

A_4 model for the quark mass matrices

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Abstract. We propose a model for the quark masses and mixings based on an A_4 family symmetry. Three scalar $SU(2)$ doublets form a triplet of A_4 . The three left-handed-quark $SU(2)$ doublets are also united in a triplet of A_4 . The right-handed quarks are singlets of A_4 . The A_4 -symmetric scalar potential leads to a vacuum in which two of the three scalar $SU(2)$ doublets have expectation values with equal moduli. Our model makes an excellent fit of the observed $|V_{ub}/V_{cb}|$. CP symmetry is respected in the charged gauge interactions of the quarks.

1 Introduction

In the standard $SU(2) \times U(1)$ gauge model for the electroweak interactions, the Yukawa couplings of the fermions to the unique scalar $SU(2)$ doublet of the model are completely arbitrary – as a matter of fact, those couplings make up almost all the free parameters of the standard model (SM). As a consequence of this fact, the SM leaves the quark masses and mixings unpredicted; although the model accounts for the existence of quark masses and mixings, their actual values remain arbitrary in the context of the SM.

Several more complex models have tried to overcome this shortcoming of the SM. However, most of those models are in reality *Ansätze*: instead of *deriving* the structure of the Yukawa couplings from some underlying symmetry of a self-consistent gauge theory, they simply *assume* the Yukawa couplings to have some aesthetically appealing pattern or texture. A model should instead rely on some flavor (family) symmetry.

If the flavor symmetry is Abelian, then all its irreducible representations (irreps) are one-dimensional and the symmetry can at most force some Yukawa couplings to vanish.¹ A non-Abelian flavor symmetry can also force non-vanishing Yukawa couplings to be interrelated among themselves through definite Clebsch–Gordan factors. Since there are three families of quarks, a most desirable non-Abelian flavor symmetry ought to have three-dimensional irreps, in order to achieve full unification of the three gen-

erations and, hence, to achieve a minimal number of independent Yukawa couplings. The smallest discrete group with a three-dimensional irrep is A_4 ,² the group of the even permutations of four objects. The group A_4 has 12 group elements, one triplet irrep $\mathbf{3}$ and three inequivalent singlet irreps $\mathbf{1}$, $\mathbf{1}'$, and $\mathbf{1}''$ ($\mathbf{1}$ is the trivial representation; the irreps $\mathbf{1}'$ and $\mathbf{1}''$ are complex-conjugate of each other). This group has, in the last few years, been used in many models for the lepton masses and mixings [3]. It has also been used in models for the quark sector [4, 5], or for both quarks and leptons simultaneously [6–8].

In this paper we suggest a model for the Yukawa couplings of the quarks based on an A_4 family symmetry. Our model has three scalar gauge- $SU(2)$ doublets united in a $\mathbf{3}$ of A_4 . The left-handed-quark $SU(2)$ doublets are also united in a $\mathbf{3}$ of A_4 . In each electric-charge sector, the three right-handed quarks are in a $\mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$ of A_4 . Thus, our model achieves a high degree of simplicity and, even, uniqueness, because it treats the three families of quarks in the same way and it treats both electric-charge sectors in the same way. Furthermore, our model does not require A_4 to be broken anywhere in the Lagrangian, not even through soft terms – the breaking of A_4 is solely spontaneous. This, too, adds to the simplicity of the model.

Surprisingly, our model is able to predict the mixing parameter $|V_{ub}/V_{cb}|$ (where V is the quark mixing, or CKM, matrix) fully right: it predicts $|V_{ub}/V_{cb}| \approx 0.088$, in agreement with the usual averages of the various phenomenological analyses. On the other hand, our model also leads to a null violation of the discrete symmetry CP in the charged gauge interactions of the quarks; thus, the observed CP violation, for instance in K^0 – \bar{K}^0 mixing, or in B_q^0 decays, should in the context of our model be

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¹ The converse of this statement also holds: any pattern of vanishing Yukawa couplings may be enforced by an Abelian flavor symmetry with an adequate spectrum of scalars [1].

² A useful list of all the discrete non-Abelian groups with 31 or less group elements is provided in [2].

explained through scalar-mediated interactions, including flavor-changing neutral Yukawa interactions.

The plan of our paper is the following. In Sect. 2 we derive the form of the quark Yukawa-coupling matrices. In Sect. 3 we study the scalar potential and the ensuing vacuum. In Sect. 4 we write down the quark mass matrices and demonstrate that in our model there is no CP violation in the CKM matrix. In Sect. 5 we explain the method that we used in the numerical analysis and give some fits and results. A short summary is provided in Sect. 6.

2 The Yukawa couplings

The gauge symmetry of the model is $SU(2) \times U(1)$. There are three scalar $SU(2)$ doublets ϕ_j ($j = 1, 2, 3$) with hypercharge $1/2$. They form a triplet $\mathbf{3}$ of the flavor symmetry A_4 . There are three left-handed-quark $SU(2)$ doublets Q_{Lj} with hypercharge $1/6$. They are united in another $\mathbf{3}$ of A_4 . There are three right-handed-quark $SU(2)$ singlets n_{Rj} with hypercharge $-1/3$ and three right-handed-quark $SU(2)$ singlets p_{Rj} with hypercharge $2/3$. The n_{R1} and p_{R1} are A_4 -invariant (they are $\mathbf{1}$'s of A_4), the n_{R2} and p_{R2} are $\mathbf{1}''$'s of A_4 , and the n_{R3} and p_{R3} are $\mathbf{1}'''$'s of A_4 . This means that there exist two non-commuting transformations T_1 and T_2 ,

$$T_1 : \begin{cases} \phi_1 \rightarrow \phi_2 \rightarrow \phi_3 \rightarrow \phi_1, \\ Q_{L1} \rightarrow Q_{L2} \rightarrow Q_{L3} \rightarrow Q_{L1}, \\ n_{R2} \rightarrow \omega n_{R2}, n_{R3} \rightarrow \omega^2 n_{R3}, \\ p_{R2} \rightarrow \omega p_{R2}, p_{R3} \rightarrow \omega^2 p_{R3}, \end{cases} \quad (1)$$

$$T_2 : \begin{cases} \phi_2 \rightarrow -\phi_2, \phi_3 \rightarrow -\phi_3, \\ Q_{L2} \rightarrow -Q_{L2}, Q_{L3} \rightarrow -Q_{L3}, \end{cases} \quad (2)$$

under which the Lagrangian is invariant. In (1), $\omega \equiv \exp(2i\pi/3) = \sqrt[3]{1} = (-1 + i\sqrt{3})/2$.

The $\mathbf{3}$ is a real representation of A_4 . Indeed, the scalar $SU(2)$ doublets with hypercharge $-1/2$

$$\tilde{\phi}_j \equiv i\tau_2 \phi_j^* \quad (3)$$

transform under T_1 and T_2 in exactly the same way as the ϕ_j , as is obvious from (1) and (2).

Given this spectrum of fields and their transformation laws under both the gauge symmetry $SU(2) \times U(1)$ and the flavor symmetry A_4 , the quark Yukawa Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} = & -y_1 (\overline{Q_{L1}}\phi_1 + \overline{Q_{L2}}\phi_2 + \overline{Q_{L3}}\phi_3) n_{R1} \\ & -y_2 (\overline{Q_{L1}}\phi_1 + \omega\overline{Q_{L2}}\phi_2 + \omega^2\overline{Q_{L3}}\phi_3) n_{R2} \\ & -y_3 (\overline{Q_{L1}}\phi_1 + \omega^2\overline{Q_{L2}}\phi_2 + \omega\overline{Q_{L3}}\phi_3) n_{R3} \\ & -y_4 (\overline{Q_{L1}}\tilde{\phi}_1 + \overline{Q_{L2}}\tilde{\phi}_2 + \overline{Q_{L3}}\tilde{\phi}_3) p_{R1} \\ & -y_5 (\overline{Q_{L1}}\tilde{\phi}_1 + \omega\overline{Q_{L2}}\tilde{\phi}_2 + \omega^2\overline{Q_{L3}}\tilde{\phi}_3) p_{R2} \\ & -y_6 (\overline{Q_{L1}}\tilde{\phi}_1 + \omega^2\overline{Q_{L2}}\tilde{\phi}_2 + \omega\overline{Q_{L3}}\tilde{\phi}_3) p_{R3} + \text{h.c.}, \end{aligned} \quad (4)$$

the six Yukawa couplings y_{1-6} being in general complex.

The scalar doublets

$$\phi_j = \begin{pmatrix} \phi_j^+ \\ \phi_j^0 \end{pmatrix}, \quad \tilde{\phi}_j = \begin{pmatrix} \phi_j^{0*} \\ -\phi_j^- \end{pmatrix} \quad (5)$$

are assumed to have vacuum expectation values (VEVs)

$$\langle 0 | \phi_1^0 | 0 \rangle = v_1 e^{-i\alpha/2}, \quad \langle 0 | \phi_2^0 | 0 \rangle = v_2 e^{i\beta/2}, \quad \langle 0 | \phi_3^0 | 0 \rangle = v_3, \quad (6)$$

where $v_{1,2,3}$ are, without loss of generality, real and non-negative. Since $\overline{Q_{Lj}} = (\overline{p_{Lj}}, \overline{n_{Lj}})$, the quark mass matrices, defined through

$$\begin{aligned} \mathcal{L}_{\text{mass}} = & -(\overline{n_{L1}} \quad \overline{n_{L2}} \quad \overline{n_{L3}}) \mathbf{M}_n \begin{pmatrix} n_{R1} \\ n_{R2} \\ n_{R3} \end{pmatrix} \\ & -(\overline{p_{L1}} \quad \overline{p_{L2}} \quad \overline{p_{L3}}) \mathbf{M}_p \begin{pmatrix} p_{R1} \\ p_{R2} \\ p_{R3} \end{pmatrix} + \text{h.c.}, \end{aligned} \quad (7)$$

are

$$\mathbf{M}_n = \mathbf{D} \begin{pmatrix} y_1 v_1 & y_2 v_1 & y_3 v_1 \\ y_1 v_2 & \omega y_2 v_2 & \omega^2 y_3 v_2 \\ y_1 v_3 & \omega^2 y_2 v_3 & \omega y_3 v_3 \end{pmatrix}, \quad (8)$$

$$\mathbf{M}_p = \mathbf{D}^* \begin{pmatrix} y_4 v_1 & y_5 v_1 & y_6 v_1 \\ y_4 v_2 & \omega y_5 v_2 & \omega^2 y_6 v_2 \\ y_4 v_3 & \omega^2 y_5 v_3 & \omega y_6 v_3 \end{pmatrix}, \quad (9)$$

where

$$\mathbf{D} \equiv \text{diag} \left(e^{-i\alpha/2}, e^{i\beta/2}, 1 \right). \quad (10)$$

The quark mass matrices in (8) and (9) are identical to the mass matrix for the charged leptons in the original A_4 model of Ma and Rajasekaran (MR) [9]. Indeed, we have adopted in our model the same A_4 representations for the quarks as MR did in their paper for the leptons. The crucial difference between the two models is that, while MR have assumed a vacuum state characterized by $v_1 = v_2 = v_3$ and $\alpha = \beta = 0$, we shall demonstrate the existence of, and employ, a vacuum state with different features.

Let the unitary matrices $U_{L,R}^{n,p}$ satisfy

$$\mathbf{U}_L^{n\dagger} \begin{pmatrix} y_1 v_1 & y_2 v_1 & y_3 v_1 \\ y_1 v_2 & \omega y_2 v_2 & \omega^2 y_3 v_2 \\ y_1 v_3 & \omega^2 y_2 v_3 & \omega y_3 v_3 \end{pmatrix} \mathbf{U}_R^n = \text{diag} (m_d, m_s, m_b), \quad (11)$$

$$\mathbf{U}_L^{p\dagger} \begin{pmatrix} y_4 v_1 & y_5 v_1 & y_6 v_1 \\ y_4 v_2 & \omega y_5 v_2 & \omega^2 y_6 v_2 \\ y_4 v_3 & \omega^2 y_5 v_3 & \omega y_6 v_3 \end{pmatrix} \mathbf{U}_R^p = \text{diag} (m_u, m_c, m_t). \quad (12)$$

Then, the quark mixing (CKM) matrix is

$$\mathbf{V} = \mathbf{U}_L^{p\dagger} \mathbf{D}^2 \mathbf{U}_L^n. \quad (13)$$

One may absorb the phases of $y_{1,2,3}$ in the overall phases of the three rows of \mathbf{U}_R^n , and similarly absorb the phases of $y_{4,5,6}$ in the matrix \mathbf{U}_R^p . Those six phases are therefore un-

physical. Thus, this model for the quark masses and mixings has ten parameters: the two phases α and β in the diagonal matrix \mathbf{D}^2 , and the eight real quantities

$$|y_1|v_3, \left| \frac{y_2}{y_1} \right|, \left| \frac{y_3}{y_1} \right|, |y_4|v_3, \left| \frac{y_5}{y_4} \right|, \left| \frac{y_6}{y_4} \right|, \frac{v_2}{v_3}, \frac{v_1}{v_3}. \quad (14)$$

As we shall see in the next section, the A_4 -symmetric scalar potential is so constrained that these ten parameters reduce to only eight.

3 The scalar potential

The most general renormalizable scalar potential invariant under the symmetry A_4 is

$$\begin{aligned} V = & \mu \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3 \right) + \lambda_1 \left(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3 \right)^2 \\ & + \lambda_2 \left[\left(\phi_1^\dagger \phi_1 \right) \left(\phi_2^\dagger \phi_2 \right) + \left(\phi_2^\dagger \phi_2 \right) \left(\phi_3^\dagger \phi_3 \right) \right. \\ & + \left. \left(\phi_3^\dagger \phi_3 \right) \left(\phi_1^\dagger \phi_1 \right) \right] + (\lambda_3 - \lambda_2) \left[\left(\phi_1^\dagger \phi_2 \right) \left(\phi_2^\dagger \phi_1 \right) \right. \\ & + \left. \left(\phi_2^\dagger \phi_3 \right) \left(\phi_3^\dagger \phi_2 \right) + \left(\phi_3^\dagger \phi_1 \right) \left(\phi_1^\dagger \phi_3 \right) \right] \\ & + \frac{\lambda_4}{2} \left\{ e^{i\epsilon} \left[\left(\phi_1^\dagger \phi_2 \right)^2 + \left(\phi_2^\dagger \phi_3 \right)^2 + \left(\phi_3^\dagger \phi_1 \right)^2 \right] + \text{h.c.} \right\}, \end{aligned} \quad (15)$$

where μ and λ_{1-4} are real. The phase ϵ is arbitrary.

We define $v \equiv \sqrt{v_1^2 + v_2^2 + v_3^2}$ and

$$\theta_1 \equiv \epsilon - \beta, \quad (16)$$

$$\theta_2 \equiv \epsilon - \alpha, \quad (17)$$

$$\theta_3 \equiv \epsilon + \alpha + \beta. \quad (18)$$

Then,

$$\begin{aligned} V_0 \equiv \langle 0 | V | 0 \rangle = & \mu v^2 + \lambda_1 v^4 + \lambda_3 (v_1^2 v_2^2 + v_2^2 v_3^2 + v_3^2 v_1^2) \\ & + \lambda_4 (v_1^2 v_2^2 \cos \theta_3 + v_2^2 v_3^2 \cos \theta_1 + v_3^2 v_1^2 \cos \theta_2). \end{aligned} \quad (19)$$

The equations for vacuum stability are

$$\begin{aligned} \frac{\partial V_0}{\partial v_1^2} = 0 = & \mu + 2\lambda_1 v^2 + \lambda_3 (v_2^2 + v_3^2) \\ & + \lambda_4 (v_2^2 \cos \theta_3 + v_3^2 \cos \theta_2), \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\partial V_0}{\partial v_2^2} = 0 = & \mu + 2\lambda_1 v^2 + \lambda_3 (v_1^2 + v_3^2) \\ & + \lambda_4 (v_1^2 \cos \theta_3 + v_3^2 \cos \theta_1), \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{\partial V_0}{\partial v_3^2} = 0 = & \mu + 2\lambda_1 v^2 + \lambda_3 (v_1^2 + v_2^2) \\ & + \lambda_4 (v_1^2 \cos \theta_2 + v_2^2 \cos \theta_1), \end{aligned} \quad (22)$$

$$\frac{\partial V_0}{\partial \alpha} = 0 = \lambda_4 v_1^2 (-v_2^2 \sin \theta_3 + v_3^2 \sin \theta_2), \quad (23)$$

$$\frac{\partial V_0}{\partial \beta} = 0 = \lambda_4 v_2^2 (-v_1^2 \sin \theta_3 + v_3^2 \sin \theta_1). \quad (24)$$

We reject possible solutions to these equations in which one of the VEVs vanishes, and also solutions in which $v_1 = v_2 =$

v_3 .³ Then, (23) and (24) yield

$$\sin \theta_j = k v_j^2, \quad (25)$$

where k is a real constant with dimension M^{-2} . Subtracting (21) and (22) from (20), one obtains

$$\begin{aligned} (v_2^2 - v_1^2) \lambda_3 + [(v_2^2 - v_1^2) \cos \theta_3 \\ + v_3^2 (\cos \theta_2 - \cos \theta_1)] \lambda_4 = 0, \\ (v_3^2 - v_1^2) \lambda_3 + [(v_3^2 - v_1^2) \cos \theta_2 \\ + v_2^2 (\cos \theta_3 - \cos \theta_1)] \lambda_4 = 0. \end{aligned} \quad (26)$$

Equations (26) constitute a Cramer system for λ_3 and λ_4 . The Cramer determinant must vanish and one hence obtains

$$\sum_{j=1}^3 a_j \cos \theta_j = 0, \quad (27)$$

where

$$a_1 \equiv v_3^4 - v_2^4 + v_1^2 v_2^2 - v_1^2 v_3^2, \quad (28)$$

$$a_2 \equiv v_1^4 - v_3^4 + v_2^2 v_3^2 - v_1^2 v_2^2, \quad (29)$$

$$a_3 \equiv v_2^4 - v_1^4 + v_1^2 v_3^2 - v_2^2 v_3^2 \quad (30)$$

satisfy

$$\sum_{j=1}^3 a_j = 0. \quad (31)$$

Equation (27) together with (25) implies

$$\begin{aligned} 0 = & -\lambda \left(a_1 \sqrt{1 - k^2 v_1^4}, a_2 \sqrt{1 - k^2 v_2^4}, a_3 \sqrt{1 - k^2 v_3^4} \right) \\ = & 4k^2 \left[4k^2 v_1^4 v_2^4 v_3^4 - \lambda (v_1^2, v_2^2, v_3^2) \right] \\ & \times (v_1^2 - v_2^2)^2 (v_1^2 - v_3^2)^2 (v_2^2 - v_3^2)^2, \end{aligned} \quad (32)$$

where [10]

$$\lambda(a, b, c) \equiv -a^4 - b^4 - c^4 + 2a^2 b^2 + 2a^2 c^2 + 2b^2 c^2. \quad (33)$$

Equation (32) has five solutions:

1. $k = 0$,
2. $k^2 = \lambda (v_1^2, v_2^2, v_3^2) / (4v_1^4 v_2^4 v_3^4)$,
3. $v_1 = v_2$,
4. $v_1 = v_3$,
5. $v_2 = v_3$.

It is easy to explore in detail solutions 1 and 2 and to show that they require $\lambda_3 = \pm \lambda_4$ (solution 1 furthermore needs $\epsilon = 0$). Thus, those solutions require a non-trivial constraint on the parameters of the potential, and that constraint is in general unstable under renormalization. Those solutions should therefore be discarded. The remaining solutions 3–5 are equivalent, since the three scalar doublets ϕ_j form a triplet of A_4 . We shall use for definiteness solu-

³ Notice that Ma and Rajasekaran opted precisely for $v_1 = v_2 = v_3$ in their model [9], which is otherwise similar to ours.

tion 3: $v_1 = v_2$ and $\theta_1 = \theta_2$, i.e. $\alpha = \beta$. Equations (20)–(24) then reduce to only three equations,

$$0 = \mu + 2\lambda_1 (2v_1^2 + v_3^2) + \lambda_3 (v_1^2 + v_3^2) + \lambda_4 [v_1^2 \cos(\epsilon + 2\alpha) + v_3^2 \cos(\epsilon - \alpha)], \quad (34)$$

$$0 = \mu + 2\lambda_1 (2v_1^2 + v_3^2) + 2\lambda_3 v_1^2 + 2\lambda_4 v_1^2 \cos(\epsilon - \alpha), \quad (35)$$

$$0 = v_1^2 \sin(\epsilon + 2\alpha) - v_3^2 \sin(\epsilon - \alpha), \quad (36)$$

which determine the three quantities v_1 , v_3 , and α .

4 The mass matrices and CP conservation

We saw in the previous section that, just as we had advertised at the end of Sect. 2, consideration of the most general A_4 -invariant scalar potential actually reduces the ten parameters of our model to only eight, since $v_2 = v_1$ and $\beta = \alpha$. The quark mass matrices of our model are therefore

$$\mathbf{M}_n = \text{diag} \left(e^{-i\alpha/2}, e^{i\alpha/2}, 1 \right) \begin{pmatrix} a & b & c \\ a & \omega b & \omega^2 c \\ ra & \omega^2 rb & \omega rc \end{pmatrix}, \quad (37)$$

$$\mathbf{M}_p = \text{diag} \left(e^{i\alpha/2}, e^{-i\alpha/2}, 1 \right) \begin{pmatrix} f & g & h \\ f & \omega g & \omega^2 h \\ rf & \omega^2 rg & \omega rh \end{pmatrix}, \quad (38)$$

where $r \equiv v_3/v_1$ and a, b, c, f, g , and h are real and positive. Then,

$$\mathbf{H}_n \equiv \mathbf{M}_n \mathbf{M}_n^\dagger = \begin{pmatrix} x & y^* e^{-i\alpha} & ry e^{-i\alpha/2} \\ ye^{i\alpha} & x & ry^* e^{i\alpha/2} \\ ry^* e^{i\alpha/2} & ry e^{-i\alpha/2} & r^2 x \end{pmatrix}, \quad (39)$$

$$\mathbf{H}_p \equiv \mathbf{M}_p \mathbf{M}_p^\dagger = \begin{pmatrix} z & w^* e^{i\alpha} & rw e^{i\alpha/2} \\ we^{-i\alpha} & z & rw^* e^{-i\alpha/2} \\ rw^* e^{-i\alpha/2} & rw e^{i\alpha/2} & r^2 z \end{pmatrix}, \quad (40)$$

where $x \equiv a^2 + b^2 + c^2$ and $z \equiv f^2 + g^2 + h^2$ are real, while $y \equiv a^2 + \omega b^2 + \omega^2 c^2$ and $w \equiv f^2 + \omega g^2 + \omega^2 h^2$ are complex. Now, computing the commutator of \mathbf{H}_p and \mathbf{H}_n one finds that it is of the form

$$[\mathbf{H}_p, \mathbf{H}_n] = \begin{pmatrix} -n & 0 & -m \\ 0 & n & -m^* \\ m^* & m & 0 \end{pmatrix}, \quad (41)$$

hence $\det[\mathbf{H}_p, \mathbf{H}_n] = 0$. Therefore, in this model there is no CP violation in the quark mixing matrix, i.e. the Jarlskog observable J [11–13] vanishes.

5 Numerical procedure and results

We have performed a global χ^2 analysis of the quark mass matrices given in the previous section – (37) and (38) – by employing the downhill simplex method [14, 15]. Table 1

specifies in its first two columns the observable quantities O_i in the form

$$O_i = \bar{O}_i \pm \sigma_i, \quad (42)$$

where \bar{O}_i is the experimental mean value of O_i and σ_i is the square root of its variance. The index $i = 1, \dots, 9$ labels the nine observables given in Table 1.

Writing \mathbf{x} for the set of the eight parameters of our model (a, b, c, f, g, h, r , and α), and $P_i(\mathbf{x})$ for the resulting predictions for each of the observables, one constructs the χ^2 function

$$\chi^2(\mathbf{x}) = \sum_{i=1}^9 \left[\frac{P_i(\mathbf{x}) - \bar{O}_i}{\sigma_i} \right]^2. \quad (43)$$

The global minimum of χ^2 represents the best possible fit of the model predictions to the experimental data.

We found an excellent fit, with $\chi^2 = 0.057$, of our model to the nine input data specified in Table 1. The input parameters of the fit are

$$a = 40.75189 \text{ MeV}, \quad (44)$$

$$b = 87.78761 \text{ MeV}, \quad (45)$$

$$c = 2.347665 \text{ MeV}, \quad (46)$$

$$f = 3941.127 \text{ MeV}, \quad (47)$$

$$g = 515.0460 \text{ MeV}, \quad (48)$$

$$h = 1.060808 \text{ MeV}, \quad (49)$$

$$r = 43.37746, \quad (50)$$

$$\alpha = 0.2251660 \text{ radians}. \quad (51)$$

Table 1. Experimental data and result of our best fit. The experimental data (average values and error bars) used in our numerical analysis are given in the second column. The data on the quark masses have been taken from [16]. The light-quark masses are renormalized at a common scale $\mu \approx 2 \text{ GeV}$. The charm- and bottom-quark masses are running masses in the $\overline{\text{MS}}$ scheme. The data on the quark mixing angles have been taken from [17]. The third column displays the values P_i predicted by our model when the values of its parameters are those in (51). The fourth column shows the number of standard deviations from the mean values, $(P_i - \bar{O}_i)/\sigma_i$, computed using the data from the second column. The value $\chi^2 = 0.057$ is the sum of the squares of the numbers in the fourth column and is dominated by the pulls of m_s and $\sin \theta_{13}$

Observable	Experimental value	Model prediction	Pull
m_d [MeV]	5 ± 2	4.977	-1.2×10^{-2}
m_s [MeV]	95 ± 25	90.545	-1.8×10^{-1}
m_b [MeV]	4200 ± 70	4200.79	$+1.1 \times 10^{-2}$
m_u [MeV]	2.25 ± 0.75	2.250	$+1.9 \times 10^{-5}$
m_c [MeV]	1250 ± 90	1250.498	$+5.5 \times 10^{-3}$
m_t [GeV]	172.5 ± 2.7	172.497	-1.2×10^{-3}
$\sin \theta_{12}$	0.2243 ± 0.0016	0.22431	$+8.1 \times 10^{-3}$
$\sin \theta_{23}$	0.0413 ± 0.0015	0.04139	$+6.0 \times 10^{-2}$
$\sin \theta_{13}$	0.0037 ± 0.0005	0.003627	-1.5×10^{-1}

Other details of the fit are given in the third and fourth columns of Table 1.

Notice that the observables in Table 1 do not include the phase δ of the quark mixing matrix. Indeed, since we already know that in our model that matrix is real, we are of course unable to fit for any $\delta \neq 0, \pi$. In our model the explanation of all the observed CP violation through a sole phase in the quark mixing matrix gets spoiled: other sources of CP violation must be found and fitted to the observed data. In our model the most likely source of CP violation will be the interactions mediated by both charged and neutral scalars, since ours is a three-Higgs-doublet model featuring, in particular, flavor-changing interactions mediated by neutral scalars.

In order to test the variation of χ^2 as a function of the value \hat{O}_i of an observable quantity O_i , we substitute in the expression for $\chi^2(\mathbf{x})$ the term $[P_i(\mathbf{x}) - \hat{O}_i]^2/(\sigma_i)^2$ by a term $[P_i(\mathbf{x}) - \hat{O}_i]^2/(0.01 \hat{O}_i)^2$. The small error assigned to \hat{O}_i in the denominator of this term guarantees that O_i gets pinned down to the value \hat{O}_i .

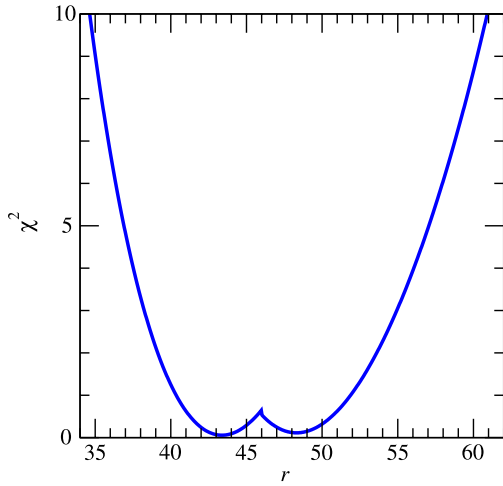


Fig. 1. χ^2 as a function of the ratio of VEVs $r \equiv v_3/v_1$

Figure 1 depicts χ^2 as a function of r , i.e. of the ratio of VEVs v_3/v_1 . We read off from that figure that only for $40 \lesssim r \lesssim 52$ can good fits be obtained; thus, the range of the ratio of VEVs is severely constrained.

In Fig. 2 (left panel), the change of χ^2 under variations of the quark mixing observable $|V_{ub}/V_{cb}|$ is shown. There is a pronounced minimum of χ^2 for $0.08 \lesssim |V_{ub}/V_{cb}| \lesssim 0.09$; this is in excellent agreement with the value obtained for that observable by the phenomenological analyses. This remarkable result of our model provides a clear-cut prediction for $|V_{ub}/V_{cb}|$. Figure 2 (right panel) gives χ^2 as a function of $|V_{td}/V_{ts}|$. We find in this case excellent fits whenever $0.14 \lesssim |V_{td}/V_{ts}| \lesssim 0.15$. Clearly, this result is correlated to our model's prediction for $|V_{ub}/V_{cb}|$, since in our model CP is conserved in quark mixing and therefore the CKM matrix is determined by only three parameters.

6 Conclusions

In this paper we have proposed a self-consistent model for the quark masses and mixings based on a family symmetry A_4 . The Yukawa-coupling matrices of our model contain, at face value, ten parameters, but, when one considers the A_4 -symmetric scalar potential in detail, one sees that two of those parameters actually disappear. In our model the family symmetry A_4 is not broken anywhere in the Lagrangian – its breaking is fully spontaneous. The model gives a perfect fit of the observed quark masses and mixing parameters, except for the fact that there is no CP violation at all in the CKM matrix. The observed CP violation should result in our model from scalar-mediated interactions, in particular flavor-changing neutral Yukawa interactions at tree level and also charged-scalar-mediated box diagrams at the one-loop level; a detailed study of those interactions should be the subject of a separate publication.

In previous A_4 models by other authors a vacuum characterized by equal VEVs, $v_1 = v_2 = v_3$, has often been employed. In our A_4 model for the quark sector, on the contrary, we use $v_1 = v_2$, while v_3 is some 43 times larger. It is

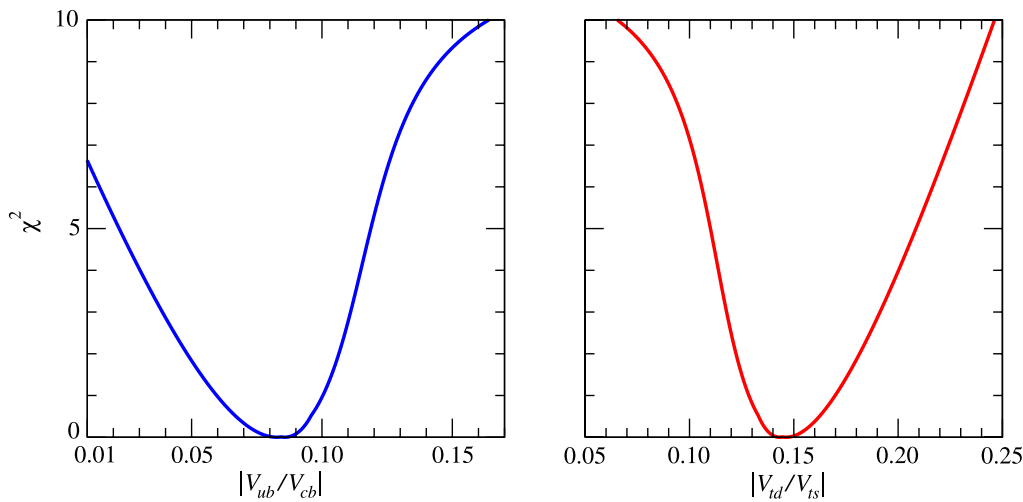


Fig. 2. χ^2 as a function of the CKM-matrix parameters $|V_{ub}/V_{cb}|$ and $|V_{td}/V_{ts}|$

not clear to us whether this vacuum may or may not produce a viable model for the lepton sector as well; this will be the subject of future investigation.

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