Fermion masses and the UV cutoff of the minimal realistic $SU(5)$ model

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We investigate the predictions for fermion masses in the minimal realistic nonsupersymmetric $SU(5)$ model with the standard model matter content. The possibility to achieve $b-\tau$ unification is studied taking into account all relevant effects. In addition, we show how to establish an upper bound on the ultraviolet cutoff Λ of the theory which is compatible with the Yukawa couplings at the grand unified scale and proton decay. We find $\Lambda \simeq 10^{17}$ GeV, to be considered a conservative upper bound on the cutoff. We also provide up-to-date values of all the fermions masses at the electroweak scale.

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I. INTRODUCTION

The hierarchy problem, unification of fundamental interactions, and the fermion mass puzzle are some of the main motivations for physics beyond the standard model (SM). In particular, if we believe in unification of electroweak and strong interactions then the so-called grand unified theories (GUTs) represent the most natural extensions of the standard model.

The first grand unified theory—the Georgi-Glashow $SU(5)$ model [1]—was introduced in 1974. In that model each generation of the SM matter is unified in the $\bar{5}$ and 10 dimensional representations, and the minimal Higgs sector is composed of two representations: 5_H and 24_H. The Georgi-Glashow (GG) model is arguably the simplest GUT. It is very predictive but it is certainly not realistic. Namely, one cannot unify the SM gauge couplings at the high scale, the neutrinos are massless, and a high-scale unification of Yukawa couplings of the down quarks and charged leptons contradicts experimental findings for the masses of those particles.

Since the simplest $SU(5)$ GUT is not realistic it requires appropriate modifications. There is a number of ways of doing that, but in order to preserve the predictivity of the theory those modifications should be minimal. With this in mind we have proposed in Ref. [2] the simplest possible extension of the Georgi-Glashow model that is in agreement with experimental observations. Phenomenological and cosmological aspects of our proposal have been analyzed subsequently in Ref. [3].

In this work we study in detail the Yukawa sector within the proposed framework, and define the upper bound on the ultraviolet (UV) cutoff Λ of the theory from the constraints imposed by proton decay lifetime measurements. We show that this UV cutoff depends on the absolute value of the tau lepton Yukawa coupling at the unification scale and not on the difference of the bottom quark to the tau lepton Yukawa couplings, as expected from bottom-tau unification. We find $\Lambda \leq 10^{17}$ GeV. Furthermore, as an essential ingredient in our study we perform an up-to-date analysis of all the fermion masses at the electroweak scale. These values are especially relevant for numerical studies of viability of various GUT models.

The paper is organized as follows: In Sec. II we describe the minimal realistic extension of the Georgi-Glashow model. In Sec. III the predictions for Yukawa couplings at the unification scale are investigated. There we also list updated values of all the fermion masses at the electroweak scale. The UV cutoff of the theory is defined and evaluated in Sec. IV. Finally, we conclude in Sec. V.

II. THE MINIMAL REALISTIC EXTENSION OF THE GEORGI-GLASHOW MODEL

The Higgs sector of the minimal realistic nonsupersymmetric $SU(5)$ model [2] is composed of the fields $24_H = (\Sigma_8, \Sigma_3, \Sigma_{(3,2)}, \Sigma_{(\bar{3},2)}, \Sigma_{24}) = (8,1,0) + (1,3,0) + (3,2,-5/6) + (\bar{3},2,5/6) + (1,1,0)$, $15_H = (\Phi_a, \Phi_b, \Phi_c) = (1,3,1) + (3,2,1/6) + (6,1,-2/3)$, and $5_H = (H, T) = (1,2,1/2) + (3,1,-1/3)$, while the matter content remains the same as in the GG model. Here we use the SM ($SU(3) \times SU(2) \times U(1)$) decomposition to set our notation. The Lagrangian of our model includes all possible terms invariant under the $SU(5)$ gauge symmetry, and accordingly includes higher-dimensional operators in order to write a consistent relation between fermion masses. (Influence of higher-dimensional operators on gauge coupling constants [4,5] is assumed to

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be negligible.) The role of the 15_H dimensional Higgs is twofold: it generates neutrino masses through a type II seesaw mechanism [6], and contributes to the unification of gauge couplings.

The possibility to achieve unification in the present context has been investigated in Refs. [2,3]. There we showed that there are only three fields which can help to achieve successful unification. Those are $\Sigma_3 \subset 24_H$, $\Phi_a \subset 15_H$, and $\Phi_b \subset 15_H$. We present in Fig. 1 the appropriate parameter space that generates successful unification of gauge couplings at one loop. It corresponds to the region bounded by the lines of constant $M_{\Phi_a} = 130$ GeV, $M_{\Phi_b} = 242$ GeV, and $M_{\Sigma_3} = M_Z$.

In fact, the maximal value of $M_{\rm GUT}$ at the two-loop level is somewhat larger than the one that corresponds to the benchmark point P1 in Fig. 1. Namely, $M_{\rm GUT} = 4.5 \times 10^{14}$ GeV for $M_{\Sigma_3} = M_{\Sigma_8} = M_Z$, $M_{\Phi_a} = 1.1 \times 10^4$ GeV, $M_{\Phi_b} = 242$ GeV, and $\alpha_{\rm GUT}^{-1} = 37.1$. With this set of values we can establish an accurate upper bound on the proton decay lifetime. In a model independent way the proton lifetime τ_p is bounded by the inequality [7]:

$$\tau_p \leq 6 \times 10^{39} \alpha_{\rm GUT}^{-2} (M_V/10^{16}\ {\rm GeV})^4 \times (0.003\ {\rm GeV}^3/\alpha)^2\ {\rm yrs}, \qquad (1)$$

where α is the matrix element, and M_V is a common mass of gauge bosons responsible for proton decay. For our purposes we set $\alpha = 0.015$ GeV^3 [8] and identify $M_V = M_{\rm GUT}$. (The main source of uncertainty in Eq. (1) comes from the matrix element α. For an up-to-date discussion on α see [9]. For a review on proton stability see [10].) Using the two-loop values of $\alpha_{\rm GUT}$ and $M_{\rm GUT}$ mentioned above we find $\tau_p^{(\text{two-loops})} \leq 1.4 \times 10^{36}$ yrs [3]. Clearly, our model could be tested or ruled out at the next generation of proton decay experiments [11].

Conversely, one can invert Eq. (1) to establish a lower limit on the GUT scale; recall, $M_{\rm GUT} = M_V$. If we take $\tau_p(p \to \pi^0 e^+) > 5.0 \times 10^{33}$ yrs [12] as experimental input, and assume $\alpha_{\rm GUT}^{-1} \simeq 37$ the region below $M_{\rm GUT} = 1.1 \times 10^{14}$ GeV is excluded by proton decay. This limit defines our benchmark point P3 in Fig. 1. Once we impose proton decay constraints on $M_{\rm GUT}$ we also get an upper bound on the scalar leptoquark mass from the benchmark point P3: $M_{\Phi_b} < 10^8$ GeV. In addition, if we consider the most natural implementation of the type II seesaw mechanism (large M_{Φ_a}) the mass of the scalar leptoquark Φ_b comes out in the phenomenologically interesting region $\mathcal{O}(10^2$–$10^3)$ GeV.

We have analyzed the unification scenario at three benchmark points P1 through P3 showed in Fig. 1. As we explained, these three points define the limits of the parameter space that yield unification of gauge couplings in the minimal realistic $SU(5)$ model at one loop. The particle content of the model also implies that the gauge coupling exhibits asymptotically free behavior between the GUT scale and the cutoff Λ. This feature is shown in Fig. 2 for the benchmark point P1. The value of $\alpha_{\rm GUT}$ at the scale M $(\Lambda > M > M_{\rm GUT})$ is given by: $\alpha_{\rm GUT}^{-1}(M) = \alpha_{\rm GUT}^{-1}(M_{\rm GUT}) + \frac{73}{12\pi} \ln\frac{M}{M_{\rm GUT}}$. In order to make any specific statements about the behavior of the gauge coupling at and above the cutoff the full structure of the more fundamental theory that represents the ultraviolet completion of our model needs to be specified. As we comment towards the end, there exists a well-defined underlying theory that

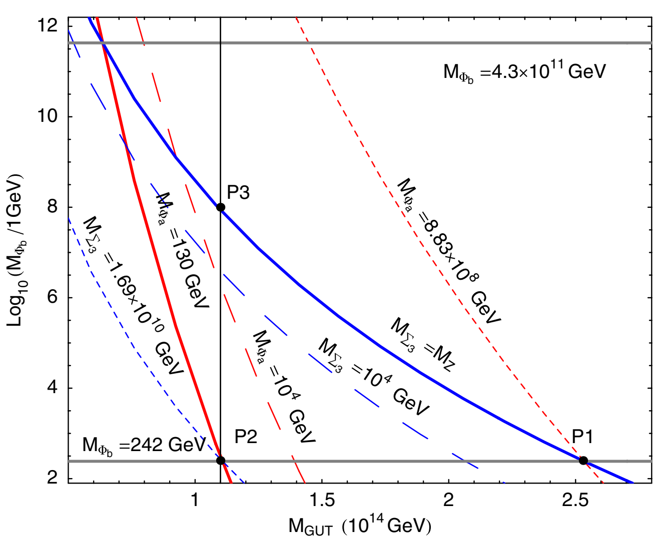

FIG. 1 (color online). The whole parameter space is shown where we can achieve gauge coupling unification. The points P1, P2, and P3 define the allowed region, and the corresponding boundary values for the masses of Φ_a, Φ_b, and Σ_3 are shown.

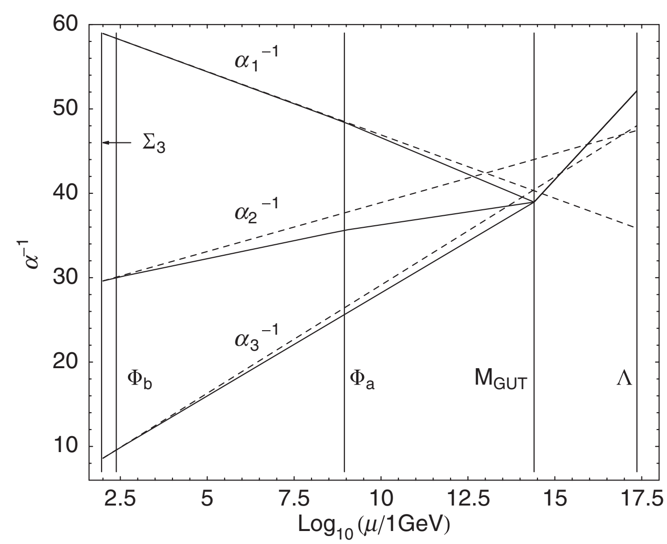

FIG. 2. One-loop gauge coupling unification at benchmark point P1 in the minimal realistic $SU(5)$ model (solid line) in comparison with the SM (dashed line). Notice the asymptotic free behavior above the GUT scale.

reproduces our model below the cutoff Λ which also supports the asymptotic behavior of the $SU(5)$ gauge coupling up to the Planck scale.

We have also shown that the minimal realistic $SU(5)$ predicts the existence of light fields. In the benchmark point P1 the Higgs field Σ_3 and the scalar leptoquark Φ_b are very light, while in the benchmark point P2 the lightest fields are Φ_a and Φ_b. Finally in the benchmark point P3 there are two light Higgses Φ_a and Σ_3. We can conclude then that the minimal realistic nonsupersymmetric $SU(5)$ model could be potentially tested at the next generation of collider experiments, particularly at the Large Hadron Collider (LHC) at CERN. The possibility to explain the baryon asymmetry in the Universe in this context has also been studied [3]. See also Ref. [13] for the possibility of probing second and third generation leptoquark parameter space with the IceCube neutrino detection facility. Since in principle all those fields could modify the predictions for the Yukawa couplings at the unification scale we will study their effect in the next section.

III. FERMION MASSES AND BOTTOM-TAU UNIFICATION

We shall investigate predictions for fermion masses in the framework of the minimal realistic $SU(5)$ model. The renormalization group equations (RGEs) for the Yukawa couplings are summarized in Appendix B. In order to compute the values of the Yukawa couplings at the GUT scale we need accurate values for the fermion masses at the electroweak scale as initial conditions for the RGEs. As a first approximation it is sufficient to consider the third generation only, and neglect the Yukawa couplings of the first and second generations in the RGE evolution. For future applications however we shall update the values of the fermion masses at the M_Z scale for all the three generations.

As input value for the top quark mass we take the latest world average from the Tevatron Electroweak Working Group [14], and extract the top quark running mass assuming that this value corresponds to the pole mass:

$$\frac{M_t}{m_t(M_t)} = 1 + \frac{4}{3}\frac{\alpha_s(M_t)}{\pi} + 10.95\left(\frac{\alpha_s(M_t)}{\pi}\right)^2 + \mathcal{O}(\alpha_s^3). \tag{2}$$

For the bottom quark mass we will adopt a conservative value at low energies $m_b(m_b) = 4.20 \pm 0.10$ GeV. This value is compatible through QCD evolution with the experimental measurement at higher energies [15], and agrees with most of the low-energy determinations [12]. Then, to obtain the initial conditions for Eq. (B6) the top and the bottom quark masses are evolved to the M_Z scale at three loops [16,17] with $\alpha_s(M_Z) = 0.1176 \pm 0.0020$ [12]. We also extract the values of the other quarks from the PDG [12] and calculate them at M_Z using three-loop RGE evolution in QCD accounting for the matching conditions over the heavy quark thresholds [16,17].

The physical lepton masses are taken also from the PDG [12]. The corresponding running masses are calculated at the M_Z scale through the relation:

$$m_l(M_Z) = M_l\left[1 - \frac{\alpha(M_Z)}{\pi}\left(1 + \frac{3}{4}\ln\frac{M_Z^2}{M_l^2}\right)\right] + \mathcal{O}(\alpha^2), \tag{3}$$

with $\alpha(M_Z)^{-1} = 127.906 \pm 0.019$. The input values for all the fermion masses and the corresponding values at the electroweak scale are summarized in Table I. These values at M_Z substantially defer and represent better reflection of our current knowledge of fermion masses from the values first evaluated in Ref. [18] and later updated in Ref. [19].

Let us now study the predictions for fermion masses at the GUT scale. The leptoquark Φ_b contributes at one loop to the running of the Yukawa couplings for charged leptons and down quarks, while the field Φ_a modifies the RGEs for charged leptons. The field Σ_3 does not couple to matter, only to the SM Higgs. It contributes to the renormalization of the mass and the couplings of the SM Higgs but does not modify the RGEs of the Yukawa couplings. The equations for the running of the Yukawa matrices Y_E and Y_D are given by:

TABLE I. Input parameters for the fermion masses and their values at the M_Z scale. Capital M denotes pole masses, while $m(\mu)$ are running masses.

	Input value	Running mass at M_Z
t	$M_t = 171.4 \pm 2.1$ GeV	$m_t(M_Z) = 170.3 \pm 2.4$ GeV
b	$m_b(m_b) = 4.20 \pm 0.10$ GeV	$m_b(M_Z) = 2.89 \pm 0.11$ GeV
c	$m_c(m_c) = 1.25 \pm 0.09$ GeV	$m_c(M_Z) = 0.63 \pm 0.08$ GeV
s	$m_s(2\text{ GeV}) = 95 \pm 25$ MeV	$m_s(M_Z) = 56 \pm 16$ MeV
u	$m_u(2\text{ GeV}) = 2.3 \pm 0.8$ MeV	$m_u(M_Z) = 1.4 \pm 0.5$ MeV
d	$m_d(2\text{ GeV}) = 5.0 \pm 2.0$ MeV	$m_d(M_Z) = 3.0 \pm 1.2$ MeV
τ	$M_\tau = 1776.99^{+0.29}_{-0.26}$ MeV	$m_\tau(M_Z) = 1746.45^{+0.29}_{-0.26}$ MeV
μ	$M_\mu = 105.658\,369\,2(94)$ MeV	$m_\mu(M_Z) = 102.728\,99(44)$ MeV
e	$M_e = 0.510\,998\,918(44)$ MeV	$m_e(M_Z) = 0.486\,661\,3(36)$ MeV

$$16\pi^2 \frac{dY_D}{d\ln\mu} = Y_D \beta_D^{\rm SM} + Y_\nu Y_\nu^\dagger Y_D \Theta(\mu - M_{\Phi_b}),$$
$$16\pi^2 \frac{dY_E}{d\ln\mu} = Y_E\bigg(\beta_E^{\rm SM} + \frac{3}{4} Y_\nu^\dagger Y_\nu \Theta(\mu - M_{\Phi_a}) + \frac{3}{2} Y_\nu^\dagger Y_\nu \Theta(\mu - M_{\Phi_b})\bigg), \tag{4}$$

where $\beta_i^{\rm SM}$ are the SM beta coefficients (see Eq. (B1)). The RGE of the up quark Yukawas is not modified with respect to the SM. The contributions of the fields Φ_a and Φ_b are due to the operator $Y_\nu \bar{5}\,\bar{5}\,15_H$ of the Yukawa potential that generates the interactions $Y_\nu \bar{l}_L^C \Phi_a l_L$ and $Y_\nu \bar{d}_R \Phi_b l_L$. In the above equation the contribution of the field Φ_a has been taken from Ref. [20]. Notice that usually in $SU(5)$ theories there is no relationship between the Yukawa couplings of neutrinos and charged fermions.

Let us analyze the scenario where the contributions of the field Φ_a and the leptoquark Φ_b have been neglected in Eq. (4). Notice that at one loop the mass of the field Φ_a in all the parameter space is much below the natural value for the seesaw mechanism ($M_{\Phi_a} = 10^{13}$–10^{14} GeV), and the Yukawa couplings for neutrinos are expected to be small. In Fig. 3 we show the one-loop evolution of the top, bottom, and tau masses, respectively, in comparison with the SM for the benchmark point P1. Since in this case we have neglected the neutrino Yukawa contributions, the RGEs in the minimal realistic $SU(5)$ model are the same as in the SM (Eq. (B6)), and the evolution of the charged fermion masses in that model is modified with respect to the SM only through the change in the evolution of the gauge couplings. Consequently, we obtain values of the bottom and tau Yukawa couplings at the GUT scale that are only 1%–2% away from the SM prediction. A similar situation will happen for the other two benchmark points. Thus, independently of the benchmark point the Yukawa coupling of the bottom quark will lie at high energies below the Yukawa coupling of the tau lepton, and therefore it is not possible to achieve unification of Y_τ and Y_b.

If we include the contributions of Φ_a and/or the leptoquark Φ_b in the running the difference between Y_τ and Y_b at the GUT scale will be increased. The reason is that both contributions are positive, and the coefficient of the neutrino Yukawa coupling in the RGE for Y_E is larger than that for Y_D. This will result into a larger tau Yukawa coupling, and as we will see in the next section into a smaller UV cutoff. In order to obtain a conservative upper bound on the UV cutoff we will focus our analysis on the first scenario where the extra contributions of the fields Φ_a and Φ_b are neglected.

IV. THE UV CUTOFF OF THE THEORY AND PROTON DECAY

In this section we study the possibility to establish an upper bound on the UV cutoff of the minimal realistic $SU(5)$ model. The relevant Yukawa potential up to order $1/\Lambda$ is defined by [2]:

$$V_{\rm Yukawa} = \epsilon_{ijklm}\bigg(10_a^{ij} Y_{ab} 10_b^{kl} 5_H^m + 10_a^{ij} Y_{ab}^{(1)} 10_b^{kl} \frac{(24_H)_n^m}{\Lambda} 5_H^n + 10_a^{ij} Y_{ab}^{(2)} 10_b^{kn} 5_H^l \frac{(24_H)_n^m}{\Lambda}\bigg) + 5_{Hi}^* 10_a^{ij} Y_{ab}^{(3)} \bar{5}_{bj} + 5_{Hi}^* \frac{(24_H)_j^i}{\Lambda} 10_a^{jk} Y_{ab}^{(4)} \bar{5}_{bk} + 5_{Hi}^* 10_a^{ij} Y_{ab}^{(5)} \frac{(24_H)_j^k}{\Lambda} \bar{5}_{bk} + \bar{5}_{ai} Y_{ab}^{(6)} \bar{5}_{bj} 15_H^{ij} + \bar{5}_{ai} Y_{ab}^{(7)} \bar{5}_{bj} \frac{5_H^i 5_H^j}{\Lambda}, \tag{5}$$

where i, j, k, l, m, and n are the $SU(5)$ indices, while a, b, and c are the family indices. Once 24_H gets a vacuum expectation value (VEV) $\langle 24_H \rangle = \sigma {\rm diag}(2, 2, 2, -3, -3)$ the GUT symmetry $SU(5)$ is broken to the SM gauge symmetry. Then, the Yukawa couplings for charged fermions read [21]:

$$Y_U = 4(Y + Y^T) - 12\frac{\sigma}{\Lambda}(Y^{(1)} + Y^{(1)T}) - 2\frac{\sigma}{\Lambda}(4Y^{(2)} - Y^{(2)T}), \tag{6}$$

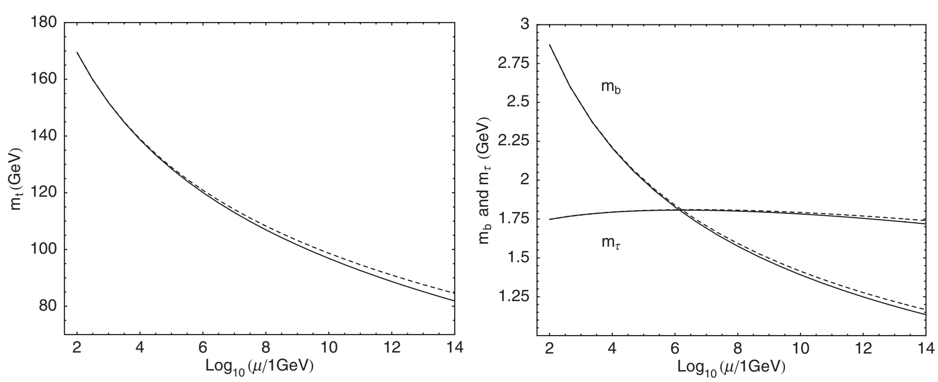

FIG. 3. One-loop evolution of the masses of the third generation in the benchmark point P1 of the minimal realistic $SU(5)$ model (solid line) and in the SM (dashed line).

$$Y_D = -Y^{(3)} + \frac{\sigma}{\Lambda}(3Y^{(4)} - 2Y^{(5)}), \tag{7}$$

$$Y_E = -Y^{(3)} + 3\frac{\sigma}{\Lambda}(Y^{(4)} + Y^{(5)}). \tag{8}$$

For neutrino masses, on the other hand, we find:

$$M_\nu = Y^{(6)}\langle\delta^0\rangle + Y^{(7)}\frac{\langle H^0\rangle^2}{\Lambda}, \tag{9}$$

where $\langle\delta^0\rangle$ and $\langle H^0\rangle$ are the VEVs of the neutral components of Φ_a and H, respectively. The Yukawa couplings for charged fermions are diagonalized as follows: $U^T Y_U U_C = Y_U^{\mathrm{diag}}$, $D^T Y_D D_C = Y_D^{\mathrm{diag}}$, and $E_C^T Y_E E = Y_E^{\mathrm{diag}}$.

Using the relevant relations for the Yukawa couplings we find:

$$Y_E - Y_D = Y^{(5)}\frac{1}{\sqrt{2\pi\alpha_{\mathrm{GUT}}}}\frac{M_V}{\Lambda}, \tag{10}$$

where we use $\sigma = \lambda/\sqrt{30}$ and $M_V = M_{\mathrm{GUT}} = \sqrt{\frac{5}{12}}\lambda g_{\mathrm{GUT}}$ with $g_{\mathrm{GUT}}^2 = 4\pi\alpha_{\mathrm{GUT}}$. For the physical Yukawa couplings the relation reads:

$$Y_E^{\mathrm{diag}} = E_C^T D^* Y_D^{\mathrm{diag}} D_C^\dagger E + E_C^T Y^{(5)} E \frac{M_V}{\sqrt{2\pi\alpha_{\mathrm{GUT}}}\Lambda}. \tag{11}$$

If we require that the theory remains perturbative at the GUT scale: $|c_{kk}| = |(E_C^T Y^{(5)} E)_{kk}|/\sqrt{4\pi} \le 1$; hence the upper bound on Λ can be parametrized as:

$$\Lambda \le \sqrt{\frac{2}{\alpha_{\mathrm{GUT}}}} \times \frac{M_{\mathrm{GUT}}}{|Y_\tau - a^{3i}Y_D^i|}. \tag{12}$$

Here $a^{ki} = (E_C^T D^*)^{ki}(D_C^\dagger E)^{ik}$. We have set $k = 3$ because in that case one finds the strongest upper bound. Notice that when $a^{3i} > 0$ ($a^{3i} < 0$) and real, the maximal value of the denominator in Eq. (12) is $Y_\tau - Y_d$ ($Y_\tau + Y_b$). Since the GUT scale in our model is very low we have to be sure that it is possible to satisfy the experimental constraints on the proton decay lifetime. In particular, before defining the upper bound on the UV cutoff we have to determine which is the pattern of the a^{ki} coefficients that is in agreement with the constraints set by nucleon decay.

Let us discuss the relation between the upper bound on Λ and proton decay. It was pointed out in Ref. [7] that in order to find the upper bound on the proton decay lifetime in the context of the minimal realistic $SU(5)$ model the following conditions have to be fulfilled:

$$E_C = DB_1, \qquad D_C = EB_2, \tag{13}$$

where

$$B_j = \begin{pmatrix} 0 & 0 & e^{i\alpha_j} \\ 0 & e^{i\beta_j} & 0 \\ e^{i\gamma_j} & 0 & 0 \end{pmatrix}. \tag{14}$$

In this case we have the maximal suppression for the proton decay channels. Now, using the above relations we get $c_{ii} = (B_1^T D^T Y^{(5)} E)_{ii}/\sqrt{4\pi}$ and the upper bound on Λ (neglecting the phases) is given by:

$$\Lambda \le \sqrt{\frac{2}{\alpha_{\mathrm{GUT}}}} \times \frac{M_{\mathrm{GUT}}}{|Y_\tau - Y_d|}. \tag{15}$$

Notice that this is the bound which corresponds to the case $a^{3i} > 0$. By definition it is consistent with fermion masses and proton decay simultaneously. Therefore, using the values of the Yukawa couplings at the GUT scale, the most conservative upper bound on Λ is coming from the scenario when we have maximal suppression of the proton decay channels. It is important to say that usually the cutoff of a GUT model is evaluated from the difference between Y_b and Y_τ [22]. However, here we have shown that the most conservative upper bound on the UV cutoff, as given by Eq. (15), is defined by the difference between Y_τ and Y_d, and not by the departure from $b - \tau$ unification.

In the previous section we have already addressed the issue of numerical values of Yukawa couplings at the GUT scale. Since the value of the tau Yukawa coupling is almost independent of the benchmark point, the actual bound on the cutoff simply depends on the ratio $M_{\mathrm{GUT}}/\sqrt{\alpha_{\mathrm{GUT}}}$. We hence summarize in Table II the values of the masses of the fields Φ_a, Φ_b, and Σ_3 at each benchmark point and the corresponding values of α_{GUT}, M_{GUT}, $Y_\tau(M_{\mathrm{GUT}})$, and the ultraviolet cutoff Λ. Notice that the most conservative upper bound on the cutoff of the theory at one-loop level is $\Lambda^{\mathrm{upper}} \simeq 10^{17}$ GeV, which could be identified with the string unification scale [23].

We briefly investigate how our analysis holds at the two-loop level. In Fig. 1, we show our results for the scenario which corresponds to the benchmark point P1. To insure the proper inclusion of boundary conditions [24] at M_{GUT} we set $\alpha_i^{-1}|_{\mathrm{GUT}} = \alpha_{\mathrm{GUT}}^{-1} - \lambda_i/(12\pi)$, where $\{\lambda_1, \lambda_2, \lambda_3\} = \{5, 3, 2\}$. (In addition to the central values given in Table I we use Cabibbo-Kobayashi-Maskawa (CKM) angles and phases from [12] and $\alpha_3(M_Z) = 0.1176 \pm 0.0020$,

TABLE II. UV cutoff for the three benchmark points. All the mass scales are in GeV.

Benchmark point	M_{Φ_a}	M_{Φ_b}	M_{Σ_3}	$\alpha_{\mathrm{GUT}}^{-1}$	M_{GUT}	$Y_\tau(M_{\mathrm{GUT}})$	Λ
P1	8.83×10^8	242	M_Z	38.9	2.53×10^{14}	0.0098	2.3×10^{17}
P2	130	242	1.69×10^{10}	38.1	1.11×10^{14}	0.0098	1.0×10^{17}
P3	6.92×10^4	8.68×10^7	M_Z	38.7	1.10×10^{14}	0.0099	9.8×10^{16}

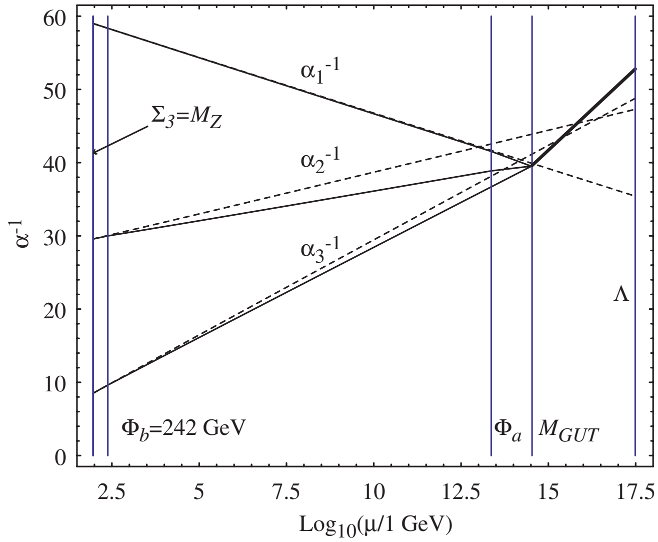

FIG. 4 (color online). The gauge coupling unification at the two-loop level for central values of low-energy observables. The two-loop SM running is presented by dashed lines. Solid lines correspond to the benchmark scenario P1 with Σ_3, Φ_b, and Φ_a below the GUT scale. Vertical lines mark the relevant scales: $M_{\Sigma_3} = M_Z$, $M_{\Phi_b} = 242$ GeV, $M_{\Phi_a} = 2.3 \times 10^{13}$ GeV, $M_{\rm GUT} = 3.4 \times 10^{14}$ GeV, and $\Lambda = 3.1 \times 10^{17}$ GeV.

$\alpha_2(M_Z) = 0.033\,816 \pm 0.000\,027$, and $\alpha_1(M_Z) = 0.016\,949 \pm 0.000\,005$ as our input.)

Comparison between Figs. 2 and 4 shows that the GUT scale is slightly larger in the two-loop case. On the other hand, Φ_a is significantly larger than in the one-loop case. And, we get $Y_\tau = 0.009\,86$ and accordingly $\Lambda = 3.1 \times 10^{17}$ GeV for the benchmark point P1, slightly above the one-loop result.

Before we summarize our results, let us address the issue of the possible origin of higher-dimensional operators we invoke in Eq. (5). After all, they play a decisive role in making our $SU(5)$ model realistic. Namely, they affect the Yukawa sector in a way that simultaneously allows for realistic charged fermion masses and efficient suppression of the gauge mediated proton decay. In particular, they modify the GUT scale relation between Y_E and Y_D that basically rules out the GG model. And, they violate another prediction of the GG model, i.e., $Y_U = Y_U^T$, which would prevent one from completely suppressing the gauge boson mediated proton decay [7]. In fact, our considerations of these modifications and proton decay constraints resulted in the upper bound on the cutoff Λ.

As we have shown, the cutoff comes out to be significantly below the Planck scale. This means that we cannot resort to the Planck scale effects to generate necessary higher-dimensional operators. It is then natural to ask for the credible renormalizable model that would effectively mimic the original proposal below the cutoff. To this end we observe that the most minimal renormalizable setup that yields the original model requires introduction of the following two matter pairs: $(5, \bar{5})$ and $(10, \overline{10})$. These pairs can clearly have gauge invariant mass terms above the GUT scale that can be identified with the scale Λ. Once these fields are integrated out the effective model below Λ would have exactly the same features as the original model considered in this paper. For example, the relevant operators that eventually modify $Y_E = Y_D$ relation are $a_i 10_i 5_H^\dagger \bar{5}$ and $b_i \bar{5}_i 24_H 5$. $(10, \overline{10})$ pair is needed to modify $Y_U = Y_U^T$. Clearly, such a simple renormalizable realization of the model would also imply asymptotic freedom of the $SU(5)$ gauge coupling between Λ and the Planck scale.

V. SUMMARY

We have investigated the predictions for fermion masses in the minimal realistic nonsupersymmetric grand unified model with the SM matter content based on $SU(5)$ gauge symmetry. We have shown that it is not possible to achieve $b - \tau$ unification in this context since the extra contributions to the running of the Yukawa couplings are always positive and larger for Y_E. We pointed out that the upper bound on the ultraviolet cutoff of the theory is $\Lambda^{\rm upper} \simeq 10^{17}$ GeV which is consistent with the predictions for Yukawa couplings and the constraints coming from proton decay. In addition, we have provided up-to-date values of all the fermions masses at the electroweak scale.

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APPENDIX A: RGES OF THE GAUGE COUPLINGS AT TWO LOOPS

The relevant equations for the running of the gauge couplings at the two-loop level take the form

$$\frac{d\alpha_i(\mu)}{d\ln\mu} = \frac{b_i}{2\pi}\alpha_i^2(\mu) + \frac{1}{8\pi^2}\sum_{j=1}^{3} b_{ij}\alpha_i^2(\mu)\alpha_j(\mu) + \frac{1}{32\pi^3}\alpha_i^2(\mu)\sum_{l=U,D,E}\mathrm{Tr}[C_{il}Y_l^\dagger Y_l]. \tag{A1}$$

The general formula for b_i and b_{ij} coefficients is given in [25]. Besides the well-known SM coefficients we have:

$$b_i^{\Sigma_3} = \begin{pmatrix} 0 \\ \frac{1}{3} \\ 0 \end{pmatrix}, \qquad b_i^{\Sigma_8} = \begin{pmatrix} 0 \\ 0 \\ \frac{1}{2} \end{pmatrix}, \qquad b_i^{\Phi_b} = \begin{pmatrix} \frac{1}{30} \\ \frac{1}{2} \\ \frac{1}{3} \end{pmatrix},$$

$$b_i^{\Phi_a} = \begin{pmatrix} \frac{3}{5} \\ \frac{2}{3} \\ 0 \end{pmatrix}, \qquad b_{ij}^{\Sigma_3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{28}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$b_{ij}^{\Sigma_8} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 21 \end{pmatrix}, \qquad b_{ij}^{\Phi_b} = \begin{pmatrix} \frac{1}{150} & \frac{3}{10} & \frac{8}{15} \\ \frac{1}{10} & \frac{13}{2} & 8 \\ \frac{1}{15} & 3 & \frac{22}{3} \end{pmatrix},$$

$$b_{ij}^{\Phi_a} = \begin{pmatrix} \frac{108}{25} & \frac{72}{5} & 0 \\ \frac{24}{5} & \frac{56}{3} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

which we incorporate at the appropriate scales. The C_{il} coefficients are [26]:

$$C_{il} = \begin{pmatrix} \frac{17}{10} & \frac{1}{2} & \frac{3}{2} \\ \frac{3}{2} & \frac{3}{2} & \frac{1}{2} \\ 2 & 2 & 0 \end{pmatrix}.$$

Obviously, Yukawa couplings enter the gauge coupling running at the two-loop level. Thus, one needs to run them as well at the one-loop level for consistency. We use the SM one-loop equations for the Yukawa couplings that can be found, for example, in Ref. [26].

APPENDIX B: RGES OF THE YUKAWA COUPLINGS

The beta functions for the evolution of the Yukawa couplings in the SM are given by

$$\begin{aligned} \beta_U^{\rm SM} &= T - G_U + \tfrac{3}{2}(Y_U^\dagger Y_U - Y_D^\dagger Y_D), \\ \beta_D^{\rm SM} &= T - G_D + \tfrac{3}{2}(Y_D^\dagger Y_D - Y_U^\dagger Y_U), \\ \beta_E^{\rm SM} &= T - G_E + \tfrac{3}{2}Y_E^\dagger Y_E, \end{aligned} \tag{B1}$$

where

$$T = \mathrm{Tr}(Y_E^\dagger Y_E + 3Y_U^\dagger Y_U + 3Y_D^\dagger Y_D), \tag{B2}$$

and

$$\begin{pmatrix} G_U \\ G_D \\ G_E \end{pmatrix} = \begin{pmatrix} \frac{17}{20} & \frac{9}{4} & 8 \\ \frac{1}{4} & \frac{9}{4} & 8 \\ \frac{9}{4} & \frac{9}{4} & 0 \end{pmatrix} \begin{pmatrix} g_1^2 \\ g_2^2 \\ g_3^2 \end{pmatrix}. \tag{B3}$$

It is convenient [27] to define the following matrices:

$$M_E = Y_E^\dagger Y_E, \qquad M_D = Y_D^\dagger Y_D, \qquad M_U = Y_U^\dagger Y_U, \tag{B4}$$

whose diagonalization is given by

$$\begin{aligned} E^\dagger M_E E &= \mathrm{diag}(Y_e^2, Y_\mu^2, Y_\tau^2), \\ D_C^\dagger M_D D_C &= \mathrm{diag}(Y_d^2, Y_s^2, Y_b^2), \\ U_C^\dagger M_U U_C &= \mathrm{diag}(Y_u^2, Y_c^2, Y_t^2), \end{aligned} \tag{B5}$$

The CKM matrix is defined through $V^{\rm CKM} = U_C^\dagger D_C$. Taking into account that $AA^\dagger = 1$, for $A = U, D, E$, one can derive RGEs for the diagonal elements of these matrices:

$$\begin{aligned} 4\pi \frac{d\alpha_i^U}{d\ln\mu^2} &= \alpha_i^U\Big[\bar{T} - \bar{G}_U + \frac{3}{2}\alpha_i^U - \frac{3}{2}\sum_j |V_{ij}^{\rm CKM}|^2 \alpha_j^D\Big], \\ 4\pi \frac{d\alpha_j^D}{d\ln\mu^2} &= \alpha_j^D\Big[\bar{T} - \bar{G}_D + \frac{3}{2}\alpha_j^D - \frac{3}{2}\sum_i |V_{ij}^{\rm CKM}|^2 \alpha_i^U\Big], \\ 4\pi \frac{d\alpha_i^E}{d\ln\mu^2} &= \alpha_i^E\Big[\bar{T} - \bar{G}_E + \frac{3}{2}\alpha_i^E\Big], \end{aligned} \tag{B6}$$

where $\bar{T} = T/(4\pi)$, $\bar{G}_l = G_l/(4\pi)$ and $\alpha_i^l = (Y_i^l)^2/(4\pi)$, with $i = 1, 2, 3$ the family index.

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