamma$

$$z^2 + y^2 = \alpha^2 a^2 + q^2 a^2 = \frac{2m}{\hbar^2} V_0\, a^2 = \lambda$$




$$z = \alpha a = \sqrt{\frac{2 m a^2}{\hbar^2}} \sqrt{|E|} \Rightarrow \text{só } z > 0$$

$\Rightarrow$ · PODE NÃO EXISTIR QQ ESTADO LIGADO

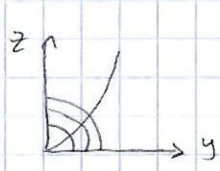
· HÁ PELO MENOS 1 ESTADO LIGADO SSE $\sqrt{\lambda} > y_0$ c/ $y_0\, t_g\, y_0 = \gamma$

$$\lambda = \frac{2m}{\hbar^2} V_0\, a^2 > y_0^2 \Rightarrow V_0 > \frac{\hbar^2}{2ma^2} y_0^2$$

II.3 $\frac{\pi}{4} = y_0\, tg\, y_0$ TEM SOL $y_0 = \frac{\pi}{4}$ PQ $\frac{\pi}{4}\, t_g(\pi/4) = \pi/4$

LOGO $\quad V_0 > \frac{\hbar^2}{2ma^2} \frac{\pi^2}{16}$

II.4 $\gamma = 0$




$\Rightarrow$ HÁ SEMPRE ESTADOS LIGADOS PARA $V_0 > 0$

$\gamma \to \infty \Rightarrow y\, tg\, y \to \infty \Rightarrow \underset{E \to \infty}{\cancel{y \to \infty}}$ OU $t_g\, y = \infty \Rightarrow y = \frac{2n-1}{2}\pi \quad n = 1, 2, \dots$

$$\Rightarrow \frac{(2n-1)^2 \pi^2}{4} = y^2 = q^2 a^2 = \frac{2ma^2}{\hbar^2} (V_0 + E)$$

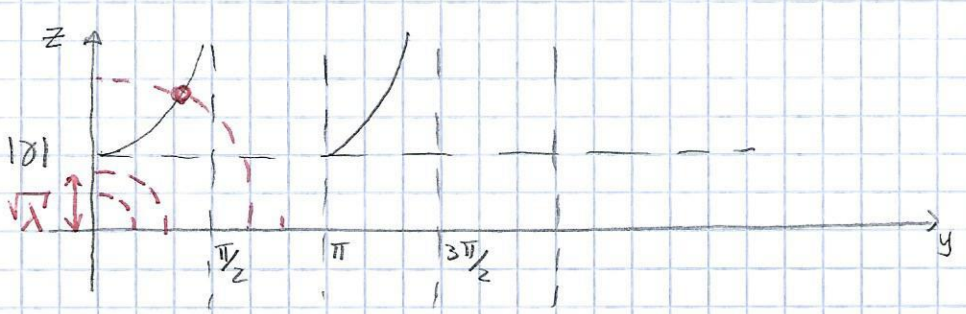
ANALISE IGUAL PARA $\forall E > -V_0$

$$\Rightarrow E = -V_0 + \frac{\hbar^2 \pi^2}{2m(2a)^2} (2n-1)^2$$

$\equiv$ SOL. PAR DE PART. NA CAIXA DE LADO $2a$ $\quad -V_0$ REDEFINIÇÃO DA ESCALA DE ENERGIA $\quad \forall V_0$

II.5 $\gamma \to -\gamma$ ou $\gamma < 0$

$$y \operatorname{tg} y + |\gamma| = z \qquad z^2 + y^2 = \lambda$$




Estado ligado só se $\sqrt{\lambda} > \gamma$

$$\Rightarrow \frac{2mV_0a^2}{\hbar^2} = \lambda > \gamma^2 \Rightarrow \boxed{V_0 > \frac{\hbar^2}{2ma^2}\gamma^2}$$

para $-V_0 < E < 0$

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III.1 $$C_0 = \int_{-\infty}^{+\infty} dx\, u_0^*(x)\, \psi(x, t=0) = \left(\frac{\beta\alpha}{\pi^2}\right)^{1/4} \int_{-\infty}^{+\infty} dx\; e^{\frac{-(\alpha+\beta)x^2/2}{-y^2}}$$

$y = \sqrt{\frac{\beta+\alpha}{2}}\, x$

$dx = \sqrt{\frac{2}{\alpha+\beta}}\, dy$

$$= \left(\frac{\beta\alpha}{\cancel{\pi^2}}\right)^{1/4} \sqrt{\frac{2}{\alpha+\beta}}\, \cancel{\sqrt{\pi}} = (\cancel{\beta^2} s)^{1/4} \sqrt{\frac{2}{(s+1)\cancel{\beta}}} = \sqrt{\frac{2}{s+1}}\; s^{1/4}$$

$$C_1 = \int_{-\infty}^{+\infty} dx\, \underbrace{u_1^*(x)}_{A}\, \underbrace{\psi(x, t=0)}_{S} \equiv 0$$

III.2 $$\frac{1}{\sqrt{2\beta}}\left[\beta x + \frac{d}{dx}\right] N e^{-\beta x^2/2} = \frac{N}{\sqrt{2\beta}} e^{-\beta x^2/2}\left[\cancel{\beta x} - \cancel{\frac{2\beta x}{2}}\right] = 0$$

III.3 $$B = \frac{s+1}{2\sqrt{s}}\frac{1}{\sqrt{2\beta}}\left(\beta\hat{x} + \frac{i}{\hbar}\hat{p}\right) + \frac{s-1}{2\sqrt{s}}\frac{1}{\sqrt{2\beta}}\left(\beta\hat{x} - \frac{i}{\hbar}\hat{p}\right) =$$

$$= \frac{\cancel{2}s}{\cancel{2}\sqrt{s}}\frac{1}{\sqrt{2\beta}}\beta\hat{x} + \frac{\cancel{2}}{\cancel{2}\sqrt{s}}\frac{1}{\sqrt{2\beta}}\frac{i}{\hbar}\hat{p}$$

$\alpha = \beta s$

$$= \frac{1}{\sqrt{2\alpha}}\left(\alpha\hat{x} + \frac{i}{\hbar}\hat{p}\right)$$

$$0 = A|0\rangle \Rightarrow 0 = \left(\beta x + \frac{d}{dx}\right) e^{-\beta x^2/2}$$

$$\downarrow \beta \to \alpha$$

$$\boxed{0 = B|\psi_0\rangle} \Leftarrow 0 = \left(\alpha x + \frac{d}{dx}\right) e^{-\alpha x^2/2}$$

Fazendo ()$^\dagger$:

$$\boxed{\langle\psi_0| B^\dagger = 0}$$

III. 4

$$4S\,[B,B^\dagger] = [(S+1)A + (S-1)A^\dagger, (S+1)A^\dagger + (S-1)A] =$$

$$= (S+1)^2 \underbrace{[A,A^\dagger]}_{1} + (S-1)^2 \underbrace{[A^\dagger, A]}_{-1}$$

$$= \cancel{S^2} + 2S + \cancel{1} - \cancel{S^2} + 2S - \cancel{1} = 4S \Rightarrow \boxed{[B,B^\dagger] = 1}$$

- $\boxed{\langle \psi_0 | \hat{x} | \psi_0 \rangle = 0 = \langle \psi_0 | \hat{p} | \psi_0 \rangle}$
  S A S   S A S

- $\begin{cases} \sqrt{2\alpha}\, B = \alpha \hat{x} + \frac{i}{\hbar} \hat{p} \\ \sqrt{2\alpha}\, B^\dagger = \alpha \hat{x} - \frac{i}{\hbar} \hat{p} \end{cases} \Rightarrow \begin{cases} \hat{x} = \frac{1}{\sqrt{2\alpha}} (B + B^\dagger) \\ \hat{p} = (-i\hbar)\sqrt{\frac{\alpha}{2}}\,(B - B^\dagger) \end{cases}$

- $\langle \psi_0 | \hat{x}^2 | \psi_0 \rangle = \frac{1}{2\alpha} \langle \psi_0 | \underbrace{B^2}_{\cancel{L}} + \underbrace{BB^\dagger}_{1 + \underbrace{B^\dagger B}_{\cancel{L}}} + \cancel{B^\dagger B} + \cancel{(B^\dagger)^2} | \psi_0 \rangle = \boxed{\frac{1}{2\alpha}}$

$$\langle \psi_0 | \hat{p}^2 | \psi_0 \rangle = -\hbar^2 \frac{\alpha}{2} \langle \psi_0 | \cancel{B^2} \underbrace{- BB^\dagger - \cancel{B^\dagger B}}_{-1 - \cancel{B^\dagger B}} + \cancel{(B^\dagger)^2} | \psi_0 \rangle = \boxed{\hbar^2 \frac{\alpha}{2}}$$

$$\hat{x}\hat{p} + \hat{p}\hat{x} \propto (B + B^\dagger)(B - B^\dagger) + (B - B^\dagger)(B + B^\dagger)$$

$$= 2B^2 - 2(B^\dagger)^2$$

$$\Rightarrow \boxed{\langle \psi_0 | \hat{x}\hat{p} + \hat{p}\hat{x} | \psi_0 \rangle \equiv 0}$$

Note: $\sigma_x^2 \sigma_p^2 \big|_{t=0} = \frac{\hbar^2}{4}$

$\Rightarrow$ MINIMUM UNCERTAINTY WAVE PACKET

III. 5

$$\langle \psi(t) | \hat{x}^2 | \psi(t) \rangle = \langle \psi_0 | (\hat{x}_H(t))^2 | \psi_0 \rangle =$$

$\langle \psi(t) | \hat{x} | \psi(t) \rangle = 0$
S A S

$$= \langle \psi_0 | \left[ \hat{x} \cos(\omega t) + \frac{\hat{p}}{m\omega} \sin(\omega t) \right]^2 | \psi_0 \rangle =$$

$$= \langle \psi_0 | \hat{x}^2 | \psi_0 \rangle \cos^2(\omega t) + \langle \psi_0 | \frac{\hat{p}^2}{m^2\omega^2} | \psi_0 \rangle \sin^2(\omega t) + \cancel{\langle \psi_0 | \frac{\hat{x}\hat{p} + \hat{p}\hat{x}}{m\omega} | \psi_0 \rangle}^{0} \sin\cos$$

$$= \underbrace{\frac{1}{2\alpha}}_{\frac{1}{2S\beta}} \cos^2(\omega t) + \underbrace{\frac{\hbar^2 \alpha}{2m^2\omega^2}}_{\frac{S\beta}{2\beta^2}} \left[ 1 - \cos^2(\omega t) \right]$$

$$= \frac{S}{2\beta} + \cos^2(\omega t) \frac{1}{2\beta} \left[ \frac{1}{S} - S \right]$$

$$\boxed{\begin{aligned} K_1 &= \frac{S}{2\beta} \\ K_2 &= \frac{S}{2\beta} \left[ \frac{1}{S^2} - 1 \right] \end{aligned}}$$