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The oblique parameters in multi-Higgs-doublet models

W. Grimus [a,*], L. Lavoura [b], O.M. Ogreid [c], P. Osland [d]

[a] *Fakultät für Physik, Universität Wien, Boltzmanngasse 5, 1090 Wien, Austria*
[b] *Universidade Técnica de Lisboa and Centro de Física Teórica de Partículas, Instituto Superior Técnico, 1049-001 Lisboa, Portugal*
[c] *Bergen University College, Bergen, Norway*
[d] *Department of Physics and Technology, University of Bergen, Postboks 7803, N-5020 Bergen, Norway*

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Abstract

We present general expressions for the oblique parameters S, T, U, V, W, and X in the $SU(2) \times U(1)$ electroweak model with an arbitrary number of scalar $SU(2)$ doublets, with hypercharge $\pm 1/2$, and an arbitrary number of scalar $SU(2)$ singlets.

1. Introduction

Definition of the oblique parameters. The oblique parameters are a useful way to parametrize the effects of new physics (NP) on electroweak observables when the following criteria are satisfied:

1. The electroweak gauge group is the standard $SU(2) \times U(1)$.
2. The NP particles have suppressed couplings to the light fermions with which experiments are performed; they couple mainly to the Standard Model (SM) gauge bosons γ, Z^0, and $W^\pm$.
3. The relevant electroweak measurements are those made at the energy scales $q^2 \approx 0$, $q^2 = m_Z^2$, and $q^2 = m_W^2$.

* Corresponding author.
E-mail addresses: walter.grimus@univie.ac.at (W. Grimus), balio@cftp.ist.utl.pt (L. Lavoura), omo@hib.no (O.M. Ogreid), per.osland@ift.uib.no (P. Osland).

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When this happens, the NP effects may be parametrized by only six quantities, which were defined by Maksymyk et al. [1], following the work by various other authors [2,3], as[1]

$$\frac{\alpha}{4s_W^2c_W^2}S=\frac{A_{ZZ}(m_Z^2)-A_{ZZ}(0)}{m_Z^2}-\left.\frac{\partial A_{\gamma\gamma}(q^2)}{\partial q^2}\right|_{q^2=0}+\frac{c_W^2-s_W^2}{c_Ws_W}\left.\frac{\partial A_{\gamma Z}(q^2)}{\partial q^2}\right|_{q^2=0}, \tag{1}$$

$$\alpha T=\frac{A_{WW}(0)}{m_W^2}-\frac{A_{ZZ}(0)}{m_Z^2}, \tag{2}$$

$$\begin{aligned}\frac{\alpha}{4s_W^2}U={}&\frac{A_{WW}(m_W^2)-A_{WW}(0)}{m_W^2}-c_W^2\frac{A_{ZZ}(m_Z^2)-A_{ZZ}(0)}{m_Z^2}\\&-s_W^2\left.\frac{\partial A_{\gamma\gamma}(q^2)}{\partial q^2}\right|_{q^2=0}+2c_Ws_W\left.\frac{\partial A_{\gamma Z}(q^2)}{\partial q^2}\right|_{q^2=0},\end{aligned} \tag{3}$$

$$\alpha V=\left.\frac{\partial A_{ZZ}(q^2)}{\partial q^2}\right|_{q^2=m_Z^2}-\frac{A_{ZZ}(m_Z^2)-A_{ZZ}(0)}{m_Z^2}, \tag{4}$$

$$\alpha W=\left.\frac{\partial A_{WW}(q^2)}{\partial q^2}\right|_{q^2=m_W^2}-\frac{A_{WW}(m_W^2)-A_{WW}(0)}{m_W^2}, \tag{5}$$

$$\frac{\alpha}{s_Wc_W}X=\left.\frac{\partial A_{\gamma Z}(q^2)}{\partial q^2}\right|_{q^2=0}-\frac{A_{\gamma Z}(m_Z^2)}{m_Z^2}. \tag{6}$$

Here, $\alpha=e^2/(4\pi)=g^2s_W^2/(4\pi)$ is the fine-structure constant, $s_W=\sin\theta_W$ and $c_W=\cos\theta_W$ are the sine and cosine, respectively, of the weak mixing angle θ_W, and the $A_{VV'}(q^2)$ are the coefficients of $g^{\mu\nu}$ in the vacuum-polarization tensors

$$\Pi^{\mu\nu}_{VV'}(q)=g^{\mu\nu}A_{VV'}\big(q^2\big)+q^\mu q^\nu B_{VV'}\big(q^2\big), \tag{7}$$

where VV' may be either $\gamma\gamma$, γZ, ZZ, or WW, and $q=(q^\alpha)$ is the four-momentum of the gauge boson.

Our definition of the oblique parameters follows [1] and allows for the case in which the NP scale is not much higher than the Fermi scale: it is not assumed that the $A_{VV'}(q^2)$ are linear functions of q^2. The original definitions [3] made that assumption and, consequently, there were only the three oblique parameters S, T, and U.

It is convenient to absorb into the oblique parameters the prefactors on the left-hand sides of Eqs. (1)–(6), by defining

$$\begin{aligned}&\bar S\equiv\frac{\alpha}{4s_W^2c_W^2}S,\qquad \bar T\equiv\alpha T,\qquad \bar U\equiv\frac{\alpha}{4s_W^2}U,\\&\bar V\equiv\alpha V,\qquad \bar W\equiv\alpha W,\qquad \bar X\equiv\frac{\alpha}{s_Wc_W}X.\end{aligned} \tag{8}$$

It should be stressed that, in the definition of an oblique parameter O, a subtraction of the SM contribution should always be understood, i.e.,

$$O=O|_{\mathrm{NP}}-O|_{\mathrm{SM}}. \tag{9}$$

[1] We follow the convention for the sign of the photon field in [4].

Therefore, the $A_{VV'}(q^2)$ that we utilize in this paper are in reality

$$A_{VV'}\left(q^2\right) = A_{VV'}\left(q^2\right)\big|_{\mathrm{NP}} - A_{VV'}\left(q^2\right)\big|_{\mathrm{SM}}. \tag{10}$$

Thus, the contributions to the $A_{VV'}(q^2)$ from loops of gauge bosons—including their longitudinal components viz. the "would-be" Goldstone bosons—cancel.

The subtraction of the SM contributions must also be used in the comparison of NP with the precision data [5]. One should note that, in such a comparison, one cannot[2] simultaneously determine from the data the SM Higgs-boson mass and the oblique parameters S and T.

Because of gauge invariance,

$$A_{\gamma\gamma}(0) = A_{\gamma Z}(0) = 0. \tag{11}$$

Therefore, X in Eq. (6) may be rewritten as

$$\bar{X} = \left.\frac{\partial A_{\gamma Z}(q^2)}{\partial q^2}\right|_{q^2=0} - \frac{A_{\gamma Z}(m_Z^2) - A_{\gamma Z}(0)}{m_Z^2}. \tag{12}$$

The parameter Δr. As a practical example of the application of the oblique parameters, we may consider Δr, defined by the relation [6] (see also [7,8])

$$G_\mu = \frac{\pi\alpha}{\sqrt{2} m_W^2 s_W^2 (1 - \Delta r)}, \tag{13}$$

where $s_W^2 \equiv 1 - m_W^2/m_Z^2$. The parameter Δr contains the loop corrections to the tree-level relation among m_W, m_Z, α, and the muon decay constant G_μ. Let us define

$$\Delta r' = \Delta r|_{\mathrm{NP}} - \Delta r|_{\mathrm{SM}}. \tag{14}$$

It is possible—provided that the NP fields have suppressed couplings to the light fermions involved in the measurements of α, G_μ, m_Z, and m_W—to express $\Delta r'$ in terms of the oblique parameters S, T, and U. Indeed, in that case $\Delta r'$ originates solely in modifications to the gauge-boson propagators, viz. [7]

$$\Delta r' = \left.\frac{\partial A_{\gamma\gamma}(q^2)}{\partial q^2}\right|_{q^2=0} + \frac{A_{WW}(0) - A_{WW}(m_W^2)}{m_W^2} - \frac{c_W^2}{s_W^2}\left[\frac{A_{ZZ}(m_Z^2)}{m_Z^2} - \frac{A_{WW}(m_W^2)}{m_W^2}\right], \tag{15}$$

where Eq. (11) has been taken into account. One then easily finds that [1]

$$\Delta r' = \frac{\alpha}{s_W^2}\left(-\frac{1}{2}S + c_W^2 T + \frac{c_W^2 - s_W^2}{4 s_W^2} U\right). \tag{16}$$

This relation is useful for a comparison of any particular NP model with the experimental data viz. the measured mass of the $W^\pm$ gauge bosons. Indeed, if one considers the measured values of α, G_μ, and m_Z^2 to constitute an experimental input to the $SU(2) \times U(1)$ gauge theory, then the relations (13) and (14) lead to the prediction of the $W^\pm$ mass

$$m_W^2 = m_W^2\big|_{\mathrm{SM}}\left(1 + \frac{s_W^2}{c_W^2 - s_W^2}\Delta r'\right). \tag{17}$$

[2] J. Erler and P. Langacker in [5], p. 119.

Historically, the parameter Δr has played an important role in the study of the SM; for instance, it has allowed the prediction of the top-quark mass before the actual observation of that particle.[3]

Aim of this paper. The purpose of this paper is to give formulae for the six oblique parameters at one-loop level[4] in an extension of the SM characterized by an enlarged scalar sector. The scalar sector of the SM consists of only one $SU(2)$ doublet, with hypercharge $1/2$. In our NP model, which we call the multi-Higgs-doublet-and-singlet model (mHDSM), there is an arbitrary number of such scalar $SU(2)$ doublets, together with an arbitrary number of scalar $SU(2)$ singlets with arbitrary hypercharges. It turns out that it is possible to derive simple, closed formulae for the oblique parameters in the mHDSM, in terms of only five functions of the masses of the scalar fields, and of the matrix elements of only two mixing matrices.

In the mHDSM, scalars with electric charges 0 or ± 1 are decoupled from scalars with any other electric charges. Therefore, in Section 2 we outline the mHDSM in which all scalar fields have electric charges 0 or ± 1, focusing especially on a general treatment of the mixing of the scalars. Section 3 contains our formulae for the oblique parameters originating in that sector of the mHDSM. In Section 4 we develop the formulae to the case in which $SU(2)$ singlets with electric charges different from 0 and ± 1 are present in the mHDSM. Section 5 summarizes the findings of this paper. A set of three appendices explains some intermediate steps of our computations; Appendix A compiles various relations satisfied by the mixing matrices of the scalars, Appendix B presents the needed Feynman integrals, and Appendix C contains the functions of the scalar masses which occur in the oblique parameters.

2. The model

We consider an $SU(2) \times U(1)$ electroweak gauge model including n_d scalar $SU(2)$ doublets ϕ_k with hypercharge $1/2$, n_c complex scalar $SU(2)$ singlets χ_j^+ with hypercharge 1, and n_n real scalar $SU(2)$ singlets χ_l^0 with hypercharge 0:

$$\begin{aligned} &\phi_k = \begin{pmatrix} \varphi_k^+ \\ \varphi_k^0 \end{pmatrix} \quad (k = 1, 2, \ldots, n_d), \qquad \chi_j^+ \quad (j = 1, 2, \ldots, n_c), \\ &\chi_l^0 \quad (l = 1, 2, \ldots, n_n). \end{aligned} \tag{18}$$

The neutral fields have vacuum expectation values (VEVs)

$$\langle 0|\varphi_k^0|0\rangle = \frac{v_k}{\sqrt{2}}, \qquad \langle 0|\chi_l^0|0\rangle = u_l, \tag{19}$$

the v_k being in general complex; the u_l are real since the χ_l^0 are real fields. As usual, we expand the neutral fields around their VEVs:

$$\varphi_k^0 = \frac{v_k + \varphi_k^{0\prime}}{\sqrt{2}}, \qquad \chi_l^0 = u_l + \chi_l^{0\prime}. \tag{20}$$

Our treatment of the scalars was previously used in [9]; it is a generalization of the treatment in [10,11]. The charged fields in (18) can be expressed in terms of the charged mass eigenfields S_a^+

[3] For references see [5], p. 526.

[4] Since we are concerned with the oblique parameters at only the one-loop level, we are allowed to use throughout the tree-level relation $m_W = c_W m_Z$.

by

$$\varphi_k^+ = \sum_{a=1}^{n} \mathcal{U}_{ka} S_a^+, \qquad \chi_j^+ = \sum_{a=1}^{n} \mathcal{T}_{ja} S_a^+, \tag{21}$$

with $n = n_d + n_c$. Similarly, the neutral fields in (20) are linear combinations of the real neutral mass eigenfields S_b^0:

$$\varphi_k^{0\prime} = \sum_{b=1}^{m} \mathcal{V}_{kb} S_b^0, \qquad \chi_l^{0\prime} = \sum_{b=1}^{m} \mathcal{R}_{lb} S_b^0, \tag{22}$$

with $m = 2n_d + n_n$. The dimensions of the matrices in Eqs. (21) and (22) are

$$\mathcal{U}\colon n_d \times n, \qquad \mathcal{T}\colon n_c \times n, \qquad \mathcal{V}\colon n_d \times m, \qquad \mathcal{R}\colon n_n \times m. \tag{23}$$

The matrices

$$\tilde{\mathcal{U}} \equiv \begin{pmatrix} \mathcal{U} \\ \mathcal{T} \end{pmatrix}, \qquad \tilde{\mathcal{V}} \equiv \begin{pmatrix} \operatorname{Re} \mathcal{V} \\ \operatorname{Im} \mathcal{V} \\ \mathcal{R} \end{pmatrix} \tag{24}$$

are the diagonalizing matrices for the mass-squared matrices of the charged and neutral scalars, respectively. The matrix $\tilde{\mathcal{U}}$ is $n \times n$ unitary, the matrix $\tilde{\mathcal{V}}$ is $m \times m$ orthogonal.

Since we are dealing with a spontaneously broken $SU(2) \times U(1)$ gauge theory, there are three unphysical Goldstone bosons, $G^\pm$ and G^0, which are "swallowed" by the $W^\pm$ and Z^0, respectively, to become their longitudinal components. For definiteness we assign to them the indices $a = 1$ and $b = 1$, respectively:

$$S_1^\pm = G^\pm, \qquad S_1^0 = G^0. \tag{25}$$

The masses of $G^\pm$ and of G^0—in a general 't Hooft (R_ξ) gauge—are arbitrary and unphysical: they cannot appear in the final formula for any observable quantity. We have checked, by computing the oblique parameters in an arbitrary 't Hooft gauge, that all terms containing those masses do indeed cancel.

In the SM, $\mathcal{T}$ and $\mathcal{R}$ do not exist and $\mathcal{U} = (1)$, $\mathcal{V} = (i, 1)$.

3. The results

The parameter T in the mHDSM was computed in our previous paper [9], where an extensive presentation of its derivation has been given. Therefore, we shall give here only the final result for that oblique parameter.

For all other five parameters, all that one needs to calculate are the functions

$$\frac{A_{VV'}(q^2) - A_{VV'}(0)}{q^2}, \tag{26}$$

for $VV' = ZZ$, WW, or γZ, and $q^2 = m_{V'}^2$, and

$$\left. \frac{\partial A_{VV}(q^2)}{\partial q^2} \right|_{q^2 = m_V^2} - \frac{A_{VV}(m_V^2) - A_{VV}(0)}{m_V^2}, \tag{27}$$

for $V = Z$ and $V = W$, and also the derivatives of $A_{\gamma\gamma}(q^2)$ and of $A_{\gamma Z}(q^2)$ at $q^2 = 0$.

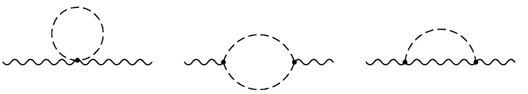

Fig. 1. Three types of Feynman diagrams occurring in the calculation of the vacuum polarizations.

For a presentation of the Lagrangian of the mHDSM in the physical basis of the scalars we refer the reader to [9]. We remark that the matrices $\mathcal{T}$ and $\mathcal{R}$ can be eliminated from the Lagrangian by making use of the unitarity of $\tilde{\mathcal{U}}$ and the orthogonality of $\tilde{\mathcal{V}}$, respectively, in Eq. (24). The Feynman diagrams which contribute to the vacuum polarizations in the mHDSM are depicted in Fig. 1.[5] Diagrams of type (a) are independent of q^2. Therefore they affect only T. It is also necessary to compute type (a) diagrams if we want to demonstrate Eq. (11) in the mHDSM, which we did; except for this purpose, type (a) Feynman diagrams are irrelevant in the computation of S, U, V, W, and X, and will henceforth not be considered in this paper.

There are no type (c) diagrams for vacuum polarizations involving either one or two photons—see the mHDSM Lagrangian in [9]. The type (c) diagrams are only relevant for A_{WW} and A_{ZZ}.

In the mHDSM all the oblique parameters, except T, are ultraviolet-finite after summation over all the Feynman diagrams but *before* subtraction of the SM expression. The parameter T, on the other hand, only becomes non-divergent after the subtraction of the SM Higgs-boson loops, as shown in [9].

We begin by quoting the result for $\bar{T}$ from [9]:

$$\begin{aligned}\bar{T} = \frac{g^2}{64\pi^2 m_W^2}\Bigg\{ & \sum_{a=2}^{n}\sum_{b=2}^{m}\big|(\mathcal{U}^\dagger\mathcal{V})_{ab}\big|^2 F\big(m_a^2, \mu_b^2\big) - \sum_{b=2}^{m-1}\sum_{b'=b+1}^{m}\big[\mathrm{Im}\big(\mathcal{V}^\dagger\mathcal{V}\big)_{bb'}\big]^2 F\big(\mu_b^2, \mu_{b'}^2\big) \\ & - 2\sum_{a=2}^{n-1}\sum_{a'=a+1}^{n}\big|(\mathcal{U}^\dagger\mathcal{U})_{aa'}\big|^2 F\big(m_a^2, m_{a'}^2\big) \\ & + 3\sum_{b=2}^{m}\big[\mathrm{Im}\big(\mathcal{V}^\dagger\mathcal{V}\big)_{1b}\big]^2\big[F\big(m_Z^2, \mu_b^2\big) - F\big(m_W^2, \mu_b^2\big)\big] \\ & - 3\big[F\big(m_Z^2, m_h^2\big) - F\big(m_W^2, m_h^2\big)\big]\Bigg\},\end{aligned} \tag{28}$$

where m_a denotes the mass of the charged scalars $S_a^\pm$ and μ_b denotes the mass of the neutral scalar S_b^0. The second term in the right-hand side (RHS) of Eq. (28) contains a sum over all pairs of *different* physical neutral scalars, i.e., $2 \leqslant b < b' \leqslant m$; similarly, the third term in that RHS contains a sum over all pairs of different *physical* charged scalars, i.e., $2 \leqslant a < a' \leqslant n$. The last term in the RHS of Eq. (28) consists of the subtraction, from the rest of $\bar{T}$, of the SM result. In that subtraction, m_h is the mass of the sole physical neutral scalar of the SM, the Higgs particle. The well-known [12] function F is given by

$$F(I, J) \equiv \begin{cases} \frac{I+J}{2} - \frac{IJ}{I-J}\ln\frac{I}{J} & \Leftarrow\ I \neq J, \\ 0 & \Leftarrow\ I = J. \end{cases} \tag{29}$$

[5] There are also tadpole diagrams, but they are irrelevant for the computation of the oblique parameters.

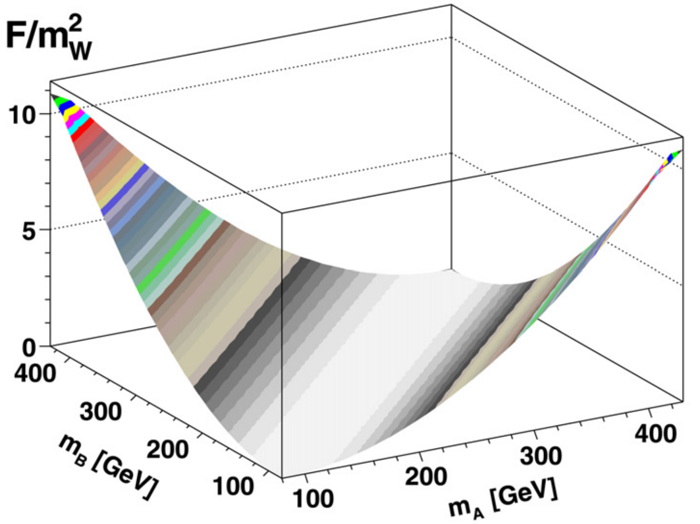

Fig. 2. $F(m_A^2, m_B^2)/m_W^2$ vs. m_A and m_B.

We depict this function in Fig. 2. Next we write down the results for $\bar{S}$, $\bar{U}$, and $\bar{X}$:

$$\begin{aligned}
\bar{S} = \frac{g^2}{384\pi^2 c_W^2} \Bigg\{ & \sum_{a=2}^{n} \big[2s_W^2 - (\mathcal{U}^\dagger\mathcal{U})_{aa}\big]^2 G\big(m_a^2, m_a^2, m_Z^2\big) \\
& + 2\sum_{a=2}^{n-1} \sum_{a'=a+1}^{n} \big|(\mathcal{U}^\dagger\mathcal{U})_{aa'}\big|^2 G\big(m_a^2, m_{a'}^2, m_Z^2\big) \\
& + \sum_{b=2}^{m-1} \sum_{b'=b+1}^{m} \big[\mathrm{Im}(\mathcal{V}^\dagger\mathcal{V})_{bb'}\big]^2 G\big(\mu_b^2, \mu_{b'}^2, m_Z^2\big) \\
& - 2\sum_{a=2}^{n} (\mathcal{U}^\dagger\mathcal{U})_{aa} \ln m_a^2 + \sum_{b=2}^{m} (\mathcal{V}^\dagger\mathcal{V})_{bb} \ln \mu_b^2 - \ln m_h^2 \\
& + \sum_{b=2}^{m} \big[\mathrm{Im}(\mathcal{V}^\dagger\mathcal{V})_{1b}\big]^2 \hat{G}\big(\mu_b^2, m_Z^2\big) - \hat{G}\big(m_h^2, m_Z^2\big) \Bigg\},
\end{aligned} \tag{30}$$

$$\begin{aligned}
\bar{U} = \frac{g^2}{384\pi^2} \Bigg\{ & \sum_{a=2}^{n} \sum_{b=2}^{m} \big|(\mathcal{U}^\dagger\mathcal{V})_{ab}\big|^2 G\big(m_a^2, \mu_b^2, m_W^2\big) \\
& - \sum_{a=2}^{n} \big[2s_W^2 - (\mathcal{U}^\dagger\mathcal{U})_{aa}\big]^2 G\big(m_a^2, m_a^2, m_Z^2\big) \\
& - 2\sum_{a=2}^{n-1} \sum_{a'=a+1}^{n} \big|(\mathcal{U}^\dagger\mathcal{U})_{aa'}\big|^2 G\big(m_a^2, m_{a'}^2, m_Z^2\big)
\end{aligned}$$

$$
\begin{aligned}
&- \sum_{b=2}^{m-1} \sum_{b'=b+1}^{m} \left[\operatorname{Im}\left(\mathcal{V}^\dagger \mathcal{V}\right)_{bb'}\right]^2 G\left(\mu_b^2, \mu_{b'}^2, m_Z^2\right) \\
&+ \sum_{b=2}^{m} \left[\operatorname{Im}\left(\mathcal{V}^\dagger \mathcal{V}\right)_{1b}\right]^2 \left[\hat{G}\left(\mu_b^2, m_W^2\right) - \hat{G}\left(\mu_b^2, m_Z^2\right)\right] \\
&\left. - \hat{G}\left(m_h^2, m_W^2\right) + \hat{G}\left(m_h^2, m_Z^2\right) \right\},
\end{aligned} \tag{31}
$$

$$
\bar{X} = -\frac{g^2 s_W}{192\pi^2 c_W} \sum_{a=2}^{n} \left[2 s_W^2 - \left(\mathcal{U}^\dagger \mathcal{U}\right)_{aa}\right] G\left(m_a^2, m_a^2, m_Z^2\right). \tag{32}
$$

An explicit SM subtraction occurs in both $\bar{S}$ and $\bar{U}$. In $\bar{X}$, on the other hand, such an explicit SM subtraction, showing the mass m_h of the SM Higgs boson, does not occur, because $\bar{X}$ relates to $A_{\gamma Z}(q^2)$, and neutral particles—like the SM Higgs boson—do not couple to the photon. Still, the SM subtraction has been performed in $\bar{X}$, as elsewhere, in order to remove, from the sum over the charged scalars $S_a^\pm$, the Goldstone-boson ($a = 1$) term. The explicit forms of the two functions $G(I, J, Q)$ and $\hat{G}(I, Q)$ are found in Eqs. (C.2) and (C.5) of Appendix C. Notice that, contrary to what happens to the function $F(I, J)$ in Eq. (29), the function $G(I, J, Q)$ does *not* vanish when its first two arguments I and J, i.e., the squared masses of the two scalar particles in the loop of a type (b) diagram, are equal. In $\bar{S}$, the terms

$$
-2 \sum_{a=2}^{n} \left(\mathcal{U}^\dagger \mathcal{U}\right)_{aa} \ln m_a^2 + \sum_{b=2}^{m} \left(\mathcal{V}^\dagger \mathcal{V}\right)_{bb} \ln \mu_b^2 - \ln m_h^2 \tag{33}
$$

are meaningful since

$$
-2 \sum_{a=2}^{n} \left(\mathcal{U}^\dagger \mathcal{U}\right)_{aa} + \sum_{b=2}^{m} \left(\mathcal{V}^\dagger \mathcal{V}\right)_{bb} - 1 = 0, \tag{34}
$$

as can be verified by using Eqs. (A.5), (A.6), (A.8), and (A.9); therefore, the terms (33) are invariant under a scaling of all the scalar masses by a common factor.

In order to write down formulae for $\bar{V}$ and $\bar{W}$ we need the functions $H(I, J, Q)$ and $\hat{H}(I, Q)$ in Eqs. (C.7) and (C.10) of Appendix C. We have

$$
\begin{aligned}
\bar{V} = \frac{g^2}{384\pi^2 c_W^2} &\left\{ \sum_{a=2}^{n} \left[2 s_W^2 - \left(\mathcal{U}^\dagger \mathcal{U}\right)_{aa}\right]^2 H\left(m_a^2, m_a^2, m_Z^2\right) \right. \\
&+ 2 \sum_{a=2}^{n-1} \sum_{a'=a+1}^{n} \left|\left(\mathcal{U}^\dagger \mathcal{U}\right)_{aa'}\right|^2 H\left(m_a^2, m_{a'}^2, m_Z^2\right) \\
&+ \sum_{b=2}^{m-1} \sum_{b'=b+1}^{m} \left[\operatorname{Im}\left(\mathcal{V}^\dagger \mathcal{V}\right)_{bb'}\right]^2 H\left(\mu_b^2, \mu_{b'}^2, m_Z^2\right) \\
&\left. + \sum_{b=2}^{m} \left[\operatorname{Im}\left(\mathcal{V}^\dagger \mathcal{V}\right)_{1b}\right]^2 \hat{H}\left(\mu_b^2, m_Z^2\right) - \hat{H}\left(m_h^2, m_Z^2\right) \right\},
\end{aligned} \tag{35}
$$

$$\bar{W} = \frac{g^2}{384\pi^2} \left\{ \sum_{a=2}^{n} \sum_{b=2}^{m} \left| \left(\mathcal{U}^\dagger \mathcal{V}\right)_{ab} \right|^2 H\left(m_a^2, \mu_b^2, m_W^2\right) + \sum_{b=2}^{m} \left[\mathrm{Im}\left(\mathcal{V}^\dagger \mathcal{V}\right)_{1b} \right]^2 \hat{H}\left(\mu_b^2, m_W^2\right) - \hat{H}\left(m_h^2, m_W^2\right) \right\}. \tag{36}$$

4. Scalar singlets with electric charge other than 0, ±1

Scalar $SU(2)$ singlets with electric charge $Q \neq 0, \pm 1$ cannot mix with the scalars discussed in the previous two sections. Moreover, they couple to the photon and Z^0 but do not couple to the $W^\pm$. Also, there are no off-diagonal couplings among different scalars with the same charge Q—for the Lagrangian, see [9]. Therefore, without loss of generality, we may confine ourselves to a single scalar $SU(2)$ singlet, with mass m and electric charge Q. In [9] we have shown that, for such a scalar,

$$\bar{T} = 0. \tag{37}$$

Since it does not couple to the $W^\pm$ gauge bosons, $A_{WW} = 0$ and therefore

$$\bar{W} = 0 \tag{38}$$

too.

The fact that there are no type (c) diagrams, and that two identical scalars occur in the loop of type (b) diagrams, greatly facilitates the computation of the other oblique parameters. The results are

$$\bar{S} = \frac{Q^2 g^2 s_W^4}{96\pi^2 c_W^2} G\left(m^2, m^2, m_Z^2\right), \tag{39}$$

$$\bar{V} = \frac{Q^2 g^2 s_W^4}{96\pi^2 c_W^2} H\left(m^2, m^2, m_Z^2\right), \tag{40}$$

and the remaining oblique parameters are proportional to $\bar{S}$:

$$\bar{U} = -c_W^2 \bar{S}, \tag{41}$$

$$\bar{X} = -\frac{c_W}{s_W} \bar{S}. \tag{42}$$

5. Conclusions

In this paper we have calculated the oblique parameters—defined in a fashion appropriate for new physics at a scale not necessarily much higher than the Fermi scale [1]—in the standard $SU(2) \times U(1)$ electroweak gauge theory supplemented by an arbitrary number of scalar $SU(2)$ doublets with hypercharge $1/2$ and scalar $SU(2)$ singlets with arbitrary hypercharges.

We have found that the oblique parameters may be written in terms of only two mixing matrices $\mathcal{U}$ and $\mathcal{V}$, which parametrize the mixing of the charge-1 and charge-0 scalars, respectively. These matrices take care simultaneously of the mixing of $SU(2)$-doublet scalars and $SU(2)$-singlet scalars, because $\mathcal{U}$ is part of a larger unitary matrix $\tilde{\mathcal{U}}$, while $\mathrm{Re}\,\mathcal{V}$ and $\mathrm{Im}\,\mathcal{V}$ are parts of a larger real orthogonal matrix $\tilde{\mathcal{V}}$. The expressions for the oblique parameters require only five functions $F(I, J)/m_W^2$, $G(I, J, Q)$, $\hat{G}(I, Q)$, $H(I, J, Q)$, and $\hat{H}(I, Q)$, of the squared scalar masses I and J and of the squared gauge-boson masses $Q = m_W^2$ or $Q = m_Z^2$. We have depicted those functions in Figs. 2–5.

We note that for a model which has a compact spectrum, in the sense that all the scalar masses—except for the mass of the Higgs particle—are close together, while they are all large compared to m_Z, all those functions are usually smaller than $F(I, J)/m_W^2$, which grows like [13] $(\sqrt{I} - \sqrt{J})^2$ and only appears in the parameter T. This is one reason why in general we expect T to be dominant in the oblique corrections.[6] The other reason for the dominance of T is the relatively large factor $g^2/(64\pi^2\alpha) = 1/(16\pi s_W^2)$ contained in T—see Eqs. (8) and (28)—which multiplies $F(I, J)/m_W^2$; the oblique parameters other than T have smaller factors.

The functions $H(I, J, Q)$ and $\hat{H}(I, Q)$ tend to zero when I/Q and J/Q grow. The function $\hat{G}(I, Q)$ grows like $\ln(I/Q)$; pure logarithms of the masses of the scalars also appear in the expression for S, and they may render that parameter relatively large even if the masses of the new scalars are all equal. The function $G(I, J, Q)$ is small for $I = J$ but it becomes sizable whenever I and J are quite far apart, even if they are both much larger than Q, i.e., than the electroweak scale.

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Appendix A. Mixing-matrix relations

In this appendix we compile some useful relations involving the mixing matrices $\mathcal{U}$ and $\mathcal{V}$.

The first set of relations follows from the unitarity of the matrix $\tilde{\mathcal{U}}$ and the orthogonality of the matrix $\tilde{\mathcal{V}}$ defined in Eqs. (24):

$$\sum_{b=1}^{m} (\mathcal{U}^\dagger \mathcal{V})_{ab} (\mathcal{V}^\dagger \mathcal{U})_{ba} = 2 (\mathcal{U}^\dagger \mathcal{U})_{aa}, \tag{A.1}$$

$$\sum_{a'=1}^{n} (\mathcal{U}^\dagger \mathcal{U})_{aa'} (\mathcal{U}^\dagger \mathcal{U})_{a'a} = (\mathcal{U}^\dagger \mathcal{U})_{aa}, \tag{A.2}$$

$$\sum_{a=1}^{n} (\mathcal{V}^\dagger \mathcal{U})_{ba} (\mathcal{U}^\dagger \mathcal{V})_{ab} = (\mathcal{V}^\dagger \mathcal{V})_{bb}, \tag{A.3}$$

$$\sum_{b'=1}^{m} \left[\mathrm{Im} (\mathcal{V}^\dagger \mathcal{V})_{bb'}\right]^2 = (\mathcal{V}^\dagger \mathcal{V})_{bb}. \tag{A.4}$$

Furthermore, since $\mathcal{U}$ is $n_d \times n$ and $\mathcal{V}$ is $n_d \times m$,

$$\sum_{a=1}^{n} (\mathcal{U}^\dagger \mathcal{U})_{aa} = n_d, \tag{A.5}$$

[6] One exception is the effective charge in atomic parity violation, in which T is multiplied by a very small factor and S may dominate [1]. Another exception is the case where only scalar singlets with electric charges other than $0, \pm 1$ are present, because then $T = 0$ [9].

$$\sum_{b=1}^{m} (\mathcal{V}^\dagger \mathcal{V})_{bb} = 2n_d. \tag{A.6}$$

A second set of relations follows from the convention of placing the vectors pertaining to the Goldstone bosons in the first columns of $\tilde{\mathcal{U}}$ and $\tilde{\mathcal{V}}$, and also from the explicit form of those vectors, namely [9,11]

$$\mathcal{U}_{k1} = \frac{v_k}{v}, \qquad \mathcal{V}_{k1} = i\frac{v_k}{v}, \quad \text{with } v \equiv \sqrt{\sum_{k=1}^{n_d} |v_k|^2} \simeq 246\ \text{GeV}. \tag{A.7}$$

Some ensuing relations are

$$(\mathcal{U}^\dagger \mathcal{U})_{11} = 1, \tag{A.8}$$

$$(\mathcal{V}^\dagger \mathcal{V})_{11} = 1, \tag{A.9}$$

$$(\mathcal{U}^\dagger \mathcal{V})_{11} = i, \tag{A.10}$$

$$(\mathcal{U}^\dagger \mathcal{U})_{a1} = 0 \quad \Leftarrow a > 1, \tag{A.11}$$

$$(\mathcal{U}^\dagger \mathcal{V})_{a1} = 0 \quad \Leftarrow a > 1, \tag{A.12}$$

$$(\mathcal{U}^\dagger \mathcal{V})_{1b} = -\operatorname{Im}(\mathcal{V}^\dagger \mathcal{V})_{1b} \quad \Leftarrow b > 1. \tag{A.13}$$

Appendix B. Feynman integrals

In this appendix we compute the Feynman integrals which arise in type (b) and type (c) Feynman diagrams.

Diagrams of type (b) lead to

$$i g^{\mu\nu} A(I, J, Q) = \bar{\mu}^{4-d} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \int_0^1 \mathrm{d}x \frac{4k^\mu k^\nu}{(k^2 - \Delta + i\varepsilon)^2}, \tag{B.1}$$

where $\bar{\mu}$ is the 't Hooft mass, d the dimension of space–time, and

$$\Delta = Qx^2 + (J - I - Q)x + I. \tag{B.2}$$

In Eq. (B.1), $Q \equiv q^2$ is the squared four-momentum of the external gauge bosons, I and J are the squared masses of the two scalar particles in the loop. Then,

$$A(I, J, Q) = \frac{1}{8\pi^2} \int_0^1 \mathrm{d}x\, \Delta(\mathrm{div} - \ln \Delta), \tag{B.3}$$

with

$$\mathrm{div} = \frac{2}{4-d} - \gamma + 1 + \ln(4\pi \bar{\mu}^2), \tag{B.4}$$

γ being the Euler–Mascheroni constant. Explicitly,

$$
\begin{aligned}
A(I, J, Q) = \frac{1}{8\pi^2} \Bigg\{ & \left(\frac{I+J}{4} - \frac{Q}{12} \right) (2\,\mathrm{div} - \ln I - \ln J) \\
& + \frac{2}{3}(I+J) - \frac{5}{18} Q - \frac{(I-J)^2}{6Q} \\
& + \left[\frac{(I-J)^2}{3Q} - I - J \right] \frac{I-J}{4Q} \ln \frac{I}{J} + \frac{r}{12Q^2} f(t,r) \Bigg\}.
\end{aligned}
\tag{B.5}
$$

The function f of

$$
t \equiv I + J - Q \quad \text{and} \quad r \equiv Q^2 - 2Q(I+J) + (I-J)^2 \tag{B.6}
$$

is given by

$$
f(t,r) \equiv \begin{cases} \sqrt{r} \ln \left| \frac{t-\sqrt{r}}{t+\sqrt{r}} \right| & \Leftarrow r > 0, \\ 0 & \Leftarrow r = 0, \\ 2\sqrt{-r} \arctan \frac{\sqrt{-r}}{t} & \Leftarrow r < 0. \end{cases} \tag{B.7}
$$

The absolute value in the argument of the logarithm takes effect only if

$$
Q > (\sqrt{I} + \sqrt{J})^2, \tag{B.8}
$$

in which case the vacuum polarization has an absorptive part; in Eq. (B.5), though, only the dispersive part is given, since it is the only one relevant for the oblique parameters.

In diagrams of type (c) the internal particles are a neutral scalar S_b^0 and a gauge boson V, which may be either $V = Z^0$ if we are computing $A_{ZZ}(q^2)$ or $V = W^\pm$ if we are computing $A_{WW}(q^2)$. We keep the notation of Eq. (B.2) but consider $I = \mu_b^2$ to be the squared mass of the neutral scalar and $J = m_V^2$ to be the squared mass of the gauge boson. Each vertex of a type (c) diagram contains one factor of the vector-boson mass m_V. In a general 't Hooft gauge, the vector-boson propagator, multiplied by m_V^2, is

$$
\frac{-m_V^2 g^{\mu\nu} + k^\mu k^\nu}{k^2 - m_V^2} - \frac{k^\mu k^\nu}{k^2 - m_G^2}, \tag{B.9}
$$

where m_G is the mass of the unphysical Goldstone boson, i.e., $m_G = m_1$ when $V = W^\pm$ and $m_G = \mu_1$ when $V = Z^0$. The first term in (B.9) leads to a Feynman integral different from the one of (B.1):

$$
i g^{\mu\nu} \bar{A}(I, J, Q) = \bar{\mu}^{4-d} \int \frac{\mathrm{d}^d k}{(2\pi)^d} \int_0^1 \mathrm{d}x \, \frac{-4 J g^{\mu\nu}}{(k^2 - \Delta + i\varepsilon)^2}. \tag{B.10}
$$

Consequently,

$$
\bar{A}(I, J, Q) = -\frac{J}{4\pi^2} \int_0^1 \mathrm{d}x \, (\mathrm{div} - 1 - \ln \Delta). \tag{B.11}
$$

Explicitly,

$$
\bar{A}(I, J, Q) = \frac{J}{8\pi^2} \left[-2\,\mathrm{div} + \ln I + \ln J - 2 + \frac{I-J}{Q} \ln \frac{I}{J} + \frac{f(t,r)}{Q} \right]. \tag{B.12}
$$

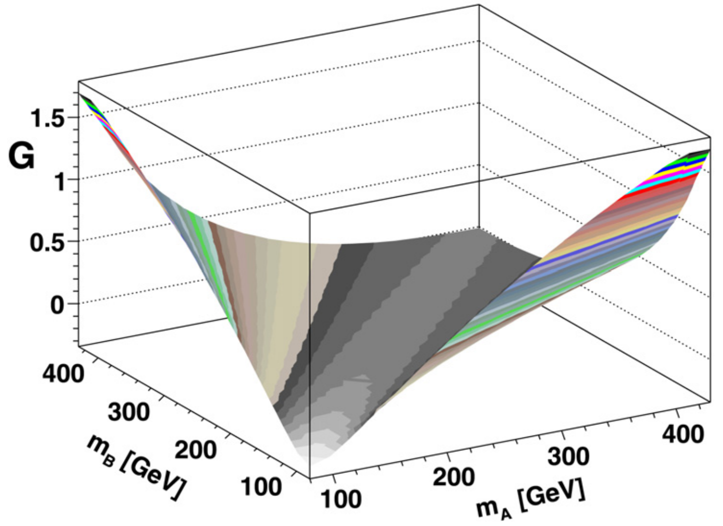

Fig. 3. $G(m_A^2, m_B^2, m_Z^2)$ vs. m_A and m_B.

Due to (B.1) and (B.9), the full expression for a type (c) diagram is

$$\bar{A}(I, J, Q) + A(I, J, Q) - A\big(I, m_G^2, Q\big). \tag{B.13}$$

The third term in Eq. (B.13) contains the unphysical mass m_G and, therefore, it cannot show up in the final results for the oblique parameters; such terms either cancel internally during the computation of the scalar contributions to the oblique parameters in the mHDSM, or they cancel out when the SM contribution is subtracted at the end of that computation.

In the limit $Q \to 0$ both $A(I, J, Q)$ and $\bar{A}(I, J, Q)$ are free from Q^{-1} divergences and can be written in terms of the function $F(I, J)$ in Eq. (29)—see [9].

Appendix C. Functions

In this appendix we derive the functions of the squared masses which actually appear in the expressions for the oblique parameters.

For terms of the form (26) stemming from type (b) diagrams, we need

$$\frac{A(I, J, Q) - A(I, J, 0)}{Q} = \frac{1}{96\pi^2}\big[2 - 2\,\mathrm{div} + \ln I + \ln J + G(I, J, Q)\big], \tag{C.1}$$

where

$$G(I, J, Q) \equiv -\frac{16}{3} + \frac{5(I+J)}{Q} - \frac{2(I-J)^2}{Q^2} + \frac{3}{Q}\left[\frac{I^2+J^2}{I-J} - \frac{I^2-J^2}{Q} + \frac{(I-J)^3}{3Q^2}\right]\ln\frac{I}{J} + \frac{r}{Q^3} f(t, r). \tag{C.2}$$

This function is shown in Fig. 3 for $Q = m_Z^2$ and for a range of values of $m_A \equiv \sqrt{I}$ and $m_B \equiv \sqrt{J}$. In the case of type (c) diagrams, we have to consider

$$\frac{\bar{A}(I,J,Q)-\bar{A}(I,J,0)}{Q}=\frac{1}{8\pi^2}\frac{J}{Q}\tilde{G}(I,J,Q), \tag{C.3}$$

where

$$\tilde{G}(I,J,Q)\equiv -2+\left(\frac{I-J}{Q}-\frac{I+J}{I-J}\right)\ln\frac{I}{J}+\frac{f(t,r)}{Q}. \tag{C.4}$$

This is ultraviolet-finite because $\bar{A}(I,J,Q)$ has no ultraviolet divergences proportional to Q. According to Eq. (B.13), the full function which appears in the computation of type (c) diagrams is

$$\begin{aligned}\hat{G}(I,Q)&\equiv G(I,Q,Q)+12\tilde{G}(I,Q,Q)\\&=-\frac{79}{3}+9\frac{I}{Q}-2\frac{I^2}{Q^2}+\left(-10+18\frac{I}{Q}-6\frac{I^2}{Q^2}+\frac{I^3}{Q^3}-9\frac{I+Q}{I-Q}\right)\ln\frac{I}{Q}\\&\quad+\left(12-4\frac{I}{Q}+\frac{I^2}{Q^2}\right)\frac{f(I,I^2-4IQ)}{Q}.\end{aligned} \tag{C.5}$$

Proceeding to terms of the form (27), one has

$$\frac{\partial A(I,J,Q)}{\partial Q}-\frac{A(I,J,Q)-A(I,J,0)}{Q}=\frac{1}{96\pi^2}H(I,J,Q), \tag{C.6}$$

which is ultraviolet-finite even for type (b) diagrams. Indeed,

$$\begin{aligned}H(I,J,Q)&\equiv 2-\frac{9(I+J)}{Q}+\frac{6(I-J)^2}{Q^2}\\&\quad+\frac{3}{Q}\left[-\frac{I^2+J^2}{I-J}+2\frac{I^2-J^2}{Q}-\frac{(I-J)^3}{Q^2}\right]\ln\frac{I}{J}\\&\quad+\left[I+J-\frac{(I-J)^2}{Q}\right]\frac{3f(t,r)}{Q^2}.\end{aligned} \tag{C.7}$$

This function is displayed in Fig. 4. Type (c) diagrams give

$$\frac{\partial \bar{A}(I,J,Q)}{\partial Q}-\frac{\bar{A}(I,J,Q)-\bar{A}(I,J,0)}{Q}=\frac{1}{8\pi^2}\frac{J}{Q}\tilde{H}(I,J,Q), \tag{C.8}$$

where

$$\begin{aligned}\tilde{H}(I,J,Q)&\equiv 4+\left(\frac{I+J}{I-J}-2\frac{I-J}{Q}\right)\ln\frac{I}{J}\\&\quad+\frac{-Q^2+3Q(I+J)-2(I-J)^2}{rQ}f(t,r).\end{aligned} \tag{C.9}$$

However, in analogy to the function $\hat{G}(I,Q)$, the function occurring in the full contribution of type (c) diagrams is

$$\begin{aligned}\hat{H}(I,Q)&\equiv H(I,Q,Q)+12\tilde{H}(I,Q,Q)\\&=47-21\frac{I}{Q}+6\frac{I^2}{Q^2}+3\left(7-12\frac{I}{Q}+5\frac{I^2}{Q^2}-\frac{I^3}{Q^3}+3\frac{I+Q}{I-Q}\right)\ln\frac{I}{Q}\\&\quad+3\left(28-20\frac{I}{Q}+7\frac{I^2}{Q^2}-\frac{I^3}{Q^3}\right)\frac{f(I,I^2-4IQ)}{I-4Q}.\end{aligned} \tag{C.10}$$

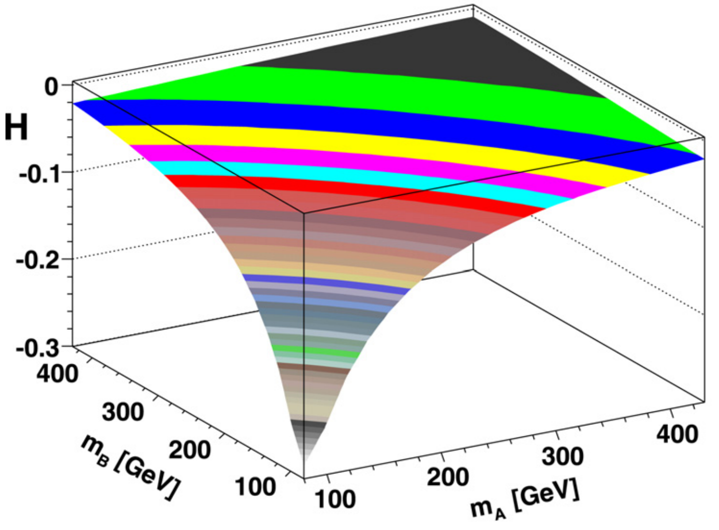

Fig. 4. $H(m_A^2, m_B^2, m_Z^2)$ vs. m_A and m_B.

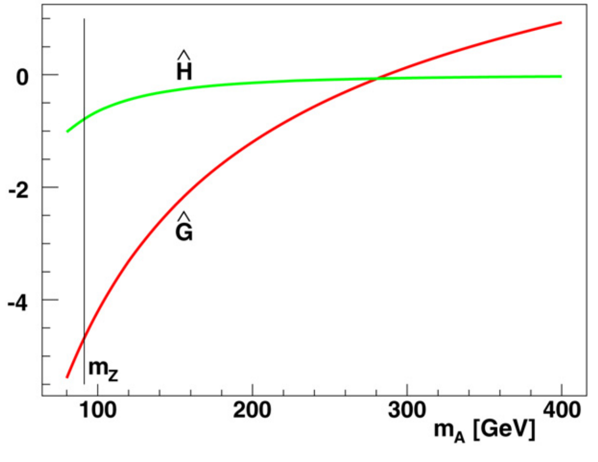

Fig. 5. $\hat{G}(m_A^2, m_Z^2)$ and $\hat{H}(m_A^2, m_Z^2)$ vs. m_A.

The functions $\hat{G}(I, Q)$ and $\hat{H}(I, Q)$ are shown in Fig. 5, for $Q = m_Z^2$ and a range of $m_A \equiv \sqrt{I}$. Asymptotically, with $\epsilon \equiv Q/I$,

$$\hat{G}(I, Q) = \left(-\frac{5}{3} + \ln\frac{1}{\epsilon}\right) - \frac{17}{2}\epsilon + \mathcal{O}(\epsilon^2), \tag{C.11}$$

$$\hat{H}(I, Q) = -\frac{1}{2}\epsilon - \frac{27}{10}\epsilon^2 + \mathcal{O}(\epsilon^3). \tag{C.12}$$

Notice that, when ϵ is very small, i.e., when the neutral-scalar masses are much larger than the Fermi scale, $\hat{H}(I, Q) \to 0$ but $\hat{G}(I, Q)$ grows logarithmically like $-\ln\epsilon$.

In $\bar{S}$, $\bar{U}$, and $\bar{X}$ there are contributions from the derivatives with respect to q^2 of the photon self-energy, and also of the mixed photon-Z^0 self-energy, evaluated at $q^2 = 0$. Those selfenergies only arise from type (b) diagrams and have two *identical* charged scalars in the loop. From equation (B.5), one obtains the very simple expression

$$\left.\frac{\partial A(I, I, Q)}{\partial Q}\right|_{Q=0} = \frac{1 - \mathrm{div} + \ln I}{48\pi^2}, \tag{C.13}$$

so that no new function beyond $F(I, J)$, $G(I, J, Q)$, $\hat{G}(I, Q)$, $H(I, J, Q)$, and $\hat{H}(I, Q)$ is needed for the oblique parameters.

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