Formulário de Mecânica Quântica

- **Poço de potencial infinito**
  $V = 0$ para $0 < x < a$ e $V = \infty$ para $x < 0$ e $x > a$. As funções próprias do operador Hamiltoniano H (i.e. da energia) são:
$$u_n(x) = \sqrt{\frac{2}{a}}\ \sin\left(\frac{n\pi}{a}x\right) \qquad , \qquad E_n = \frac{\pi^2\hbar^2}{2ma^2}\ n^2\ .$$

- **Poço de potencial infinito simétrico**
  $V = 0$ para $-a/2 < x < a/2$ e $V = \infty$ para $x < -a/2$ e $x > a/2$. As funções próprias do operador Hamiltoniano H (i.e. da energia) são ($n = 1, 2, 3, \ldots$):
$$\begin{aligned} u_n^-(x) &= \sqrt{\frac{2}{a}}\ \sin\left(\frac{2n\pi}{a}x\right) & E_n^- &= E_0\ (2n)^2 \\ u_n^+(x) &= \sqrt{\frac{2}{a}}\ \cos\left[(2n-1)\,\frac{\pi}{a}x\right] & E_n^+ &= E_0\ (2n-1)^2 \end{aligned} \qquad E_0 = \frac{\pi^2\hbar^2}{2ma^2}\ .$$

- **Primitivas para os problemas do poço infinito**
$$\int dy\ \sin^2(y) = \frac{1}{2}y - \frac{1}{4}\sin(2y)$$
$$\int dy\ \sin(ny)\sin(my) = \frac{1}{2(m-n)}\ \sin[y(m-n)] - \frac{1}{2(m+n)}\ \sin[y(m+n)] \quad ; \quad m \neq n$$
$$\int dy\ y\sin^2(ny) = \frac{y^2}{4} - \frac{\sin(2ny)y}{4n} - \frac{\cos(2ny)}{8n^2}$$
$$\int dy\ y\sin(ny)\sin(my) = \frac{1}{2}\left(\frac{\cos((m-n)y)}{(m-n)^2} - \frac{\cos((m+n)y)}{(m+n)^2} + \frac{y\sin((m-n)y)}{m-n} - \frac{y\sin((m+n)y)}{m+n}\right)\ ;\ m \neq n$$

- **Oscilador harmónico: Polinómios de Hermite**
  As funções próprias são
$$u_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}}\ H_n(y)\ e^{-y^2/2}$$
  onde $y = \sqrt{\frac{m\omega}{\hbar}}x$ e os primeiros polinómios de Hermite, $H_n(y)$, são:
$$\begin{aligned} H_0 &= 1 & H_1 &= 2y \\ H_2 &= 4y^2 - 2 & H_3 &= 8y^3 - 12y \\ H_4 &= 16y^4 - 48y^2 + 12 & H_5 &= 32y^5 - 160y^3 + 120y \end{aligned}$$
  As energias são dadas por
$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega \quad n = 0, 1, 2, 3, \ldots$$

- **Oscilador harmónico: Operadores A e A^+**
$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2x^2 = \hbar\omega\left(A^+A + \frac{1}{2}\right)$$
  onde
$$A = \sqrt{\frac{m\omega}{2\hbar}}x + i\frac{p}{\sqrt{2m\omega\hbar}}, \quad A^+ = \sqrt{\frac{m\omega}{2\hbar}}x - i\frac{p}{\sqrt{2m\omega\hbar}}$$
  com
$$\left[A, A^+\right] = 1$$
  As relações inversas são
$$x = \sqrt{\frac{\hbar}{2m\omega}}\left(A + A^+\right), \quad p = -i\sqrt{\frac{\hbar m\omega}{2}}\left(A - A^+\right)$$
  Os estados correctamente normalizados são
$$|n\rangle = \frac{1}{\sqrt{n!}}\left(A^+\right)^n|0\rangle$$
  com $A\,|0\rangle = 0$ e
$$A\,|n\rangle = \sqrt{n}\,|n-1\rangle\,, \quad A^+\,|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

- **Momento Angular e Harmónicas Esféricas**

$$[L_i, L_j] = i\hbar\epsilon_{ijk}L_k, \quad L_\pm = L_x \pm iL_y, \quad [L_+, L_-] = 2\hbar L_z, \quad [L_z, L_\pm] = \pm\hbar L_\pm$$
$$L^2\,|l,m\rangle = \hbar^2 l(l+1)\,|l,m\rangle\,, \quad L_z\,|l,m\rangle = \hbar m\,|l,m\rangle\,, \quad L_\pm\,|l,m\rangle = \hbar\sqrt{l(l+1)-m(m\pm1)}\,|l,m\pm1\rangle$$

As primeiras harmónicas esféricas são:

$$\begin{aligned} Y_{00} &= \frac{1}{\sqrt{4\pi}} & Y_{11} &= -\sqrt{\frac{3}{8\pi}}\,e^{i\varphi}\,\sin\theta \\ Y_{10} &= \sqrt{\frac{3}{4\pi}}\cos\theta & Y_{22} &= \sqrt{\frac{15}{32\pi}}\,e^{i2\varphi}\,\sin^2\theta \\ Y_{21} &= -\sqrt{\frac{15}{8\pi}}\,e^{i\varphi}\,\sin\theta\cos\theta & Y_{20} &= \sqrt{\frac{5}{16\pi}}\,\big(3\cos^2\theta-1\big) \end{aligned}$$

com

$$Y_{l,-m} = (-1)^m Y^*_{lm} \qquad \text{e} \qquad \int d\Omega\,|Y_{lm}(\theta,\varphi)|^2 = 1$$

- **Equação Radial**

Para um potencial esfericamente simétrico a equação radial é

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2}\Big(V(r)-E\Big)\right]R(r) = 0$$

Fazendo a mudança de variável $u(r) = r\ R(r)$ temos

$$\left[\frac{d^2}{dr^2} - \frac{l(l+1)}{r^2} - \frac{2\mu}{\hbar^2}\Big(V(r)-E\Big)\right]u(r) = 0$$

- **Átomo de Hidrogénio e Funções Radiais**

A solução geral é

$$\psi_{nlm}(r,\theta,\varphi) = R_{nl}(r)Y_{lm}(\theta,\varphi), \quad \text{com energias} \quad E_n = -\frac{1}{2}mc^2\alpha^2\frac{1}{n^2}, \quad \alpha^{-1} = 137.036$$

As primeiras funções radiais são

$$\begin{aligned} R_{10} &= 2\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0} & R_{20} &= 2\left(\frac{1}{2a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right) e^{-r/2a_0} \\ R_{21} &= \frac{1}{\sqrt{3}}\left(\frac{1}{2a_0}\right)^{3/2}\frac{r}{a_0}\,e^{-r/2a_0} & R_{30} &= 2\left(\frac{1}{3a_0}\right)^{3/2}\left(1-\frac{2r}{3a_0}+\frac{2r^2}{27a_0^2}\right) e^{-r/3a_0} \\ R_{31} &= \frac{4\sqrt{2}}{3}\left(\frac{1}{3a_0}\right)^{3/2}\frac{r}{a_0}\left(1-\frac{r}{6a_0}\right) e^{-r/3a_0} & R_{32} &= \frac{2\sqrt{2}}{27\sqrt{5}}\left(\frac{1}{3a_0}\right)^{3/2}\left(\frac{r}{a_0}\right)^2 e^{-r/a_0} \end{aligned}$$

Os valores médios de algumas potências de r

$$\begin{aligned} \left\langle r^k \right\rangle &\equiv \int_0^\infty dr\,r^{2+k}\,R^2_{nl}(r) & \langle r\rangle &= \frac{a_0}{2}\left[3n^2 - l(l+1)\right] & & \\ \left\langle r^2 \right\rangle &= \frac{a_0^2n^2}{2}\left[5n^2+1-3l(l+1)\right] & \left\langle \frac{1}{r}\right\rangle &= \frac{1}{a_0n^2} & a_0 &= \frac{\hbar}{mc\alpha} \\ \left\langle \frac{1}{r^2}\right\rangle &= \frac{2}{a_0^2n^3(2l+1)} & \left\langle \frac{1}{r^3}\right\rangle &= \frac{2}{a_0^3n^3l(2l+1)(l+1)} & & \end{aligned}$$

- **Átomos Hidrogenóides**

Para os átomos com Z protões e um electrão, basta fazer

$$\alpha \to Z\alpha, \quad \text{e portanto} \quad a_0 \to \frac{1}{Z}a_0,\ E_n = -\frac{1}{2}mc^2(Z\alpha)^2\frac{1}{n^2}$$

- **Integrais Oscilador Harmónico e Átomo Hidrogénio**

$$\int_0^\infty x^{\alpha-1}e^{-x} = \Gamma(\alpha), \quad \int_0^\infty x^{2\alpha-1}e^{-x^2} = \frac{1}{2}\Gamma(\alpha), \quad \Gamma(\alpha+1) = \alpha\Gamma(\alpha), \quad \Gamma(1) = 1, \quad \Gamma(\frac{1}{2}) = \sqrt{\pi}$$

- **Spin** $1/2$
  1. Os vectores próprios do operador $S_n = \vec{S}\cdot\vec{n}$, onde, $\vec{n} = \sin\theta\cos\varphi\vec{e}_x + \sin\theta\sin\varphi\vec{e}_y + \cos\theta\vec{e}_z$, são

$$\psi(S_n = +\hbar/2) = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix} \qquad \psi(S_n = -\hbar/2) = \begin{pmatrix} \sin\frac{\theta}{2} \\ -e^{i\varphi}\cos\frac{\theta}{2} \end{pmatrix}$$

  2. As matrizes de Pauli são

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- **Adição de Momento Angular**

$$\vec{J} = \vec{J}_1 + \vec{J}_2, \quad J_i^2 \left|j_1, m_1\right\rangle = \hbar^2 j_i(j_i+1)\left|j_1, m_1\right\rangle, \quad J^2\left|j, m_j\right\rangle = \hbar^2 j(j+1)\left|j, m_j\right\rangle, \quad J_z\left|j, m_j\right\rangle = \hbar m_j\left|j, m_j\right\rangle$$

  1. Os valores possíveis para j são

$$j_1 + j_2, j_1 + j_2 - 1, \ldots, |j_1 - j_2|$$

  2. Qualquer estado $|j, m_j\rangle$ se pode exprimir como uma combinação linear dos produtos dos estados $|j_1, m_1\rangle$ e $|j_2, m_2\rangle$ na seguinte forma

$$|j, m_j\rangle = \sum_{m_j = m_1 + m_2} C(j, m_j; j_1, m_1, j_2, m_2)\,|j_1, m_1\rangle\,|j_2, m_2\rangle$$

  onde $C(j, m_j; j_1, m_1, j_2, m_2)$ são os coeficientes de Clebsch-Gordon e estão dados na Fig. 1 para os valores mais baixos de j_1 e j_2.

  3. A relação inversa é (os coeficientes de Clebsch-Gordon são reais)

$$|j_1, m_1\rangle\,|j_2, m_2\rangle = \sum_j C(j, m_j; j_1, m_1, j_2, m_2)\,|j, m_j\rangle$$

  4. Para o átomo de Hidrogénio ($\vec{J} = \vec{L} + \vec{S}$)

$$\begin{aligned} \psi_{l+1/2, m_j} &= \sqrt{\frac{l + m_j + 1/2}{2l+1}} Y_{l, m_j - 1/2}\,\chi^+ + \sqrt{\frac{l + 1/2 - m_j}{2l+1}} Y_{l, m_j + 1/2}\,\chi^- \\ \psi_{l-1/2, m_j} &= -\sqrt{\frac{l + 1/2 - m_j}{2l+1}} Y_{l, m_j - 1/2}\,\chi^+ + \sqrt{\frac{l + m_j + 1/2}{2l+1}} Y_{l, m_j + 1/2}\,\chi^- \end{aligned}$$

- **Teoria de Perturbações**

  No caso não degenerado, para uma perturbação com Hamiltoniano H_1,

$$\Delta E_n^{(1)} = \langle\phi_n|H_1|\phi_n\rangle, \quad \Delta E_n^{(2)} = \sum_{k\neq n} \frac{|\langle\phi_n|H_1|\phi_k\rangle|^2}{E_n^{(0)} - E_k^{(0)}}, \quad |\psi_n\rangle = |\phi_n\rangle + \sum_{k\neq n} \frac{\langle\phi_k|H_1|\phi_n\rangle}{E_n^{(0)} - E_k^{(0)}}\,|\phi_k\rangle$$

- **Constantes Físicas**

  $\hbar = 1.05457266(63)\times 10^{-34}$ Js

  $e = 1.60217733(49)\times 10^{-19}$ C

  $c = 1/\sqrt{\epsilon_0\mu_0} = 2.99792458\times 10^8$ m/s

  $\mu_0 = 4\pi\times 10^7$ H/m

  $m_e = 9.1093897(54)\times 10^{-31}$ kg $= 0.510998918(44)$ MeV

  $m_p = 1.6726231(10)\times 10^{-27}$ kg $= 938.272029(80)$ MeV

  $m_n = 1.6749286(1)\times 10^{-27}$ kg $= 939.565360(81)$ MeV

  $hc = 1240$ nm.eV, $\quad \hbar c = 197.35$ nm.eV

  $a_0 = 0.5291772108(18)\times 10^{-10}$ m

  $\alpha^{-1} = 137.03599911(46)$

  $\mu_B = \dfrac{e\hbar}{2m_e} = 5.788\times 10^{-5}$ eV/T

35. CLEBSCH-GORDAN COEFFICIENTS, SPHERICAL HARMONICS, AND d FUNCTIONS

Note: A square-root sign is to be understood over *every* coefficient, *e.g.*, for −8/15 read $-\sqrt{8/15}$.

Notation:

	J	J	...
	M	M	...
m_1 m_2			
m_1 m_2	Coefficients		
. .			

$$Y_1^0 = \sqrt{\frac{3}{4\pi}}\cos\theta$$

$$Y_1^1 = -\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{i\phi}$$

$$Y_2^0 = \sqrt{\frac{5}{4\pi}}\Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

$$Y_2^1 = -\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\, e^{i\phi}$$

$$Y_2^2 = \frac{1}{4}\sqrt{\frac{15}{2\pi}}\sin^2\theta\, e^{2i\phi}$$

$$Y_\ell^{-m} = (-1)^m Y_\ell^{m*}$$

$$d^\ell_{m,0} = \sqrt{\frac{4\pi}{2\ell+1}}\, Y_\ell^m\, e^{-im\phi}$$

$$\langle j_1 j_2 m_1 m_2 | j_1 j_2 J M\rangle = (-1)^{J-j_1-j_2}\langle j_2 j_1 m_2 m_1 | j_2 j_1 J M\rangle$$

1/2×1/2

m_1	m_2	1 +1
+1/2	+1/2	1

m_1	m_2	1 0	0 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	1 −1
−1/2	−1/2	1

1×1/2

m_1	m_2	3/2 +3/2
+1	+1/2	1

m_1	m_2	3/2 +1/2	1/2 +1/2
+1	−1/2	1/3	2/3
0	+1/2	2/3	−1/3

m_1	m_2	3/2 −1/2	1/2 −1/2
0	−1/2	2/3	1/3
−1	+1/2	1/3	−2/3

m_1	m_2	3/2 −3/2
−1	−1/2	1

2×1

m_1	m_2	3 +3
+2	+1	1

m_1	m_2	3 +2	2 +2
+2	0	1/3	2/3
+1	+1	2/3	−1/3

m_1	m_2	3 +1	2 +1	1 +1
+2	−1	1/15	1/3	3/5
+1	0	8/15	1/6	−3/10
0	+1	2/5	−1/2	1/10

m_1	m_2	3 0	2 0	1 0
+1	−1	1/5	1/2	3/10
0	0	3/5	0	−2/5
−1	+1	1/5	−1/2	3/10

m_1	m_2	3 −1	2 −1	1 −1
0	−1	2/5	1/2	1/10
−1	0	8/15	−1/6	−3/10
−2	+1	1/15	−1/3	3/5

m_1	m_2	3 −2	2 −2
−1	−1	2/3	1/3
−2	0	1/3	−2/3

m_1	m_2	3 −3
−2	−1	1

1×1

m_1	m_2	2 +2
+1	+1	1

m_1	m_2	2 +1	1 +1
+1	0	1/2	1/2
0	+1	1/2	−1/2

m_1	m_2	2 0	1 0	0 0
+1	−1	1/6	1/2	1/3
0	0	2/3	0	−1/3
−1	+1	1/6	−1/2	1/3

m_1	m_2	2 −1	1 −1
0	−1	1/2	1/2
−1	0	1/2	−1/2

m_1	m_2	2 −2
−1	−1	1

2×1/2

m_1	m_2	5/2 +5/2
+2	+1/2	1

m_1	m_2	5/2 +3/2	3/2 +3/2
+2	−1/2	1/5	4/5
+1	+1/2	4/5	−1/5

m_1	m_2	5/2 +1/2	3/2 +1/2
+1	−1/2	2/5	3/5
0	+1/2	3/5	−2/5

m_1	m_2	5/2 −1/2	3/2 −1/2
0	−1/2	3/5	2/5
−1	+1/2	2/5	−3/5

m_1	m_2	5/2 −3/2	3/2 −3/2
−1	−1/2	4/5	1/5
−2	+1/2	1/5	−4/5

m_1	m_2	5/2 −5/2
−2	−1/2	1

3/2×1/2

m_1	m_2	2 +2
+3/2	+1/2	1

m_1	m_2	2 +1	1 +1
+3/2	−1/2	1/4	3/4
+1/2	+1/2	3/4	−1/4

m_1	m_2	2 0	1 0
+1/2	−1/2	1/2	1/2
−1/2	+1/2	1/2	−1/2

m_1	m_2	2 −1	1 −1
−1/2	−1/2	3/4	1/4
−3/2	+1/2	1/4	−3/4

m_1	m_2	2 −2
−3/2	−1/2	1

3/2×1

m_1	m_2	5/2 +5/2
+3/2	+1	1

m_1	m_2	5/2 +3/2	3/2 +3/2
+3/2	0	2/5	3/5
+1/2	+1	3/5	−2/5

m_1	m_2	5/2 +1/2	3/2 +1/2	1/2 +1/2
+3/2	−1	1/10	2/5	1/2
+1/2	0	3/5	1/15	−1/3
−1/2	+1	3/10	−8/15	1/6

m_1	m_2	5/2 −1/2	3/2 −1/2	1/2 −1/2
+1/2	−1	3/10	8/15	1/6
−1/2	0	3/5	−1/15	−1/3
−3/2	+1	1/10	−2/5	1/2

m_1	m_2	5/2 −3/2	3/2 −3/2
−1/2	−1	3/5	2/5
−3/2	0	2/5	−3/5

m_1	m_2	5/2 −5/2
−3/2	−1	1

$$d^j_{m',m} = (-1)^{m-m'} d^j_{m,m'} = d^j_{-m,-m'}$$

$$d^1_{0,0} = \cos\theta \qquad d^{1/2}_{1/2,1/2} = \cos\frac{\theta}{2} \qquad d^1_{1,1} = \frac{1+\cos\theta}{2}$$

$$d^{1/2}_{1/2,-1/2} = -\sin\frac{\theta}{2} \qquad d^1_{1,0} = -\frac{\sin\theta}{\sqrt{2}}$$

$$d^1_{1,-1} = \frac{1-\cos\theta}{2}$$

2×3/2

m_1	m_2	7/2 +7/2
+2	+3/2	1

m_1	m_2	7/2 +5/2	5/2 +5/2
+2	+1/2	3/7	4/7
+1	+3/2	4/7	−3/7

m_1	m_2	7/2 +3/2	5/2 +3/2	3/2 +3/2
+2	−1/2	1/7	16/35	2/5
+1	+1/2	4/7	1/35	−2/5
0	+3/2	2/7	−18/35	1/5

m_1	m_2	7/2 +1/2	5/2 +1/2	3/2 +1/2	1/2 +1/2
+2	−3/2	1/35	6/35	2/5	2/5
+1	−1/2	12/35	5/14	0	−3/10
0	+1/2	18/35	−3/35	−1/5	1/5
−1	+3/2	4/35	−27/70	2/5	−1/10

m_1	m_2	7/2 −1/2	5/2 −1/2	3/2 −1/2	1/2 −1/2
+1	−3/2	4/35	27/70	2/5	1/10
0	−1/2	18/35	3/35	−1/5	−1/5
−1	+1/2	12/35	−5/14	0	3/10
−2	+3/2	1/35	−6/35	2/5	−2/5

m_1	m_2	7/2 −3/2	5/2 −3/2	3/2 −3/2
0	−3/2	2/7	18/35	1/5
−1	−1/2	4/7	−1/35	−2/5
−2	+1/2	1/7	−16/35	2/5

m_1	m_2	7/2 −5/2	5/2 −5/2
−1	−3/2	4/7	3/7
−2	−1/2	3/7	−4/7

m_1	m_2	7/2 −7/2
−2	−3/2	1

3/2×3/2

m_1	m_2	3 +3
+3/2	+3/2	1

m_1	m_2	3 +2	2 +2
+3/2	+1/2	1/2	1/2
+1/2	+3/2	1/2	−1/2

m_1	m_2	3 +1	2 +1	1 +1
+3/2	−1/2	1/5	1/2	3/10
+1/2	+1/2	3/5	0	−2/5
−1/2	+3/2	1/5	−1/2	3/10

m_1	m_2	3 0	2 0	1 0	0 0
+3/2	−3/2	1/20	1/4	9/20	1/4
+1/2	−1/2	9/20	1/4	−1/20	−1/4
−1/2	+1/2	9/20	−1/4	−1/20	1/4
−3/2	+3/2	1/20	−1/4	9/20	−1/4

m_1	m_2	3 −1	2 −1	1 −1
+1/2	−3/2	1/5	1/2	3/10
−1/2	−1/2	3/5	0	−2/5
−3/2	+1/2	1/5	−1/2	3/10

m_1	m_2	3 −2	2 −2
−1/2	−3/2	1/2	1/2
−3/2	−1/2	1/2	−1/2

m_1	m_2	3 −3
−3/2	−3/2	1

2×2

m_1	m_2	4 +4
+2	+2	1

m_1	m_2	4 +3	3 +3
+2	+1	1/2	1/2
+1	+2	1/2	−1/2

m_1	m_2	4 +2	3 +2	2 +2
+2	0	3/14	1/2	2/7
+1	+1	4/7	0	−3/7
0	+2	3/14	−1/2	2/7

m_1	m_2	4 +1	3 +1	2 +1	1 +1
+2	−1	1/14	3/10	3/7	1/5
+1	0	3/7	1/5	−1/14	−3/10
0	+1	3/7	−1/5	−1/14	3/10
−1	+2	1/14	−3/10	3/7	−1/5

m_1	m_2	4 0	3 0	2 0	1 0	0 0
+2	−2	1/70	1/10	2/7	2/5	1/5
+1	−1	8/35	2/5	1/14	−1/10	−1/5
0	0	18/35	0	−2/7	0	1/5
−1	+1	8/35	−2/5	1/14	1/10	−1/5
−2	+2	1/70	−1/10	2/7	−2/5	1/5

m_1	m_2	4 −1	3 −1	2 −1	1 −1
+1	−2	1/14	3/10	3/7	1/5
0	−1	3/7	1/5	−1/14	−3/10
−1	0	3/7	−1/5	−1/14	3/10
−2	+1	1/14	−3/10	3/7	−1/5

m_1	m_2	4 −2	3 −2	2 −2
0	−2	3/14	1/2	2/7
−1	−1	4/7	0	−3/7
−2	0	3/14	−1/2	2/7

m_1	m_2	4 −3	3 −3
−1	−2	1/2	1/2
−2	−1	1/2	−1/2

m_1	m_2	4 −4
−2	−2	1

$$d^{3/2}_{3/2,3/2} = \frac{1+\cos\theta}{2}\cos\frac{\theta}{2}$$

$$d^{3/2}_{3/2,1/2} = -\sqrt{3}\,\frac{1+\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,2} = \Big(\frac{1+\cos\theta}{2}\Big)^2$$

$$d^{3/2}_{3/2,-1/2} = \sqrt{3}\,\frac{1-\cos\theta}{2}\cos\frac{\theta}{2} \qquad d^2_{2,1} = -\frac{1+\cos\theta}{2}\sin\theta$$

$$d^{3/2}_{3/2,-3/2} = -\frac{1-\cos\theta}{2}\sin\frac{\theta}{2} \qquad d^2_{2,0} = \frac{\sqrt{6}}{4}\sin^2\theta \qquad d^2_{1,1} = \frac{1+\cos\theta}{2}(2\cos\theta - 1)$$

$$d^{3/2}_{1/2,1/2} = \frac{3\cos\theta - 1}{2}\cos\frac{\theta}{2} \qquad d^2_{2,-1} = -\frac{1-\cos\theta}{2}\sin\theta \qquad d^2_{1,0} = -\sqrt{\frac{3}{2}}\sin\theta\cos\theta$$

$$d^{3/2}_{1/2,-1/2} = -\frac{3\cos\theta + 1}{2}\sin\frac{\theta}{2} \qquad d^2_{2,-2} = \Big(\frac{1-\cos\theta}{2}\Big)^2 \qquad d^2_{1,-1} = \frac{1-\cos\theta}{2}(2\cos\theta + 1) \qquad d^2_{0,0} = \Big(\frac{3}{2}\cos^2\theta - \frac{1}{2}\Big)$$

Figure 35.1: The sign convention is that of Wigner (*Group Theory*, Academic Press, New York, 1959), also used by Condon and Shortley (*The Theory of Atomic Spectra*, Cambridge Univ. Press, New York, 1953), Rose (*Elementary Theory of Angular Momentum*, Wiley, New York, 1957), and Cohen (*Tables of the Clebsch-Gordan Coefficients*, North American Rockwell Science Center, Thousand Oaks, Calif., 1974). The coefficients here have been calculated using computer programs written independently by Cohen and at LBNL.

Figure 1: Coeficientes de Clebsch-Gordon para $j_i = 1/2, 1, 3/2, 2$. Fonte: Particle Data Group web page, `http://pdg.lbl.gov/2007/reviews/clebrpp.pdf`