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Covariant calculation of the nucleon and nucleon $\rightarrow \Delta$ form factor*

F. Gross[1,**], G. Ramalho[1,2,***], M. T. Peña[2,†]

[1] Thomas Jefferson National Accelerator Facility, Newport News, VA, USA
[2] Instituto Superior Técnico, Centro de Física Teórica de Partículas, Lisboa, Portugal

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Abstract All four nucleon electromagnetic form factors can be very well described by a manifestly covariant nucleon wave function with *zero* orbital angular momentum. The same model gives a qualitative description of deep inelastic scattering. The results for the G_M^* form factor of the $N \rightarrow \Delta$ transition are consistent with other quark models.

The recent JLab polarization transfer results [1] showed accurately [2] that the ratio of the electric G_{Ep} to magnetic G_{Mp} form factors of the proton is not constant as Q^2, the square of the momentum transfer, varies. Within the light-cone formalism, this lack of scaling was seen as proof that the proton wavefunction *must* have orbital angular momentum components $L > 0$. Do these results mean that the proton is not round? For an answer we refer the reader to previous work [3]. Here we simply show that it is possible to construct a pure S-wave model of the nucleon and the Δ excitation.

We used the manifestly covariant spectator theory to model both baryons as systems of three constituent quarks. The wavefunction has a pure S-wave nonrelativistic $SU(2) \times SU(2)$ limit and is built from a basis of fixed-polarization states. Both these states and the corresponding matrix elements for the current are covariant. For details, see [4]. The baryon with four-momentum P and mass

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** *E-mail address*: gross@jlab.org
*** *E-mail address*: ramalho@jlab.org
† *E-mail address*: teresa@fisica.ist.utl.pt

Correspondence: F. Gross, Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA

M is described by a wave function for an off-shell quark and an on-mass-shell diquark-like cluster. The diquark has its four-momentum k constrained by its on-mass-shell condition. The diquark-quark system has diquark isospin 0 and 1 components $\phi_I^0 = \xi^{0*}\chi^I$ and $\phi_I^1 = -(1/\sqrt{3})\tau \cdot \xi^{1*}\chi^I = (1/\sqrt{6})[\tau_-\xi_+^1 - \tau_+\xi_-^1 - \sqrt{2}\,\tau_3\xi_0^1]\chi^I$, where $\tau_\pm = \tau_x \pm i\tau_y$ are the isospin raising and lowering operators, $I = \pm 1/2$ is the isospin of the quark (or nucleon), and χ and ξ are respectively the third quark and the diquark isospin. For the Δ, $\phi \to \tilde{\phi}$, involving a $\frac{1}{2} \to \frac{3}{2}$ isospin transition matrix, instead of τ. For the spin part of the wavefunction, we use a basis of "fixed-axis" polarization vectors. The polarization of the diquark is first defined in the rest frame of the baryon, by an expansion in four-vectors $\varepsilon_0(\lambda)$ with angular momentum projections $\lambda = \{1, 0, -1\}$ along the z-axis. When the bound state is moving, we choose the z-axis to be in the direction of the motion. Then the bound state four-momentum is $P = \{\mathcal{E}_p, 0, 0, p\}$, where $\mathcal{E}_p = \sqrt{M^2 + p^2}$, and the boosted basis states are

$$\varepsilon_p^\mu(\pm) = Z^\mu{}_\nu\, \varepsilon_0^\nu(\pm) = \mp \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ \pm i \\ 0 \end{bmatrix}, \qquad \varepsilon_p^\mu(0) = Z^\mu{}_\nu\, \varepsilon_0^\nu(0) = \frac{1}{M} \begin{bmatrix} p \\ 0 \\ 0 \\ \mathcal{E}_p \end{bmatrix}. \tag{1}$$

The bound state wavefunction Ψ is a matrix element of the quark annihilation operator. Omitting the isospin part, the axial vector diquark contribution to the wave function (a scalar diquark contribution was considered too) is $\Psi(P, k; \varepsilon_p, \rho) \equiv \langle k, \varepsilon_p | q(0) | P, \rho \rangle = \varepsilon_p^{*\alpha}(\lambda) \Gamma_\alpha(P, k) u(P, \rho)$, where $q(0)$ is the quark field operator and $u(P, \rho)$ is a nucleon spinor with helicity ρ. In the applications we have chosen the simple form $\Gamma_\alpha(P, k) = \phi_N(P, k)\gamma^5\gamma_\alpha$, where $\phi_N(P, k) = \phi_N(P \cdot k)$ is a scalar function. Analogously, the S-state Delta wave-function is $\Psi_\Delta(P, k) = -\phi_\Delta(P, k)\epsilon_P^{\beta*} w_\beta(P)$, where w_β is the Rarita-Schwinger vector spinor. The calculation of the current matrix element requires the polarization vectors in the initial and final state to be in a collinear frame. Neglecting exchange currents, the matrix element of the nucleon current has the form

$$J^\mu = \langle P_+ | j^\mu(q) | P_- \rangle = \int_k \bar{u}(P_+) \mathcal{A}^\mu_{\alpha\beta}(P_-, q, k) u(P_-) D^{\alpha\beta}_{+-}(P_-, q), \tag{2}$$

where P_- and $P_+ = P_- + q$ are the momenta of the incoming and outgoing baryons and $\mathcal{A}^\mu_{\alpha\beta}(P_-, q, k) = \overline{\Gamma}_\alpha(P_+, k) j^\mu(q) \Gamma_\beta(P_-, k)$ contains information on the more elementary constituent quark charges and magnetic moments. The polarization sum is $D^{\alpha\beta}_{+-}(P_-, q) = \sum_\lambda \varepsilon_+^\alpha(\lambda) \varepsilon_-^{*\beta}(\lambda)$. Its general form is

$$\begin{aligned} D^{\mu\nu}_{+-}(P_-, q) = \sum_\lambda \varepsilon_+^\mu \varepsilon_-^\nu = a_1 \left(-g^{\mu\nu} + \frac{P_-^\mu P_+^\nu}{b} \right) \\ + a_2 \left(P_- - \frac{bP_+}{M_+^2} \right)^\mu \left(P_+ - \frac{bP_-}{M_-^2} \right)^\nu \end{aligned} \tag{3}$$

with

$$b = P_+ \cdot P_-, \qquad a_1 = 1 \qquad \text{and} \qquad a_2 = -\frac{M_+ M_-}{b(M_+ M_- + b)}.$$

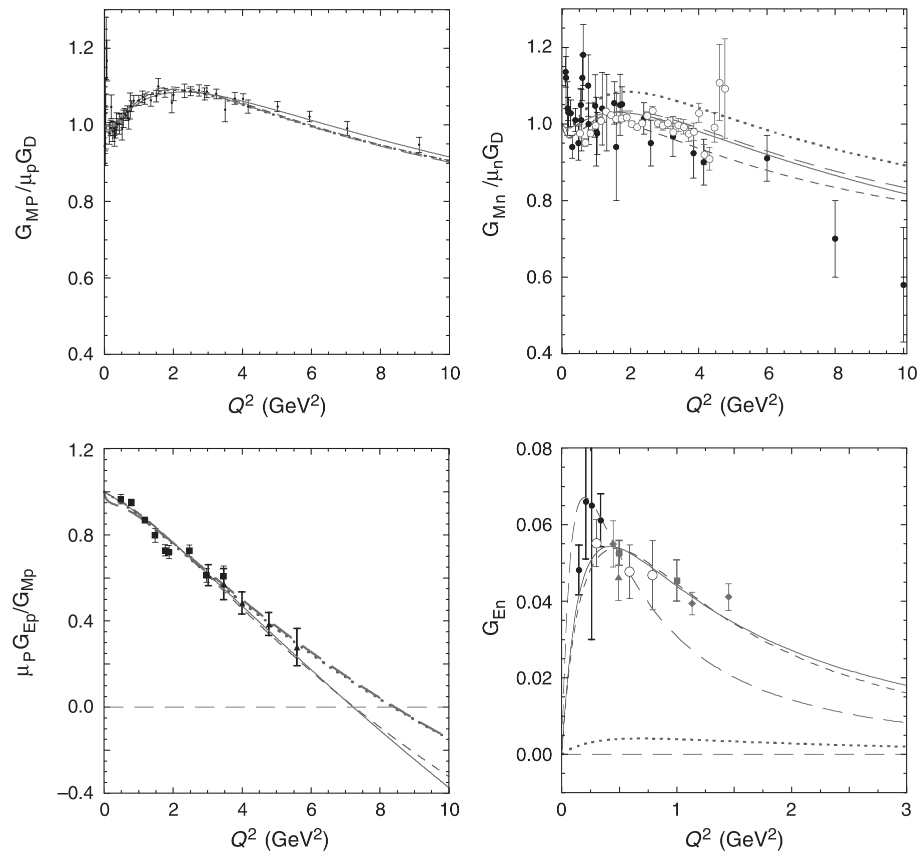

Fig. 1 Data for the nucleon form factors [1] compared with four models: Models I (dotted line) and II (short dashed line) are based on vector meson dominance for the quark current. Models III (long dashed line) and IV (solid line) include the two-pion cut contribution from ref. [5]

From Fig. 1 we conclude that the data do not require the proton to be deformed, or that it contains components with $L > 0$. Still the data give interesting new information about the constituent quark form factors. We predict that G_{Ep} changes sign near $Q^2 \approx 8\,\mathrm{GeV}^2$. The N-Δ transition form factors require $L > 0$ components: if both the nucleon *and* the Δ are spherical $G_C^* = G_E^* = 0$ contrarily to experiment and in agreement with other calculations. Also, as other quark models do, this one explains only about 60% of G_M^* at $Q^2 = 0$. The model is simple but it can be used to study phenomena near the quark-hadron transition, since a qualitative description of deep inelastic scattering was also achieved.

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