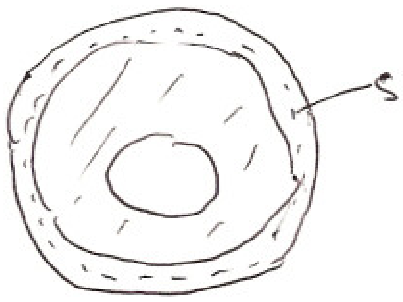
1º teste TEO – Versão B (3/11/2006)

a) Usando a lei de Gauss para uma superfície S



dentro do condutor externo e sabendo que o campo no condutor é nulo vem

$$\oint_S (\vec{D}\cdot\vec{n})\,dS = 0 \Rightarrow 2Q + Q_{int} = 0$$

logo

$$\boxed{Q_{int} = -2Q}$$

Como

$$-Q = Q_{int} + Q_{ext} \Rightarrow Q_{ext} = -Q - Q_{int} = Q$$

logo

$$\boxed{Q_{ext} = +Q}$$

b) $0<r<r_1$: $\vec{E} = \vec{D} = \vec{P} = 0$ campo nulo dentro do condutor

$r_1<r<r_2$:

$$\oint_S (\vec{D}\cdot\vec{n})\,dS = 2Q$$

$$|\vec{D}|4\pi r^2 = 2Q \Rightarrow \boxed{\vec{D} = \frac{2Q}{4\pi}\frac{1}{r^2}\vec{e}_r}$$

Logo $\boxed{\vec{E} = \frac{2Q}{4\pi\varepsilon}\frac{1}{r^2}\vec{e}_r}$ e $\boxed{\vec{P} = \frac{\varepsilon-\varepsilon_0}{\varepsilon}\frac{2Q}{4\pi}\frac{1}{r^2}\vec{e}_r}$

$r_2 < r < r_3$ Condutor $\Rightarrow$ $\vec{E} = 0 = \vec{D} = \vec{P}$

$r > r_3$

$$\int_S (\vec{D} \cdot \vec{n})\, dS = (+2Q - Q) = Q$$

logo

$$\boxed{\vec{D} = \frac{Q}{4\pi} \frac{1}{r^2} \vec{e}_r}$$

$$\boxed{\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r^2} \vec{e}_r}$$

$$\boxed{\vec{P} = 0}$$

c) $$\phi(\vec{r}) = \int_r^\infty \vec{E} \cdot d\vec{r} = \frac{Q}{4\pi\varepsilon_0} \frac{1}{r} \qquad r \geq r_3$$

d) em r_1 $\vec{n} = -\vec{e}_r$. logo

$$\sigma' = \vec{P} \cdot \vec{n} = -\frac{\varepsilon - \varepsilon_0}{\varepsilon} \frac{2Q}{4\pi} \frac{1}{r_1^2} < 0$$

1º teste teo – Versão F (3/11/2006)

"Igual" à versão B. Soluções:

a) $\boxed{Q_{int} = 2Q}$ $\boxed{Q_{ext} = -Q}$

b) $0<r<r_1$ e $r_1<r_2<r_3$: $\vec{E} = \vec{D} = \vec{P} = 0$

$r_1<r<r_2$: $\boxed{\vec{E} = -\frac{2Q}{4\pi\varepsilon}\frac{1}{r^2}\vec{e}_r}$; $\boxed{\vec{D} = -\frac{2Q}{4\pi}\frac{1}{r^2}\vec{e}_r}$

$\boxed{\vec{P} = -\frac{\varepsilon-\varepsilon_0}{\varepsilon}\frac{2Q}{4\pi}\frac{1}{r^2}\vec{e}_r}$

$r>r_3$: $\boxed{\vec{E} = -\frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\vec{e}_r}$; $\boxed{\vec{D} = -\frac{Q}{4\pi}\frac{1}{r^2}\vec{e}_r}$

$\boxed{\vec{P} = 0}$

c) $\boxed{\phi = -\frac{Q}{4\pi\varepsilon_0}\frac{1}{r}}$

d) $\boxed{\sigma' = \frac{\varepsilon-\varepsilon_0}{\varepsilon}\frac{2Q}{4\pi}\frac{1}{r_1^2} > 0}$