Unifying gauge couplings at the string scale

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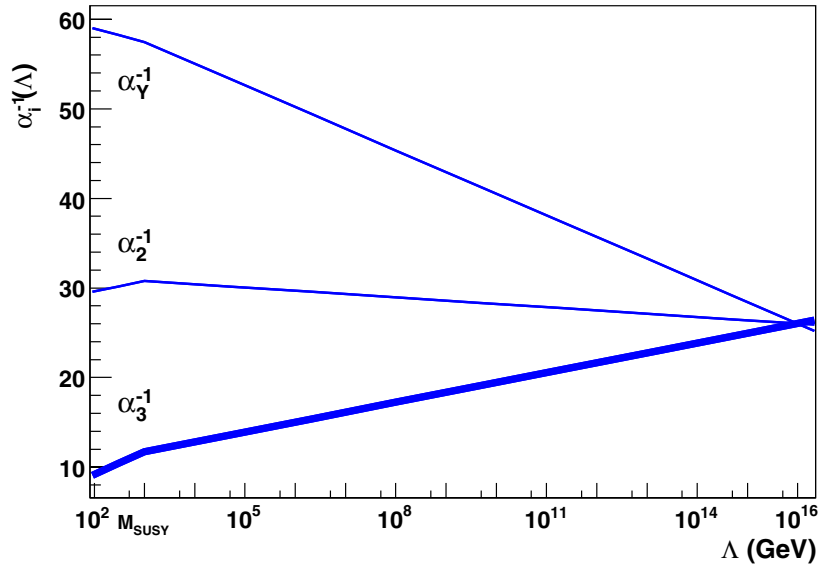E-mail: `david.costa@ist.utl.pt, gonzalez@cftp.ist.utl.pt`

Abstract. Using the current precision electroweak data, we look for the minimal particle content which is necessary to add to the standard model in order to have a complete unification of gauge couplings and gravity at the weakly coupled heterotic string scale. We find that the addition of a vector-like fermion at an intermediate scale and a non-standard hypercharge normalization are in general sufficient to achieve this goal at two-loop level. Requiring the extra matter scale to be below the TeV scale, it is found that the addition of three vector-like fermion doublets with a mass around 700 GeV yields a perfect string-scale unification, provided that the affine levels are $(k_Y , k_2 , k_3) = (13/3 , 1 , 2)$, as in the $SU(5) \times SU(5)$ string-GUT. Furthermore, if supersymmetry is broken at the unification scale, the Higgs mass is predicted in the range 125 GeV - 170 GeV, depending on the precise values of the top quark mass and $\tan \beta$ parameter.
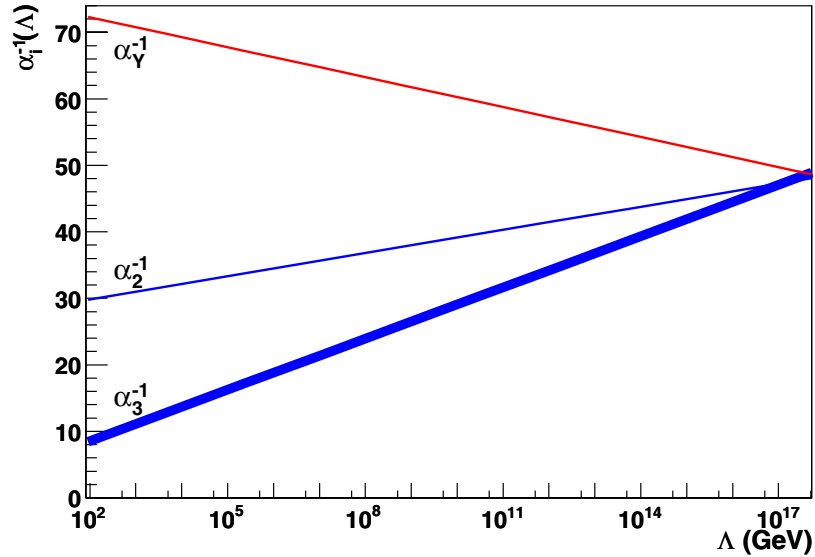
1. Introduction

Unification of gauge couplings has always been one of the few solid pieces of evidence in favor of supersymmetry. It is well known that the extrapolation of low-energy data within the framework of the MSSM yields an almost perfect unification of gauge couplings at the scale $\Lambda_{MSSM} \approx 2 \times 10^{16}$ GeV (see Figure 1), which is lower than the typical string scale, $\Lambda_S \gtrsim 10^{17}$ GeV. The resolution of this discrepancy has been the subject of many studies and several paths to unification have been proposed [1, 2, 3]. On the other hand, it is remarkable that, in the non-supersymmetric SM, the one-loop g_2 and g_3 gauge couplings already unify at a scale $\Lambda_{SM} \approx 10^{17}$ GeV (see Figure 2), which is close to the unification scale predicted by the string theory. In this case, gauge coupling unification could be achieved for a hypercharge normalization $k_Y \approx 13/10$ [1]. However, if two-loop effects are taken into account, the above scale should be at most $\Lambda_{SM} \approx 4 \times 10^{16}$ GeV [4], which is one order of magnitude smaller than the expected string scale. For high-scale supersymmetry breaking, it has been recently shown that gauge coupling unification can be achieved at about 2×10^{16} GeV in axion models with SM vector-like fermions [5], or at 10^{16-17} GeV in the SM with suitable normalizations of the $U(1)_Y$, which can be realized in specific orbifold GUTs [6]. Nevertheless, the unification scale in all of these cases is somehow below the expected string scale.

The phenomenology of $E_8 \times E_8$ heterotic string theory [7] exhibits many of the attractive features of the low-energy physics that we see today. In particular, the four-dimensional standard model (SM) gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ and its generations can be easily incorporated. String theory also offers an elegant explanation for the doublet-triplet splitting problem [8]. Moreover, the unification of gauge couplings and gravity is an intrinsic property of heterotic string theory. Remarkably, unification of couplings is a prediction of string theories

Figure 1. Gauge coupling running in the MSSM.

Figure 2. Gauge coupling running in the SM with $k_Y = 13/10$.

even without any grand unified theory (GUT) below the Planck scale. Indeed, gauge and gravitational couplings unify at tree level as [9]

$$\alpha_{\text{string}} = \frac{2\,G_N}{\alpha'} = k_i\,\alpha_i \,, \tag{1}$$

where $\alpha_{\text{string}} = g_{\text{string}}^2/4\pi$ is the string-scale unification coupling constant, G_N is the Newton constant, α' is the Regge slope, $\alpha_i = g_i^2/4\pi\,(i = Y, 2, 3)$ are the gauge couplings and k_i are the so-called affine or Kač-Moody levels at which the group factors $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ are

Table 1. Kač-Moody levels for several possible string-GUT models; $F = 1, 2, \ldots$ stands for the number of families in that particular model [10].

Group		k_Y	k_2	k_3
$SU(5)$, $SO(10)$, $SU(15)$, E_6, E_8, $[SU(3)]^3 \times Z_3$, $SU(16)$, $SU(8) \times SU(8)$, $SO(18)$	Canonical	5/3	1	1
$[SU(3)]^4 \times Z_4$		5/3	1	2
$SU(5) \times SU(5)$, $SO(10) \times SO(10)$		13/3	1	2
$[SU(6)]^3 \times Z_3$		14/3	3	1
$[SU(6)]^4 \times Z_4$		19/3	3	2
E_7		2/3	2	1
$[SU(4)]^3 \times Z_3$		11/3	1	1
$[SU(2F)]^3 \times Z_4$		(6F-4)/3	F	1
$[SU(2F)]^4 \times Z_4$		(9F-8)/3	F	2

realized in the four-dimensional string. The appearance of non-standard affine levels k_i plays an important role in string theories. While the non-Abelian factors k_2 and k_3 should be positive integers, the Abelian factor k_Y can take a priori any arbitrary value, only constrained to be $k_Y > 1$ for the right-handed electron to have a consistent hypercharge assignment. Furthermore, these factors determine the value of the mixing angle $\sin \theta_W$ at the string scale.

It may be possible that the string compactifies in four dimensions not to the SM group, but to a simple group which acts as a unified group. In this case $\Lambda_{\text{GUT}} = \Lambda_S$ and the Kač-Moody levels are fixed by the group structure,

$$k_i = \frac{\text{Tr } T^2}{\text{Tr } T_i^2},$$
(2)

where T is a generator of the subgroup G_i properly normalized over a representation R of the string-GUT group and T_i is the same generator but normalized over the representation of the subgroup embedded into R [10]. For illustration, the Kač-Moody levels for different possible string-GUT models are presented in Table 1.

Since string theory relates a dimensionless gauge coupling to a dimensionful gravitational coupling, Equation (1) itself predicts the unification scale $\Lambda = g_{\text{string}} M_P$, where $M_P = 1.22 \times 10^{19}$ GeV is the Planck mass. This scale is lowered by the inclusion of one-loop string

effects and in the weak coupling limit one finds [11]

$$\Lambda = g_{\text{string}}\Lambda_S \,, \tag{3}$$

where Λ_S is given by

$$\Lambda_S = \frac{e^{(1-\gamma)/2}\, 3^{-3/4}}{4\pi}\, M_P \approx 5.27 \times 10^{17} \text{ GeV}\,, \tag{4}$$

$\gamma \approx 0.577$ is the Euler constant. It has also been noted [12, 13] that the unification scale in the strong coupling limit can be much lower than the perturbative result given by Equation (3). Yet, it is not clear whether unification is a robust prediction in this case.

Next, using the current precision electroweak data, we study the problem of gauge coupling unification within string theory, with the aim to look for the minimal particle content which is necessary to add to the SM in order to achieve unification at the weakly coupled heterotic string scale [14].

2. One-loop analysis

The evolution of the gauge coupling constants at one loop is governed by the renormalization group equations (RGE)

$$\alpha_i^{-1}(\mu) = \alpha_{i\,Z}^{-1} - \frac{b_i}{2\pi}\, \log\frac{\mu}{M_Z}\,, \tag{5}$$

where $\alpha_{i\,Z} \equiv \alpha_i(M_Z)$ and the β-function coefficients b_i are given by

$$b_i = \frac{1}{3}\sum_R [\, s(R)\, N_i(R)\,] - \frac{11}{3}\, C_2(G_i)\,, \tag{6}$$

for non-supersymmetric theories. The function $s(R)$ is 1 for complex scalars, 2 for chiral fermions and 4 for vector-like fermions. The Casimir group invariant for the adjoint representation, $C_2(G_n)$, is n for $SU(n)$ groups and null for a $U(1)$ group. The functions $N_i(R)$ encode the group structure contributions as follows

$$N_i(R) = T_i(R)\prod_{j\neq i} d_j(R)\,, \tag{7}$$

where $d_i(R)$ is the dimension of the representation concerning the invariant subgroup G_i and $T_i(R)$ is the Dynkin index which, in our convention, is $1/2$ for the fundamental representations of $SU(n)$ groups and y^2 for the $U(1)_Y$ group. We use the convention that the hypercharge $Y = Q - T_{3L}$. In particular, for the SM with N generations and n_H complex Higgs doublets one finds

$$b_Y = \frac{20}{9}\, N + \frac{n_H}{6}\,, \quad b_2 = \frac{4}{3}\, N + \frac{n_H}{6} - \frac{22}{3}\,, \quad b_3 = \frac{4}{3}\, N - 11\,. \tag{8}$$

Let us now examine the one-loop running of the gauge couplings. The unified coupling constant α_{string} at the scale Λ is expressed in terms of the $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ gauge couplings and the corresponding affine levels k_i through Equations (1)-(4). Thus, at the unification scale Λ, Equation (5) implies

$$\alpha_{i\,Z}^{-1} = k_i\, \alpha_{\text{string}}^{-1} + \frac{b_i}{2\pi}\, \log\frac{\Lambda}{M_Z}\,, \tag{9}$$

with the additional constraint

$$\alpha_{\text{string}} = \frac{1}{4\pi}\left(\frac{\Lambda}{\Lambda_S}\right)^2\,, \tag{10}$$

which reflects the stringy nature of the unification.

These equations can be analytically solved to determine the scale Λ. We obtain

$$\left(\frac{\Lambda_S}{\Lambda}\right)^2 = -\frac{b_i}{16\pi^2 k_i} W_{-1}\left[-\left(\frac{4\pi \Lambda_S}{M_Z}\right)^2 \frac{k_i}{b_i} e^{-4\pi/(b_i \alpha_{iZ})}\right], \tag{11}$$

where $W_{-1}(x)$ is the $k = -1$ real branch of the Lambert W_k function [15].

In our numerical calculations we shall use the following electroweak input data at the Z boson mass scale $M_Z \simeq 91.2$ GeV [16, 17]:

$$\begin{aligned}
\alpha^{-1}(M_Z) &= 128.91 \pm 0.02\,, \\
\sin^2\theta_W(M_Z) &= 0.23120 \pm 0.00015\,, \\
\alpha_s(M_Z) &= 0.1182 \pm 0.0027\,,
\end{aligned} \tag{12}$$

for the fine structure constant α, the weak mixing angle θ_W and the strong coupling constant α_s, respectively. The top quark pole mass M_t^{pole} is taken as [18]

$$M_t^{\text{pole}} = 178.0 \pm 4.3 \text{ GeV}\,, \tag{13}$$

and the Higgs vacuum expectation value $v = 174.1$ GeV.

Using the SM coefficients b_i given in Equation (8) and assuming $k_2 = k_3 = 1$, we obtain from Equation (11), $\Lambda \approx 2.7 \times 10^{17}$ GeV, which in turn implies $\alpha_{\text{string}} = 0.021$. Substituting these values into Equation (9) we find $\alpha_s(M_Z) \simeq 0.1239$, a value which is clearly outside the experimental range given in Equation (12). The above result already indicates that the string-scale unification of gauge couplings requires either non-perturbative (or higher-order perturbative) string effects to lower the unification scale or extra matter particles to modify the RGE evolution of the gauge couplings. It is precisely the second possibility that we will consider here.

Anticipating a possible string-GUT compactification scenario, we shall restrict our analysis to the inclusion of fermions in real irreducible representations. The addition of chiral fermions leads in general to anomalies and their masses are associated to the electroweak symmetry breaking, which imposes further constraints. Also, the introduction of new light scalars requires additional fine-tunings. Thus, we shall consider the following fermionic states [19]:

$$\begin{aligned}
Q &= (\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{2})_{-1/6}\,, & L &= (\mathbf{1},\mathbf{2})_{-1/2} + (\mathbf{1},\mathbf{2})_{1/2}\,, \\[4pt]
U &= (\mathbf{3},\mathbf{1})_{2/3} + (\bar{\mathbf{3}},\mathbf{1})_{-2/3}\,, & D &= (\mathbf{3},\mathbf{1})_{-1/3} + (\bar{\mathbf{3}},\mathbf{1})_{1/3}\,, \\[4pt]
E &= (\mathbf{1},\mathbf{1})_{-1} + (\mathbf{1},\mathbf{1})_{1}\,, & X &= (\mathbf{3},\mathbf{2})_{-5/6} + (\bar{\mathbf{3}},\mathbf{2})_{5/6}\,, \\[4pt]
G &= (\mathbf{8},\mathbf{1})_{0}\,, & V &= (\mathbf{1},\mathbf{3})_{0}\,.
\end{aligned} \tag{14}$$

These can naturally appear in extensions of the SM as a part of some incomplete GUT multiplets. They are present, for instance, in the $\mathbf{5} + \bar{\mathbf{5}}$, $\mathbf{10} + \overline{\mathbf{10}}$ and $\mathbf{24}$ irreducible representations of $SU(5)$. The addition of such matter states gives corrections to the b_i coefficients in the gauge coupling running. Denoting by Δ_i these corrections, one has

$$\Delta_Y = \frac{2}{9} n_Q + \frac{2}{3} n_L + \frac{16}{9} n_U + \frac{4}{9} n_D + \frac{4}{3} n_E + \frac{50}{9} n_X\,, \tag{15}$$

$$\Delta_2 \;=\; 2\,n_Q + \frac{2}{3}\,n_L + 2\,n_X + \frac{4}{3}\,n_V \;, \tag{16}$$

$$\Delta_3 \;=\; \frac{4}{3}\,n_Q + \frac{2}{3}\,n_U + \frac{2}{3}\,n_D + \frac{4}{3}\,n_X + 2\,n_G \;, \tag{17}$$

where n_r denotes the number of multiplets belonging to the irreducible representations r given in Equation (14). The string unification conditions (9) also get modified,

$$\alpha_{i\,Z}^{-1} \;=\; k_i\,\alpha_{\text{string}}^{-1} \;+\; \frac{b_i}{2\pi}\,\log\frac{\Lambda}{M_Z} \;+\; \frac{\Delta_i}{2\pi}\,\log\frac{\Lambda}{M} \;, \tag{18}$$

where M is the new-physics threshold. Notice that we assume a common mass scale for the extra matter content, once we are interested in minimal scenarios which could lead to a successful unification. The solution of the above equations is now given by

$$\left(\frac{\Lambda_S}{\Lambda}\right)^2 \;=\; -\frac{1}{16\pi^2\,\rho}\,W_{-1}\!\left[-\left(\frac{4\pi\,\Lambda_S}{M_Z}\right)^2 \rho\,e^{-4\pi\,\eta}\right] \;, \tag{19}$$

for the unification scale and

$$\frac{M}{M_Z} \;=\; \left(\frac{\Lambda}{M_Z}\right)^{\rho'-1} e^{-2\pi\eta'} \;, \tag{20}$$

for the threshold, where

$$
\begin{aligned}
\rho &\equiv \frac{\Delta_3\,k_2 - \Delta_2\,k_3}{\Delta_3\,b_2 - \Delta_2\,b_3} \;, \quad &\eta &\equiv \frac{\Delta_3\,\alpha_{2\,Z}^{-1} - \Delta_2\,\alpha_{3\,Z}^{-1}}{\Delta_3\,b_2 - \Delta_2\,b_3} \;, \\[2mm]
\rho' &\equiv \frac{b_3\,k_2 - b_2\,k_3}{\Delta_3\,k_2 - \Delta_2\,k_3} \;, \quad &\eta' &\equiv \frac{k_3\,\alpha_{2\,Z}^{-1} - k_2\,\alpha_{3\,Z}^{-1}}{k_3\,\Delta_2 - k_2\,\Delta_3} \;.
\end{aligned}
\tag{21}
$$

Finally, the hypercharge normalization k_Y is determined from Equation (18):

$$k_Y \;=\; \alpha_{\text{string}}\left[\alpha_{1\,Z}^{-1} - \frac{b_Y}{2\pi}\,\log\frac{M}{M_Z} - \frac{\Delta_Y}{2\pi}\,\log\frac{\Lambda}{M}\right] \;. \tag{22}$$

Using Equations (19)-(22), it is straightforward to obtain all the possible solutions that lead to the string-scale unification of couplings at one-loop order. Here we present only those which are minimal, i.e. those which require the addition of a single extra particle with a mass scale M. The results are given in Table 2. There exist 3 minimal solutions, namely, $n_U = 1$, $n_D = 1$ and $n_G = 1$, which correspond to the addition of an up-type or down-type vector-like fermion or one gluino-type fermion, respectively, with quantum numbers as given in Equations (14). In all three cases the presence of a non-canonical hypercharge normalization, $k_Y \neq 5/3$, is required. We have taken the non-Abelian affine levels k_2 and k_3 to be equal to 1 or 2, which are the preferred values from the string-model building viewpoint [1]. We also notice that no minimal solution was found with $k_2 \neq k_3$.

3. Two-loop gauge coupling unification

To perform a more precise analysis of string unification, a two-loop RGE study becomes necessary. We make use of the two-loop RGEs of gauge couplings [20], which include the one-loop Yukawa coupling running and take properly into account the new physics contributions and

Table 2. Minimal extra matter content which leads to string-scale unification at one loop. The results for the new-physics threshold M, the unification scale Λ and the hypercharge affine level k_Y are presented for the central values given in Equation (12).

	$n_U = 1$		$n_D = 1$		$n_G = 1$	
	$k_{2,3} = 1$	$k_{2,3} = 2$	$k_{2,3} = 1$	$k_{2,3} = 2$	$k_{2,3} = 1$	$k_{2,3} = 2$
$M\,(\text{GeV})$	6.8×10^{15}	1.3×10^{15}	6.8×10^{15}	1.3×10^{15}	7.9×10^{16}	5.8×10^{16}
$\Lambda\,(\text{GeV})$	2.7×10^{17}	3.8×10^{17}	2.7×10^{17}	3.8×10^{17}	2.7×10^{17}	3.8×10^{17}
k_Y	1.24	2.44	1.26	2.49	1.26	2.50

Table 3. Minimal solutions which lead to string-scale unification at two-loop order. We use the central values for the electroweak input data given in Equations (12) and (13).

	$n_U = 1$		$n_D = 1$		$n_G = 1$	
	$k_{2,3} = 1$	$k_{2,3} = 2$	$k_{2,3} = 1$	$k_{2,3} = 2$	$k_{2,3} = 1$	$k_{2,3} = 2$
$M\,(\text{GeV})$	7.2×10^{12}	1.5×10^{12}	7.1×10^{12}	1.4×10^{12}	8.2×10^{15}	6.1×10^{15}
$\Lambda\,(\text{GeV})$	2.7×10^{17}	3.8×10^{17}	2.7×10^{17}	3.8×10^{17}	2.7×10^{17}	3.8×10^{17}
k_Y	1.20	2.35	1.25	2.47	1.26	2.50

threshold. In Table 3 we present the two-loop results for the minimal one-loop solutions given in Table 2. As in the one-loop case, no solution was found with $k_2 \neq k_3$.

It turns out that the unification scale Λ and the hypercharge normalization are not very sensitive to higher order corrections. This can be readily seen by comparing the one-loop results of Equations (19) and (22) with the two-loop values numerically obtained (see Table 3). On the other hand, the new-physics threshold M can be significantly altered by such corrections. In particular, we notice that while at one loop the solutions $n_U = 1$ and $n_D = 1$ require an intermediate scale of the order of $10^{15} - 10^{16}$ GeV, this scale is lowered to $10^{12} - 10^{13}$ GeV at two-loop order. One may ask whether such an intermediate mass scale could be naturally generated. In principle, it might be due to the possible presence of nonrenormalizable higher-order operators or could be associated with an approximate global symmetry, such as a chiral symmetry of Peccei-Quinn type.

We have also searched for minimal solutions where the new matter states have a mass scale below the TeV scale. Seven solutions were found, which are listed in Table 4. All of them require the non-Abelian affine levels to be $k_2 = 1$ and $k_3 = 2$. Of particular interest is the first solution with three vector-like fermion doublets, i.e. $n_Q = 3$. Not only it yields a perfect string-scale

Table 4. Minimal extra particle content with a mass below the TeV scale, which leads to unification at two-loop order. We use the electroweak input data given in Equations (12) and (13). The non-Abelian affine levels are $k_2 = 1$ and $k_3 = 2$ in all cases. The quantities in brackets reflect the effects of the $\alpha_s(M_Z)$ uncertainty.

	M (GeV)	Λ (GeV)	k_Y
$n_Q = 3$	$[\,653\,,\,823\,]$	5.2×10^{17}	$[\,4.27\,,\,4.37\,]$
$n_Q = 2\,,n_X = 1$	$[\,676\,,\,852\,]$	5.2×10^{17}	$[\,1.98\,,\,2.00\,]$
$n_Q = 2\,,n_V = 1$	$[\,459\,,\,587\,]$	4.6×10^{17}	$[\,3.37\,,\,3.42\,]$
$n_Q = 1\,,n_X = 1\,,n_V = 1$	$[\,475\,,\,607\,]$	4.6×10^{17}	$[\,1.60\,,\,1.61\,]$
$n_Q = 1\,,n_V = 2$	$[\,351\,,\,452\,]$	4.1×10^{17}	$[\,2.81\,,\,2.84\,]$
$n_X = 1\,,n_V = 2$	$[\,363\,,\,468\,]$	4.1×10^{17}	$[\,1.37\,,\,1.37\,]$
$n_V = 3$	$[\,283\,,\,367\,]$	3.8×10^{17}	$[\,2.43\,,\,2.44\,]$

unification at $g_{\text{string}} \approx 1$, but also, for $\alpha_s(M_Z) = 0.119$ and $M = 710$ GeV, it implies the hypercharge normalization $k_Y = 13/3$, thus suggesting an $SU(5) \times SU(5)$ or $SO(10) \times SO(10)$ string-GUT compactification [21, 10].

4. Higgs boson mass

In the string landscape [22], the supersymmetry breaking scale can be high and the SM (with, eventually, some residual matter content) is the simplest effective theory all the way down to low energies. In this scenario, the mass of the yet undiscovered Higgs boson appears to be the most relevant parameter. In general, supersymmetric models contain one pair of Higgs doublets H_u and H_d. The combination $\phi \equiv \sin\beta\, H_u - \cos\beta\, i\sigma_2 H_d^*$ is typically chosen as the fine-tuned SM Higgs doublet ϕ with a small mass term. If supersymmetry is broken at the string scale, the Higgs boson quartic coupling λ at the unification scale is then given by

$$\lambda(\Lambda) = \frac{1}{4}\left[\, g^2(\Lambda) + g'^{\,2}(\Lambda)\,\right]\cos^2 2\beta = \pi\,\alpha_{\text{string}}\left(\frac{1}{k_Y} + \frac{1}{k_2}\right)\cos^2 2\beta\,. \tag{23}$$
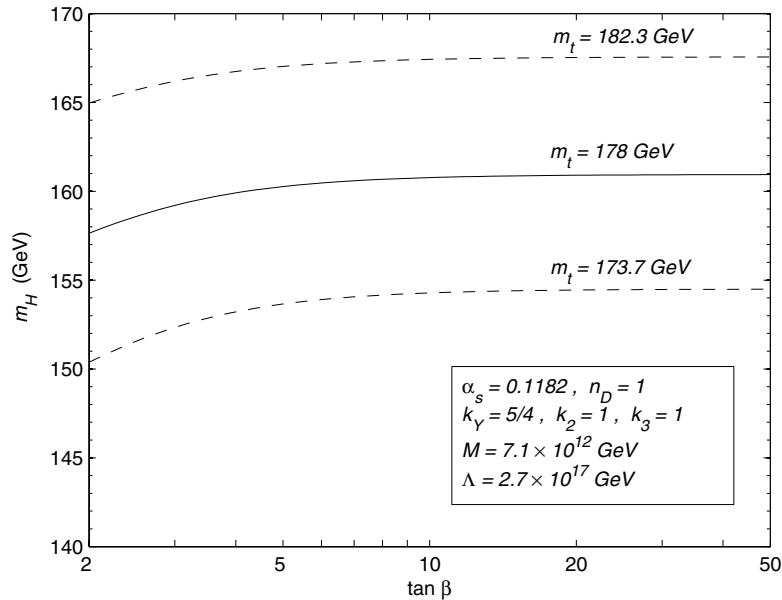
After evolving this coupling down to the electroweak scale, one can calculate the Higgs boson mass m_H by minimizing the one-loop effective potential,

$$V = -m^2\,(\phi^\dagger\phi) + \frac{\lambda}{2}\,(\phi^\dagger\phi)^2 + 3\,\alpha_t^2\,(\phi^\dagger\phi)^2\left[\log\frac{4\pi\,\alpha_t\,(\phi^\dagger\phi)}{Q^2} - \frac{3}{2}\right]\,, \tag{24}$$

which includes top quark radiative corrections. Here m^2 is the Higgs mass parameter, $\alpha_t = y_t^2/4\pi$ is the top quark coupling and the scale Q is chosen at $Q^2 = m_H^2$. The resulting Higgs mass can

Figure 3. The prediction for the Higgs boson mass in the SM extended with one down-type vector-like fermion. The predicted Higgs mass for the other two solutions given in Table 3 ($n_U = 1$ and $n_G = 1$) is similar to the one depicted in the figure.

be written in the following simple analytical form

$$m_H^2 \;=\; 12\,v^2\,\alpha_t^2\,W_0\left(\frac{\pi}{3\,\alpha_t}\,e^{\frac{\lambda}{6\,\alpha_t^2}}\right)\;, \tag{25}$$
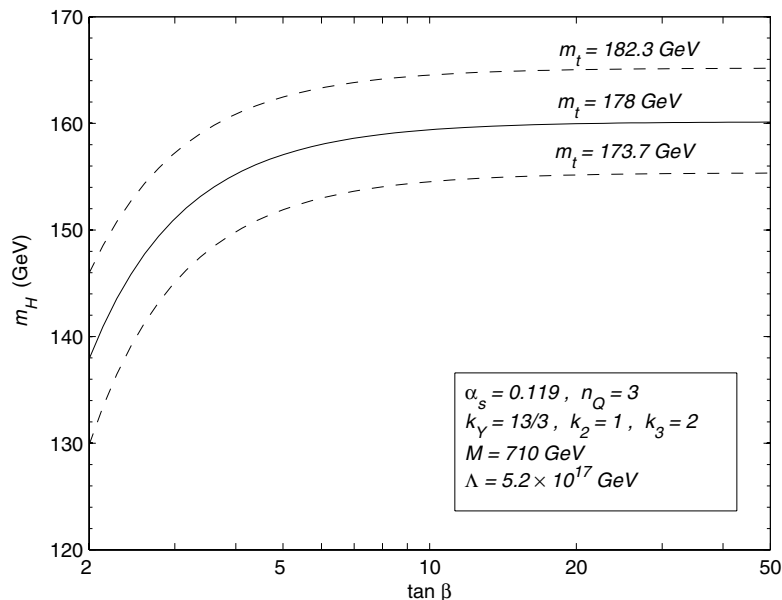
where $W_0(x)$ is the principal branch of the Lambert W function.

The predictions for the Higgs mass are presented in Figs. 3 and 4, for the minimal string unification solutions found in the previous section (cf. Tables 3 and 4). If we vary m_t within the 1σ range given in Equation (13) and $\tan\beta$ from 2 to 50, the predicted Higgs boson mass will range from 150 GeV to 167 GeV for the solutions $n_{U,D,G} = 1$, while for the solution $n_Q = 3$ the predicted mass varies in the range from 130 GeV to 165 GeV. If we take into account the presently allowed $\alpha_s(M_z)$ uncertainty, these intervals are slightly larger and we find $125 \text{ GeV} \lesssim m_H \lesssim 170 \text{ GeV}$. Future colliders will have the potential for the discovery of a Higgs boson with a mass in the above range [23].

5. Conclusion

String theory offers us a consistent framework for the unification of all the fundamental interactions including gravity. For a weakly coupled heterotic string, the unification scale is expected around 5×10^{17} GeV, which is too high to be achieved in the SM or MSSM, even with a non-canonical normalization of the hypercharge. A possible way to reconcile the GUT and string scales is the addition of new matter states to the particle spectrum. In this talk we have presented some minimal solutions based on the introduction of vector-like fermions. Working at two-loop order, three minimal solutions were found, which correspond to the presence at an intermediate scale of an up-type, down-type or gluino-type fermion with affine levels $k_2 = k_3 = 1$ and $k_Y \approx 6/5, 5/4, 63/50$, respectively.

Figure 4. The predicted Higgs boson mass in the SM extended with three vector-like fermion doublets.

Another interesting issue is the existence of new particles with masses relatively close to the electroweak scale. Imposing the new-physics threshold to be below the TeV scale, we have found several minimal solutions for string-scale unification. All of them require at least three new matter states. It is remarkable that the addition of three vector-like fermion doublets ($n_Q = 3$) yields unification at the string scale Λ_S for $(k_Y, k_2, k_3) = (13/3, 1, 2)$. These values are consistent with the affine levels of an $SU(5) \times SU(5)$ string-GUT (see Table 1). In this case, the strong coupling constant at the M_Z scale is $\alpha_s(M_Z) = 0.119$, with all the other electroweak input data given at their central values.

The string landscape allows for a high-scale supersymmetry breaking. If supersymmetry is broken at the string scale, most of its problems, such as fast dimension-five proton decay, excessive flavor and CP violation and stringent constraints on the Higgs mass, are avoided. In this scenario, the Higgs boson mass is predicted in the range 125 GeV $\lesssim m_H \lesssim 170$ GeV, for the minimal string unification solutions presented here.

Acknowledgments

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