Partículas Elementares (2015/2016)

2-Rutherford, Secção-eficaz decaimento beta, o neutrão

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Lisboa, 09/2015

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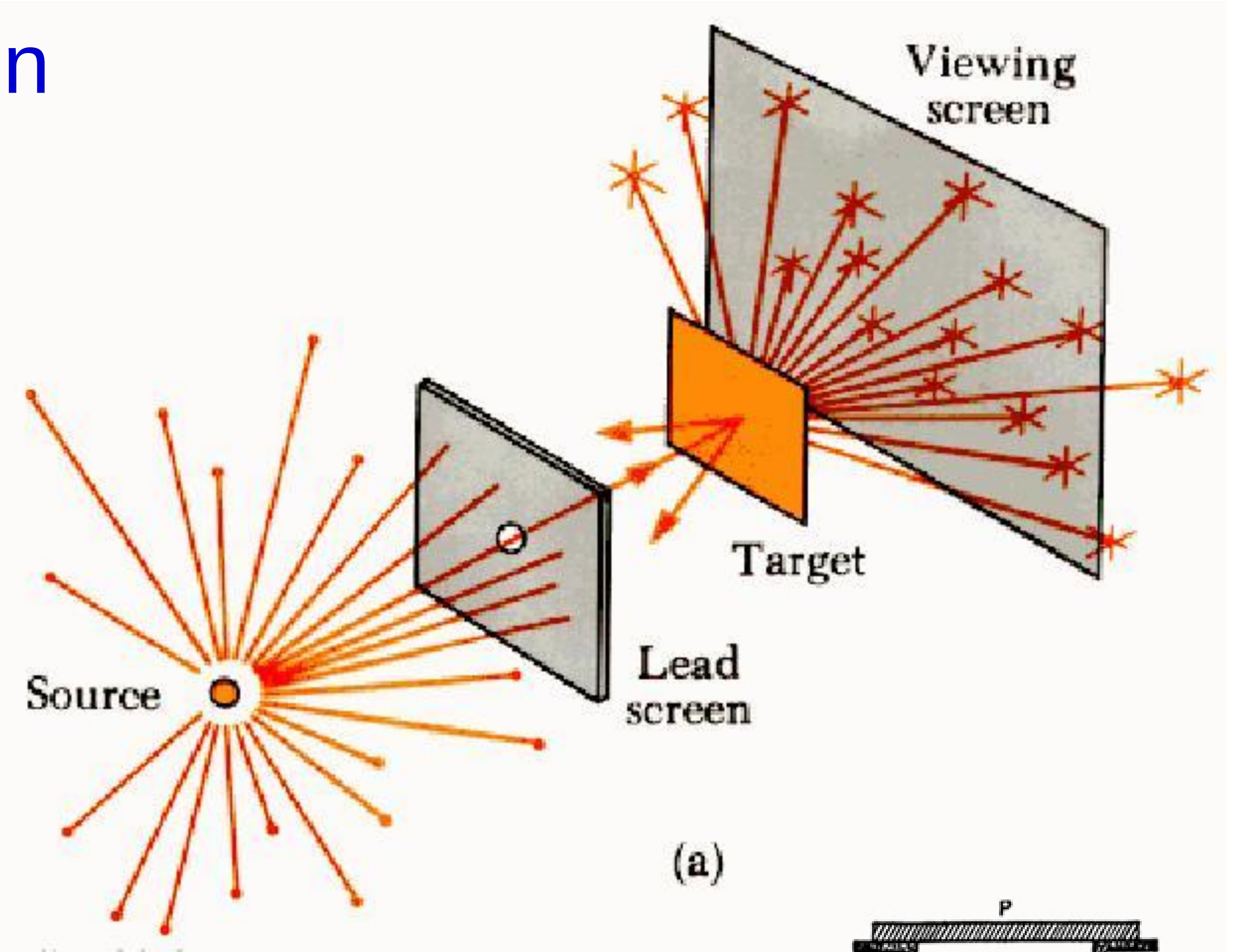
Imagin one atom

Dalton – mechanical connections
Philip Lenard – electric dipoles
Hantora Nagoaka – Saturnian model
Thompson – plum pudding
Rutherford – very small positive nucleus

Geiger- Marsden experiment



1/8000 α particles suffer deflections ≥ 90°

1909

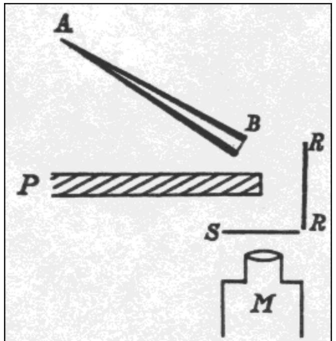


Fig. 1.

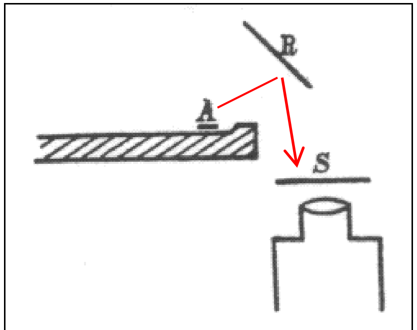
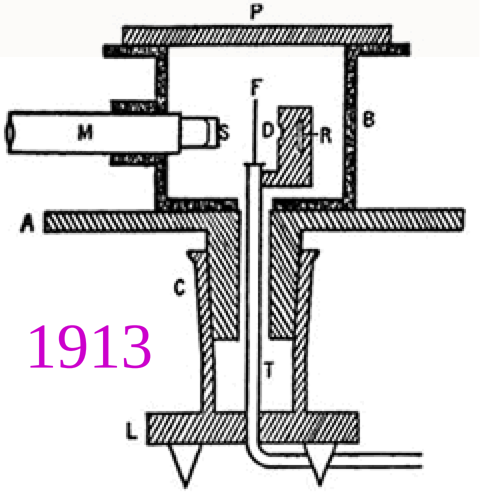
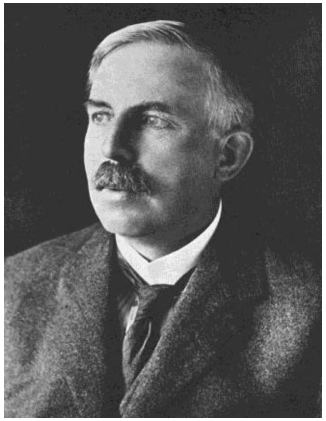


Fig. 3.

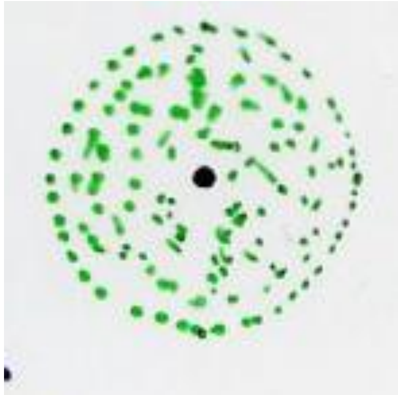
1913



Rutherford Model



Small nuclei inside the atom!



Atom size r ~ 10^{-10} m

Nuclei size r ~ 10^{-15} m

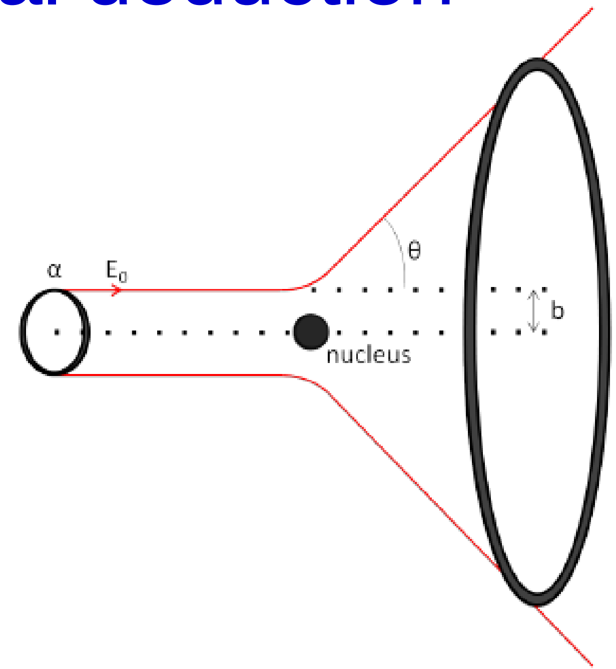
$$N(\theta) \propto \frac{1}{\sin^4(\theta/2)} \frac{Z^2}{E_0^2}$$

Rutherford Formula: Classical deduction

E_0 – beam kinetic energy

b – impact parameter

θ – scattering angle

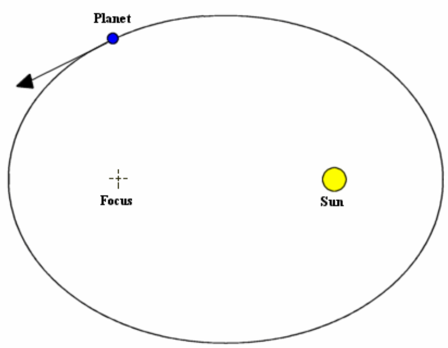


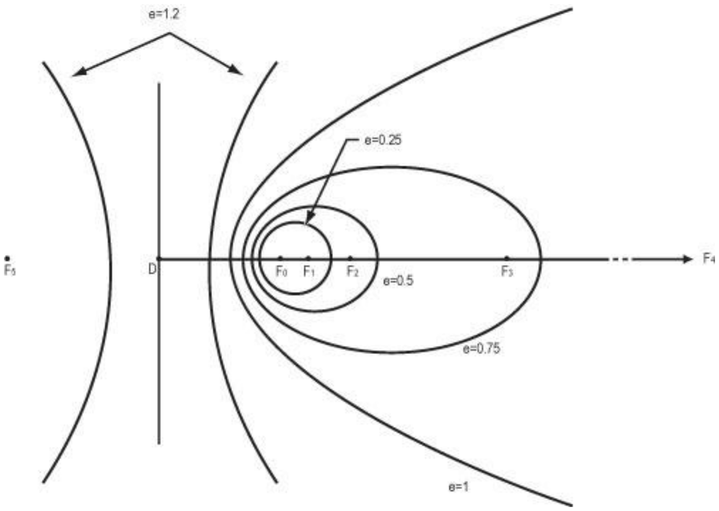
Conservation of Energy and angular momentum determines well defined trajectories (Kepler)

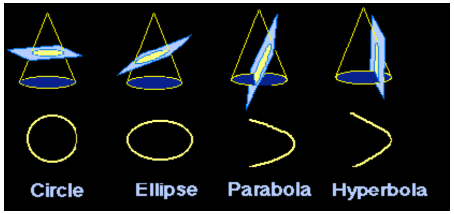
$$b = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q_1Q_2}{2E_0}\cot\left(\frac{\theta}{2}\right) \qquad (2.1)$$

1ª Lei de Kepler

Órbitas elípticas (Sol no foco)







$$\begin{cases} E = \dfrac{1}{2} m\dot{r}^2 + \dfrac{L^2}{2mr^2} - \dfrac{\alpha}{r} \\ L^2 = \left(mr^2 \dfrac{d\theta}{dt}\right)^2 \end{cases}$$

$$\left(\frac{1}{r^2}\frac{dr}{d\theta}\right)^2 = \frac{1}{r^2} + \frac{2m\alpha^2}{L^2 r} + \frac{2mE}{L^2}$$

$$\varepsilon = \sqrt{1 + \frac{2EL^2}{m\alpha^2}} \; ; \; \lambda = \frac{L^2}{m\alpha}\frac{1}{1+\varepsilon}$$

$$r(\theta) = \frac{\lambda(1+\varepsilon)}{1 + \varepsilon\cos(\theta - \theta_0)}$$

	ε	**λ**	**α**	**E**
elipse	[0,1]	> 0	> 0	< 0
parábola	1	> 0	> 0	= 0
hipérbole	> 1	> 0 ou < 0	> 0 ou < 0	> 0

Rutherford Formula: Classical deduction

If the number of beam particles per unit of transverse area n_{beam} is not a function of the transverse coordinates b and ϕ (the beam is uniform and wide with respect to the target size), the differential number of particles as a function of b is:

$$\frac{dN}{db} = 2\pi b \, n_{beam} \,. \tag{2.2}$$

Expressing the differential number of particles as a function of the scattering angle θ:

$$\frac{dN}{d\theta} = \frac{dN}{db}\frac{db}{d\theta} \tag{2.3}$$

we obtain using equation 2.1:

$$\frac{dN}{d\theta} = \pi \left(\frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{2E_0} \right)^2 \frac{\cos\left(\frac{\theta}{2}\right)}{\sin^3\left(\frac{\theta}{2}\right)} n_{beam} \tag{2.4}$$

or, in terms of the solid angle Ω,($d\Omega = 2\pi \sin\theta d\theta$):

$$\frac{dN}{d\Omega} = \left(\frac{1}{4\pi\epsilon_0} \frac{Q_1 Q_2}{4E_0} \right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)} n_{beam}, \tag{2.5}$$

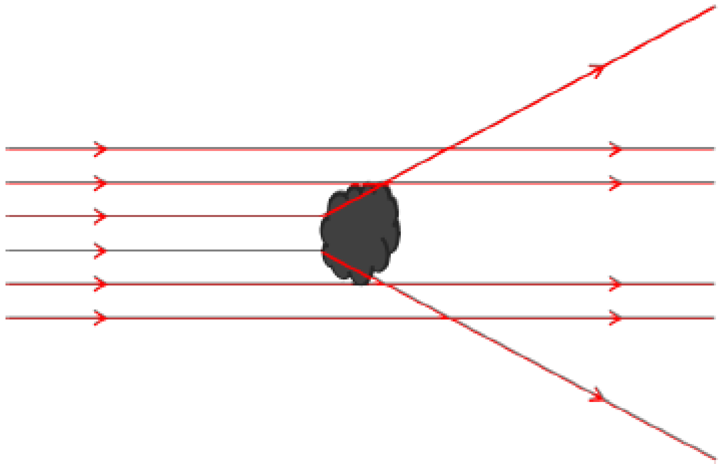
Rutherford Formula!!!

Cross-section (σ)

Interaction of one particle beam with a single object target

$$\sigma_{tot} = \frac{N_{int}}{n_{beam}}$$

$$\sigma_{tot} = \frac{W_{int}}{J} \qquad J = \rho_{beam} v$$



$\sigma \sim$ "target projected area"

barn = 10^{-24} cm^2

mb = 10^{-27} cm^2

μb = 10^{-30} cm^2

Elastic cross-section - σ_{el}

"input particles" = "output particles"

Inelastic cross-section - σ_{in}

"input particles" ≠ "output particles"

Total cross-section - σ_{tot}

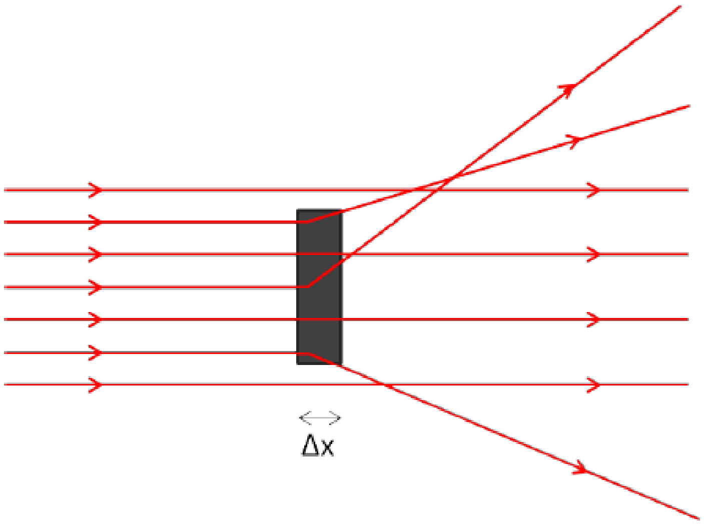
$\sigma_{tot} = \sigma_{el} + \sigma_{in}$

Cross-section (σ)

Interaction of one particle beam with a "composed" target

$$\sigma_{tot} = \frac{W_{int}}{J \cdot N_t}$$

$$N_t = \mathcal{N} \frac{\rho \Delta x}{w_a}$$ (x 1cm^2)



$\mathcal{N}$ – Avogadro number

ρ – Target specific mass

w_a – atomic weight

Luminosity

$\mathcal{L} = J \cdot N_t$

Total number of interactions

$$N_{tot} = \sigma_{tot} \int_T \mathcal{L} dt.$$

Neglecting the beam absorption within the target

Interaction lengths

$$\frac{\Delta N}{N} = \frac{N_{int}}{n_{beam}} = \frac{W_{int}}{J} = N_t\,\sigma = \frac{N_{Avog}\,\rho}{A}\Delta x\;\sigma$$

$$\frac{dN}{dx} = -\frac{\sigma\,N_{Avog}\,\rho}{A}\,N \quad \Rightarrow \quad N = N_0\,e^{-\frac{x}{L_{int}}}$$

$$L_{int} = \frac{A}{\sigma_{inl}\,N_{Avog}\,\rho}\;(cm) \qquad L_{int} = \frac{A}{\sigma_{inl}\,N_{Avog}}\;(g\;cm^{-2})$$

$$L_{coll} = \frac{A}{\sigma_{total}\,N_{Avog}\,\rho}\;(cm) \qquad L_{coll} = \frac{A}{\sigma_{tot}\,N_{Avog}}\;(g\;cm^{-2})$$

L_{int} nuclear (g cm^{-2})

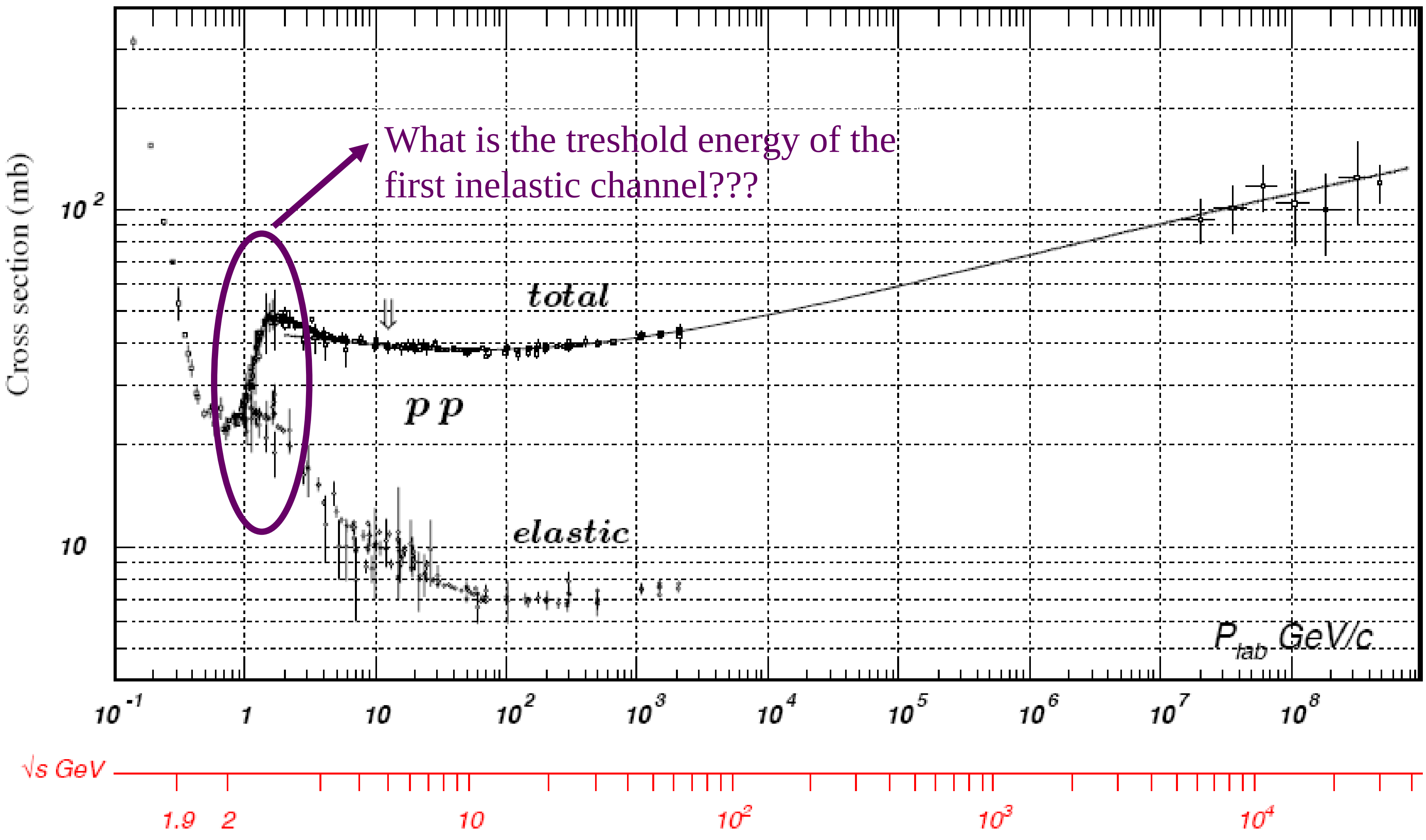
$L_{int}(H_2) = 50.8$
$L_{int}(D_2) = 54.7$
$L_{int}(Be) = 75.2$
$L_{int}(C) = 86.3$
$L_{int}(Xe) = 169$
$L_{int}(Pb) = 194$

L_{coll} nuclear (g cm^{-2})

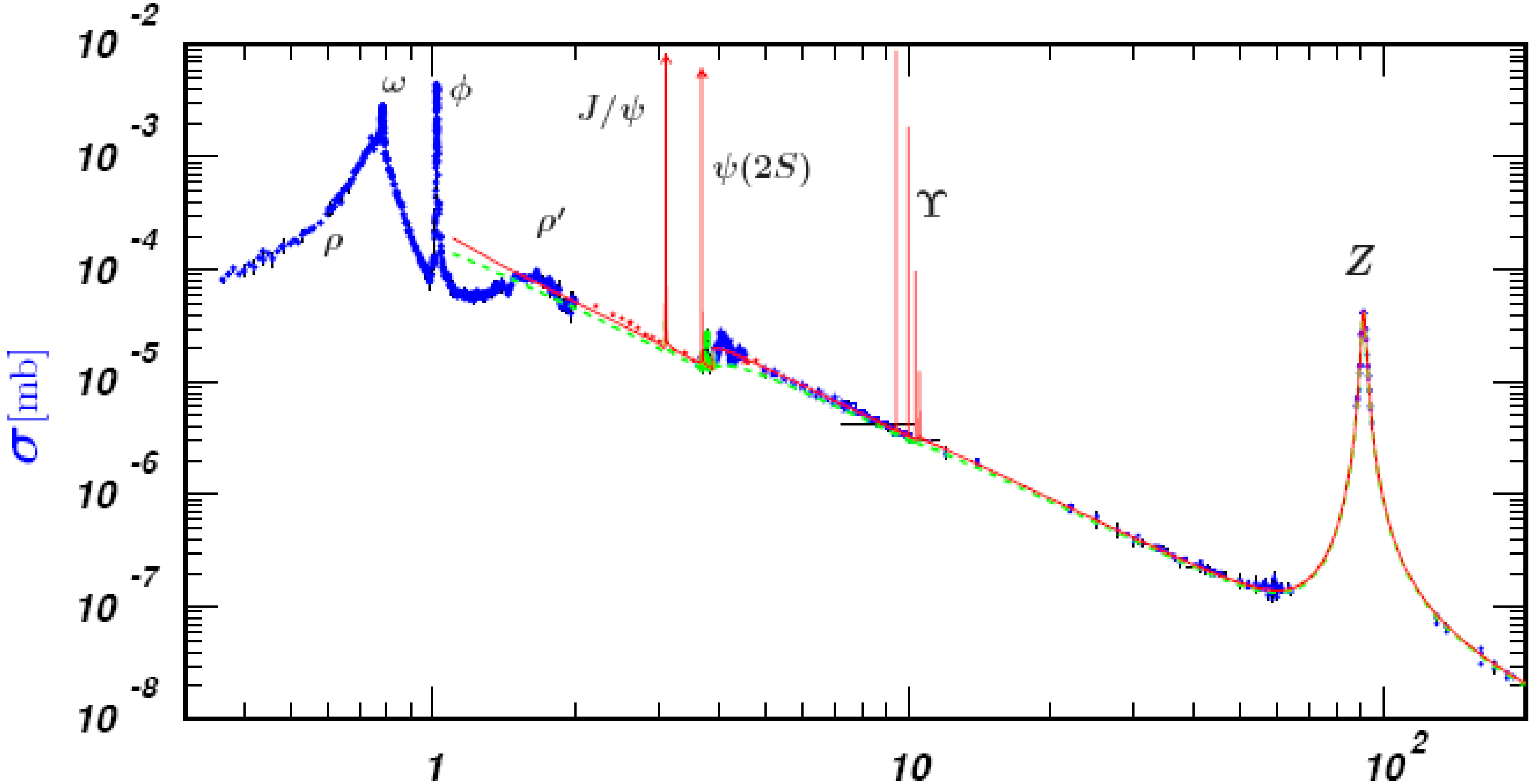
$L_{coll}(H_2) = 43.3$
$L_{col}l(D_2) = 45.7$
$L_{coll}(Be) = 55.8$
$L_{coll}(C) = 60.2$
$L_{coll}(Xe) = 103$
$L_{coll}(Pb) = 116$

How does L_{xx} depends on A?

proton-proton cross-section

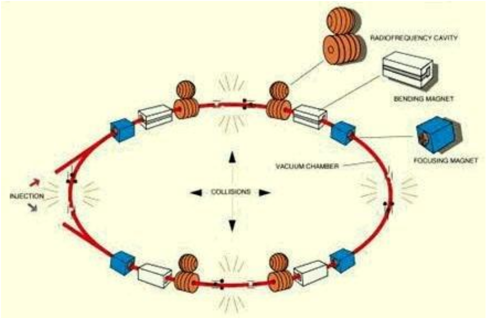


$e^+e^- \rightarrow$ hadrons cross-section

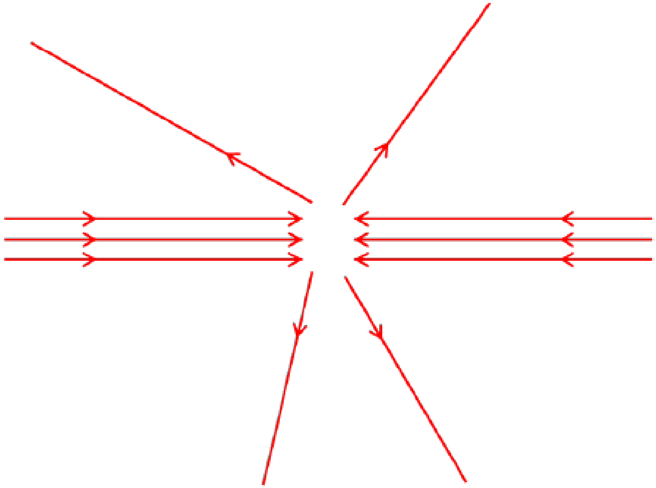


Why $\sigma_{e+e-}(\sqrt{s}) \searrow$ while $\sigma_{pp}(\sqrt{s}) \nearrow$? ? ?

Luminosity in beam-beam collisions (colliders)



$$\mathcal{L} = \frac{N_1 N_2}{A_T} N_b\, f$$



N_1 , N_2 – number of particles in the crossing bunches

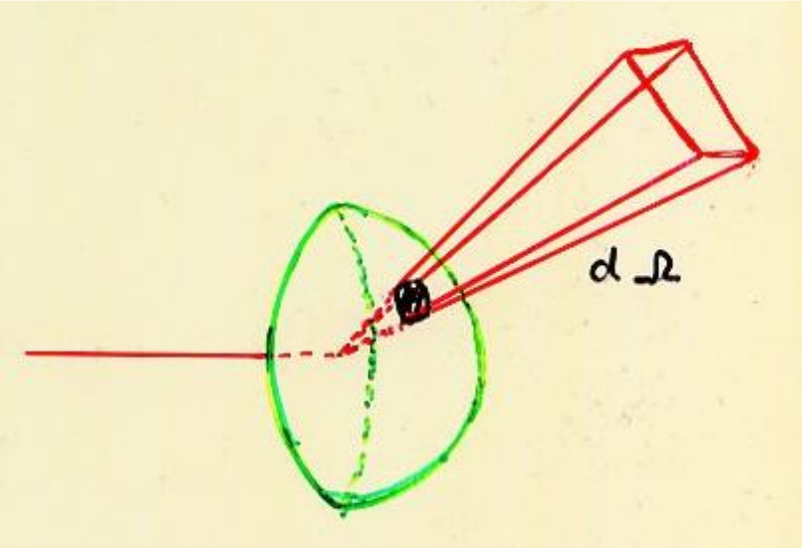
N_b – number of bunches per beam

A_t – intersection transverse area

f – beam revolution frequency

Differential cross-section

$$\frac{d\sigma(\theta,\phi)}{d\Omega} = \frac{1}{\mathcal{L}}\frac{dW_{int}(\theta,\phi)}{d\Omega}$$



$$\sigma_{tot} = \int\int \frac{d\sigma(\theta,\phi)}{d\Omega} d\cos\theta$$

$$d\,\Omega = d\phi\,\sin(\theta)d\theta$$

In an e.m. interaction what is the value of σ_{tot} ???

3. Cross-section fixed target

Consider a fixed target experiment with a monochromatic p beam with a energy of 20 GeV and a 2 m length liquid hydrogen (H_2) target (ρ= 60 kg m^{-3}). In the detector placed just behind the target were measured beam fluxes of 7 10^6 protons/s and 10^7 protons/s respectively with the target full and empty. Determine the proton-proton total cross section at this energy and its statistical error:

a) Without taking into account the attenuation of the beam inside the target
b) Taking into account the attenuation of the beam inside the target

4. LHC collisions

The LHC running parameters in 2012 were: Number of bunches = 1400; Time between bunches = 50 ns; Number protons per bunch = 1.1 10^{11}; Beam width (sigma) at the crossing point = 16 μm

a) Determine the maximum instantaneous Luminosity of the LHC in 2012

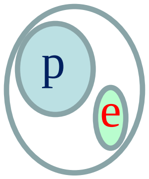
b) Determine the number of interactions per collision (σ_{pp}~100 mb)

c) Determine the maximum number of Higgs bosons decaying into 2 photons ((σ_H~21pb; $BR_{H\gamma\gamma}$= 2,28 x 10^{-3})) which could have been produced in 2012 in the LHC . Compare this number to the real number of detected Higgs in this particular decay mode reported by the LHC collaborations (~400). Discuss the difference knowing that the integrated Luminosity of the LHC (Luminosity integrated over the time) during 2012 was around 20 fb^{-1}.

O neutrão

~ 1920 Rutherford

A/Z ~ 2

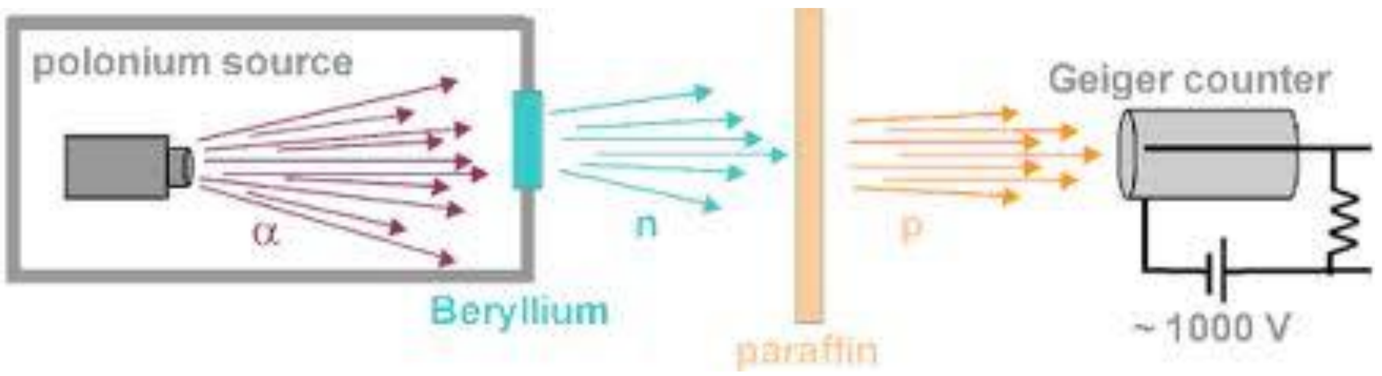


?

~ 1930 Bothe and Becker

Produção de “radiação” neutra penetrante e não ionizante no bombardeamento de Be com partículas α

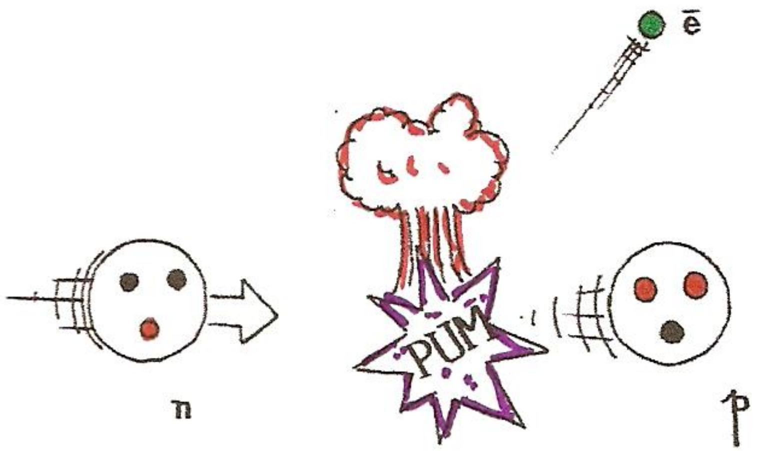
~ 1932 Chadwick



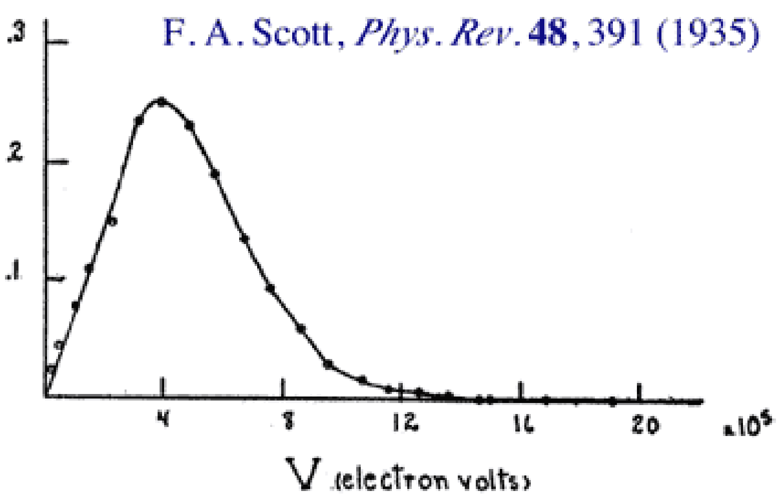
James Chadwick

m_n ~ 938 ± 1.8 MeV (939.57 Mev)

O neutrão decai…

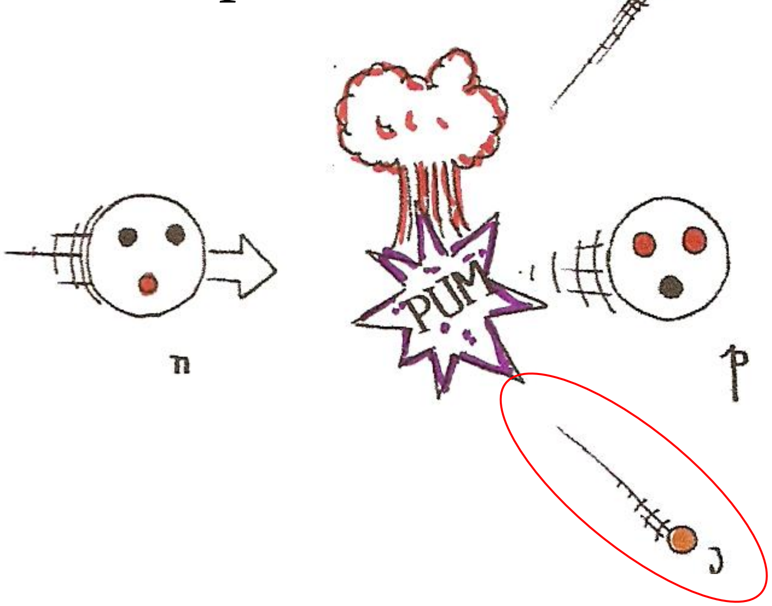


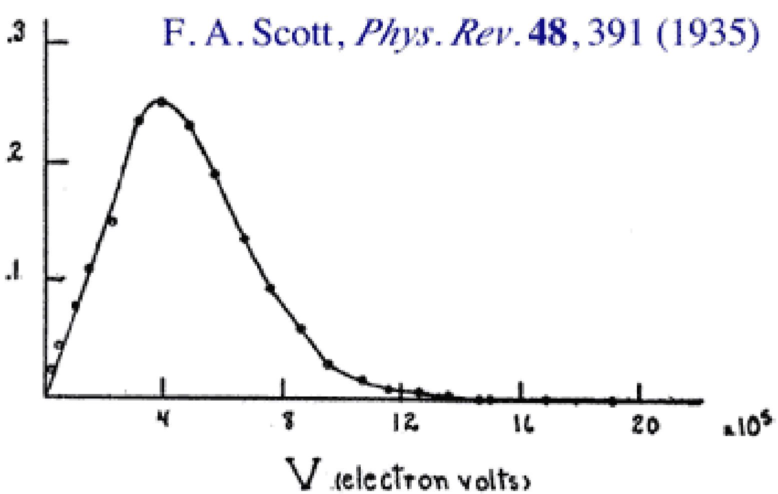
conservação E e $\vec{P}$???



O neutrão decai…

$$n \rightarrow p\, e^- \bar{\nu}$$



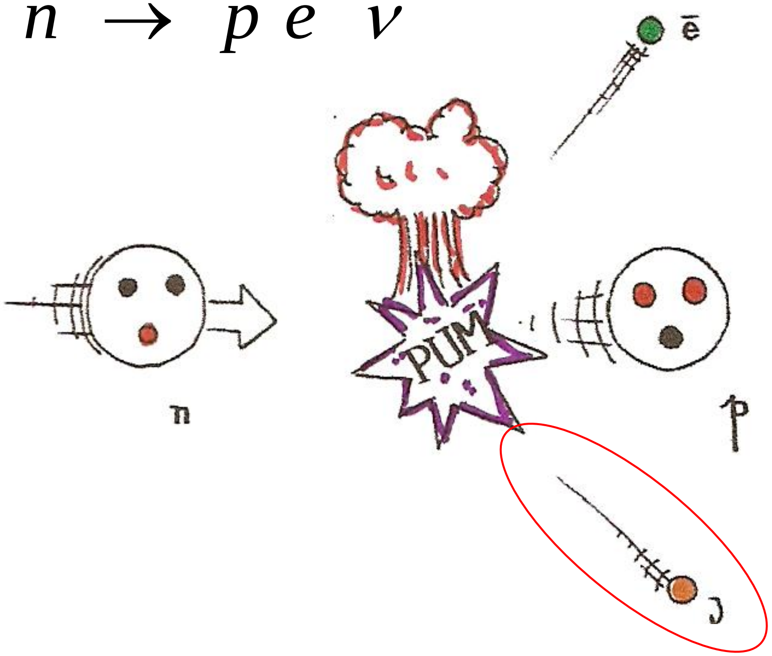


Pauli (1930)

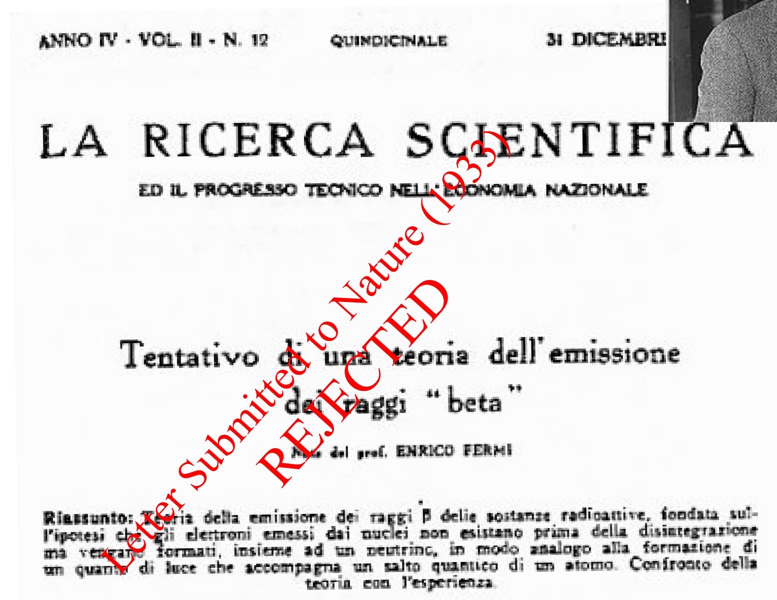
O neutrino !!!

A força fraca

$$n \rightarrow p\ e^{-}\ \bar{\nu}$$



E. Fermi



ANNO IV - VOL. II - N. 12 QUINDICINALE 31 DICEMBRE

LA RICERCA SCIENTIFICA

ED IL PROGRESSO TECNICO NELL'ECONOMIA NAZIONALE

Tentativo di una teoria dell'emissione dei raggi "beta"

Nota del prof. ENRICO FERMI

Riassunto: Teoria della emissione dei raggi β delle sostanze radioattive, fondata sull'ipotesi che gli elettroni emessi dai nuclei non esistano prima della disintegrazione ma vengano formati, insieme ad un neutrino, in modo analogo alla formazione di un quanto di luce che accompagna un salto quantico di un atomo. Confronto della teoria con l'esperienza.

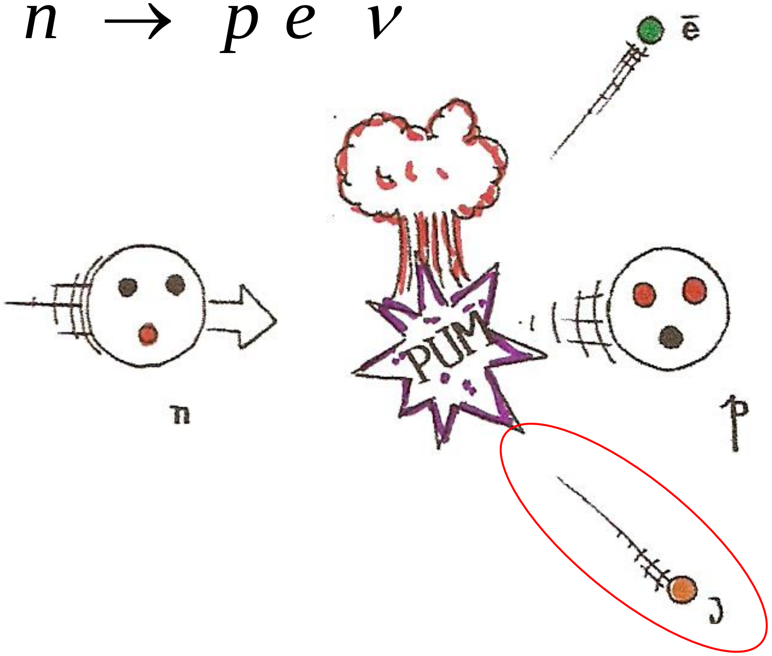
Letter Submitted to Nature (1933)

REJECTED

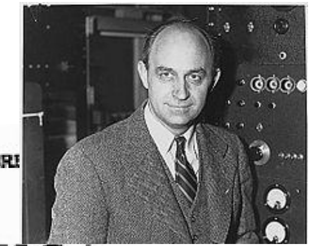
Nuovo Cimento and Zeitschrift fur Physik

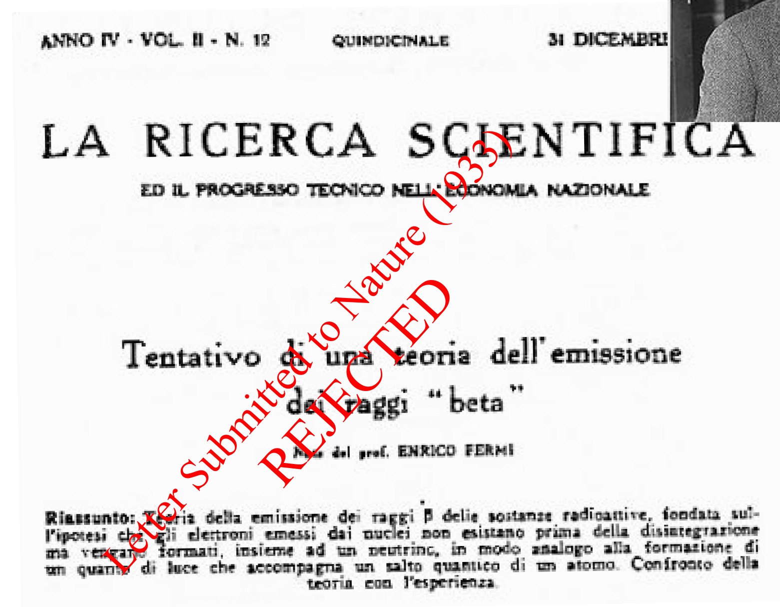
A força fraca

$$n \rightarrow p\ e^{-}\ \overline{\nu}$$



E. Fermi





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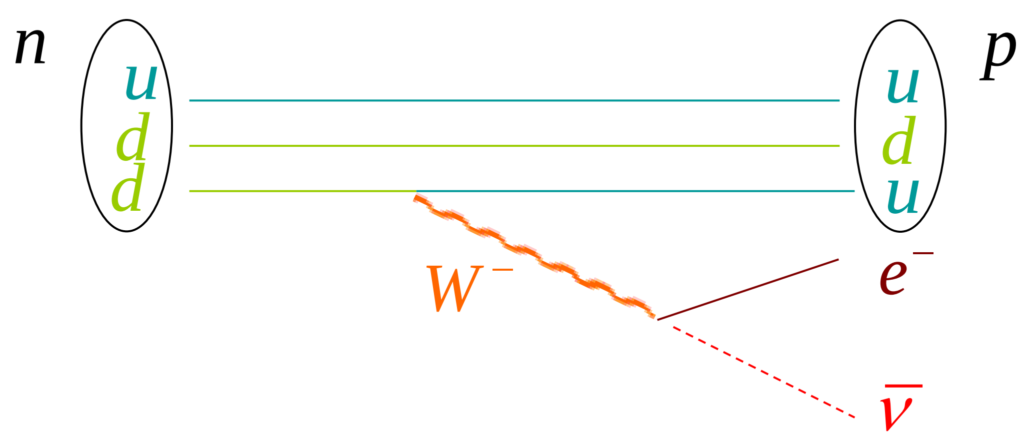
Nuovo Cimento and Zeitschrift fur Physik

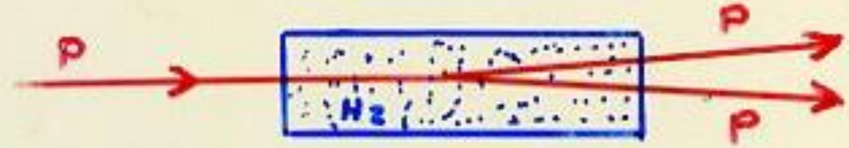
A visão moderna !

Um quark d transforma-se num quark u emitindo um bosão W que "decai" num par electrão, anti-neutrino



Secção eficaz : exemplos (I)

Alvo fixo



Um feixe de 10^6 protões com um momento $p_{lab} = 2$ GeV/c incide num alvo de H_2 ($\rho = 0.063$ g cm^{-3}) com 1 m de comprimento. O número de colisões elásticas observadas é de $7\ 10^4$ ($\varepsilon = 1$!!!). Qual é a secção eficaz elástica pp a 2 GeV/c.

Resolução (I)

$$\sigma_{el} = \frac{N_{int}}{N_{inc} \times \ell_{alvo} \times n^o\,alvos/u.vol.}$$

$$= \frac{N_{int}}{N_{inc} \times \ell_{alvo} \times \frac{N_0\,\rho}{W}}$$

$$= \frac{7\ 10^4}{10^6 \times 100 \times 6\ 10^{23} \times \frac{0.063}{1}}$$

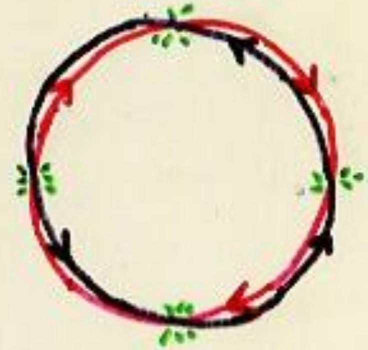
$$\simeq 2\ 10^{-26}\ cm^2$$

$$\simeq 20\ mb$$

A secção eficaz total pp a 2 GeV/c é ~ 40 mb ou seja por cada 100 protões incidentes existem ~14 interacções das quais ~7 elásticas.

A secção eficaz depende da energia! (fig.)

Secção eficaz : exemplos (II)



Anel de colisão

Os valores "nominais" de LEP I
são os seguintes:

Nº partículas por feixe : $N = 1.71 \; 10^{12}$

Nº de "bunches" : $N_b = 4$

Freq. de cruzamento : $f = 10.8$ KH

Dimensão horizontal do feixe: $\sigma_x = 250 \; \mu m$

" vertical do feixe: $\sigma_y = 15 \; \mu m$

92 μs numa volta
23 μs um cruz.

Calcule a luminosidade "nominal" de LEP I e o número de Z_0 hadrónicos ($\sigma_{had} = 30$ nb, $\sqrt{s} = M_Z$) que se obtêm numa hora de funcionamento no pico do Z_0

Resolução (II)

$$\mathcal{L} = \frac{f N^2}{4\pi \sigma_x \sigma_y N_b}$$

$$\simeq 1.7 \; 10^{31} \; cm^{-2} s^{-1}$$

$$N_{Z0} = \sigma \, \mathcal{L} \, t$$

$$= 30 \; 10^{-33} \times 1.7 \; 10^{31} \times 3600$$

$$\simeq 1800$$

1990 max.

$\mathcal{L} \sim 10^{31} cm^{-2} s^{-1}$

total 1990 / Exp.

$\sim$ 150.000

1992

$\sim$ 1.200.000

Main bibliography (2015/2016)

"Modern Particle Physics"
Mark Thomson
Cambridge University Press (2013)

Complementary bibliography

- "O Modelo Standard das Interações Eletrofracas"
  Jorge Romão, http://porthos.ist.utl.pt/ftp/textos/ElectroWeakSM.pdf
- "An introduction to particle and Astroparticle Physics"
  Alessandro De Angelis, Mário Pimenta, Springer (already available!)
- "Introduction to Elementary Particles"
  David Griffiths
  John Wiley and Sons
  (1st edition -1987, 2nd edition 2008)
- "Introduction to Elementary Particle Physics"
  Alessandro Bettini
  Cambridge University Press (2008)
- "Introducção à Teoria de Campo (ITC)"
  Jorge Romão, http://porthos.ist.utl.pt/ftp/textos/itc.pdf

Avaliação da cadeira de Partículas Elementares (2015/2016)

A avaliação consta de duas componentes:
Testes/Exame (75%)
Artigo (25%)

Testes/Exame

Esta componente comporta dois testes e/ou um exame com duas datas. A data do segundo teste coincide com a primeira data do exame. Na segunda data de exame os alunos podem optar por recuperar um dos testes. A nota final será a melhor nota obtida via teste ou via exame. Cada teste tem a duração de 1.5 horas. O exame tem a duração de 3h. Será autorizada a consulta de um formulário com uma folha A4 e do PDG. O primeiro teste será **no sábado 24/10.**

Artigo

Esta componente comporta a realização (individual ou num grupo de dois alunos) de um pequeno artigo (4 paginas) com o formato de um “proceeding” de conferencia em que se discuta teórica e experimentalmente uma observável em física de partículas e/ou astropartículas. Sugestões de temas serão dadas até 9/10, os temas serão escolhidos até 9/11 e o artigo entregue até 17/12.