Flavour physics of the RS model with KK masses reachable at LHC

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Abstract

The version of the higher-dimensional Randall-Sundrum (RS) model with matter in the bulk, which addresses the gauge hierarchy problem, has additional attractive features. In particular, it provides an intrinsic geometrical mechanism that can explain the origin of the large mass hierarchies among the Standard Model fermions. Within this context, a good solution for the gauge hierarchy problem corresponds to low masses for the Kaluza-Klein (KK) excitations of the gauge bosons. Some scenarios have been proposed in order to render these low masses (down to a few TeV) consistent with precision electroweak measurements. Here, we give specific and complete realizations of this RS version with small KK masses, down to 1 TeV, which are consistent with the entire structure of the fermions in flavour space: (1) all the last experimental data on quark/lepton masses and mixing angles (including massive neutrinos of Dirac type) are reproduced, (2) flavour changing neutral current constraints are satisfied and (3) the effective suppression scales of non-renormalizable interactions (in the physical basis) are within the bounds set by low energy flavour phenomenology. Our result, on the possibility of having KK gauge boson modes as light as a few TeV, constitutes one of the first theoretical motivations for experimental searches of direct signatures at the LHC collider, of this interesting version of the RS model which accommodates fermion masses.

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1 Introduction

The old idea of the existence of additional spatial dimensions [1, 2], a fundamental ingredient for string theories, has recently received a renewed interest due to several proposals for universal extra dimension models [3] (with all Standard Model fields propagating in extra dimensions), brane universe models [4]-[13] (with Standard Model fields living in our 3-dimensional spatial subspace) and intermediate models [14, 15, 16] (with gauge and Higgs bosons in the bulk, fermions being confined at fixed points along extra dimensions).

Amongst the brane models, the one suggested by Randall and Sundrum (RS) [12, 13], and its different extensions, has attracted particular attention. A considerable advantage of the RS scenario is that it addresses the so-called gauge hierarchy problem (i.e. the huge discrepancy between the gravitational and the electroweak scale) without introducing any new energy scale in the fundamental theory.

In a variant of the original RS set-up, which matured over the years [17]-[21], all the Standard Model (SM) particles except the Higgs boson (to ensure that the gauge hierarchy problem does not re-emerge) have been promoted to bulk fields rather than brane fields.

This RS version possesses the three following important phenomenological characteristics. First, unification of the gauge couplings at high scale is possible within a Grand Unified Theory (GUT) framework [22]-[26]. Secondly, from the cosmological point of view, there exists a viable Kaluza-Klein (KK) WIMP candidate [27] for the dark matter of universe [28, 29]. Finally, this RS version provides a totally new physical interpretation [30, 31] for the origin of the large mass hierarchy prevailing among all different flavours and types of SM fermions [1]. Such an interpretation of the whole SM fermion mass hierarchy is attractive, as it does not rely on the presence of any new symmetry in the short-distance theory, in contrast with the usual Froggatt-Nielsen mechanism [35] where a flavour symmetry is required. As a matter of fact, this interpretation is purely geometrical: it is based on the possibility of different localizations for SM fermions along extra dimension, depending on their flavour and type. In such a scenario, the quark masses and CKM mixing angles can be effectively accommodated [36, 37, 38], as well as the lepton masses and MNS mixing angles in both cases where neutrinos acquire Majorana masses (via either dimension five operators [39] or the see-saw mechanism [40]) and Dirac masses (see [41], and, [42] for order unity Yukawa couplings leading to mass hierarchies essentially generated by the higher-dimensional mechanism).

In the present article, we will elaborate concrete, complete and coherent realizations of the RS scenario, with bulk matter, which simultaneously: *(i)* reproduce all the last experimental data on quark/lepton masses and mixings (in the minimal case of Dirac neutrino masses where right handed neutrinos are added to SM fields) through the above geometrical mechanism *(ii)* satisfy the strongest Flavour Changing Neutral Current (FCNC) constraints (FCNC effects will be calculated, including new ones) for masses of the first KK gauge boson excitation down to $m_{KK} = 1\text{TeV}$ [2] *(iii)* generate non-renormalizable operator scales in agreement with low energy

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[1] Within the RS context, other higher-dimensional mechanisms [17, 32, 33, 34] apply specifically to neutrinos in order to explain their lightness compared to the rest of SM fermions.

[2] In our notation, $m_{KK} = m_{KK}^{(1)}(\gamma) = m_{KK}^{(1)}(gluon)$ is the mass of first KK excitation of the photon and gluon, $m_{KK}^{(1)}(Z^0) = (m_{KK}^2 + m_{Z^0}^2)^{1/2}$ and $m_{KK}^{(1)}(W^{\pm}) = (m_{KK}^2 + m_{W^{\pm}}^2)^{1/2}$.

phenomenology (a realistic analysis in the physical basis will be performed) *(iv)* are compatible with the model in reference [43] where the electroweak (EW) gauge symmetry is enhanced to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ so that the conditions issued from EW precision measurements can be fulfilled for $m_{KK} \gtrsim 3\text{TeV}$ *(v)* respect all the remaining constraints, e.g. the intrinsic theoretical bound on the curvature of the AdS_5 space.

In preliminary works [37, 41], where realizations of the RS scenario (with SM bulk fields) were constructed in order to simultaneously create correct SM fermion masses/mixings and obey the FCNC constraints, the mass of first KK excitation of gauge bosons was taken to be high: $m_{KK} = 10\text{TeV}$. In this way, the FCNC effects, due to the significant flavour dependence of fermion locations (needed to generate SM fermion mass hierarchies), were suppressed because in fact they are conveyed by the exchange of KK states of the gauge bosons (see Section 3 for details). Here, in contrast, we will show that the data on SM fermion masses/mixings can be compatible with FCNC bounds for m_{KK} values down to 1TeV.

Our result, of a conceivable light KK gauge boson excitation (1TeV), is important in the sense that it reopens the possibility, for the attractive version of the RS model generating SM fermion masses, to be tested at the Large Hadron Collider (LHC) [44] with a centre-of-mass energy at 14TeV. For such a test to be possible, a scenario should apply relaxing the severe EW precision constraints, e.g. the ones proposed in [43] or [45, 46, 47]. Assuming the scenario in [43] with a left-right gauge structure, one can expect to obtain some signatures of the RS model at LHC via KK gauge boson exchanges, since these KK states can be as light as $\sim 3\text{TeV}$ (limit from EW bounds).

Moreover, this result, i.e. the possibility of having low KK gauge field masses, is in favour of a good solution for the gauge hierarchy problem. Indeed, lower KK masses correspond typically to an effective gravity scale on our brane which is closer to the electroweak scale.

The organization of this paper is as follows. In Section 2, we describe consistent realizations of the RS scenario which generate the correct SM fermion mass hierarchies. Then in Section 3, the FCNC effects appearing in these RS realizations are computed and we show that those fulfil well the relevant experimental constraints for m_{KK} values down to 1TeV. Our method to obtain so small m_{KK} values remaining acceptable is also exposed there. Other experimental constraints, like those coming from precision EW data, are discussed in Section 4. In Section 5, we calculate the effective suppression scales of four dimensional operators in physical basis and demonstrate that, within the above RS realizations, the suppression scale values induce FCNC process amplitudes in agreement with experimental bounds. Finally, in Section 6, we summarize and discuss the impacts of our results.

2   Generation of mass hierarchies

2.1   The RS set-up

We consider the RS scenario with all SM fields residing in the bulk, except the Higgs boson which is confined on the TeV-brane (see below). Recall that the RS scenario consists of a 5-dimensional theory where the extra spatial dimension (denoted by y) is compactified over a $S^1/\mathbb{Z}_2$ orbifold with radius R_c ($-\pi R_c \leq y \leq \pi R_c$). Gravity

also propagates inside the bulk and the extra dimension is bordered by two 3-branes with tensions $\Lambda_{(y=0,\pi R_c)}$ (vacuum energy densities) tuned such that,

$$\Lambda_{(y=0)} = -\Lambda_{(y=\pi R_c)} = -\Lambda/k = 24kM_5^3, \tag{1}$$

Λ being the bulk cosmological constant, M_5 the fundamental 5-dimensional gravity scale and k a RS characteristic energy scale (see below). Within this framework, there exists a solution to the 5-dimensional Einstein's equations respecting 4-dimensional Poincaré invariance. It corresponds to a zero mode of the graviton localized on the positive tension brane (3-brane at $y = 0$) and to the non-factorisable metric:

$$ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \text{ with } \sigma(y) = k|y|, \tag{2}$$

where x^μ [$\mu = 1,\ldots,4$] are the coordinates for the familiar 4 dimensions and $\eta_{\mu\nu} = diag(-1,1,1,1)$ is the 4-dimensional flat metric. The bulk geometry, associated to this metric, is a slice of Anti-de-Sitter (AdS_5) space with curvature radius $1/k$.

We now discuss the energy scales that will be considered. While on the brane at $y = 0$ (Planck-brane) the effective gravity scale is equal to the (reduced) Planck mass: $M_{Pl} = 1/\sqrt{8\pi G_N} = 2.44 \ 10^{18}\text{GeV}$ ($G_N \equiv$ Newton constant), on the other brane at $y = \pi R_c$ (TeV-brane) the gravity scale,

$$M_\star = w \ M_{Pl}, \tag{3}$$

is suppressed by the exponential 'warp' factor $w = e^{-\pi k R_c}$. We see from Eq.(3) that for a small extra dimension $R_c \simeq 11/k$ (the k value being typically close to M_5), one finds $w \sim 10^{-15}$ so that $M_\star = \mathcal{O}(1)\text{TeV}$, thus solving the gauge hierarchy problem. For these values of R_c, M_5 is close to the effective 4-dimensional gravity scale M_{Pl}:

$$M_{Pl}^2 = \frac{M_5^3}{k}(1 - e^{-2\pi k R_c}). \tag{4}$$

From the phenomenological point of view, each one of the models in [43, 45, 46, 47], designed for softening the constraints from EW precision data, permits a value for m_{KK} that can be as small as $\sim 3\text{TeV}$. Hence, one can take a maximal m_{KK} value of 10TeV. This value corresponds to:

$$kR_c = 10.11 \tag{5}$$

Indeed, the maximal value of m_{KK} ($m_{KK} = 2.45 \ k \ e^{-k\pi R_c}$) is determined by the kR_c value and the theoretical bound (guarantying that the solution for the metric can be trusted) on the 5-dimensional curvature scalar R_5,

$$|R_5| = |-20k^2| < M_5^2, \tag{6}$$

which, together with relation (4), leads to: $k < 0.105 \ M_{Pl}$.

The value chosen in Eq.(5) gives rise to the effective gravity scale: $M_\star = 39.2\text{TeV}$ (see Eq.(3)). This energy scale is close to the electroweak symmetry breaking scale even if it is not exactly identical. Furthermore, in the context of model in reference [43] with a left-right gauge structure, the needed fine tuning on Higgs boson mass (having a dominant loop contribution coming from KK mode exchanges) is estimated to be of the order of 1% in the mass-squared.

2.2 SM fermion masses and mixings

● **5-dimensional masses:** In order to produce the SM fermion mass hierarchies via the higher-dimensional mechanism mentioned in Section 1, the SM (zero mode) fermions must possess different localizations along the extra dimension. Hence, each type of SM fermion field Ψ_i ($i = \{1, 2, 3\}$ being the family index) is coupled to a distinct 5-dimensional mass m_i in the fundamental theory:

$$\int d^4x \int dy \ \sqrt{G} \ m_i \bar{\Psi}_i \Psi_i, \tag{7}$$

where $G = e^{-8\sigma(y)}$ ($\sigma(y)$ is defined in Eq.(2)) is the determinant of the RS metric (see [48] for another mechanism of fermion confinement along the extra dimension). A necessary condition to modify the location of SM fermions is that the masses m_i have a non-trivial dependence on the fifth dimension, more precisely a 'kink' profile [5, 49]. For example, these masses could be vacuum expectation values of some scalar fields. An attractive possibility is to take [50]:

$$m_i = c_i \ \frac{d\sigma(y)}{dy} = \pm \ c_i \ k, \tag{8}$$

the c_i being new dimensionless parameters (note that this structure for the mass does not conflict with the $\mathbb{Z}_2$ symmetry of the $S^1/\mathbb{Z}_2$ orbifold). Then the 5-dimensional fermion fields decompose as (n labelling the tower of KK excitations),

$$\Psi_i(x^\mu, y) = \frac{1}{\sqrt{2\pi R_c}} \sum_{n=0}^{\infty} \psi_i^{(n)}(x^\mu) f_n^i(y), \tag{9}$$

admitting the following solution for the zero mode wave function along extra dimension [17, 30],

$$f_0^i(y) = \frac{e^{(2-c_i)\sigma(y)}}{N_0^i}, \tag{10}$$

where the normalization factor reads as,

$$N_0^{i \ 2} = \frac{e^{2\pi k R_c(1/2 - c_i)} - 1}{2\pi k R_c(1/2 - c_i)}. \tag{11}$$

From Eq.(10), we see that when c_i increases the zero mode of associated fermion gets more localized toward the Planck-brane.

● **Mass matrices:** In the present framework, as in the SM, fermions acquire a Dirac mass through a Yukawa coupling to the Higgs boson. This coupling reads as (starting from the 5-dimensional action),

$$\int d^4x \int dy \ \sqrt{G} \left(Y_{ij}^{(5)} \ H \bar{\Psi}_{+i} \Psi_{-j} + h.c. \right) = \int d^4x \ M_{ij} \ \bar{\psi}_{Li}^{(0)} \psi_{Rj}^{(0)} + h.c. + \dots \tag{12}$$

The $Y_{ij}^{(5)}$ are the 5-dimensional Yukawa coupling constants and the dots stand for KK mass terms. The effective 4-dimensional mass matrix is obtained after integrating:

$$M_{ij} = \int dy \ \sqrt{G} \ \frac{Y_{ij}^{(5)}}{2\pi R_c} \ H f_0^i(y) f_0^j(y). \tag{13}$$

As discussed in Section 1 and 2.1, the Higgs profile has, typically, a shape peaked at the TeV-brane. Let us assume the exponential form:

$$H = H_0 \, e^{4k(|y| - \pi R_c)}, \tag{14}$$

which is motivated by the equation of motion for a bulk scalar field [51]. Based on the $W^{\pm}$ boson mass expression, the amplitude H_0 can be expressed as a function of kR_c and the 5-dimensional weak gauge coupling constant $g^{(5)}$. The Yukawa coupling constants are chosen almost universal: $Y_{ij}^{(5)} = \kappa_{ij} g^{(5)}$ with $0.9 \leq |\kappa_{ij}| \leq 1.1$, following the philosophy adopted for example in [36, 39, 42], so that the quark/lepton mass hierarchies are mainly governed by the geometrical mechanism considered. We assume that the fermion mass matrix in Eq.(13) is real. In order to reproduce CP violating observables, one would have to introduce complex phases in Yukawa couplings. For a treatment of CP physics in the RS scenario with bulk matter, see [52]-[54].

Here, we consider the minimal massive neutrino scenario where three right handed neutrinos are added to the SM field content so that neutrinos acquire Dirac masses. There are no Majorana mass terms for the right handed neutrinos as we impose lepton number symmetry. Our motivation for imposing lepton number symmetry is to stabilize the proton: as a matter of fact, it seems that there exist no quark/lepton localizations which simultaneously fit fermion masses and generate effective non-renormalizable operator scales inducing an acceptable proton life time [36, 37].
The explicit expression of effective 4-dimensional Dirac mass matrix (13) was given in [42]. This mass matrix is only a function of κ_{ij}, kR_c and c_i parameters. Hence, the dependences of down-quark, up-quark, charged lepton and neutrino Dirac mass matrices read respectively as,

$$M_{ij}^d = M_{ij}^d(\kappa_{ij}^d, kR_c, c_i^Q, c_j^d) \quad \text{and} \quad M_{ij}^u = M_{ij}^u(\kappa_{ij}^u, kR_c, c_i^Q, c_j^u),$$

$$M_{ij}^l = M_{ij}^l(\kappa_{ij}^l, kR_c, c_i^L, c_j^l) \quad \text{and} \quad M_{ij}^\nu = M_{ij}^\nu(\kappa_{ij}^\nu, kR_c, c_i^L, c_j^\nu). \tag{15}$$

$\kappa_{ij}^{d,u,l,\nu}$ are associated respectively to the down-quark, up-quark, charged lepton and neutrino Yukawa couplings, $c_j^{d,u,l,\nu}$ parameterize the 5-dimensional masses for right handed fermions and $c_i^{Q,L}$ for fields belonging to quark/lepton $SU(2)_L$ doublets.
For the considered fermion locations (depending on c_i), the mixings between zero modes of quarks/leptons and their first KK modes (localized at the TeV-brane), induced by the Yukawa couplings, are insignificant (see [37] for details). Indeed, the KK fermion states decouple, for m_{KK} values of the order of the TeV scale as chosen here, since their masses (also depending on c_i [55]) are larger than m_{KK}. As a consequence, the SM fermion masses and mixing angles can be reliably calculated from the matrix (13) for the zero modes as the mass corrections due to KK modes are neglectable (even at the one loop-level [36, 56]). Even for the top quark, which has the largest wave function overlap with the Higgs boson and thus also with the KK quark excitations, these mass corrections are not significant compared to the uncertainty on its own mass (see below). Another consequence is that the variation of the effective number of neutrinos contributing to the Z^0 boson width, induced by the mixings between the zero and KK states of neutrinos [32], is in agreement with its experimental limit (see [41] for precisions).

• **Experimental data:** In order to be rigorous, one should specify that the fermion masses (13) are running masses at the cutoff energy scale of effective 4-dimensional theory (which is in the TeV range). This is a common scale, close to the electroweak scale, at which there is no influence from flavour dependent evolution of Yukawa couplings on the fermion mass hierarchy. The theoretical predictions for charged lepton masses, quark masses and mixing angles, derived from the matrices in Eq.(15), will be fitted with the associated values taken at the Z^0 boson mass scale (*c.f.* Appendix A). In order to take into account the effect of renormalization group from the Z^0 mass scale up to the TeV cutoff scale, we assume an uncertainty of 5% [3] on the charged lepton masses, quark masses and mixing angles (this effect is of a few percent for charged leptons between pole masses and TeV scale [57]). This significant uncertainty agrees with our philosophy of not fixing the fundamental parameters at a high-level accuracy. For similar reasons, we will consider the experimental data on neutrino masses and lepton mixing angles only at the 4σ level (*c.f.* Appendix A).

One must also impose the experimental limits on absolute neutrino masses. In our case of Dirac neutrino masses, the relevant limits are the ones extracted from tritium beta decay experiments (*c.f.* Appendix A) which hold irrespective of the nature of neutrino mass.

• **Obtained solutions:** In Appendix B, we present 3 points [A,B,C] ([X,Y,Z]) of parameter space constituted by $\{\kappa_{ij}^{d,u}; c_k^{Q,d,u}\}$ ($\{\kappa_{ij}^{l,\nu}; c_k^{L,l,\nu}\}$) which reproduce (via matrices (15)) all current experimental data on quark (lepton) masses and mixing angles, summarized in Appendix A, with the accuracy discussed in the previous paragraph.

In fact, for the points A, B and C, the parameter c_3^u can lie respectively in the range $[0.30, 0.35]$, $[0.00, 0.15]$ and $[-0.40, -0.08]$ where the quark masses and mixings are still reproduced with the allowed accuracy. These ranges correspond to variations of the top quark mass inside its uncertainty interval ((*c.f.* Appendix A).

Next we comment on the obtained c_i values in Appendix B. First, we observe that the absolute values of c_i are close to each other. In other terms, for fundamental parameters of almost the same order of magnitude, the higher dimensional mechanism generates strong hierarchies among the physical quark/lepton masses. This important result means that the SM fermion mass hierarchy problem is really addressed. Secondly, all the $|c_i|$ values are close to unity, which is desirable for two reasons. The first reason is that in this case the 5-dimensional masses $|m_i|$ in Eq.(8) are close to the scale k (being of a similar order as the gravity scale M_5). In other words, no other energy scale, with a significantly different value, is introduced. Thus the RS model maintains its quasi uniqueness of order of magnitude for the various energy scales. The other reason is that the values of $|c_i|$ (defined by Eq.(8)) can be chosen safely if,

$$|c_i| < \sqrt{20}. \tag{16}$$

This follows from condition $|m_i| < M_5$ and constraint (6).

Concerning the Yukawa coupling constants obtained in Appendix B, we note that a certain accuracy is required for some of them. Nevertheless, this accuracy can be lowered by choosing other Yukawa couplings with a higher precision.

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[3]When the error (see Appendix A) on renormalized masses and mixing angles at the Z^0 mass scale exceeds 5%, we admit an uncertainty equal to this error.

3 FCNC constraints

3.1 FCNC origin

Within the SM, there are no FCNC's at tree level, and most loop-induced FCNC effects are extremely small. In contrast, within the context of RS model with bulk matter, FCNC processes can be induced at tree level by exchanges of KK excitations of neutral gauge bosons. Indeed, these KK states possess FC couplings as we will discuss now.

Let us consider the action of the effective 4-dimensional coupling between SM fermions $\psi_i^{(0)}(x^\mu)$ and KK excitations of any neutral gauge boson $A_\mu^{(n)}(x^\mu)$. In the interaction (or weak) basis, it reads as,

$$S_{gauge} = g_L^{SM} \int d^4x \sum_{n=1}^{\infty} \bar{\psi}_{Li}^{(0)} \, \gamma^\mu \, \mathcal{C}_{Lij}^{(n)} \, \psi_{Lj}^{(0)} \, A_\mu^{(n)} \; + \; \{L \leftrightarrow R\}, \qquad (17)$$

where $g_{L/R}^{SM}$ is the relevant SM gauge coupling constant and $\mathcal{C}_{Lij}^{(n)}$ the 3×3 diagonal matrix $diag(C^{(n)}(c_1), C^{(n)}(c_2), C^{(n)}(c_3))$. The coefficient $C^{(n)}(c_i)$ quantifies the wave function overlap along extra dimension between SM fermions ($f_0^i(y)$) and the nth KK mode of the neutral gauge boson. This coefficient corresponds to the coefficient $C_{00n}^{f\bar{f}A}$ in reference [55]. The action in Eq.(17) can be rewritten in the mass (or physical) basis (indicated by the prime):

$$S_{gauge} = g_L^{SM} \int d^4x \sum_{n=1}^{\infty} \bar{\psi}_{L\alpha}^{(0)\,\prime} \, \gamma^\mu \, V_{L\alpha\beta}^{(n)} \, \psi_{L\beta}^{(0)\,\prime} \, A_\mu^{(n)} \; + \; \{L \leftrightarrow R\}, \qquad (18)$$

where,

$$V_L^{(n)} = U_L^\dagger \, \mathcal{C}_L^{(n)} \, U_L, \qquad (19)$$

U_L being the unitary matrix of basis transformation for left handed fermions and α, β being flavour indexes. In conclusion, the non-universality of the effective coupling constants $g_{L/R}^{SM} \times C^{(n)}(c_i)$ between KK modes of the gauge fields and the three SM fermion families (which have different locations along y), in the weak basis, induces non-vanishing off-diagonal elements for matrix $V_{L/R}^{(n)}$, in the physical basis, giving rise to FC couplings (see Eq.(18)).

3.2 Small FCNC effects with low KK masses

The mass hierarchies and mixings of SM fermions require different values for the c_i parameters (Section 2.2), or equivalently different fermion locations, which induce FCNC effects at tree level (Section 3.1). These FCNC effects can be suppressed by choosing the c_i values within a certain type of configuration, as we will discuss now. Thus, the FCNC bounds can be respected even for some low KK masses (FCNC reactions being due to KK mode exchanges).

The idea is to search for c_i configurations reproducing fermion masses, where the c_i parameters, being of a same type (for instance of type c^d or c^L) and with different values for each generation, are larger than about 0.5 (for example: $c_1^d = 0.7$, $c_2^d = 0.8$ and $c_3^d = 0.9$). Indeed, in this c_i value domain, the coupling constants $g_{L/R}^{SM} \times C^{(n)}(c_i)$ are quasi universal among three families since the overlap between any KK gauge

state and a fermion is almost independent of the fermion localization (*c.f.* [55] with conventions such that their parameter ν is identified as our $-c$). Therefore, the FC couplings of the KK states of the neutral gauge bosons, appearing in the physical basis (see previous subsection), almost vanish.

With respect to the third family, associated to the heaviest fermion, it is difficult to find a configuration where the c_3 value for each type of fermion is either similar to the c_1 and c_2 values, or, higher than around 0.5 (heavy fermions should typically correspond to small c values to be localized near the TeV-brane where the Higgs field is confined) at the same time as first two c_1 and c_2 [4]. However, this is compensated by the fact that, for the third fermion generation, FCNC constraints are less severe [30] [5].

As a matter of fact, all c_i values presented in Appendix B have been obtained accordingly to two main criteria: (I) they reproduce the quark/lepton masses and mixing angles, as discussed in previous section (II) they resemble the c_i configurations described above. Thus, the six points of parameter space given in Appendix B verify the various experimental FCNC constraints with KK neutral gauge boson masses as low as $m_{KK} = 1\mathrm{TeV}$, as we are going to show in the following (including FCNC effects induced at one loop level).

• $l_\alpha \to l_\beta l_\gamma l_\gamma$ **decay:** First, we study the pure leptonic reactions which are free from hadronic uncertainties. In the present framework, the FCNC leptonic decay channels for charged leptons μ^- and τ^- are induced via processes of type $l_\alpha \to l_\beta Z/\gamma^{(n)\star} \to l_\beta l_\gamma l_\delta$ (where $_{\alpha,\beta,\gamma,\delta}$ are flavour indexes), i.e. by exchanges of virtual KK excitations of the Z^0 boson or photon which have FC couplings. For instance, the analytical expressions for the widths of these decay channels have been calculated in [64], within a model-independent analysis of constraints on new physics (based on effective lagrangian techniques), as a function of the elements of the leptonic FC matrix, here denoted by $V_{L/R\alpha\beta}^{(n)}$, in the KK gauge field action (18). This matrix $V_{L/R}^{(n)}$ is completely determined for each point X,Y,Z of parameter space given in Appendix B. Indeed, each parameter set X,Y,Z fixes the charged lepton mass matrix M^l (see Eq.(15)) and thus the matrix $U_{L/R}^l$ (which diagonalizes $M^l M^{l\dagger}/M^{l\dagger} M^l$) involved in $V_{L/R}^{l(n)}$ (see Eq.(19)).

In Table 1, we show the values of the branching ratios for all possible FCNC lepton decay channels induced by exchanges of the first KK excitation of the Z^0 boson and photon (effects of higher KK states are discussed below). We have derived these values with $m_{KK} = 1\mathrm{TeV}$ for the case Y in Appendix B. We see in this table that all branching ratios are lower than their experimental upper limit, as wanted. Similarly, the branching ratios for the two other cases X and Z, given in appendix, also satisfy all relevant experimental bounds. In addition, we notice from Table 1 that the amplitudes for the processes $\tau^- \to e^- \mu^+ \mu^-$ and $\tau^- \to e^+ \mu^- \mu^-$, for example, are different. The former involves the FC coupling $Z/\gamma^{(1)}\bar{e}\tau$ (fixed by $V_{L/R13}^{l(1)}$) whereas the latter involves $Z/\gamma^{(1)}\bar{\mu}\tau$ (fixed by $V_{L/R23}^{l(1)}$).

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[4]Illustrative examples of this feature are all the values of c_3^u for points A,B,C and c_3^L for points Y,Z in Appendix B.

[5]This comment on the third fermion family can be reformulated from a predictive point of view as follows. In RS scenarios generating SM fermion mass hierarchies, fermion locations for the third family generally do not correspond to the configurations described in text above. Therefore, FCNC effects involving third family are typically larger. This can be observed in various tables of this section.

Process	$Z^{(1)}$	$\gamma^{(1)}$	Experimental bound
$B(\mu^- \to e^- e^+ e^-)$	$1.4\ 10^{-14}$	$9.4\ 10^{-14}$	$1.0\ 10^{-12}$
$B(\tau^- \to e^- e^+ e^-)$	$1.1\ 10^{-12}$	$8.5\ 10^{-12}$	$2.9\ 10^{-6}$
$B(\tau^- \to \mu^- \mu^+ \mu^-)$	$5.5\ 10^{-8}$	$3.0\ 10^{-7}$	$1.9\ 10^{-6}$
$B(\tau^- \to e^- \mu^+ \mu^-)$	$9.0\ 10^{-13}$	$6.6\ 10^{-12}$	$1.8\ 10^{-6}$
$B(\tau^- \to e^+ \mu^- \mu^-)$	$1.5\ 10^{-17}$	$9.7\ 10^{-17}$	$1.5\ 10^{-6}$
$B(\tau^- \to \mu^- e^+ e^-)$	$7.0\ 10^{-8}$	$3.8\ 10^{-7}$	$1.7\ 10^{-6}$
$B(\tau^- \to \mu^+ e^- e^-)$	$2.4\ 10^{-22}$	$2.1\ 10^{-21}$	$1.5\ 10^{-6}$

Table 1: Branching ratios for all FCNC lepton decay channels induced by exchanges of the first KK excitations $Z^{(1)}$ and $\gamma^{(1)}$, for $m_{KK} = 1$TeV and point Y of Appendix B. The associated upper experimental bounds at 90% *C.L.*, taken from [60], are also shown.

Decay channel	X	Y	Z	Experimental bound
$B(Z^0 \to e^\pm \mu^\mp)$	$2.1\ 10^{-17}$	$3.7\ 10^{-15}$	$3.2\ 10^{-15}$	$1.7\ 10^{-6}$
$B(Z^0 \to e^\pm \tau^\mp)$	$3.3\ 10^{-15}$	$1.7\ 10^{-12}$	$1.8\ 10^{-12}$	$9.8\ 10^{-6}$
$B(Z^0 \to \mu^\pm \tau^\mp)$	$2.1\ 10^{-14}$	$1.0\ 10^{-7}$	$7.6\ 10^{-8}$	$1.2\ 10^{-5}$

Table 2: Branching ratios for FC leptonic decays induced by $Z^0 - Z^{(1)}$ mixing, for $m_{KK} = 1$TeV and the 3 points X,Y,Z of Appendix B, with relevant upper experimental bounds at 95% *C.L.* (from [60]).

In the present discussion on FCNC constraints, we do not present FCNC rates associated to exchanges of higher KK excitations $(n = 2, \dots)$ of gauge bosons $Z/\gamma^{(n)}$, as those are much weaker than the $Z/\gamma^{(1)}$ contributions to FCNC rates. The reason is that, compared to the $Z/\gamma^{(1)}$, the $Z/\gamma^{(n=2,\cdots)}$ masses are larger and their absolute couplings to SM fermions, proportional to $|C^{(n)}(c_i)|$, are smaller whatever the fermion location parameter c_i is (even getting smaller typically as the KK level (n) increases) as shown clearly in [55]. For example for the same case Y as in Table 1, we find a branching ratio, of the decay channel $\tau^- \to \mu^- e^+ e^-$ (receiving the largest $Z/\gamma^{(1)}$ contributions) induced by the $Z^{(2)}$ $(\gamma^{(2)})$ exchange, equal to $1.2\ 10^{-14}$ $(6.2\ 10^{-14})$ for an identical $m_{KK} = 1$TeV which leads to $m_{KK}^{(2)}(\gamma) = (5.57/2.45)m_{KK} = 2.27$TeV.

• $Z^0 \to l_\alpha l_\beta$ **decay:** The mixing between Z^0 boson and modes $Z^{(n)}$ gives rise to the FC lepton decay channels $Z^0 \to l_\alpha \bar{l}_\beta$ $[\alpha \neq \beta]$. From the general formalism described in [65], for FC effects due to an additional heavy Z' boson, one can easily deduce the width expressions for these leptonic decays in terms of matrix $V_{L/R\alpha\beta}^{(n)}$. We find the corresponding branching ratio values in Table 2. This table shows that, for $m_{KK} = 1$TeV, the cases X,Y,Z do not conflict with the experimental bounds on rates of Z^0 FC decays into leptons. Finally, we note that for $m_{KK} = 1$TeV, the $Z^0 - Z^{(1)}$ mixing angle is given by $\sin^2 \theta \approx 7\ 10^{-5}$.

• $P^0 - \bar{P}^0$ **mixing:** Next, we study FCNC reactions in the hadron sector, starting by processes with $\Delta F = 2$: The KK gauge field exchanges at tree level generate mass splittings in neutral pseudo-scalar meson systems. The mass splitting Δm_P between flavour eigenstates for a meson P was given in [65] as a function of meson

Contribution	A	B	C
Δm_K $(Z^{(1)})$	$1.1\ 10^{-21}$	$1.7\ 10^{-21}$	$8.8\ 10^{-21}$
Δm_K $(\gamma^{(1)})$	$3.0\ 10^{-23}$	$4.4\ 10^{-23}$	$2.3\ 10^{-22}$

Table 3: Mass splitting (in GeV) for K^0 meson generated by $Z^{(1)}$ and $\gamma^{(1)}$, with $m_{KK} = 1$TeV and the 3 points A,B,C of Appendix B.

Contribution	A	B	C
Δm_B $(Z^{(1)})$	$3.9\ 10^{-18}$	$7.6\ 10^{-18}$	$1.6\ 10^{-17}$
Δm_B $(\gamma^{(1)})$	$1.0\ 10^{-19}$	$2.0\ 10^{-19}$	$4.2\ 10^{-19}$

Table 4: Mass splitting (in GeV) for B^0 meson generated by $Z^{(1)}$ and $\gamma^{(1)}$, with $m_{KK} = 1$TeV and the 3 points A,B,C of Appendix B.

decay constant f_P and off-diagonal elements of matrix $V_{L/R}^{(n)}$. In Table 3, we present the values for mass splitting of the kaon induced by $Z^{(1)}$ and $\gamma^{(1)}$ exchanges: the results, obtained for $m_{KK} = 1$TeV, show that the 3 cases A,B,C of Appendix B give contributions smaller than the experimental uncertainty which reads as $\Delta m_K = [3.483 \pm 0.006]\ 10^{-15}$ GeV [60].

Similarly, in Table 4, we give the values for mass splitting of the B meson. These values show that the 3 cases A,B,C lead to contributions which do not saturate the measured value: $\Delta m_B = [3.304 \pm 0.046]\ 10^{-13}$ GeV [60].

The mass splittings for D meson are given in Table 5. The values obtained for the cases A,B,C are in perfect agreement with the experimental limit, $\Delta m_D < 4.6\ 10^{-14}$ GeV at 95% $C.L.$ [60].

Mass splittings in meson systems are also produced by exchanges of KK gluon excitations mediating FC. These Δm_P contributions are larger than the excited EW gauge boson ones, due to the high strength of the strong interaction. However, considering for example the B^0 meson, the KK gluon contribution to mass splitting can remain well below the experimental error on Δm_B, as we are going to see. The first KK gluon contribution to the mass splitting for the B^0 meson, which has a mass (of 5279.4MeV) much larger than the QCD-scale [66], can be estimated [67] and yields respectively $\Delta m_B = \{1.5; 3.0; 6.2\} \times 10^{-16}$GeV for cases A,B,C with $m_{KK} = 1$TeV. These values are about three orders of magnitude above the $\gamma^{(1)}$ contributions (see Table 4) but are still about one order below the experimental uncertainty on Δm_B which is $\pm 4.6\ 10^{-15}$GeV (see text above).

$\bullet$ $\mu - e$ **conversion:** Some FCNC reactions involve both quarks and leptons. The exchanges of neutral KK gauge fields mediating FC lead to coherent $\mu - e$ conversion in muonic atoms. The SINDRUM II collaboration at PSI has carried out a program of experiments to search for $\mu - e$ conversion in various nuclei [68] and the best exclusion limit obtained comes from the Titanium reaction [69]:

$$B(\mu^- + Ti \to e^- + Ti) = \frac{\Gamma(\mu^- + Ti \to e^- + Ti)}{\Gamma_{\text{CAPT}}} < 6.1\ 10^{-13} \quad \text{at } 90\%\ C.L., \quad (20)$$

Γ_{CAPT} being the total nuclear muon capture rate in Ti which is measured with a good precision (see [70] for a nuclear-model-independent bound on the vertex $Z' - e - \mu$). The expression for the branching ratio $B(\mu^- + Ti \to e^- + Ti)$ can be

Contribution	A	B	C
$\Delta m_D\ (Z^{(1)})$	$5.5\ 10^{-20}$	$1.9\ 10^{-19}$	$3.4\ 10^{-19}$
$\Delta m_D\ (\gamma^{(1)})$	$1.5\ 10^{-21}$	$4.9\ 10^{-21}$	$8.8\ 10^{-21}$

Table 5: Mass splitting (in GeV) for D^0 meson generated by $Z^{(1)}$ and $\gamma^{(1)}$, with $m_{KK} = 1\text{TeV}$ and the 3 points A,B,C of Appendix B.

Contribution	A/X	B/Y	C/Z
$B(Z^{(1)})$	$9.5\ 10^{-16}$	$7.5\ 10^{-14}$	$4.7\ 10^{-14}$
$B(\gamma^{(1)})$	$1.6\ 10^{-15}$	$4.8\ 10^{-14}$	$1.5\ 10^{-14}$

Table 6: Branching fraction for $\mu^- + Ti \rightarrow e^- + Ti$ induced by $Z^{(1)}$ or $\gamma^{(1)}$ exchange, with $m_{KK} = 1\text{TeV}$ and for 3 representative combinations of the points listed in Appendix B.

deduced from global analysis in [65] where the FC amplitudes of an additional Z' boson are calculated (taking into account the $Z^0 - Z'$ mixing). One obtains this branching ratio as a function of a nuclear form factor, the Titanium atomic (Z=22) and neutron (N=26) numbers and the matrix elements $V^{l(1)}_{L/R12}$, $V^{u(1)}_{L/R11}$ and $V^{d(1)}_{L/R11}$. We find that the value of this branching ratio respects the bound in Eq.(20) for any set of quark parameters A,B,C combined with any set X,Y,Z for leptons and with $m_{KK} = 1\text{TeV}$, as can be checked from Table 6 where values are given for examples of combinations.

- $K^0 \rightarrow l_\alpha l_\beta$ **decay:** Tight experimental constraints apply on certain (semi-) leptonic FCNC decay amplitudes for mesons. First, the decay channels of type $K^0_L \rightarrow l_\alpha \bar{l}_\beta$ receive contributions from processes involving KK gauge boson excitations. The K^0_L branching fraction associated to such contributions is directly derived from the general results of the systematic survey of lowest-dimension effective interactions (as manifestations of heavy physics) performed in [64]: we obtain the branching ratio $B(K^0_L \rightarrow l_\alpha \bar{l}_\beta)$ as function of the $B(K^+ \rightarrow \nu_\mu \bar{\mu})$ value and the matrix elements $V^{d(1)}_{L/R12}$, $V^{l(1)}_{L/R\alpha\beta}$. In our framework, the computed values of $B(K^0_L \rightarrow l_\alpha \bar{l}_\beta)$ are in agreement with the associated experimental values (either much smaller than the measurement error or lower than the existing bound) for all sets of quark parameters in the cases A,B,C combined with any set in X,Y,Z for leptons and for $m_{KK} = 1\text{TeV}$, as can be observed in Table 7 in which values are presented for some characteristic examples of combinations.

- $K^+ \rightarrow \pi^+ \nu\nu$ **decay:** The exchange of $Z^{(1)}$ contributes to the $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ FCNC decay (with an implicit summation over the indexes α, β of final state neutrinos $\nu_\alpha \bar{\nu}_\beta$ including $\alpha \neq \beta$ channels). The SM contribution to this decay leads to a branching fraction of $B_{SM} = (0.4\ \text{to}\ 1.2)\ 10^{-10}$ in agreement with the experimental result: $B_{exp} = [1.6^{+1.8}_{-0.8}]\ 10^{-10}$ [60]. Hence, the maximal allowed value for the ratio B_{RS}/B_{SM} (B_{RS} representing the branching fraction of $K^+ \rightarrow \pi^+ \nu\bar{\nu}$ induced in the RS model by the $Z^{(1)}$ exchange) is typically between 1.8 and 3.6. This ratio B_{RS}/B_{SM} can be expressed [66, 71] in terms of the matrix elements $V^{d(1)}_{L/R12}$, $V^{\nu(1)}_{L\alpha\beta}$. We find that the B_{RS}/B_{SM} value is clearly below the limit discussed above, for all the combinations of A,B,C with X,Y,Z and $m_{KK} = 1\text{TeV}$. In Table 8, we give the

Branching ratio	A/X	B/Y	C/Z	Experimental value
$B(K_L \to e^+ e^-)$	$4.9\ 10^{-14}$	$1.1\ 10^{-14}$	$1.7\ 10^{-14}$	$[9^{+6}_{-4}]\ 10^{-12}$
	$3.6\ 10^{-13}$	$8.0\ 10^{-14}$	$1.3\ 10^{-13}$	
$B(K_L \to \mu^+ \mu^-)$	$4.9\ 10^{-14}$	$8.5\ 10^{-15}$	$1.1\ 10^{-14}$	$[7.27 \pm 0.14]\ 10^{-9}$
	$3.6\ 10^{-13}$	$6.2\ 10^{-14}$	$8.4\ 10^{-14}$	
$B(K_L \to e^{\pm} \mu^{\mp})$	$1.7\ 10^{-26}$	$4.5\ 10^{-24}$	$2.1\ 10^{-23}$	$< 4.7\ 10^{-12}$
	$1.7\ 10^{-25}$	$4.0\ 10^{-23}$	$1.9\ 10^{-22}$	

Table 7: Branching ratio of decays $K_L \to l_\alpha \bar{l}_\beta$ for $m_{KK} = 1$TeV and the 3 combinations A/X,B/Y,C/Z of points given in Appendix B. The first number corresponds to the $Z^{(1)}$ contribution and the second one is for $\gamma^{(1)}$. In last column, we also provide each measured branching fraction value with its uncertainty, as well as the experimental upper limit at 90% $C.L.$ in the case of the FC final state $e^{\pm} \mu^{\mp}$ [60].

Contribution	A/X	B/Y	C/Z
$R(K^+ \to \pi^+ \nu \bar{\nu})$	$6.7\ 10^{-6}$	$1.1\ 10^{-6}$	$1.3\ 10^{-6}$

Table 8: Ratio $R = B_{RS}/B_{SM}$ for the decay channel $K^+ \to \pi^+ \nu \bar{\nu}$ (induced only by the $Z^{(1)}$ exchange in case of the RS model) for $m_{KK} = 1$TeV and the same 3 combinations of points given by Appendix B.

B_{RS}/B_{SM} value for the same examples of parameter combinations as before.

- $l_\alpha \to l_\beta \gamma$ **decay:** One loop neutral current penguin diagrams, exchanging a $Z/\gamma^{(1)}$ and a charged lepton, induce FC radiative decays into the photon: $l_\alpha \to l_\beta \gamma$ [$\alpha \neq \beta$]. The amplitudes of such diagrams can be expressed, using the formalism in [65], in terms of the matrices $V_{L/R}^{l(1)}$. In Table 9, the branching ratios of various decay channels $l_\alpha \to l_\beta \gamma$ are given for $m_{KK} = 1$TeV and the 3 lepton parameter sets X,Y,Z: we see that none of those values conflicts with experimental bounds.

Next, we discuss another type of contribution to the FC decay channel $l_\alpha \to l_\beta \gamma$ [$\alpha \neq \beta$]. In the SM extension where neutrinos have (Dirac) masses, this radiative decay is mediated by the exchange of a $W^{\pm}$ gauge boson and a neutrino at one loop-level. In this case, the source of FC resides in the lepton mixing matrix $V_{MNS} = U_L^{l\dagger} U_L^{\nu}$. Within this scenario, the rate for the FC decay $l_\alpha \to l_\beta \gamma$ is suppressed by the GIM cancellation mechanism [72] which is ensured, simultaneously, by the unitarity of V_{MNS} and the quasi-degeneracy of the 3 neutrino masses (relatively to the $W^{\pm}$ mass).
In the RS model with bulk matter, loop contributions of KK neutrino excitations [41] to $l_\alpha \to l_\beta \gamma$ invalidate [73] the GIM cancellation. Indeed, these excitations have KK masses which are not negligible (and thus not quasi-degenerate in family space) compared to $m_{W^{\pm}}$. The GIM mechanism is also invalidated by the loop contributions of the KK $W^{\pm(n)}$ modes which couple (KK level by level), in the 4-dimensional theory, via an effective lepton mixing matrix of type $V_{MNS}^{eff} = U_L^{l\dagger} \mathcal{C}_L^{(n)} U_L^{\nu}$ being non-unitary due to the non-universality of $\mathcal{C}_L^{(n)} = diag(C_m^{(n)}(c_1^L), C_m^{(n)}(c_2^L), C_m^{(n)}(c_3^L))$, where $m \geq 0$ is the exchanged neutrino KK level index and $C_m^{(n)} = C_{0mn}^{f\bar{f}A}$ (in the notation of [55]). However, for the cases considered in Appendix B, the three c_i^L values are equal (for case X) or almost equal (for Y and Z). Remember that only the c_i^L values play

Branching ratio	X	Y	Z	Experimental limit
$B(\mu^- \to e^- \gamma)$	$1.1\ 10^{-13}$	$1.4\ 10^{-13}$	$1.1\ 10^{-13}$	$1.2\ 10^{-11}$
	$8.3\ 10^{-13}$	$1.0\ 10^{-12}$	$8.3\ 10^{-13}$	
$B(\tau^- \to e^- \gamma)$	$1.4\ 10^{-14}$	$2.9\ 10^{-14}$	$1.4\ 10^{-14}$	$2.7\ 10^{-6}$
	$3.3\ 10^{-14}$	$1.1\ 10^{-13}$	$3.3\ 10^{-14}$	
$B(\tau^- \to \mu^- \gamma)$	$4.9\ 10^{-11}$	$6.8\ 10^{-11}$	$4.9\ 10^{-11}$	$1.1\ 10^{-6}$
	$2.1\ 10^{-11}$	$3.0\ 10^{-11}$	$2.1\ 10^{-11}$	

Table 9: Branching ratio for all decays $l_\alpha \to l_\beta \gamma$ $[\alpha \neq \beta]$ for $m_{KK} = 1$TeV and the 3 points X,Y,Z of Appendix B. First value is associated to the $Z^{(1)}$ contribution while the second one is for $\gamma^{(1)}$. In last column, we give the experimental upper limit at 90% C.L. for each decay channel [60].

a rôle here, as the leptons coupling to $W^\pm$ must be left handed. Thus, the GIM cancellation mechanism is restored for these cases, KK level by level [74], in the process $l_\alpha \to l_\beta \gamma$. Indeed, the two arguments given above do not hold anymore. First, for three (almost) equal c_i^L values, the 3 family masses of the excited neutrino states at a common KK level, exchanged in the loop, are (quasi-)degenerate with respect to $m_{W^\pm}$ [55]. Secondly, for (almost) identical c_i^L's, the effective matrix $V_{MNS}^{eff} = U_L^{l\dagger} \mathcal{C}_L^{(n)} U_L^\nu$ of KK $W^{\pm(n)}$ modes (almost) verifies $V_{MNS}^{eff} V_{MNS}^{eff\dagger} \propto \mathbb{1}_{3\times3}$ since: (i) $\mathcal{C}_L^{(n)} = diag(C_m^{(n)}(c_1^L), C_m^{(n)}(c_2^L), C_m^{(n)}(c_3^L))$ is (quasi-)universal (ii) U_L^ν is nearly unitary (m)-KK level by level, as the neutrino masses are much smaller than their KK excitation masses, i.e. no significant mixings are induced among different neutrino KK levels. We conclude that for the parameter space described in Appendix B, the dominant contributions to the widths of FC decay $l_\alpha \to l_\beta \gamma$ must originate from the exchanges of KK neutral gauge fields discussed before (see also the next discussion on $b \to s\gamma$).

• $b \to s\gamma$ **decay:** Similarly, one loop penguin diagrams, exchanging a neutral KK gauge field and a down quark, contribute to the FC radiative partonic decay: $b \to s\gamma$. The experimental measurement of $R_{b\to s\gamma}$ yields [60],

$$R_{b\to s\gamma} \equiv \frac{\Gamma(B \to X_s\gamma)}{\Gamma(B \to X_c e\bar{\nu}_e)} = 3.39^{+0.62}_{-0.54}\ 10^{-3}, \tag{21}$$

where the Γ's denote the widths. For the SM expectation, one has $R_{b\to s\gamma}^{SM} = 3.23 \pm 0.09\ 10^{-3}$ [60]. Therefore, the contribution of the KK gauge fields to $R_{b\to s\gamma}$ cannot exceed $8.7\ 10^{-4}$. The b quark mass is much higher than the QCD-scale. Thus, long-range strong interaction effects are not expected to be important in the decay $B \to X_s\gamma$ [66]. Hence, the ratio $R_{b\to s\gamma}$ is usually approximated by,

$$R_{b\to s\gamma} \simeq \frac{\Gamma(b \to s\gamma)}{\Gamma(b \to c e\bar{\nu}_e)}, \tag{22}$$

The expression for this ratio, as a function of the $V_{L/R}^{d(1)}$ matrix elements and phase-space factors, can be easily deduced from previous study [65]: the values of the KK gauge field contributions to the ratio in Eq.(22), obtained for quark parameter sets A,B,C and $m_{KK} = 1$TeV, are much smaller than the experimental bound discussed above, as exhibits Table 10.

Contribution	A	B	C
$R_{b \to s\gamma}$ $(Z^{(1)})$	$9.4 \ 10^{-16}$	$7.0 \ 10^{-16}$	$2.6 \ 10^{-16}$
$R_{b \to s\gamma}$ $(\gamma^{(1)})$	$1.8 \ 10^{-16}$	$8.0 \ 10^{-17}$	$1.9 \ 10^{-17}$

Table 10: Contributions to the ratio $R_{b \to s\gamma}$ (see text) from $Z^{(1)}$ and $\gamma^{(1)}$ exchanges, for $m_{KK} =$ 1TeV and the 3 points A,B,C listed in Appendix B.

We finish this part by discussing the contribution to $b \to s\gamma$ coming from the exchange of a $W^{\pm (n)}$ $[n = 0, 1 \ldots]$ gauge field and an up quark (or its KK excitations) at one loop-level. Analogous arguments as those employed in the discussion on $l_\alpha \to l_\beta \gamma$ apply here. Hence, the major contributions to the $b \to s\gamma$ rate should not come from the $W^{\pm (n)}$ exchange for the cases considered here, where all c_i^Q values are exactly equal between all families (*c.f.* Appendix B).
Nevertheless, in the quark sector, there are deviations to the restoration of the GIM cancellation discussed above, due to the fact that the top quark mass cannot be totally neglected relatively to the KK up-type quark excitation scales. Indeed, this fact leads to a mass shift of the KK top quark mode from the rest of the KK up-type quark modes, and thus eliminates the degeneracy among 3 family masses of the up quark excitations at fixed KK level (with regard to $m_{W^{\pm (n)}}$). Moreover, this means that the Yukawa interaction with the Higgs boson induces a substantial mixing of the top quark KK tower members among themselves [75, 76].
As an example, the data on $b \to s\gamma$ are accommodated with $m_{KK} \simeq 1$TeV for $c_3^Q = 0.4$ (which is close to the values in the cases B and C of Appendix B), as shown in [74] using numerical methods for the diagonalization of a large dimensional mass matrix and taking into account the top quark mass effects described previously.

4 Other constraints

4.1 EW precision data

EW precision measurements place restrictions on the RS model [55, 71, 74, 77]-[80] (with bulk matter) since deviations from EW precision observables arise in the framework of this model. Indeed, the mixing between the top quark and its KK excitations (discussed above) results in a shift of the ratio $m_{W^\pm}/m_{Z^0}$ from the value obtained within the SM. Moreover, mixing between the EW gauge bosons and their KK modes induces modifications of the boson masses/couplings. The authors in [77] have found that a good fit of EW precision observables, including the ρ parameter, can be obtained with $m_{KK} \approx 11$TeV. In [55], a global analysis based on a large set of EW observables has yielded a lower bound on m_{KK} varying typically between 0 and 20TeV for a universal value $|c_i| < 1$. If the weak gauge boson masses and couplings are treated simultaneously, one obtains the conservative bound $m_{KK} \gtrsim 10$TeV, for a universal c_i value lying inside the interval $[-1, 1]$ (and for $10^{-2} < k/M_5 < 1$) [79].

Different specific models have been proposed in literature in order to soften this lower bound on m_{KK} coming from EW precision data. The motivation was to address the little hierarchy problem, i.e. the smallness of the EW symmetry breaking scale compared to m_{KK}. In this sense, the EW bound on m_{KK} is model-dependent. For example, models with brane-localized kinetic terms for fermions [81] or gauge

bosons [82] allow to relax the lower bound on m_{KK} down to a few TeV (see [45, 46] for gauge boson kinetic terms and [47] for fermion terms). The introduction of brane-localized kinetic terms changes the KK wave functions so that our results, presented here, cannot be directly translated to such models by a simple rescaling with the appropriate powers of m_{KK}. A different type of model, with a left-right EW gauge structure in the bulk and already mentioned [43], also allows for softening the EW bound on m_{KK}, thanks to the bulk custodial isospin gauge symmetry arising in this context. Our realization of the RS model with bulk matter is compatible with this type of models, allowing to combine all EW precision data with a value for $m_{KK} \gtrsim 3\text{TeV}$. We discuss this in more detail in the next paragraph.

In the model of [43][6], with the EW gauge symmetry enhanced to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, the usual EW gauge group $SU(2)_L \times U(1)_Y$ is recovered through the breaking of both $SU(2)_R$ and $U(1)_{B-L}$ on the Planck-brane (scenario II) and possibly a small breaking of $SU(2)_R$ in the bulk (scenario I). Furthermore, the right handed fermions are promoted to $SU(2)_R$ doublet fields, with the new (non physical) component having no zero mode. Hence, for instance in the quark sector, the right handed $c_i^{d,u}$ parameters describe now the locations of $SU(2)_R$ doublets, however, the total number of $c_i^{Q,d,u}$ parameters remains identical. Within this context, our configurations of c_i parameters can be in agreement with a reasonable fit of the EW precision data for $m_{KK} \simeq 3\text{TeV}$, as shown in the following from results of [43].

• δg_Z^b **shift:** The coupling g_Z^b of the Z^0 boson to the b quark is measured with high accuracy. Experimentally, this is done through the width ratio $R_b = \Gamma(b\bar{b})/\Gamma(hadronic)$ [60]. For $m_{KK} = 3\text{TeV}$ and $c_3^Q = 0.200; 0.370; 0.413$, corresponding to the cases A;B;C respectively, the shift in the coupling (obtained from formula (5.9) in [43]) is $\delta g_Z^{b_L}/g_Z^{b_L} = 1.7\%; 0.8\%; 0.6\%$ which respects the upper limit of $\sim 1\%$ as imposed by precision EW results. The left handed b_L quark (with $c_3^Q < 0.5$ as dictated by the top quark mass) is considered, since its effective couplings to KK gauge fields are much larger than the couplings of the b_R quark (with $c_3^d > 0.5$, systematically).

• **S parameter:** The value of the "oblique" parameter S is found [43] to be typically $0.26 - 0.15$ for $m_{KK} = 3 - 4\text{TeV}$ (when $c_3^Q, c_3^u < 0.5$ and $c_3^d > 0.5$). For lower m_{KK} values, S is too large in order to reasonably fit EW precision data (independently of T).

• **T parameter:** In scenario I (mentioned above), for $kR_c \sim 10$, $m_{KK} \simeq 3\text{TeV}$ and the above range of S, the T parameter reaches values required to fit the EW measurements [43]. In scenario II, the correct T values required for above S range can be generated radiatively from top loops. Indeed, using expression (6.4) of [43], we find $T_{\text{KK top}} \simeq 0.15$ for $c_3^u \approx 0$, $c_3^Q = 0.37$ (as in our case B) and $m_{KK} = 3\text{TeV}$ (the involved $m_{KK}^{(1)}(t_L)$ mass being fixed by c_3^Q and m_{KK}).

4.2 Universality limits

Especially for low KK masses, mixing between the zero modes of SM fermions and their KK excitations induce a loss of flavour universality for the effective quark/lepton couplings to neutral gauge bosons. Indeed, the existence of these mixings causes a loss of unitarity (in zero mode fermion flavour space) with regard to the matrices

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[6]The Higgs phenomenology in left-right symmetric RS extensions was analysed in [83].

responsible for the transformation from weak to physical basis. The largest deviation, induced by such effects, from the SM value of the fermion couplings to the Z^0 boson arises for the top quark.

Under the hypothesis that the LHC measures the top coupling to the Z^0 with a precision of 5% (an accuracy of a few percent is expected to be reached in the LHC performances) *and* that the result coincides with the SM prediction, an experimental lower limit could be placed on the mass $m_{KK}^{(1)}(t)$ of first KK top quark excitation. This hypothetical limit, obtained in [84], is $m_{KK}^{(1)}(t) \gtrsim 1 - 4\text{TeV}$ for a universal c_i value in the range $[0, 0.5]$, corresponding to a less severe bound on m_{KK} which is systematically smaller than $m_{KK}^{(1)}(t)$. The other indirect constraints of this type, not involving the top quark, are less restrictive.

4.3 Muon magnetic moment

The anomalous magnetic moment of the muon is a well known model building constraint on theories beyond SM. In the RS framework, this magnetic moment receives contributions from the loop exchanges of KK excitations. The experimental world average measurement of $(g - 2)_\mu$ translates into the upper limit $c_i \lesssim 0.70$ for $1\text{TeV} < m_{KK} < 10\text{TeV}$, assuming a universal value for all the c_i parameters [56]. Because of this simplification assumption, i.e. c_i universality, this upper limit does not strictly apply to our realistic RS scenario, where the values of c_i parameters are flavour and type dependent.

The authors of [56] have also examined the perturbativity condition on effective Yukawa coupling constants from which they have deduced the constraint $c_i \lesssim 0.77$, still under the hypothesis of a universal c_i value.

5 Non-renormalizable operators

In models of low gravity scale yielding a low cutoff, the impact of non-renormalizable operators is dramatically amplified. This constitutes a serious challenge for model building. Within the RS framework, the fundamental value of 5-dimensional gravity scale in the bulk (where SM fields propagate, in the present RS set-up) M_5 is close to the high M_{Pl} value (*c.f.* Eq.(4)). However, one should ask whether effective 4-dimensional non-renormalizable interactions, determined by field overlaps along the fifth dimension, are sufficiently suppressed.

Let us, explicitly, express the effective 4-dimensional energy scale $(Q_{\alpha\beta\gamma\delta})$ of four fermion operators, relevant for FC reactions, in the physical basis. We start from the generic four fermion operator in the fundamental theory, assuming M_5 as the characteristic energy scale and taking all dimensionless coupling constants λ_{ijkl} equal to unity:

$$\int d^4x \int dy \sqrt{G} \, \frac{\lambda_{ijkl}}{M_5^3} \, \bar{\Psi}_i \Psi_j \bar{\Psi}_k \Psi_l = \int d^4x \, \frac{1}{Q_{\alpha\beta\gamma\delta}^2} \, \bar{\psi}_\alpha^{(0)\,\prime} \psi_\beta^{(0)\,\prime} \bar{\psi}_\gamma^{(0)\,\prime} \psi_\delta^{(0)\,\prime} + \dots \quad (23)$$

We remind that $_{i,j,k,l}$ are family indexes of the weak basis and $_{\alpha,\beta,\gamma,\delta}$ flavour indexes of the mass basis. The dots stand for KK excitation coupling terms. The expression for the effective energy scale $Q_{\alpha\beta\gamma\delta}$ in mass basis, obtained after the integration over

y and using Eq.(4), reads as (with an implicit sum over i, j, k, l)

$$\frac{1}{Q^2_{\alpha\beta\gamma\delta}} = \frac{U^\dagger_{\alpha i} U_{j\beta} U^\dagger_{\gamma k} U_{l\delta}}{\Lambda^2_{ijkl}} \tag{24}$$

where the matrices $U_{i\alpha}$ are responsible for the basis transformation of SM fermions (see Section 3.1) and the 4-dimensional energy scale Λ_{ijkl} is given by

$$\frac{1}{\Lambda^2_{ijkl}} = \frac{\lambda_{ijkl}}{N^i_0 N^j_0 N^k_0 N^l_0} \frac{1 - e^{-2\pi k R_c}}{2\pi^2 (kR_c)^2 M^2_{Pl}} \frac{e^{\pi k R_c (4 - c_i - c_j - c_k - c_l)} - 1}{4 - c_i - c_j - c_k - c_l}. \tag{25}$$

The normalization factors N^i_0 were defined by Eq.(11).

In the following subsections, we calculate the effective scale $Q_{\alpha\beta\gamma\delta}$, numerically, for the various types of four fermion operators contributing to FCNC processes: we will show that, for the 6 sets of parameters given in Appendix B which fix both the $U_{i\alpha}$ matrices and N^i_0 factors (and thus $Q_{\alpha\beta\gamma\delta}$), the obtained $Q_{\alpha\beta\gamma\delta}$ values induce different FCNC effects respecting all associated experimental constraints.

5.1   Lepton FC decays

Some four fermion operators induce the leptonic three-body decays of FCNC type $l_\alpha \rightarrow l_\beta l_\gamma l_\delta$, at a rate given approximately by $\Gamma \simeq m^5_{l_\alpha}/Q^4_{\alpha\beta\gamma\delta}$ (omitting the phase space factors) as deduced from Eq.(23). In Table 11, we explicitly present all the kinds of higher dimensional operators contributing to such decay channels. We restrict ourselves to operators which originate from EW gauge invariant terms and have allowed chirality configurations. These operators are written in terms of zero mode fields in the mass basis. For each one of these operators, we show, in the same table, the numerical value of the associated scale $Q_{\alpha\beta\gamma\delta}$ obtained from our theoretical expression in Eq.(24)-(25) for the point Y of parameter space. Our conclusion, here, is that all effective $Q_{\alpha\beta\gamma\delta}$ scale values obtained are well above their experimental lower limit. Indeed, the constraint on the branching ratio $B(\mu^- \rightarrow e^- e^+ e^-) < 1.0\ 10^{-12}$ ($B(\tau^- \rightarrow l_\beta l_\gamma l_\delta) \lesssim 2\ 10^{-6}$) [60], considered previously in Table 1, translates into an experimental limit $Q_{2111} > 2.6\ 10^6$GeV ($Q_{3\beta\gamma\delta} \gtrsim 5\ 10^4$GeV). The same results hold for the X and Z cases, i.e. the $Q_{\alpha\beta\gamma\delta}$ values systematically satisfy these experimental limits.

5.2   Meson mass splittings

Other types of four fermion operators contribute to the mass splitting in neutral pseudo-scalar meson systems. In Table 12, we give the gauge invariant forms, allowed by chirality, of dimension-six operators contributing to the Δm_K mass splitting (in terms of the zero mode quarks in the physical basis). For each operator, we also give the value of the corresponding scale Q_{1122} for the 3 sets A,B,C. With regard to this table, we observe that the values found for Q_{1122} satisfy the experimental bound $Q_{1122} > 5\ 10^6$GeV [30, 37] imposed by constraints on $K^0 - \bar{K}^0$ mixing (studied in Section 3.2).

Similarly, for the cases A,B,C, the values obtained for the Q_{1133} scale of the $(\bar{d}^c d)(\bar{b} b^c)$ operator contributing to Δm_B are respectively equal to 2.4 10^6, 4.8 10^6, 7.0 10^6GeV for left handed states, and 1.3 10^{11}, 6.6 10^{10}, 4.0 10^{10}GeV for the right

Decay channel	Operator	$\bar{l}^c_L l_L \bar{l}_L l^c_L$	$\bar{l}^c_R l_R \bar{l}_R l^c_R$
$\mu^- \to e^- e^+ e^-$	$\bar{\mu}^c e \bar{e} e^c$	$2.0\ 10^{12}$	$6.1\ 10^{12}$
$\tau^- \to e^- e^+ e^-$	$\bar{\tau}^c e \bar{e} e^c$	$9.4\ 10^{10}$	$1.5\ 10^{12}$
$\tau^- \to \mu^- \mu^+ \mu^-$	$\bar{\tau}^c \mu \bar{\mu} \mu^c$	$4.1\ 10^7$	$2.6\ 10^{11}$
$\tau^- \to e^- \mu^+ \mu^-$	$\bar{\tau}^c \mu \bar{\mu} e^c$	$7.4\ 10^8$	$6.8\ 10^{11}$
$\tau^- \to e^+ \mu^- \mu^-$	$\bar{\tau}^c e \bar{\mu} \mu^c$	$7.4\ 10^8$	$6.8\ 10^{11}$
$\tau^- \to \mu^- e^+ e^-$	$\bar{\tau}^c e \bar{\mu} e^c$	$1.7\ 10^{10}$	$1.1\ 10^{12}$
$\tau^- \to \mu^+ e^- e^-$	$\bar{\tau}^c \mu \bar{e} e^c$	$1.7\ 10^{10}$	$1.1\ 10^{12}$

Table 11: Four fermion operator types inducing the different FCNC lepton decays, together with their associated effective 4-dimensional $Q_{\alpha\beta\gamma\delta}$ scale value (in units of GeV) for the point Y of Appendix B. This value is given for the two possible chirality configurations of each operator, as indicated in the last two columns. We use conventions for Dirac spinors meaning that chirality projection acts first, then charge conjugation second and Dirac bar third: $\bar{l}^c_{L/R} = \overline{(l_{L/R})^c}$.

Operator	A	B	C
$\bar{d}^c_L d_L \bar{s}_L s^c_L$	$5.9\ 10^7$	$1.2\ 10^8$	$1.7\ 10^8$
$\bar{d}^c_R d_R \bar{s}_R s^c_R$	$4.1\ 10^{12}$	$2.1\ 10^{12}$	$1.3\ 10^{12}$

Table 12: Q_{1122} energy scale (in GeV) of the operators contributing to Δm_K for the 3 points A,B,C of Appendix B. Recall that in our spinorial notation, one has for the down quark: $\bar{d}^c_{L/R} = \overline{(d_{L/R})^c}$.

handed ones. These values are all within (although close to, for the left handed states) their experimental bound: $Q_{1133} > 2\ 10^6 \text{GeV}$.
The 3 cases A,B,C also give rise to Q_{2211} values, for the operator $(\bar{c}^c c)(\bar{u} u^c)$, which are clearly in agreement with the experimental constraints coming from $D^0 - \bar{D}^0$ mixing.

5.3 Muon electron conversion

Certain non-renormalizable operators involving both quarks and leptons can lead to $\mu - e$ conversion in muonic atoms. Indeed, the operators presented in Table 13 generate this conversion. On this table, we also show the corresponding effective Q_{1112} values computed for three characteristic combinations involving the quark parameter sets A,B,C and the lepton sets X,Y,Z. One can check there that the values obtained for Q_{1112} are well within the experimental constraint $Q_{1112} > 10^5 \text{GeV}$ originating from the exclusion limit on $B(\mu^- + Ti \to e^- + Ti)$ discussed in Eq.(20). In fact, any combination of A,B,C together with X,Y,Z leads to acceptable values for the Q_{1112} mass scale.

6 Conclusion

From the study on the RS model (with bulk matter) presented here, we obtain the following main conclusion. Regardless of the details of the model, there exist certain types of configuration for the fermion localizations which, simultaneously, reproduce the fermion mass hierarchies and mixings, and, generate FCNC effects within the present experimental limits for low KK gauge boson masses. The impact of this

Operator	A/X	B/Y	C/Z
$\bar{e}_R d_L \bar{d}_L \mu_R$	$1.7\ 10^{11}$	$7.2\ 10^{10}$	$6.8\ 10^{10}$
$\bar{e}_L d_R \bar{d}_R \mu_L$	$1.3\ 10^{11}$	$8.2\ 10^{11}$	$9.3\ 10^{11}$
$\bar{e}_R u_L \bar{u}_L \mu_R$	$3.6\ 10^{11}$	$1.6\ 10^{11}$	$1.5\ 10^{11}$
$\bar{e}_L u_R \bar{u}_R \mu_L$	$2.6\ 10^9$	$1.2\ 10^{11}$	$1.7\ 10^{11}$

Table 13: Effective scale Q_{1112} (in GeV) of the four operators contributing to coherent $\mu - e$ conversion, for 3 combinations of sets A,B,C and X,Y,Z taken from Appendix B.

conclusion is important, for two reasons:
First, this new possibility of the existence of light KK gauge boson states constitutes one of the first motivations for experimental searches of gauge boson excitations at the next coming high energy colliders [7].
Second, the possibility of having low KK gauge boson masses allows for a good solution of the gauge hierarchy problem within the RS scenario. As a matter of fact, low m_{KK} masses permit large values for kR_c and thus small $M_\star$ gravity scale values, close to the EW scale [8].

In a detailed analysis, we have constructed complete realizations of the RS scenario that address the gauge hierarchy problem, reproduce all the present data on quark/lepton masses and mixing angles (for the case of Dirac neutrinos), induce FCNC process amplitudes satisfying the experimental bounds for KK masses down to $m_{KK} = 1\text{TeV}$, generate acceptable effective suppression scales for non-renormalizable operators in the physical basis (for the parameter product in Eq.(5)) and, finally, can be compatible with some RS extensions suggested in the literature, respecting precision EW constraints with $m_{KK} \gtrsim 3\text{TeV}$.

In other words, we have shown that the attractive version of the RS model, providing a geometrical interpretation for the huge SM fermion mass hierarchies, does not necessarily conflict with the existence of small KK gauge field masses around 3TeV. Thus, in particular, it should induce diverse characteristic signatures potentially detectable at LHC. Indeed, even if a precise experimental investigation would be needed to prove the feasibility of such a detection, a preliminary study performed in [55], under the simplification assumption of a unique universal c_i value (which clearly prevents the creation of quark/lepton mass hierarchies), already obtained the following indicative results: the Tevatron Run II (with an integrated luminosity of $\mathcal{L} = 2fb^{-1}$) is capable of testing masses up to $m_{KK} \simeq 1\text{TeV}$ via a direct search for the first KK excited gauge boson exchanges, while the expected LHC sensitivity (for $\mathcal{L} = 100fb^{-1}$) on m_{KK} can reach values up to about 6TeV.

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[7]Most of the previous phenomenological works on RS model signatures at colliders, found in the literature, were dedicated to processes exchanging the KK excitations of graviton in the original RS set-up with all SM fields trapped at the TeV-brane.

[8]For example, with $m_{KK} = 1\text{TeV}$ (in agreement with FCNC constraints, as we have shown), the theoretical condition (6) on k dictates the maximum kR_c value to be 10.83 leading to $M_\star = 4\text{TeV}$, which is almost of the same order as the EW scale.

Acknowledgments

Appendix

A   Experimental data

At the Z^0 boson mass scale $Q = m_{Z^0}$, the renormalized charged lepton masses are [58]

$$m_e = 0.48684727 \pm 1.4 \ 10^{-7} \text{ MeV}$$
$$m_\mu = 102.75138 \pm 3.3 \ 10^{-4} \text{ MeV} \tag{A.1}$$
$$m_\tau = 1.74669^{+0.00030}_{-0.00027} \text{ GeV}$$

the quark masses are [58]

$$m_d = 4.69^{+0.60}_{-0.66} \text{ MeV} ; \quad m_u = 2.33^{+0.42}_{-0.45} \text{ MeV}$$
$$m_s = 93.4^{+11.8}_{-13.0} \text{ MeV} ; \quad m_c = 677^{+56}_{-61} \text{ MeV} \tag{A.2}$$
$$m_b = 3.00 \pm 0.11 \text{ GeV} ; \quad m_t = 181 \pm 13 \text{ GeV}$$

and the three CKM matrix parameters are [58]

$$\begin{aligned} |V_{us}| &= 0.2205 \pm 0.0018 \\ |V_{cb}| &= 0.0373 \pm 0.0018 \\ |V_{ub}/V_{cb}| &= 0.08 \pm 0.02. \end{aligned} \tag{A.3}$$

Next, we give the current data on neutrino masses and lepton mixings. A general three-flavour fit to the world's global neutrino data sample has been performed in [59]: the data sample used in this analysis includes the results from solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K) experiments. The values for oscillation parameters obtained in this analysis at the 4σ level are contained in the intervals

$$6.8 \leq \Delta m_{21}^2 \leq 9.3 \quad [10^{-5} \text{eV}^2],$$

$$1.1 \leq \Delta m_{31}^2 \leq 3.7 \quad [10^{-3} \text{eV}^2] \tag{A.4}$$

where $\Delta m_{21}^2 \equiv m_{\nu_2}^2 - m_{\nu_1}^2$ and $\Delta m_{31}^2 \equiv m_{\nu_3}^2 - m_{\nu_1}^2$ are the differences of squared neutrino mass eigenvalues, and

$$0.21 \leq \sin^2 \theta_{12} \leq 0.41,$$

$$0.30 \leq \sin^2 \theta_{23} \leq 0.72,$$

$$\sin^2 \theta_{13} \leq 0.073 \tag{A.5}$$

where θ_{12}, θ_{23} and θ_{13} are the three mixing angles of the convenient form of parameterization for the lepton mixing matrix (MNS matrix) now adopted as standard by the Particle Data Group [60]. Furthermore, the data on tritium beta decay [61] provided by the Mainz [62] and Troitsk [63] experiments give rise to the following upper bounds at 95% $C.L.$,

$$m_\beta \leq 2.2 \text{ eV} \quad [\text{Mainz}],$$

$$m_\beta \leq 2.5 \text{ eV} \quad [\text{Troitsk}] \tag{A.6}$$

with the effective mass m_β defined by $m_\beta^2 = \sum_{i=1}^3 |U_{ei}|^2 m_{\nu_i}^2$, U_{ei} denoting the lepton mixing matrix elements.

B Points of parameter space

We give here 3 complete sets [A,B,C] of parameters, namely all Yukawa coupling constants and 5-dimensional masses (see Section 2.2 for notations and conventions), reproducing the present quark masses and mixing angles (which are summarized in Appendix A):

$$[A]$$

$$c_1^Q = 0.2 \; ; \qquad c_2^Q = 0.2 \; ; \qquad c_3^Q = 0.2$$
$$c_1^d = 0.728 \; ; \qquad c_2^d = 0.740 \; ; \qquad c_3^d = 0.628$$
$$c_1^u = 0.62 \; ; \qquad c_2^u = 0.62 \; ; \qquad c_3^u = 0.35$$

$$\kappa_{ij}^d = \begin{pmatrix} 1.0 & 1.0 & 1.01 \\ 1.1 & -0.9 & 0.952 \\ 1.0 & 1.0 & 1.067 \end{pmatrix} \qquad \kappa_{ij}^u = \begin{pmatrix} 1.0 & 0.9 & 1.03 \\ 1.1 & 1.0 & 0.9 \\ 1.0 & 0.9 & 1.1 \end{pmatrix}$$

$$[B]$$

$$c_1^Q = 0.37 \; ; \qquad c_2^Q = 0.37 \; ; \qquad c_3^Q = 0.37$$
$$c_1^d = 0.716 \; ; \qquad c_2^d = 0.728 \; ; \qquad c_3^d = 0.615$$
$$c_1^u = 0.607 \; ; \qquad c_2^u = 0.607 \; ; \qquad c_3^u = 0.050$$

$$\kappa_{ij}^d = \begin{pmatrix} 1.0 & 1.0 & 1.017 \\ 1.1 & -0.9 & 0.96 \\ 1.0 & 1.0 & 1.075 \end{pmatrix} \qquad \kappa_{ij}^u = \begin{pmatrix} 1.0 & 0.9 & 1.029 \\ 1.1 & 1.0 & 0.9 \\ 1.0 & 0.9 & 1.1 \end{pmatrix}$$

$$[C]$$

$$c_1^Q = 0.413 \; ; \qquad c_2^Q = 0.413 \; ; \qquad c_3^Q = 0.413$$
$$c_1^d = 0.703 \; ; \qquad c_2^d = 0.721 \; ; \qquad c_3^d = 0.608$$
$$c_1^u = 0.6 \; ; \qquad c_2^u = 0.6 \; ; \qquad c_3^u = -0.4$$

$$\kappa_{ij}^d = \begin{pmatrix} 1.0 & 1.0 & 1.017 \\ 1.1 & -0.9 & 0.96 \\ 1.0 & 1.0 & 1.075 \end{pmatrix} \qquad \kappa_{ij}^u = \begin{pmatrix} 1.0 & 0.9 & 1.029 \\ 1.1 & 1.0 & 0.9 \\ 1.0 & 0.9 & 1.1 \end{pmatrix}$$

the Yukawa coupling indexes i and j corresponding respectively to the line and column, exactly as in Eq.(12)-(13).

Now, we present 3 sets [X,Y,Z] of parameters creating the current data on lepton masses and mixings (*c.f.* Appendix A):

$$[X]$$

$$c_1^L = -1.5 \; ; \qquad c_2^L = -1.5 \; ; \qquad c_3^L = -1.5$$
$$c_1^l = 0.760 \; ; \qquad c_2^l = 0.833 \; ; \qquad c_3^l = 0.667$$
$$c_1^\nu = 1.512 \; ; \qquad c_2^\nu = 1.513 \; ; \qquad c_3^\nu = 1.468$$

$$\kappa_{ij}^l = \begin{pmatrix} 0.9 & 1.0 & 1.1 \\ 1.0 & 1.1 & 1.1 \\ -1.1 & 0.9 & 0.9 \end{pmatrix} \quad \kappa_{ij}^\nu = \begin{pmatrix} -1.1 & -0.9 & -1.1 \\ -1.1 & 1.0 & -1.1 \\ -0.9 & 0.9 & 0.9 \end{pmatrix}$$

$$[Y]$$

$$c_1^L = 0.200 \; ; \quad c_2^L = 0.200 \; ; \quad c_3^L = 0.261$$
$$c_1^l = 0.737 \; ; \quad c_2^l = 0.696 \; ; \quad c_3^l = 0.647$$
$$c_1^\nu = 1.496 \; ; \quad c_2^\nu = 1.503 \; ; \quad c_3^\nu = 1.463$$

$$\kappa_{ij}^l = \begin{pmatrix} 1.0 & 1.0 & 1.0 \\ 1.0 & 1.003 & 1.0 \\ -0.9 & 1.0 & 1.0 \end{pmatrix} \quad \kappa_{ij}^\nu = \begin{pmatrix} -1.0 & -1.1 & -1.0 \\ -1.1 & 1.0 & -1.1 \\ -1.0 & 1.0 & 0.9 \end{pmatrix}$$

$$[Z]$$

$$c_1^L = 0.35 \; ; \quad c_2^L = 0.35 \; ; \quad c_3^L = 0.39$$
$$c_1^l = 0.728 \; ; \quad c_2^l = 0.694 \; ; \quad c_3^l = 0.636$$
$$c_1^\nu = 1.49 \; ; \quad c_2^\nu = 1.49 \; ; \quad c_3^\nu = 1.45$$

$$\kappa_{ij}^l = \begin{pmatrix} 1.0 & 1.0 & 1.0 \\ 1.0 & 1.0035 & 1.0 \\ -0.9 & 1.0 & 1.0 \end{pmatrix} \quad \kappa_{ij}^\nu = \begin{pmatrix} -1.0 & -1.1 & -1.0 \\ -1.1 & 1.0 & -1.1 \\ -1.0 & 1.0 & 0.9 \end{pmatrix}$$

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