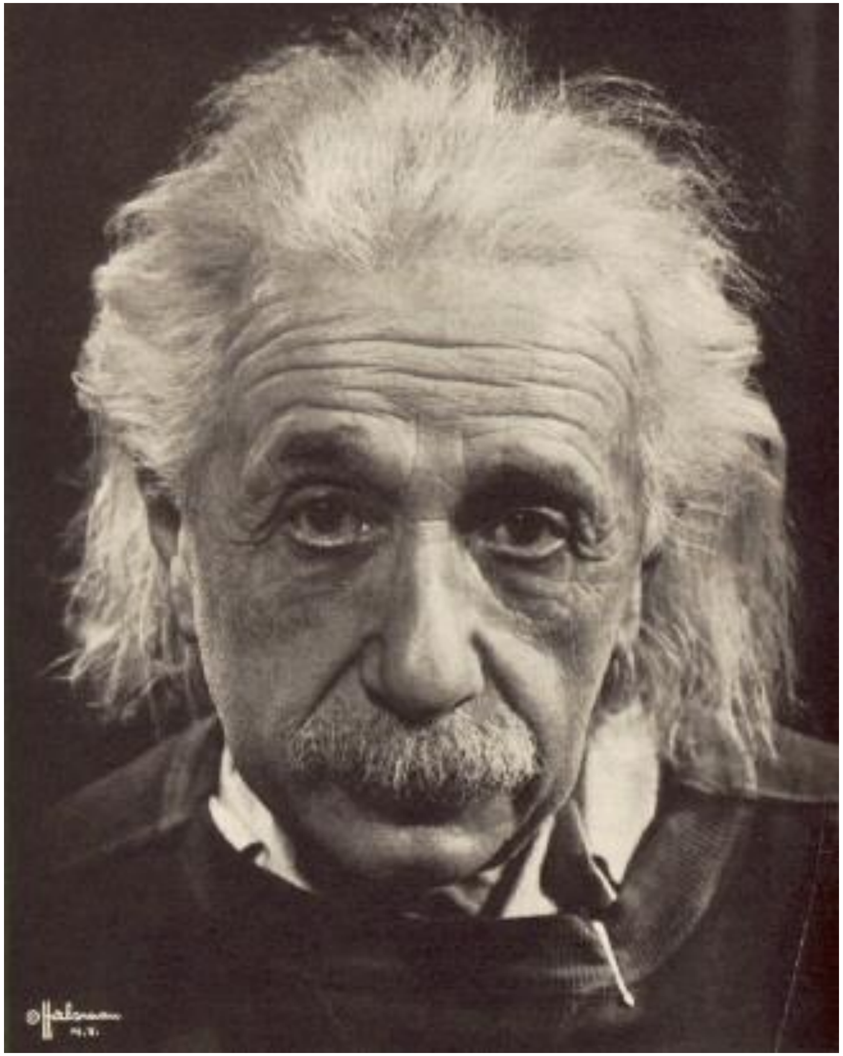
Partículas Elementares (2015/2016)

3- Special Relativity, Units

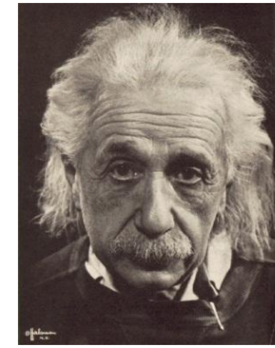
Mário Pimenta
Lisboa, 09/2015
pimenta@lip.pt

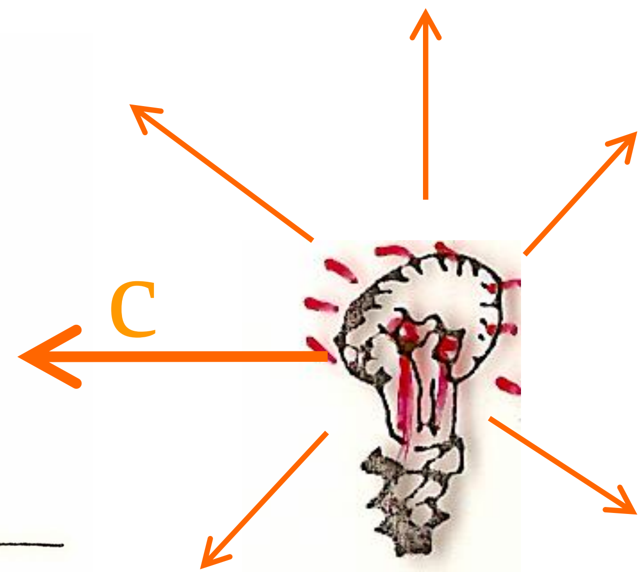


- The laws of physics are the same for all observers in uniform motion relative to one another
- The speed of light in a vacuum is the same for all observers, regardless of their relative motion or of the motion of the source of the light.

Albert Einstein 1879-1955

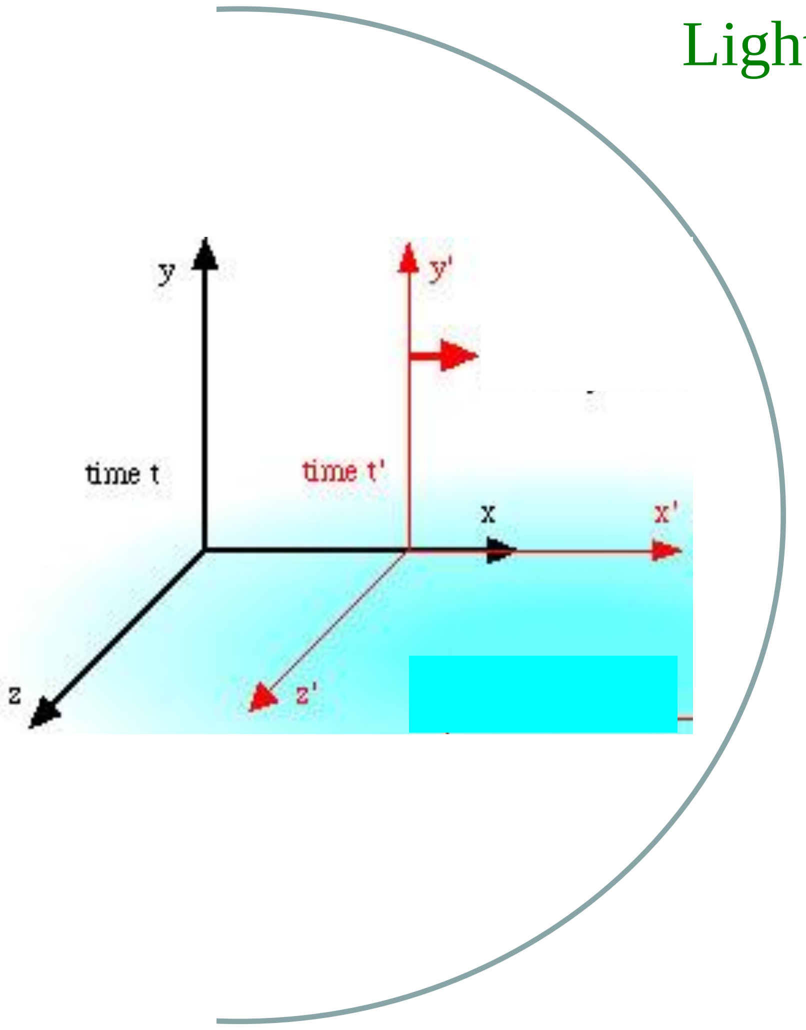
Special relativity





$$c + v = c\ !$$

Invariance of c

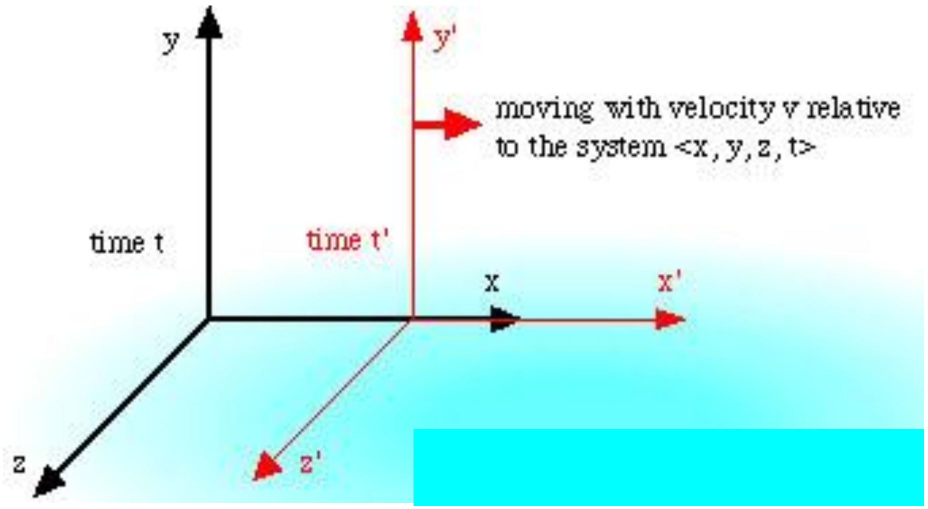


Light emitted at $x = x' = t = t' = 0$

$$(ct)^2 - (x^2 + y^2 + z^2) = 0$$

$$(ct')^2 - (x'^2 + y'^2 + z'^2) = 0$$

The Lorentz transformations



$$x' = (x - v\,t) / \sqrt{1 - v^2 / c^2}$$

$$y' = y$$

$$z' = z$$

$$t' = (t - v\,x / c^2) / \sqrt{1 - v^2 / c^2}$$

$$\beta = v / c$$

$$\tau = c\,t$$

$$\gamma = 1 / \sqrt{1 - \beta^2}$$

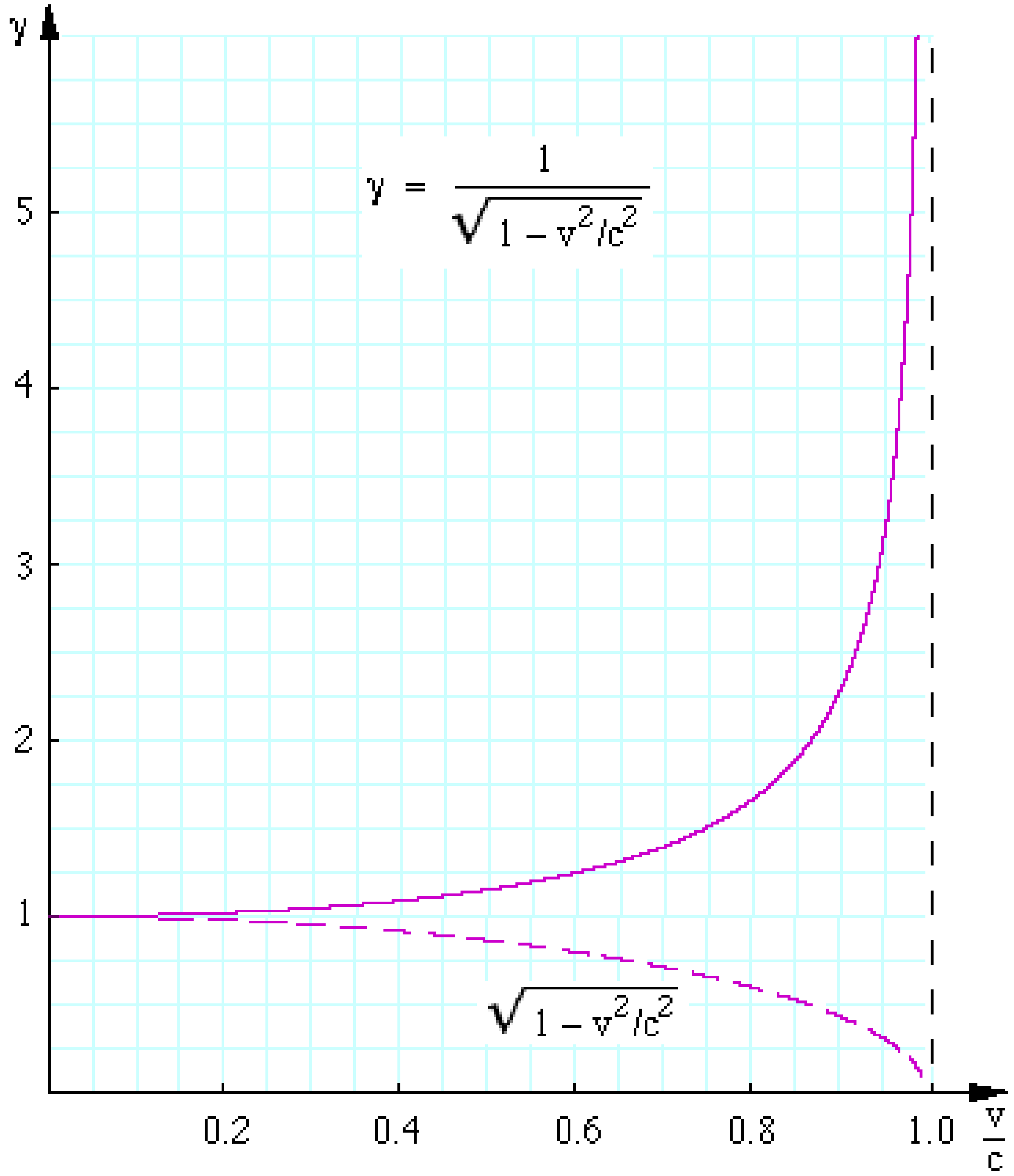
$$x' = \gamma\,(x - \beta\,\tau)$$

$$y' = y$$

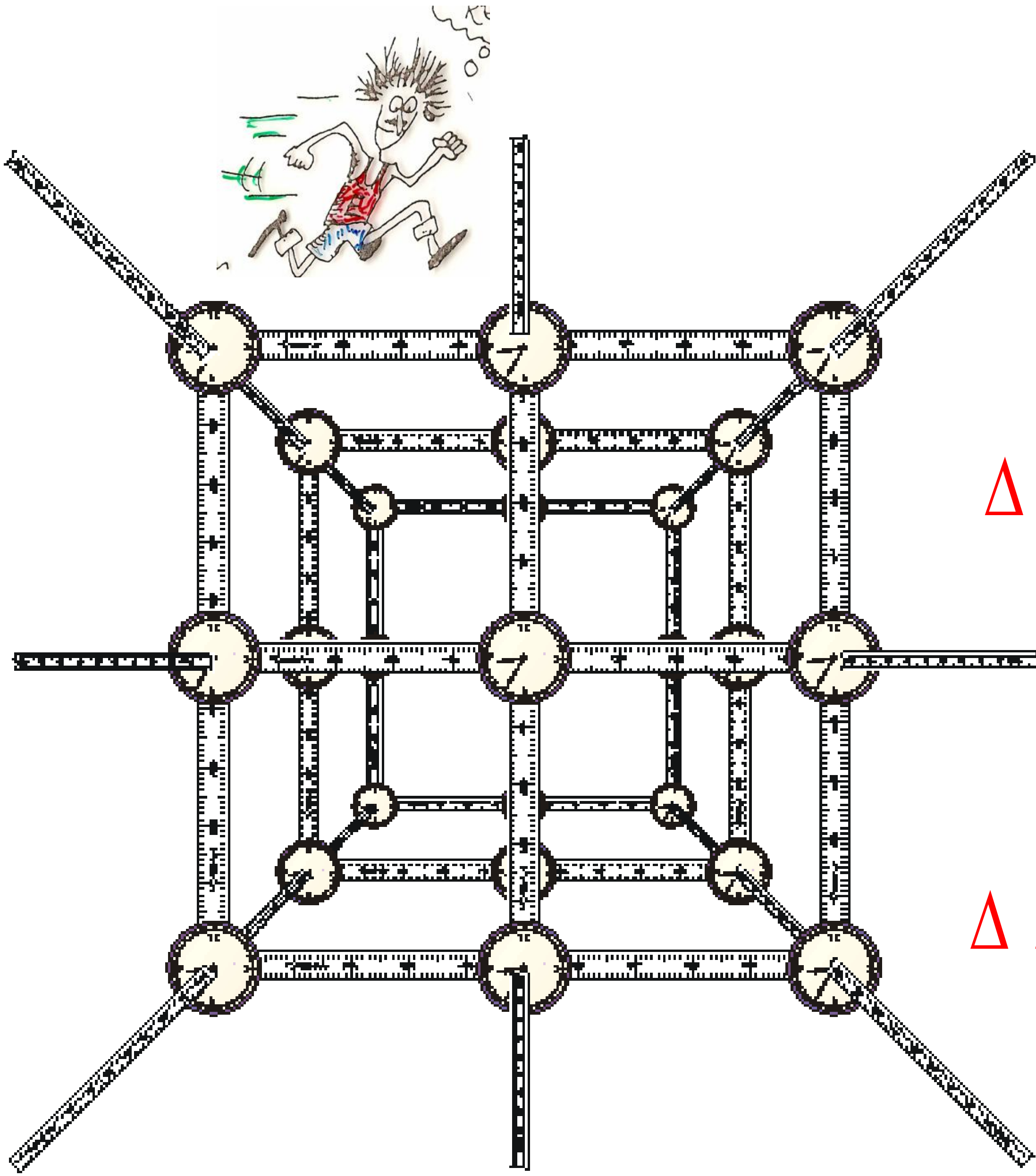
$$z' = z$$

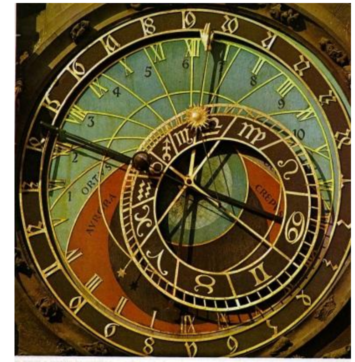
$$\tau' = \gamma\,(\tau - \beta\,x)$$

The γ factor

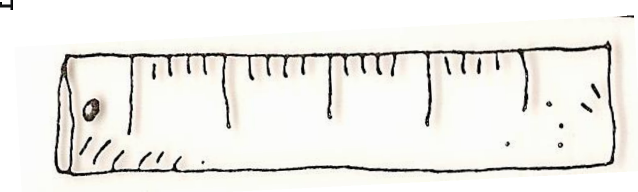

γ
5
4
3
2
1
$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$
$\sqrt{1 - v^2/c^2}$
0.2
0.4
0.6
0.8
1.0
$\frac{v}{c}$

Space and time interval are not invariant !



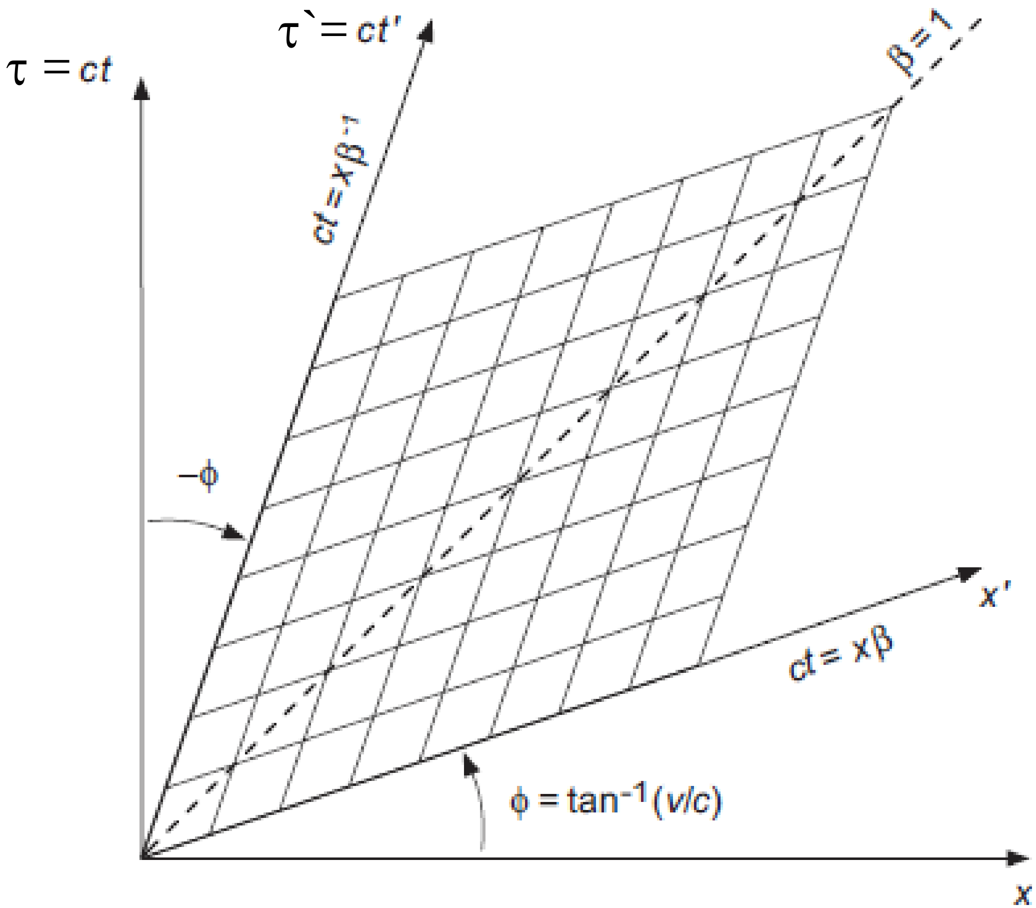


$$\Delta T = \Delta T' / \sqrt{1 - v^2 / c^2}$$



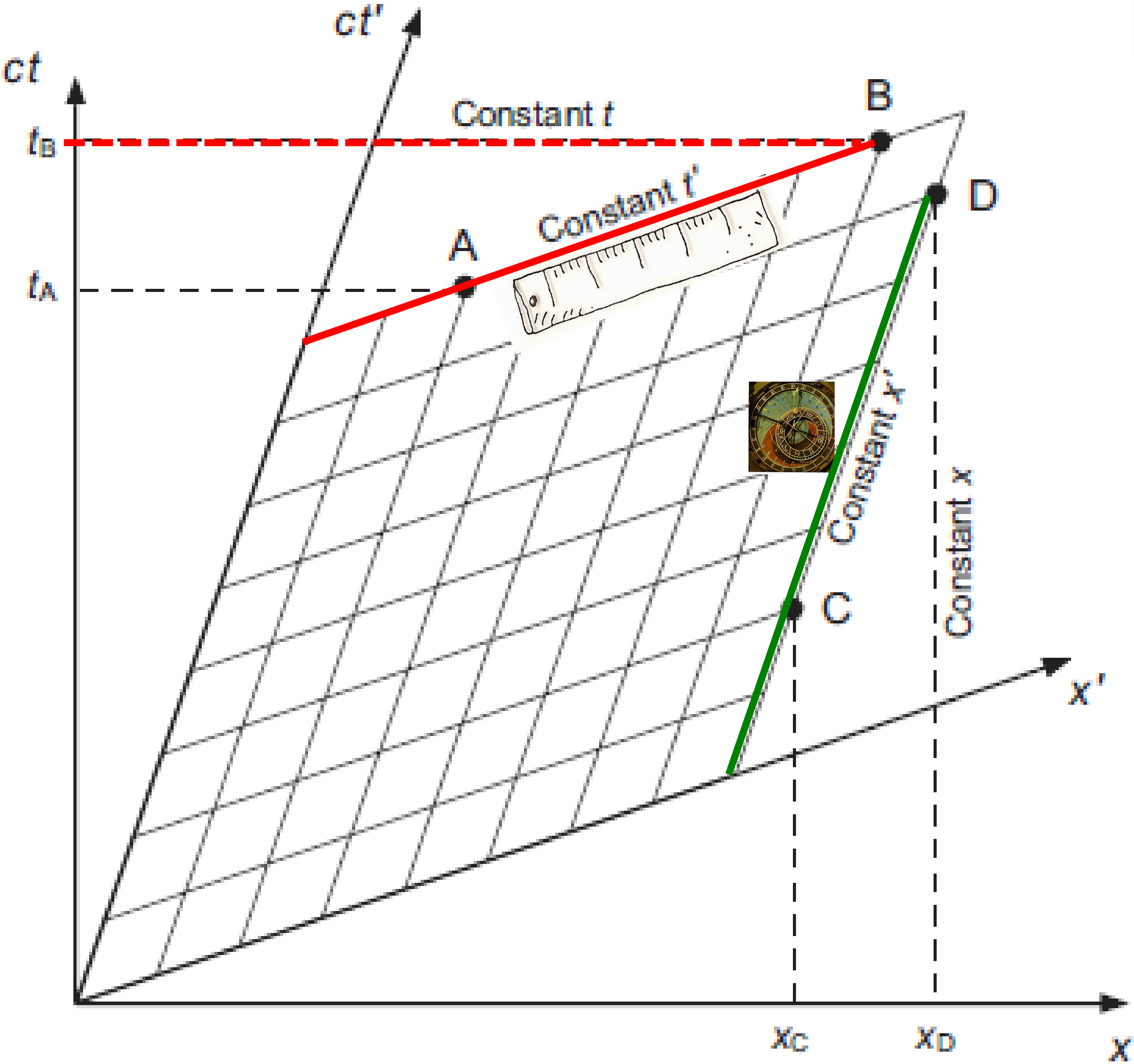
$$\Delta L = \Delta L' \sqrt{1 - v^2 / c^2}$$

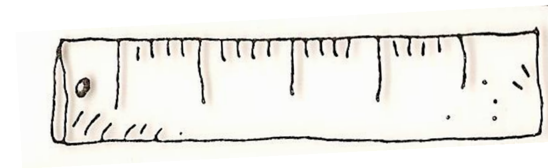
Graphical representation of the Lorentz transformations



Time and space axis are rotated in opposite directions by an angle $\theta = \arctan(\beta)$

Clocks and rulers in S'





$$\Delta x' = L'$$

$$\Delta x = L + \beta \Delta \tau$$

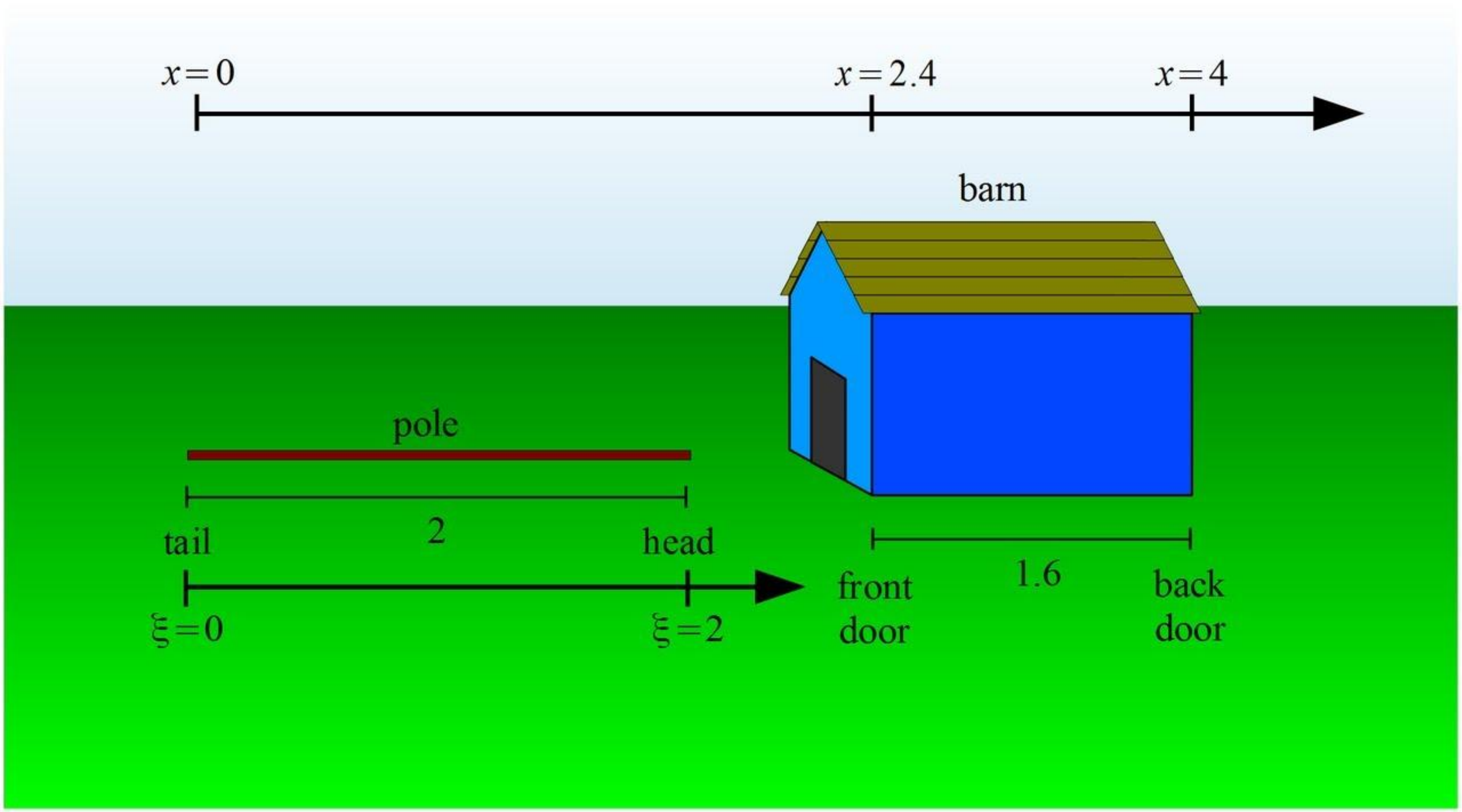
$$L = L' / \gamma$$

$$\tau_0^2 = \tau^2 - (\Delta x)^2$$

$$\tau = \gamma \tau_0$$

“barn-pole” paradox

Can a *pole* that is longer than a *barn* completely fit inside the *barn*?



Muons at Earth surface !

$$\tau_\mu = 2.19703 \pm 0.00004 \; 10^{-6} s$$

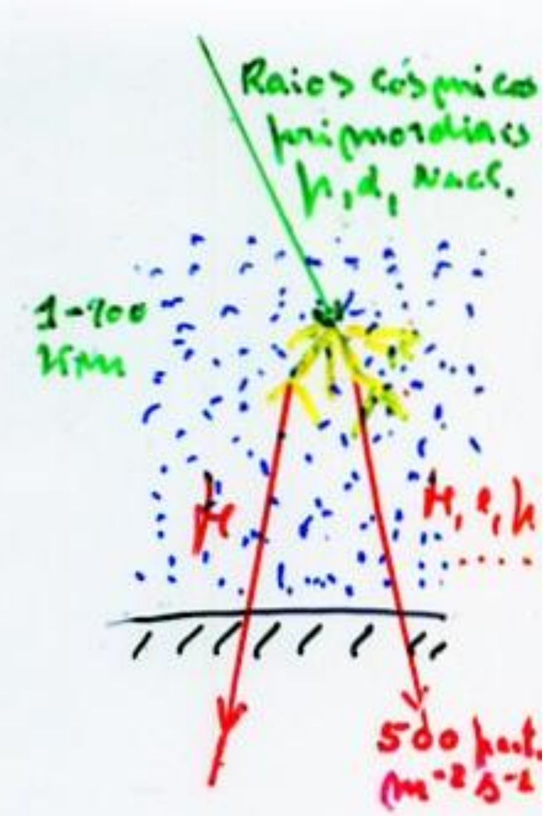
$$c\tau_\mu = 658.65 \; m \; !!!$$

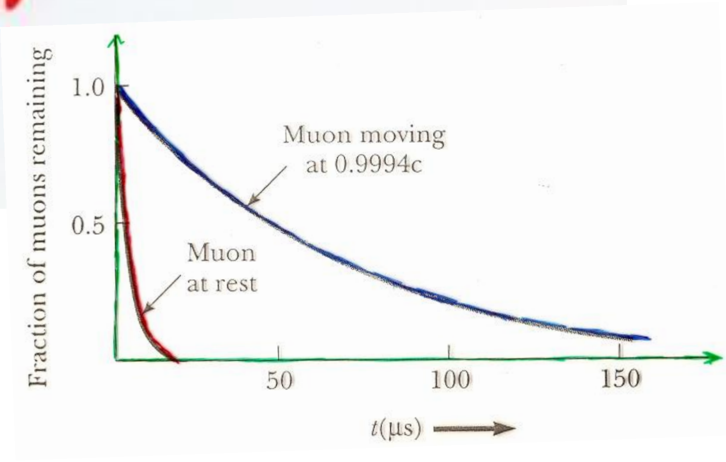
Dilatação do tempo $\Delta T = \gamma \, \Delta T'$

$$= E/m \, \Delta T'$$

$$m_\mu = 105.658387 \pm 0.000034 \; MeV$$

$$\gamma \sim 10 - 10^3 \; (E \sim 1 - 100 \; GeV) \; !!!$$





Four vectors

$$X = \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}$$

Norm of X

$$X^2 = (ct)^2 - (x^2 + y^2 + z^2)$$

$$X^2 = g_{\mu\nu} X^\mu X^\nu$$

Minkowski metric

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Internal product (four-vectors)

Minkowski metric

$$AB = a^0 b^0 - \vec{a}.\vec{b}$$

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$AB = g_{\mu\nu} a^\mu b^\nu$$

Scalars → invariants

Lorentz transformations

$$X^{\mu'} = \Lambda^{\mu}_{\nu} X^{\nu}$$

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

Space-time interval

$$\begin{pmatrix} \Delta ct \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} = \begin{pmatrix} ct_1 \\ x_1 \\ y_1 \\ z_1 \end{pmatrix} - \begin{pmatrix} ct_0 \\ x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

is a four vector

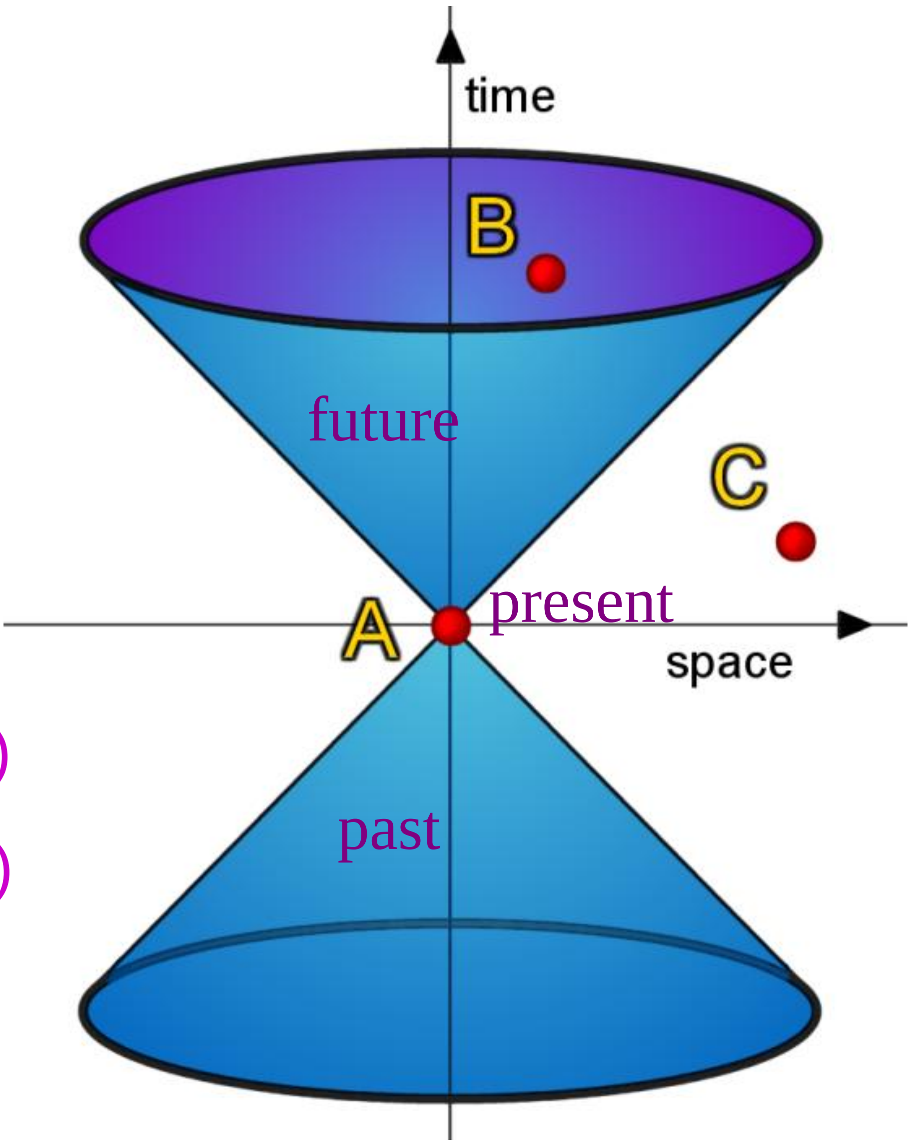
$s^2 = \Delta^2(ct) - (\Delta^2 x + \Delta^2 y + \Delta^2 z)$ is a Lorentz invariant!

Light cone

$s^2 > 0$ time-like interval

$s^2 < 0$ space-like interval

$s^2 = 0$ light interval



if $s^2 > 0$: $\sqrt{(s^2)}$ = proper time (τ_0)

If $s^2 < o$: $\sqrt{(-s^2)}$= rest length (L_0)

Velocity and momentum 4 vectors

$$U = \frac{\Delta X}{\Delta t_0} = \gamma \left[\frac{c\Delta t}{\Delta t}, \frac{\Delta x}{\Delta t}, \frac{\Delta y}{\Delta t}, \frac{\Delta z}{\Delta t} \right]$$

$$= \left[\gamma \, c, \; \gamma \, \vec{u} \right]$$

$$P = m\,U = \left[\gamma \, m \, c, \; \gamma \, m \, \vec{u} \right]$$

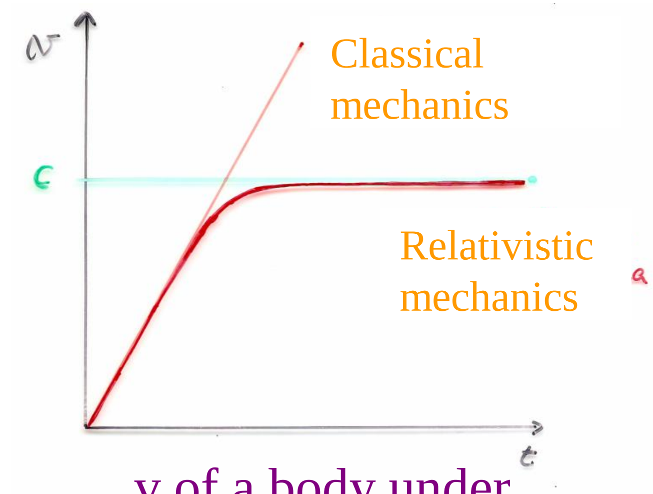
$$= \left[\frac{E}{c}, \; \vec{p} \right]$$

Energy and momentum

$$E = \gamma \; m \; c^2$$

$$\vec{p} = \gamma \; m \; \vec{v}$$

m - particle mass



v of a body under a constant force

Kinetic energy

$$E_k = \int \frac{d(\gamma \; m \; v)}{dt} dx = \gamma \; m \; c^2 - m \; c^2$$

Low $\beta \Rightarrow E_k = m \; c^2(\gamma - 1) \approx m \; c^2 (1 + \frac{v^2}{2c^2} + \ldots - 1) \approx \frac{1}{2} m \; v^2$

Energy-momentum Lorentz transformations

$$\begin{pmatrix} E'/c \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E/c \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

Useful relations

$$\vec{p} = \gamma m v$$

$$E = \gamma m c^2$$

$$p = \begin{bmatrix} E/c \\ \vec{p} \end{bmatrix}$$

$$p^2 = (E/c)^2 - |\vec{p}|^2 = m c^2$$

$$E^2 = |\vec{p}|^2 c^2 + m^2 c^4$$

$$\gamma = E/(m c^2)$$

$$v/c = \vec{p}/(E/c)$$

Sistema Natural $c = 1$

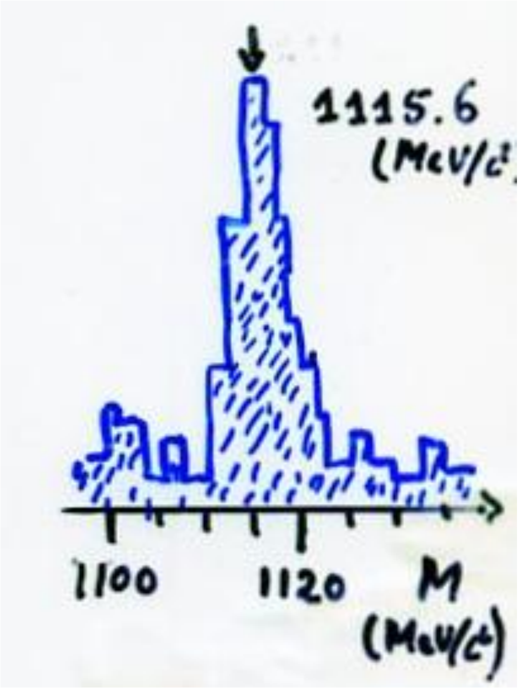
$$E^2 = p^2 + M^2$$

$$\gamma = E/M$$

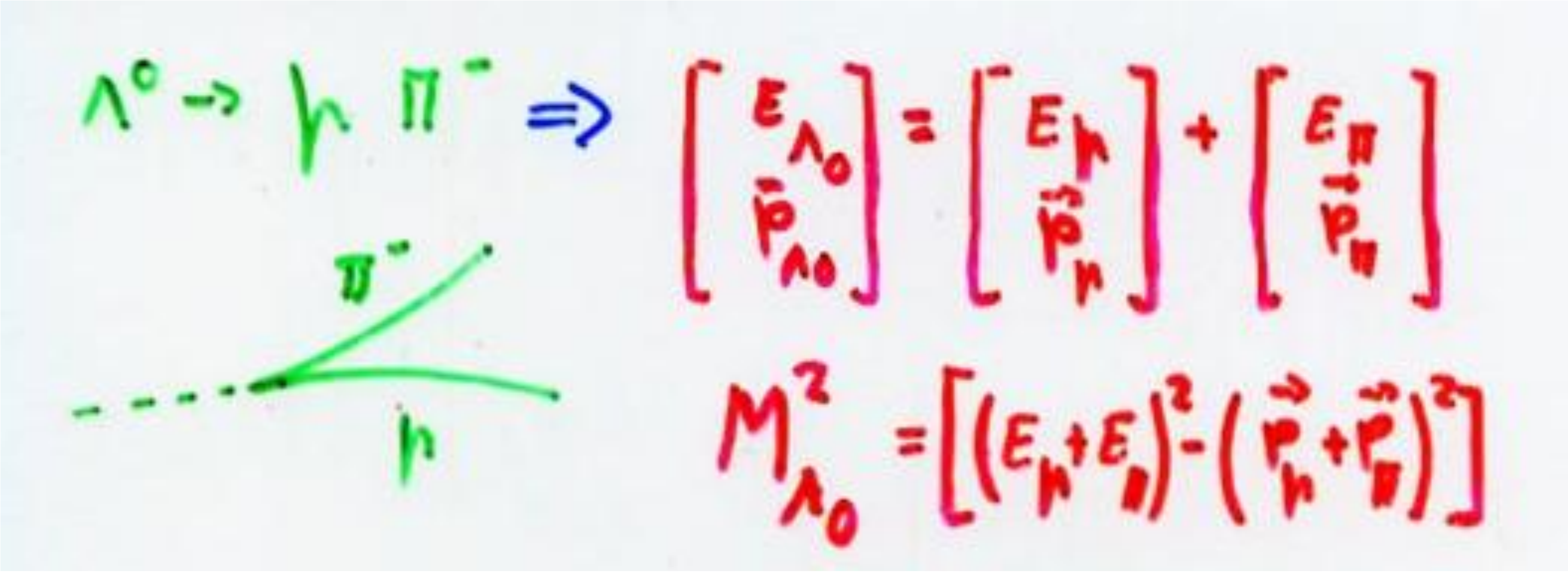
$$\beta = p/E$$

Two-body Invariant mass

Λ^0 discovery



Natural Units



Two-body center-of-mass energy

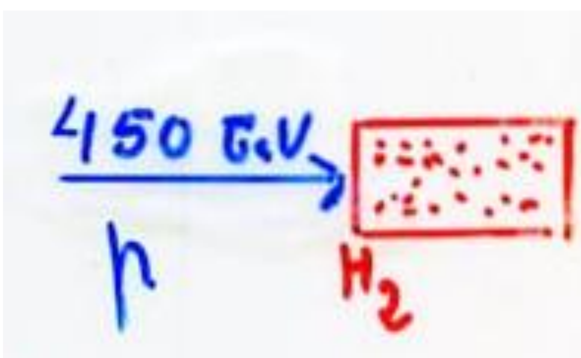
Natural Units

$$[(E_{CM},0)]^2 = [(E_{Lab},P_{Lab}) + (M,0)]^2$$

$$E_{CM}^2 = (E+M)^2 - P_{Lab}^2$$

$$= 2M^2 + 2E_1 M$$

$$E_{CM} \simeq \sqrt{2 P_{Lab} M}$$



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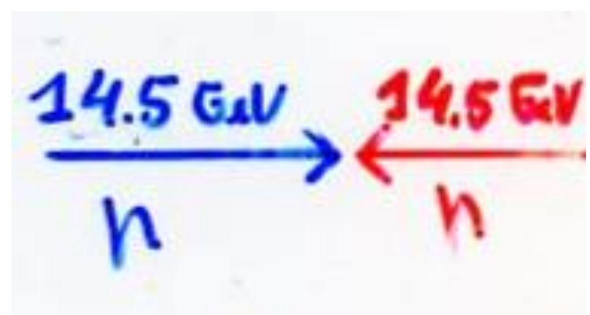


Photon conversion

Consider the conversion of one photon in one electron-positron pair. Determine the minimal energy that the photon has to have in order that this conversion would be possible if the photon is in presence of:

a) one proton
b) one electron
c) no charged particle is around

π^0 decay

Consider the decay of a π^0 into $\gamma\gamma$ (P_π = 100 GeV/c).Determine:

a) The minimal and the maximal angles that the two photons may have in the Laboratory frame ($P\ \pi^0$ = 100 GeV/c).

b) The probability of having one of the photon with an energy less than E^0 ($(E_\pi/2 - P_\pi/2) < E^0 < E_\pi/2$) in the Laboratory frame.

c) Same as a) but considering now that the decay of the π^0 is into e^+e^-.

d) The maximum momentum that the π^0 may have in order that the maximal angle in its decay into $\gamma\gamma$ and in e^+e^- would be the same.

π^- decay

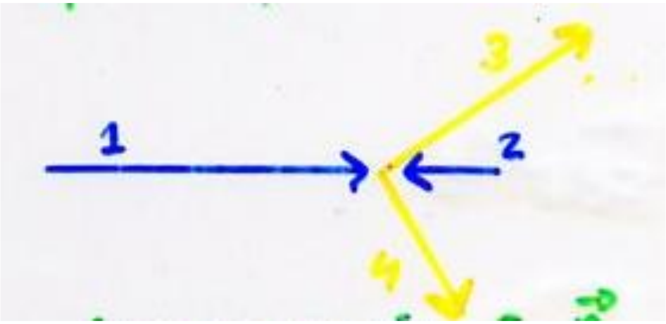
Consider the decay of a flying π^- into $\mu^- \bar{\nu}$ and suppose that the $\bar{\nu}$ was emitted along the flight line of the π^-. Determine:

a) The energy and momentum of the μ^- and of the $\bar{\nu}$ in the π^- frame.
b) The energy and momentum of the μ^- and of the $\bar{\nu}$ in the Laboratory frame (P_{π^-} = 100 GeV/c).
c) Same as b) but considering now that was the μ^- that was emitted along the flight line of the π^-.

Mandelstam variables

s $\quad (p_1 + p_2)^2$

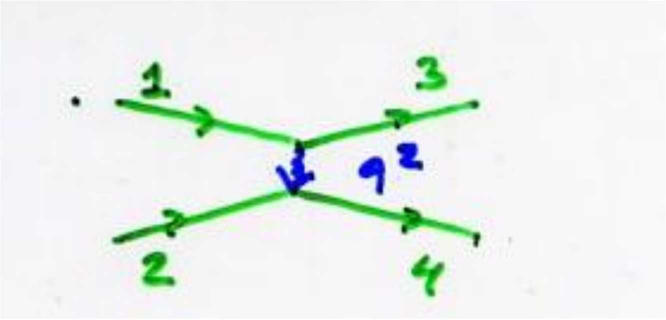
E_{CM}^2



$t, q^2 \quad (p_1 - p_3)^2$

$\approx - 4E_1E_3 \ \sin^2 (\theta/2)$

$$\left(\frac{d\sigma}{d\,\Omega}\right)_{Rutherford} \propto \frac{1}{q^4}$$



u $\quad (p_1 - p_4)^2$

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$

Units

International system of units (SI) four fundamental units:

- a unit of length (the meter, m)
- a unit of time (the second, s),
- a unit of mass (the kilogram, kg)
- and a unit of charge (the coulomb, C)

Particle Physics scales :

- Length 1 fm = 10^{-15} m
- Charge 1 |e| = 1.60 10^{-19} C
- Energy 1 MeV = 1.60 10^{-13} J
- Mass 1 MeV/c^2 = 1.78 10^{-30} kg

Two fundamental constants:

- c ~ 3.00 10^{8} m s^{-1}
- $\hbar$ ~ 1.05 10^{-34} J s

New Systems of Units

- c, $\hbar$, GeV

- Natural Units, NU (c = $\hbar$ = 1)

In NU a single unit can be used (the Energy) and all the *c* and *h* disappear from the formulas !!!

		SI	h, c, GeV	Natural
u. massa	M	Kg	GeV/c^2	GeV
u. comprimento	L	m	$\hbar c/GeV$	GeV^{-1}
u. tempo	T	s	$\hbar/GeV$	GeV^{-1}

Conversion factors

$$
\begin{aligned}
1\ \mathrm{m} &= \frac{1\mathrm{m}}{\hbar c} \simeq 5.10 \times 10^{12}\ \mathrm{MeV}^{-1} \\
1\ \mathrm{s} &= \frac{1\mathrm{s}}{\hbar} \simeq 1.52 \times 10^{21}\ \mathrm{MeV}^{-1} \\
1\ \mathrm{kg} &= 1\mathrm{J}/c^2 \simeq 5.62 \times 10^{29}\ \mathrm{MeV}
\end{aligned}
$$

Conversion of Units (SI to NU)

A quantity with dimensions in SI $M^p L^q T^r$ has NU dimension $E^{(p-q-r)}$

	mks			NU
Quantity	p	q	r	n
Mass	1	0	0	1
Length	0	1	0	-1
Time	0	0	1	-1
Action ($\hbar$)	1	2	-1	0
Velocity (c)	0	1	-1	0
Momentum	1	1	-1	1
Energy	1	2	-2	1

The conversion factors are then:

$$Q_{NU} = Q_{SI}\left(5.62\ 10^{29}\frac{MeV}{kg}\right)^p\left(5.10\ 10^{12}\frac{MeV^{-1}}{m}\right)^q\left(1.52\ 10^{21}\frac{MeV^{-1}}{s}\right)^r$$

Conversion of Units (NU to SI)

In Natural Units all the c and $\hbar$ disappear in the formulas and to recover them it is necessary to perform a dimensional analysis

For instance cross sections are in NU expressed usually in GeV^{-2} but physically it is an area and then their dimensions in SI is L^2

$$1\ GeV^{-2} = \frac{(\hbar c)^2}{(10^9 eV)^2} = 3.89\ 10^{-32} m^2$$

$$1\ GeV^{-2} = 0.389\ mb$$

The unit of length L

$$1\ GeV^{-1} = \frac{\hbar c}{10^9 eV} = 0.1974\ fm$$

The unit of time T

$$1\ GeV^{-1} = \frac{\hbar}{10^9 eV} = 6.582\ 10^{-25} s$$

Electromagnetism in NU

$$\frac{e^2}{4\pi\varepsilon_0}$$ has in SI the dimension of J.m

$$\frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137}$$ is a dimensionless quantity

Lorentz-Heaviside convention

$$\varepsilon_0 = \mu_0 = c = 1$$

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$ The fine-structure constant

$$e = \sqrt{4\pi\alpha} = 0.303$$ A pure number !

Planck scale

Gravity meets Quantum Mechanics

Schwarzschild radius ~ Compton wavelength

$$R_S \approx \lambda_C \quad \longrightarrow \quad \frac{2 G_N m_P}{c^2} \approx \frac{h}{mc}$$

$$m_P \approx \sqrt{\frac{\hbar c}{G_N}}$$

$$l_P \approx \frac{h}{m_P c} \approx \sqrt{\frac{\hbar G_N}{c^3}}$$

$$t_P \approx \frac{l_P}{c} \approx \sqrt{\frac{\hbar G_N}{c^5}}$$

$m_P \sim 2.18\ 10^{-8}$ Kg

$l_P \sim 1.6\ 10^{-35}$ m

$t_P \sim 5.4\ 10^{-44}$ s

m_P can be derived using dimensional analysis

Units

1. Determine in Natural Units
   a) your own dimensions (height, weight, age)
   b) The mean lifetime of the muon (τ_μ=2.2 10^{-6} s)
   c) the Compton wavelength

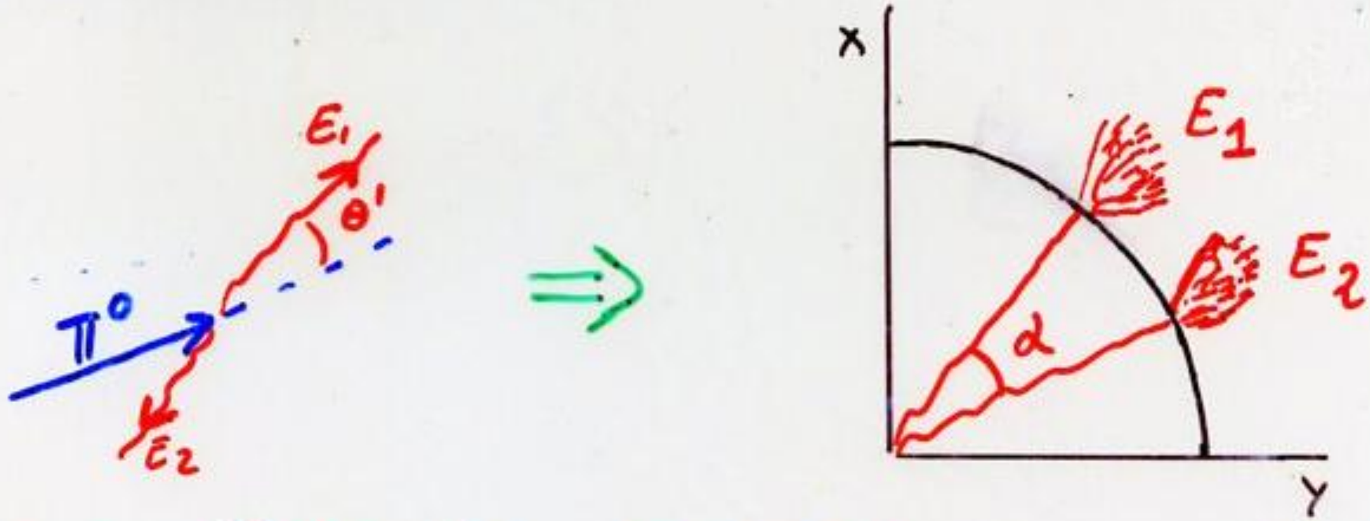
2. In NU and in SI the expression of the muon life time is given respectively by:

$$\tau_\mu = \frac{192\ \pi^3}{G_F{}^2 m_\mu{}^5} \qquad \tau_\mu = \frac{192\ \pi^3\ \hbar^7}{G_F{}^2 m_\mu{}^5 c^4}$$

   where G_F is the Fermi constant

   a) Is the Fermi constant dimensionless? If not compute its dimension in NU and SI
   b) Obtain the conversion factor for transforming G_F from SI to NU

π^0 decay



$$E_{\gamma_1} = \gamma \frac{M_\pi}{2} (1 + \beta \cos\theta')$$

$$E_{\gamma_2} = \gamma \frac{M_\pi}{2} (1 - \beta \cos\theta')$$

$\gamma = E_\pi / M_\pi$

$\beta = P_\pi / E_\pi$

$$\alpha_{min} = \frac{2 M_\pi}{P_\pi}$$

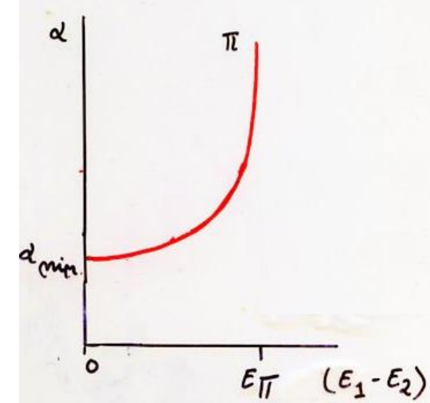
$$P_{E_\gamma < E'} = \frac{2 E'}{P_\pi}$$

5 GeV/c

$\alpha_{min} \sim 0.054$ Rad

DELPHI BARREL $\sim$ 12 cm

$P_{E_\gamma < 0.5} \sim 20\%$!



Velocity transformations

$$v_x = dx \ / dt \ = c \ dx \ / d\tau$$

$$v'_x = dx' / dt' = c \ dx' / d\tau'$$

$$\beta = V / c$$

$$x = \gamma \ (x' + \beta \ \tau')$$

$$y = y'$$

$$z = z'$$

$$\tau = \gamma \ (\tau' + \beta \ x')$$

$$v_x = c \frac{\gamma(dx' + \beta \ d\tau')}{\gamma(d\tau' + \beta \ dx')} = \frac{v'_x + V}{1 + V \ v'_x / c^2}$$

$$v_y = c \frac{dy'}{\gamma(d\tau' + \beta \ dx')} = \frac{v'_y}{\gamma(1 + V \ v'_x / c^2)}$$

$$v_z = c \frac{dz'}{\gamma(d\tau' + \beta \ dx')} = \frac{v'_z}{\gamma(1 + V \ v'_x / c^2)}$$

$$v'_x = c \rightarrow v_x = c \ !!!$$

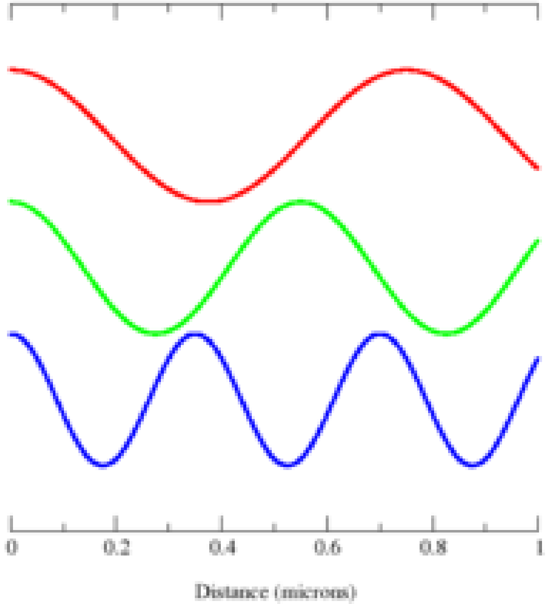
$$V \ \cong 0 \rightarrow v \ = v' + V \ !!!$$

The photon - γ

$m_\gamma = 0$

$E_\gamma = p_\gamma c$

$E_\gamma = h\nu = hc/\lambda$



Compton effect

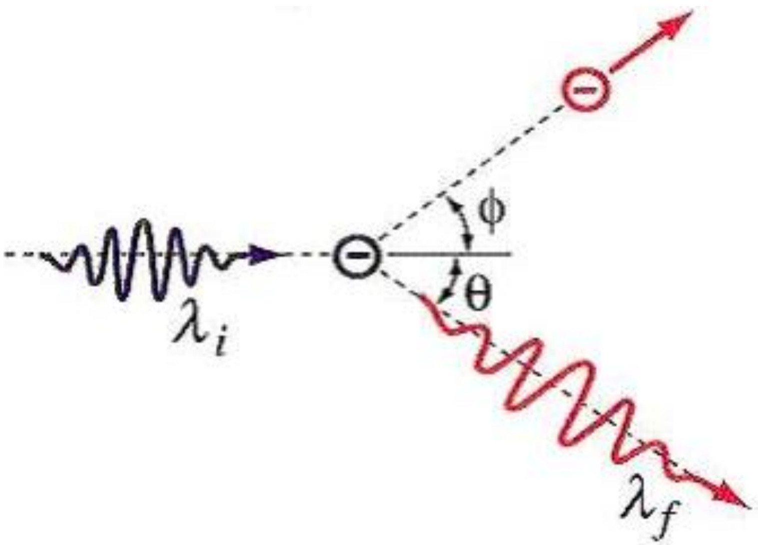
Energy- momentum conservation

$hc/\lambda_i + m_e c^2 = hc/\lambda_f + \sqrt{(m_e c^2)^2 + (p_e c)^2}$

$h/\lambda_i = h/\lambda_f \cos(\theta) + p_e \cos(\Phi)$

$0 = h/\lambda_f \sin(\theta) + p_e \sin(\Phi)$

$\lambda_f - \lambda_i = h/(m_e c)\ (1 - \cos(\theta))$



GZK threshold

The Cosmic Microwave Background fills the Universe with photons with a peak energy of 0.37 meV and a density of $\rho \sim 10^6\ m^{-3}$. Determine:

a) The minimal energy (Known as the GZK threshold) that a proton should have in order that the reaction may occur.
b) The interaction length of such protons in the Universe considering a mean cross section above the threshold of 0.6 mb.

Production at the Bevatron

The anti-protons were first produced in Laboratory in proton proton fixed target collisions at the Bevatron.

a) Describe the minimal reaction able to produce in such collisions
b) Determine the minimal energy that the proton beam had to have in order that were produced considering that the target protons have a Fermi momentum of around 150 MeV.