Apêndice A

How to do Calculations in Particle Physics

A.1 Introduction

In particle physics most of the experimental results are either cross sections or decay widths. So it is very important do learn how to go from theory (the Lagrangian) to these quantities to be able to compare with the experiment.

In this course we have learned how to do these calculations in some cases. As these are dispersed in various chapters it might be useful to collect here all the information. We follow the conventions, methods and notation of Chapters 2, 4, 7 and 9 and of the recommended book for this course by Mark Thomson [5].

A.2 Fermi Golden Rule

The decay rates and cross sections are given in Eq. (2.12) for the decays

$$\Gamma = \frac{1}{2m_1}\, S \int |\mathcal{M}|^2\, (2\pi)^4 \delta^4(p_1 - \sum_{i=2}^{n} p_i) \prod_{j=2}^{n} \frac{d^3 p_j}{(2\pi)^3 2p_j^0} \tag{A.1}$$

and in Eq. (2.27) for the cross section

$$\sigma = \frac{1}{4\sqrt{(p_1\cdot p_2)^2 - m_1^2 m_2^2}}\, S \int |\mathcal{M}|^2\, (2\pi)^4 \delta^4(p_1 + p_2 - \sum_{i=3}^{n} p_i) \prod_{j=2}^{n} \frac{d^3 p_j}{(2\pi)^3 2p_j^0} \tag{A.2}$$

A.3 The CM Reference Frame

In this course we have considered only decays $1 \to 2 + 3$ in the rest frame of the decaying particle and cross sections for processes of the type

$$1 + 2 \to 3 + 4 \tag{A.3}$$

in the CM frame, that is with the kinematics,

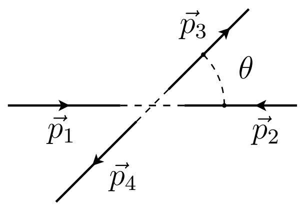


Figura A.1: CM kinematics

In this case Eqs. (A.1) and (A.2) simplify and we get (see Eq. (2.25) and Eq. (2.39)),

$$\Gamma = \frac{S}{8\pi m_1^2} \, |\vec{p}_2| \; |\mathcal{M}|^2 \tag{A.4}$$

and

$$\frac{d\sigma}{d\Omega} = \frac{S}{64\pi^2 s} \frac{|\vec{p}_3|}{|\vec{p}_1|} |\mathcal{M}|^2 \tag{A.5}$$

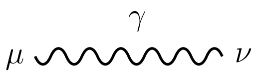
where $|\vec{p}_2|$, $|\vec{p}_1|$ $|\vec{p}_3|$ can be easily obtained in the CM frame, see for instance, Eq. (2.47). In these equations the factor S is a symmetry factor for identical particles in the final state. For instance, for the decay $A \to B + B$ it would be

$$S = \frac{1}{2!} \tag{A.6}$$

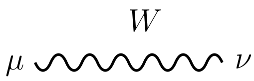
A.4 Feynman Rules

These are the rules to write the invariant amplitude $\mathcal{M}$. We will not repeat them here, they were given for the model ABC with scalars fields in Sec. 2.5 and for a model with spin 1/2 particles, like in QED, in Sec. 4.4. We will give here only the Feynman rules for the propagators and vertices of the standard model that we will use in our calculations. A complete description of the Feynman rules for the standard model can be found in Romão and Silva [35].

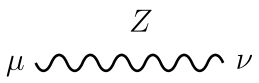
A.4.1 Propagators



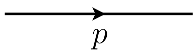
$$-i\frac{g_{\mu\nu}}{k^2} \tag{A.7}$$



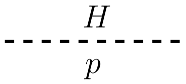
$$-i\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2}}{k^2 - M_W^2} \tag{A.8}$$



$$-i\frac{g_{\mu\nu}-\frac{k_\mu k_\nu}{M_Z^2}}{k^2-M_Z^2} \tag{A.9}$$



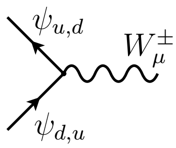
$$\frac{i(\not{p}+m_f)}{p^2-m_f^2} \tag{A.10}$$



$$\frac{i}{p^2-M_H^2} \tag{A.11}$$

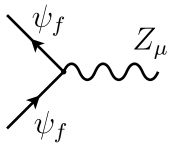
A.4.2 Vertices

Charged Current

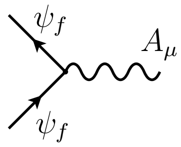


$$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1-\gamma_5}{2} \tag{A.12}$$

Neutral Current



$$-i\frac{g}{\cos\theta_W}\gamma_\mu\left(g_V^f-g_A^f\gamma_5\right)$$

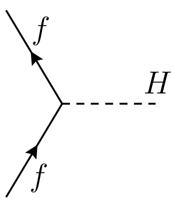


$$-ieQ_f\gamma_\mu \tag{A.13}$$

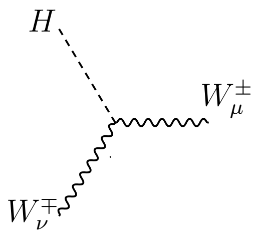
where

$$g_V^f=\frac{1}{2}T_f^3-Q_f\sin^2\theta_W,\quad g_A^f=\frac{1}{2}T_f^3\ . \tag{A.14}$$

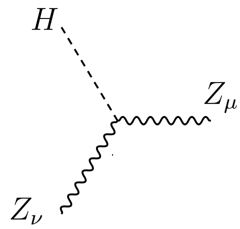
Higgs Interactions



$$-i\,\frac{g}{2}\,\frac{m_f}{M_W} \tag{A.15}$$



$$ig\,M_W\,g_{\mu\nu} \tag{A.16}$$

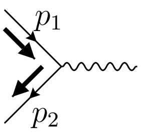


$$i\,\frac{g}{\cos\theta_W}\,M_Z\,g_{\mu\nu} \tag{A.17}$$

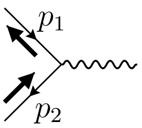
A.5 Results for the Helicity Currents

When we have particles with spin we have to sum over all the polarizations in the final state and take the average in the initial sate (for unpolarized beams). The general procedure to do this is known as the Casimir's trick and leads to the calculation of traces of the Dirac γ matrices. This lies beyond the scope of this course[1]. Here we just discuss a particular case when all fermions (spin 1/2) are massless. In this case we have shown in Sec. 4.6 that we can use helicity amplitudes to evaluate the fermionic currents. We summarize here the results. In the expressions below, θ is the angle between particle 3 and 1 in the CM as given in Fig. A.1.

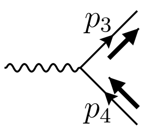
A.5.1 s-channel



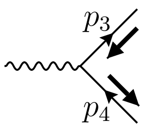
$$J_{u_1v_2}(\uparrow,\downarrow) = \sqrt{s}\ (0,-1,-i,0) \tag{A.18}$$



$$J_{u_1v_2}(\downarrow,\uparrow) = \sqrt{s}\ (0,-1,i,0) \tag{A.19}$$



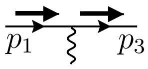
$$J_{u_3v_4}(\uparrow,\downarrow) = \sqrt{s}\ (0,-\cos\theta,i,\sin\theta) \tag{A.20}$$



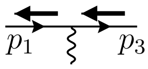
$$J_{u_3v_4}(\downarrow,\uparrow) = \sqrt{s}\ (0,-\cos\theta,-i,\sin\theta) \tag{A.21}$$

[1] For a complete discussion of this general case see Ref. [2].

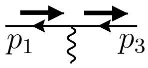
A.5.2 t-channel



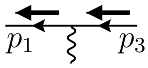
$$J_{u_1u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}, i\sin\frac{\theta}{2}, \cos\frac{\theta}{2}\right) \tag{A.22}$$



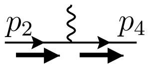
$$J_{u_1u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, \cos\frac{\theta}{2}\right) \tag{A.23}$$



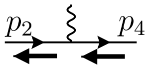
$$J_{v_1v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}, i\sin\frac{\theta}{2}, \cos\frac{\theta}{2}\right) \tag{A.24}$$



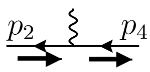
$$J_{v_1v_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2}, \sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, \cos\frac{\theta}{2}\right) \tag{A.25}$$



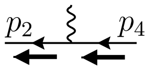
$$J_{u_2u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, i\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}\right) \tag{A.26}$$



$$J_{u_2u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}\right) \tag{A.27}$$



$$J_{v_2v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, i\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}\right) \tag{A.28}$$



$$J_{v_2v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}, -i\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}\right) \tag{A.29}$$

A.5.3 u-channel

$$J_{u_1u_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}, -i\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\right) \tag{A.30}$$

$$J_{u_1u_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(-\sin\frac{\theta}{2}, \cos\frac{\theta}{2}, -i\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}\right) \tag{A.31}$$

$$J_{u_2u_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(-\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}, i\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\right) \tag{A.32}$$

$$J_{u_2u_3}(\downarrow,\downarrow) = \sqrt{s}\,\left(\sin\frac{\theta}{2}, \cos\frac{\theta}{2}, i\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}\right) \tag{A.33}$$

$$J_{v_1v_4}(\uparrow,\uparrow) = \sqrt{s}\,\left(-\sin\frac{\theta}{2}, \cos\frac{\theta}{2}, i\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}\right) \tag{A.34}$$

$$J_{v_1v_4}(\downarrow,\downarrow) = \sqrt{s}\,\left(\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}, i\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\right) \tag{A.35}$$

$$J_{v_2v_3}(\uparrow,\uparrow) = \sqrt{s}\,\left(\sin\frac{\theta}{2}, \cos\frac{\theta}{2}, -i\cos\frac{\theta}{2}, -\sin\frac{\theta}{2}\right) \tag{A.36}$$

$$J_{v_2 v_3}(\downarrow,\downarrow) = \sqrt{s}\ \left(-\sin\frac{\theta}{2}, -\cos\frac{\theta}{2}, -i\cos\frac{\theta}{2}, \sin\frac{\theta}{2}\right) \tag{A.37}$$

A.6 Simple Examples

In this section we collect all the calculations that we did in Chapters 4, 7 and 9. We will not repeat the discussion just the calculations.

A.6.1 $e^- + e^+ \to \mu^- + \mu^+$ in QED

This process was studied in Sec. 4.6. For low energy ($\sqrt{s} \ll M_Z$) we can neglect the diagram with a Z boson and then we have only one Feynman diagram, shown in Fig. A.2. The amplitude for this process is

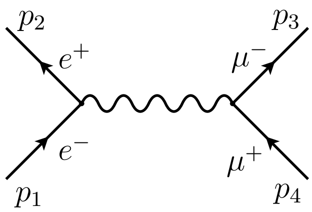


Figura A.2: $e^- + e^+ \to \mu^- + \mu^+$ scattering in QED.

$$\mathcal{M} = i\,\overline{v}(p_2)(ie\gamma^\mu)u(p_1)\ \frac{-i\ g_{\mu\nu}}{(p_1+p_2)^2}\ \overline{u}(p_3)(ie\gamma^\nu)v(p_4) \tag{A.38}$$

$$= -\frac{e^2}{s}\overline{v}(p_2)\gamma^\mu u(p_1)\ \overline{u}(p_3)\gamma_\mu v(p_4)\ . \tag{A.39}$$

Due to the chirality properties of the QED interaction, instead of sixteen possible spin combinations we have only four non- zero currents, two for the initial state and two for the final state. These were already given in Eqs. (A.18)-(A.21) (and also em Sec. 4.6). Therefore we get,

$$\begin{aligned}\mathcal{M}(\uparrow\downarrow;\uparrow\downarrow) =& -\frac{e^2}{s}J_{u_1 v_2}(\downarrow,\uparrow)\cdot J_{u_3 v_4}(\uparrow,\downarrow)\\ =& -\frac{e^2}{s}\left[\sqrt{s}(0,-1,-i,0)\right]\cdot\left[\sqrt{s}(0,-\cos\theta,i,\sin\theta)\right]\\ =& \frac{e^2}{s}s\ (1+\cos\theta) \equiv 4\pi\alpha\,(1+\cos\theta)\end{aligned} \tag{A.40}$$

Similarly

$$|\mathcal{M}(\uparrow\downarrow;\uparrow\downarrow)|^2 = |\mathcal{M}(\downarrow\uparrow;\downarrow\uparrow)|^2 = (4\pi\alpha)^2\,(1+\cos\theta)^2 \tag{A.41}$$

$$|\mathcal{M}(\uparrow\downarrow;\downarrow\uparrow)|^2 = |\mathcal{M}(\downarrow\uparrow;\uparrow\downarrow)|^2 = (4\pi\alpha)^2\,(1-\cos\theta)^2 \tag{A.42}$$

and

$$\left\langle|\mathcal{M}_{fi}|^2\right\rangle = \frac{1}{4}\,(4\pi\alpha)^2\left[2(1+\cos\theta)^2+2(1-\cos\theta)^2\right] \tag{A.43}$$

$$= (4\pi\alpha)^2\,(1+\cos^2\theta) \tag{A.44}$$

Finally for the cross section, using Eq. (A.5), we get

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s}\left\langle|\mathcal{M}|^2\right\rangle = \frac{\alpha^2}{4s}(1+\cos^2\theta) \tag{A.45}$$

and the total cross section is obtained after integration in the angles to give,

$$\sigma = \frac{4\pi\alpha^2}{3s} \tag{A.46}$$

A.6.2 Bhabha scattering

The process $e^- + e^+ \to e^- + e^+$ is known as Bhabha scattering. For this process we have, in QED, the two diagrams of Fig. A.3 where there is relative a minus sign

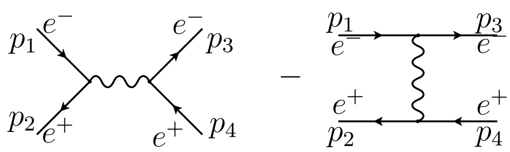
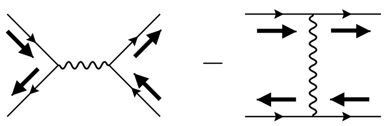
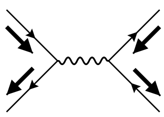
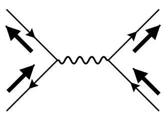


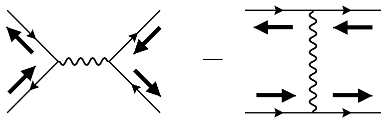
Figura A.3: Diagrams for Bhabha

between the two diagrams (rule 10 in the Feynman rules for QED, Sec. 4.4). We have only six possible helicity combinations shown below,

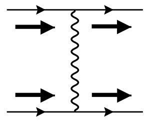
$\mathcal{M}(\uparrow,\downarrow;\uparrow,\downarrow) =$  (A.47)

$\mathcal{M}(\uparrow,\downarrow;\downarrow,\uparrow) =$  (A.48)

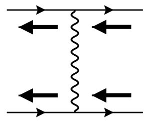
$\mathcal{M}(\downarrow,\uparrow;\uparrow,\downarrow) =$  (A.49)

$\mathcal{M}(\downarrow,\uparrow;\downarrow,\uparrow) =$  (A.50)

$$\mathcal{M}(\uparrow,\uparrow;\uparrow,\uparrow) = \tag{A.51}$$



$$\mathcal{M}(\downarrow,\downarrow;\downarrow,\downarrow) = \tag{A.52}$$



The general amplitude for Bhabha scattering can then be written in the form

$$\mathcal{M}(h_1,h_2;h_3,h_4) = -\frac{e^2}{s} J_{u_1 v_2}(h_1,h_2)\cdot J_{u_3 v_4}(h_3,h_4) + \frac{e^2}{t} J_{u_1 u3}(h_1,h_3)\cdot J_{v_2 v_4}(h_2,h_4) \tag{A.53}$$

Using Eqs. (A.22) and (A.22) and summing the six non-zero helicity amplitudes we get finally

$$\left\langle |\mathcal{M}|^2 \right\rangle = 2e^4 \left[\frac{t^2+(s+t)^2}{s^2} + \frac{s^2+(s+t)^2}{t^2} + 2\frac{(s+t)^2}{st} \right] \tag{A.54}$$

$$= 2e^4 \left[\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)} - \frac{2\cos^4(\theta/2)}{\sin^2(\theta/2)} + \frac{1+\cos^2\theta}{2} \right] \tag{A.55}$$

where

$$t = -\frac{s}{2}(1+\cos\theta) = -s\cos^2\frac{\theta}{2}, \quad u = -\frac{s}{2}(1-\cos\theta) = -s\sin^2\frac{\theta}{2} \tag{A.56}$$

Using Eq. (A.5), we get for the differential cross section,

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s} \left[\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)} - \frac{2\cos^4(\theta/2)}{\sin^2(\theta/2)} + \frac{1+\cos^2\theta}{2} \right] . \tag{A.57}$$

A.6.3 Decay $Z \to f\overline{f}$

Consider now the decay of $Z \to f\overline{f}$. The Feynman diagram is given in Fig. A.4.

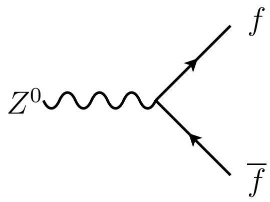


Figura A.4: Z decay into $\overline{f}$.

Applying the Feynman rules we get for the amplitude

$$\mathcal{M} = g_Z\, \epsilon_\mu(k,\lambda)\; \overline{u}(p_3)\gamma^\mu \left(g_V^f - g_A^f \gamma_5 \right) v(p_4) \tag{A.58}$$

where we have defined the shorthand notation,

$$g_Z = \frac{g}{\cos\theta_W} \ . \tag{A.59}$$

In order to simplify matters and also because it is a very good approximation, ($M_Z \gg m_f$), we will neglect all the fermion masses, and use the techniques of the helicity amplitudes explained in Chapter 4. The Z boson is a spin 1 particle with mass, and therefore has three polarizations. In the rest frame of the Z the polarization vectors for these three cases can be written as

$$\begin{aligned}
\epsilon_+^\mu =& -\frac{1}{\sqrt{2}}(0,1,i,0), && S_z=+1,\ h=+1 \\
\epsilon_-^\mu =& \frac{1}{\sqrt{2}}(0,1,-i,0), && S_z=-1,\ h=-1 \\
\epsilon_L^\mu =& (0,0,0,1), && S_z=0,\ h=0
\end{aligned} \tag{A.60}$$

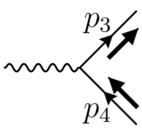
On the other hand we can write

$$\begin{aligned}
g_V^f - g_A^f\gamma_5 =& \left(g_V^f - g_A^f\gamma_5\right)(P_L+P_R) \\
=& (g_V^f+g_A^f)P_L + (g_V^f-g_A^f)P_R \equiv g_L^f P_L + g_R^f P_R
\end{aligned} \tag{A.61}$$

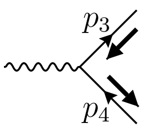
with

$$g_L^f \equiv g_V^f + g_A^f, \quad g_R^f \equiv g_V^f - g_A^f \tag{A.62}$$

As in the massless limit chirality equals helicity, this means that we can have only two possible helicity combinations,



$$J_{u_3v_4}(\uparrow,\downarrow) = \sqrt{s}\ (0,-\cos\theta,i,\sin\theta) \tag{A.63}$$



$$J_{u_3v_4}(\downarrow,\uparrow) = \sqrt{s}\ (0,-\cos\theta,-i,\sin\theta) \tag{A.64}$$

We therefore obtain ($\sqrt{s}=M_Z$)

$$\mathcal{M}(+;\uparrow,\downarrow) = g_Z\, g_R^f\ \epsilon_+\cdot J_{u_3v_4}(\uparrow,\downarrow) = g_Z\, g_R^f\ M_Z\frac{1}{\sqrt{2}}(1+\cos\theta) \tag{A.65}$$

$$\mathcal{M}(-;\uparrow,\downarrow) = g_Z\, g_R^f\ \epsilon_-\cdot J_{u_3v_4}(\uparrow,\downarrow) = g_Z\, g_R^f\ M_Z\frac{1}{\sqrt{2}}(1-\cos\theta) \tag{A.66}$$

$$\mathcal{M}(L;\uparrow,\downarrow) = g_Z\, g_R^f\ \epsilon_L\cdot J_{u_3v_4}(\uparrow,\downarrow) = g_Z\, g_R^f\ M_Z\sin\theta \tag{A.67}$$

$$\mathcal{M}(+;\downarrow,\uparrow) = g_Z\, g_L^f\ \epsilon_+ \cdot J_{u_3 v_4}(\downarrow,\uparrow) = -g_Z\, g_L^f\ M_Z \frac{1}{\sqrt{2}}(1-\cos\theta) \tag{A.68}$$

$$\mathcal{M}(-;\downarrow,\uparrow) = g_Z\, g_L^f\ \epsilon_- \cdot J_{u_3 v_4}(\downarrow,\uparrow) = -g_Z\, g_L^f\ M_Z \frac{1}{\sqrt{2}}(1+\cos\theta) \tag{A.69}$$

$$\mathcal{M}(L;\downarrow,\uparrow) = g_Z\, g_L^f\ \epsilon_L \cdot J_{u_3 v_4}(\downarrow,\uparrow) = g_Z\, g_L^f\ M_Z \sin\theta \tag{A.70}$$

Therefore we get,

$$\langle|\mathcal{M}|^2\rangle = \frac{1}{3}\sum_{\text{spins}}|\mathcal{M}|^2 \tag{A.71}$$

$$\begin{aligned}
&= \frac{1}{3}\left[|\mathcal{M}(+;\uparrow,\downarrow)|^2 + |\mathcal{M}(-;\uparrow,\downarrow)|^2 + |\mathcal{M}(L;\uparrow,\downarrow)|^2\right.\\
&\quad \left.+ |\mathcal{M}(+;\downarrow,\uparrow)|^2 + |\mathcal{M}(-;\downarrow,\uparrow)|^2 + |\mathcal{M}(L;\downarrow,\uparrow)|^2\right]\\
&= \frac{2}{3} g_Z^2 \left(g_R^{f\,2} + g_L^{f\,2}\right)\\
&= \frac{4}{3}\left(\frac{g}{\cos\theta_W}\right)^2 M_Z^2 \left[g_V^{f\,2} + g_A^{f\,2}\right]
\end{aligned} \tag{A.72}$$

For the total width we get

$$\Gamma = \frac{M_Z}{12\pi}\left(\frac{g}{\cos\theta_W}\right)^2 \left[g_V^{f\,2} + g_A^{f\,2}\right] \tag{A.73}$$

This result is normally presented in terms of the Fermi constant,

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \left(\frac{g}{\cos\theta_W}\right)^2 \frac{1}{8M_Z^2} \tag{A.74}$$

where we have used the standard model relation for the W and Z masses,

$$M_W = M_Z \cos\theta_W \tag{A.75}$$

Therefore we get

$$\Gamma = \frac{2G_F M_Z^3}{3\sqrt{2}\pi}\left[g_V^{f\,2} + g_A^{f\,2}\right]\ . \tag{A.76}$$

A.6.4 Scattering $e^-\overline{\nu}_e \to \mu^-\overline{\nu}_\mu$

As another example we consider the $e^-\overline{\nu}_e \to \mu^-\overline{\nu}_\mu$ scattering in the CM. In lowest order in perturbation theory we have the Feynman diagram of Fig. A.5. The amplitude is given by,

$$\mathcal{M} = i\left(\frac{ig}{\sqrt{2}}\right)^2 \overline{v}(p_2)\gamma^\mu \frac{1-\gamma_5}{2} u(p_1) \frac{-ig_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2 + iM_W\Gamma_W} \overline{u}(p_3)\gamma^\nu \frac{1-\gamma_5}{2} v(p_4) \tag{A.77}$$

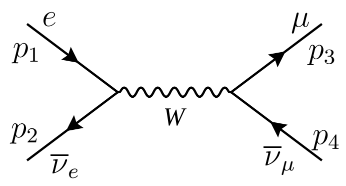


Figura A.5: scattering $e^-\overline{\nu}_e \to \mu^-\overline{\nu}_\mu$.

where $q = p_1 + p_2$ and Γ_W is the day width of the W. Using the fact that we are neglecting the fermion masses the term in the numerator of the W boson propagator proportional to the momenta vanishes after application of the Dirac equation, see Sec. 9.6.2. Making use of the relation $G_F/\sqrt{2} = g^2/8M_W^2$, we further simplify the expression

$$\mathcal{M} = -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{s - M_W^2 + iM_W\Gamma_W}\overline{v}(p_2)\gamma^\mu P_L u(p_1)\overline{u}(p_3)\gamma^\mu P_L v(p_4)\ . \tag{A.78}$$

From the structure of Eq. (A.78) we immediately see that the only non-zero helicities are those shown in Fig. A.6 Therefore we get only one helicity combination,

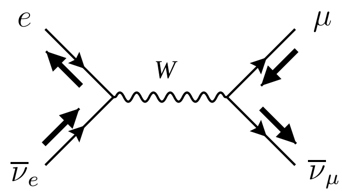


Figura A.6: Helicities for $e^-\overline{\nu}_e \to \mu^-\overline{\nu}_\mu$.

$$\begin{aligned}
\mathcal{M}(\downarrow,\uparrow;\downarrow,\uparrow) =& -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{s - M_W^2 + iM_W\Gamma_W}J_{u_1v_2}(\downarrow,\uparrow)\cdot J_{u_3v_4}(\downarrow,\uparrow)\\
=& -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{s - M_W^2 + iM_W\Gamma_W}\sqrt{s}(0,-1,i,0)\cdot\sqrt{s}(0,-\cos\theta,-i,\sin\theta)\\
=& -\frac{4G_F}{\sqrt{2}}\frac{M_W^2}{s - M_W^2 + iM_W\Gamma_W}s\,(1+\cos\theta)\ .
\end{aligned} \tag{A.79}$$

Now we obtain

$$\begin{aligned}
\langle|\mathcal{M}|^2\rangle =& \frac{1}{2}|\mathcal{M}(\downarrow,\uparrow;\downarrow,\uparrow)|^2\\
=& 4G_F^2\frac{M_W^4}{(s-M_W^2)^2 + M_W^2\Gamma_W^2}\ s^2(1+\cos\theta)^2
\end{aligned} \tag{A.80}$$

We get therefore for the differential cross section in the CM frame

$$\frac{d\sigma}{d\Omega} = \frac{G_F^2 s}{16\pi^2} \frac{M_W^4}{(s - M_W^2)^2 + M_W^2 \Gamma_W^2} (1 + \cos\theta)^2 \tag{A.81}$$

After integration over the angles we get finally,

$$\sigma = \frac{1}{3} \frac{G_F^2 s}{\pi} \frac{M_W^4}{(s - M_W^2)^2 + M_W^2 \Gamma_W^2} \tag{A.82}$$

In Sec. 9.6.2 we discussed the low and high energy limit of this result.

A.6.5 Scattering $\mu^- \overline{\nu}_\mu \to e^- \overline{\nu}_e$

This process was considered in Sec. 7.5.2 in the context of the current-current V-A theory. The process is described by the Feynman diagram of Fig. A.7 to which

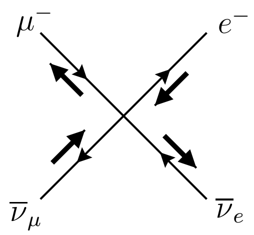


Figura A.7: Diagram para $\mu^- + \overline{\nu}_\mu \to e^- + \overline{\nu}_e$.

corresponds the amplitude

$$\mathcal{M} = \frac{4G_F}{\sqrt{2}} \, \overline{v}(p_2)\gamma^\mu P_L u(p_1) \, \overline{u}(p_3)\gamma_\mu P_L v(p_4) \tag{A.83}$$

This is exactly equal to Eq. (A.78) in the limit $\sqrt{s} \ll M_W$. Therefore the result for the cross section will be the same in the same limit.