

Softly broken lepton number $L_e - L_\mu - L_\tau$ with non-maximal solar neutrino mixing

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Abstract

We consider the most general neutrino mass matrix which leads to $\theta_{13} = 0$, and present the formulae needed for obtaining the neutrino masses and mixing parameters in that case. We apply this formalism to a model based on the lepton number $\bar{L} = L_e - L_\mu - L_\tau$ and on the seesaw mechanism. This model needs only one Higgs doublet and has only two right-handed neutrino singlets. Soft \bar{L} breaking is accomplished by the Majorana mass terms of the right-handed neutrinos; if the \bar{L} -conserving and \bar{L} -breaking mass terms are of the same order of magnitude, then it is possible to obtain a consistent \bar{L} model with a solar mixing angle significantly smaller than 45° . We show that the predictions of this model, $m_3 = 0$ and $\theta_{13} = 0$, are invariant under the renormalization-group running of the neutrino mass matrix.

1. Introduction

In recent times there has been enormous experimental progress in our knowledge of the mass-squared differences and of the mixing of light neutrinos—for a review see, for instance, [1]. Unfortunately, to this progress on the experimental and phenomenological—i.e. neutrino oscillations [2] and the MSW effect [3]—fronts there has hardly been a counterpart in our theoretical understanding of neutrino masses and lepton mixing—for a review see, for instance, [4].

It has been conclusively shown that the lepton mixing matrix is substantially different from the quark mixing matrix. Whereas the solar mixing angle, θ_{12} , is large— $\theta_{12} \sim 33^\circ$ —and the atmospheric mixing angle, θ_{23} , could even be maximal, the third mixing angle, θ_{13} , is small—there is only an upper bound on it which, according to [1], is given at the 3σ level by $\sin^2 \theta_{13} < 0.047$. The true magnitude of θ_{13} will be crucial in the future experimental exploration of lepton mixing, and it is also important for our theoretical understanding of that mixing—see, for instance, [5].

In this paper, we contemplate the possibility that at some energy scale a flavour symmetry exists such that θ_{13} is exactly zero. It has been shown [6] that, in the basis where the charged-lepton mass matrix is diagonal, the most general neutrino mass matrix which yields $\theta_{13} = 0$ is given, apart from a trivial phase convention [6], by

$$\mathcal{M}_\nu = \begin{pmatrix} X & \sqrt{2}A \cos(\gamma/2) & \sqrt{2}A \sin(\gamma/2) \\ \sqrt{2}A \cos(\gamma/2) & B + C \cos \gamma & C \sin \gamma \\ \sqrt{2}A \sin(\gamma/2) & C \sin \gamma & B - C \cos \gamma \end{pmatrix}, \quad (1)$$

with parameters X , A , B and C which are in general complex. The mass matrix (1), but not necessarily the full Lagrangian, enjoys a \mathbb{Z}_2 symmetry [6, 7]—see also [8]—defined by

$$S(\gamma)\mathcal{M}_\nu S(\gamma) = \mathcal{M}_\nu, \quad (2)$$

with an orthogonal 3×3 matrix

$$S(\gamma) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma & \sin \gamma \\ 0 & \sin \gamma & -\cos \gamma \end{pmatrix} \quad (3)$$

which satisfies $S(\gamma) = [S(\gamma)]^T$ and $[S(\gamma)]^2 = \mathbf{1}$. We may remove two unphysical phases from (1), e.g. by choosing X and A to be real, and then there are seven real parameters in that mass matrix. Those seven parameters correspond to the following seven physical quantities: the three neutrino masses $m_{1,2,3}$, the solar and atmospheric mixing angles and two Majorana phases. The only prediction of the mass matrix (1) is $\theta_{13} = 0$; however, that prediction entails the non-observability of the Dirac phase in lepton mixing—in general there are *nine* physical quantities in the neutrino masses and mixings.

Expressed in terms of the parameters³ of \mathcal{M}_ν , one obtains masses given by [6, 10]

$$m_3 = |B - C| \quad (4)$$

and

$$m_{1,2}^2 = \frac{1}{2}[|X|^2 + |D|^2 + 4|A|^2 \mp \sqrt{(|X|^2 + |D|^2 + 4|A|^2)^2 - 4|XD - 2A^2|^2}], \quad (5)$$

with

$$D \equiv B + C; \quad (6)$$

while the mixing angles are given by

$$\theta_{23} = |\gamma/2| \quad (7)$$

and

$$(m_2^2 - m_1^2) \sin 2\theta_{12} = 2\sqrt{2}|X^*A + A^*D|, \quad (8)$$

$$(m_2^2 - m_1^2) \cos 2\theta_{12} = |D|^2 - |X|^2. \quad (9)$$

The only Majorana phase which—for $\theta_{13} = 0$ —plays a role in neutrinoless $\beta\beta$ decay is the phase $\Delta = \arg[(U_{e2}/U_{e1})^2]$, where U_{e1} and U_{e2} are matrix elements of the lepton mixing (PMNS [2, 11]) matrix U . The phase Δ is given by [12]

$$8 \operatorname{Im}(X^*D^*A^2) = m_1 m_2 (m_1^2 - m_2^2) \sin^2 2\theta_{12} \sin \Delta. \quad (10)$$

The other Majorana phase is practically unobservable [13]⁴.

³ The procedure for obtaining the neutrino masses and the lepton mixing matrix from the parameters of a fully general neutrino mass matrix has been given in [9].

⁴ When $|X| = |D|$ and $X^*A = -A^*D$ one should use, instead of (5) and (8)–(10), $m_1 = m_2 = \sqrt{|X|^2 + 2|A|^2}$, $\Delta = \pi$ and $\cos 2\theta_{12} = |X|/m_1$.

In specific models with $\theta_{13} = 0$, the neutrino mass matrix (1) is further restricted:

- The \mathbb{Z}_2 model of [10], which is based on the non-Abelian group $O(2)$ [14], yields maximal atmospheric neutrino mixing, i.e. $\gamma = \pi/2$, and has six physical parameters.
- The D_4 model of [15], which is based on the discrete group D_4 , also has $\gamma = \pi/2$ and, in addition, it predicts $XC = A^2$. The number of parameters is four—in that model the Majorana phases are expressible in terms of the neutrino masses and of the solar mixing angle.
- The softly-broken- D_4 model of [7] is a generalization of the D_4 model: the atmospheric mixing angle is undetermined, but $XC = A^2$ still holds.
- The seesaw model of the first line of table 1 of [16], which is based on the Abelian group \mathbb{Z}_4 , reproduces matrix (1) without restrictions.

In this paper, we consider the $U(1)$ symmetry generated by the lepton number $\bar{L} \equiv L_e - L_\mu - L_\tau$ [17]. It is well known that exact \bar{L} symmetry enforces $\theta_{13} = 0$ (with $X = B = C = 0$ in (1)), while an approximate \bar{L} symmetry tends to produce either a solar mixing angle too close to 45° or a solar mass-squared difference too close to the atmospheric mass-squared difference [18]. A possible way out of this dilemma is to assume a significant contribution to U from the diagonalization of the charged-lepton mass matrix [19]; another possibility is a significant breaking of \bar{L} [20]. Here we discuss a \bar{L} model, first proposed in [21], which makes use of the seesaw mechanism [22] with *only two* right-handed neutrino singlets ν_R . The $U(1)_{\bar{L}}$ symmetry is softly broken in the Majorana mass matrix of the ν_R , but—contrary to what was done in [21]—the soft breaking is assumed here to be rather ‘strong’, in order to achieve a solar mixing angle significantly smaller than 45° . The model presented in section 2 predicts a mass matrix (1) with $B = C$, i.e. it predicts $m_3 = 0$ together with $\theta_{13} = 0$. We will show in section 3 that these predictions *are stable* under the renormalization-group running from the seesaw scale down to the electroweak scale. Next, we show in section 4 that our model does *not* provide enough leptogenesis to account for the observed baryon-to-photon ratio of the universe. We end in section 5 with our conclusions.

2. The model

The lepton number $\bar{L} = L_e - L_\mu - L_\tau$ has a long history in model building [17, 18]. In this paper, we rediscuss the model of [21], which has *only one Higgs doublet*, ϕ , and two right-handed neutrinos, ν_{R1} and ν_{R2} , with the following assignments of the quantum number \bar{L} :

$$\bar{L} \left| \begin{array}{cccccc} \nu_e, e & \nu_\mu, \mu & \nu_\tau, \tau & \nu_{R1} & \nu_{R2} & \phi \\ 1 & -1 & -1 & 1 & -1 & 0 \end{array} \right. \quad (11)$$

This model is a simple extension of the standard model which incorporates the seesaw mechanism [22]. The right-handed neutrino singlets have a Majorana mass term

$$\mathcal{L}_M = -\frac{1}{2} \bar{\nu}_R M_R C \bar{\nu}_R^T + \text{h.c.}, \quad (12)$$

(where C is the charge-conjugation matrix in spinor space) with

$$M_R = \begin{pmatrix} R & M \\ M & S \end{pmatrix}. \quad (13)$$

The elements of the matrix M_R are of the heavy seesaw scale. The entry M in M_R is compatible with \bar{L} symmetry, while the entries R and S break that lepton number softly. The breaking of

\bar{L} is soft since the Majorana mass terms have dimension 3. Because of the $U(1)$ symmetry associated with \bar{L} , the neutrino Dirac mass matrix has the structure [21]⁵

$$M_D = \begin{pmatrix} a & 0 & 0 \\ 0 & b' & b'' \end{pmatrix}. \quad (14)$$

Then the effective Majorana mass matrix of the light neutrinos is given by the seesaw formula

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D = \frac{1}{M^2 - RS} \begin{pmatrix} Sa^2 & -Mab' & -Mab'' \\ -Mab' & Rb'^2 & Rb'b'' \\ -Mab'' & Rb'b'' & Rb''^2 \end{pmatrix}. \quad (15)$$

In the case of \bar{L} conservation, i.e. when $R = S = 0$, we have $m_1 = m_2$ and θ_{12} is 45° ; this is a well known fact. Non-zero mass parameters R and S induce $\Delta m_\odot^2 \equiv m_2^2 - m_1^2 \neq 0$ and allow a non-maximal solar mixing angle⁶. The phases of a , b' and b'' are unphysical; in the following we shall assume those parameters to be real and positive. The only physical phase is [21]

$$\alpha \equiv \arg(R^* S^* M^2). \quad (16)$$

CP conservation is equivalent to α being a multiple of π . Defining $d \equiv M^2 - RS$ and $b \equiv \sqrt{b'^2 + b''^2}$, we see that (15) has the form (1) with

$$X = \frac{Sa^2}{d}, \quad A = -\frac{Mab}{\sqrt{2}d}, \quad B = C = \frac{Rb^2}{2d}, \quad (17)$$

and

$$\cos \frac{\gamma}{2} = \frac{b'}{b}, \quad \sin \frac{\gamma}{2} = \frac{b''}{b}. \quad (18)$$

As mentioned in the introduction, $C = B$ and therefore $m_3 = 0$, while X , A and B are independent parameters. Since $m_3 = 0$ the neutrino mass spectrum displays inverted hierarchy. The present model has five real parameters— $|X|$, $|A|$, $|B|$, γ and α —which correspond to the physical observables $m_{1,2}$, θ_{12} , θ_{23} and the Majorana phase Δ (the second Majorana phase is unphysical in this case because $m_3 = 0$).

Let us now perform a consistency check by using all the available input from the neutrino sector. We have the following observables at our disposal: the effective Majorana mass in neutrinoless $\beta\beta$ decay $m_{\beta\beta}$, the solar mass-squared difference Δm_\odot^2 , the atmospheric mass-squared difference Δm_{atm}^2 , the solar mixing angle θ_{12} and the atmospheric mixing angle θ_{23} . In a three-neutrino scenario the definition of Δm_{atm}^2 is not unique; we define $\bar{m}^2 \equiv (m_1^2 + m_2^2)/2$ and use $\Delta m_{\text{atm}}^2 \simeq \bar{m}^2$, which is valid in this model because of the inverted mass hierarchy. The relation $\gamma = 2\theta_{23}$ plays no role in the following discussion, which consists in determining the four parameters $|X|$, $|A|$, $|D| = 2|B|$, and α as functions of the four observables $m_{\beta\beta}$, Δm_\odot^2 , Δm_{atm}^2 and θ_{12} . We note that, because of (16) and (17), $\alpha = \arg(D^* X^* A^2)$.

We first note that, in (1), $m_{\beta\beta}$ is just given by $|X|$

$$|X| = m_{\beta\beta}. \quad (19)$$

We then use (9) to write

$$|D|^2 = m_{\beta\beta}^2 + \Delta m_\odot^2 \cos 2\theta_{12}. \quad (20)$$

From (5), $\bar{m}^2 = 2|A|^2 + (|X|^2 + |D|^2)/2$, hence

$$|A|^2 = \frac{1}{2}(\bar{m}^2 - m_{\beta\beta}^2 - \frac{1}{2}\Delta m_\odot^2 \cos 2\theta_{12}). \quad (21)$$

⁵ We are assuming, without loss of generality, the charged-lepton mass matrix to be already diagonal.

⁶ The case of non-zero R and S has also been considered in [23].

Since $|A| \geq 0$ we have the bound

$$m_{\beta\beta}^2 \leq \bar{m}^2 - \frac{1}{2} \Delta m_\odot^2 \cos 2\theta_{12}. \quad (22)$$

In order to find α we start from (8), writing

$$(\Delta m_\odot^2)^2 \sin^2 2\theta_{12} = 8|A|^2(|X|^2 + |D|^2 + 2|X||D| \cos \alpha). \quad (23)$$

We define

$$p \equiv \frac{2m_{\beta\beta}^2}{\Delta m_\odot^2 \cos 2\theta_{12}}, \quad (24)$$

$$\rho \equiv \frac{2\bar{m}^2}{\Delta m_\odot^2 \cos 2\theta_{12}}, \quad (25)$$

and obtain

$$\cos \alpha = \left[-(1+p) + \frac{\tan^2 2\theta_{12}}{2(\rho-1-p)} \right] \frac{1}{\sqrt{p(p+2)}}. \quad (26)$$

Thus, we have expressed all the parameters of the model in terms of physical quantities.

The parameter ρ is known and it is quite large: using the mean values of the mixing parameters [1] $\Delta m_\odot^2 \sim 8.1 \times 10^{-5} \text{ eV}^2$, $\Delta m_{\text{atm}}^2 \sim 2.2 \times 10^{-3} \text{ eV}^2$ and $\sin^2 \theta_{12} \sim 0.30$, we find $\rho \sim 136$. The parameter p , on the other hand, is unknown. Equation (26) requires that a non-zero range $[p_-, p_+]$ for p exists for which the right-hand side of that equation lies in between -1 and $+1$. One finds that

$$p_\pm = \frac{\rho}{2} \left(1 + \cos^2 2\theta_{12} \pm \sin^2 2\theta_{12} \sqrt{1 - \frac{1}{\rho^2 \cos^2 2\theta_{12}}} \right) - 1. \quad (27)$$

Since ρ is large this can be approximated by

$$p_- \simeq \rho \cos^2 2\theta_{12} - 1, \quad p_+ \simeq \rho - 1, \quad (28)$$

or

$$\bar{m}^2 \cos^2 2\theta_{12} - \frac{\Delta m_\odot^2 \cos 2\theta_{12}}{2} \lesssim m_{\beta\beta}^2 \lesssim \bar{m}^2 - \frac{\Delta m_\odot^2 \cos 2\theta_{12}}{2}. \quad (29)$$

In this approximation, the upper bound on $m_{\beta\beta}$ coincides with the one in (22). With good accuracy we have in this model

$$\sqrt{\Delta m_{\text{atm}}^2 \cos 2\theta_{12}} \lesssim m_{\beta\beta} \lesssim \sqrt{\Delta m_{\text{atm}}^2}. \quad (30)$$

This is one of the predictions of the model. Thus, if the claim $m_{\beta\beta} > 0.1 \text{ eV}$ of [24] is confirmed, then the present model will be ruled out since $\sqrt{\Delta m_{\text{atm}}^2} \sim 0.047 \text{ eV}$.

From (8), (9) and (17) we find the following expression for the solar mixing angle:

$$\tan 2\theta_{12} = \frac{2|M|ab}{|R|b^2 - |S|a^2} \frac{||R|b^2 + |S|a^2 e^{i\alpha}|}{|R|b^2 + |S|a^2}. \quad (31)$$

This equation shows that non-maximal solar neutrino mixing is easily achievable when $|R|$ and $|S|$ are of the same order of magnitude as $|M|$. This is what we mean with ‘strong’ soft \bar{L} breaking, namely that the Majorana mass terms which violate \bar{L} softly (i.e. R and S) and the one which conserves \bar{L} (i.e. M) are of the same order of magnitude⁷.

One may ask whether it is possible to evade this feature and assume $|R|, |S| \ll |M|$. In that case, since experimentally $\tan 2\theta_{12} \simeq 2.3$, and since the second fraction in the right-hand

⁷ In [21] we assumed $|R|, |S| \ll |M|$ and ended up with almost-maximal solar mixing, which was still allowed by the data at that time.

side of (31) cannot be larger than 1, we would conclude that $b/a \sim |M/R|$. But then $|R|b^2$ would be much larger than $|S|a^2$ and therefore $|D| \gg |X|$ which, from (19) and (20), means that $m_{\beta\beta}^2 \ll \Delta m_{\odot}^2 \cos 2\theta_{12}$. This contradicts our previous finding that $m_{\beta\beta}^2$ must be of the order of magnitude of Δm_{atm}^2 . We thus conclude that the hypothesis $|R|, |S| \ll |M|$ is incompatible with the experimental data.

3. Radiative corrections

We have not yet taken into account the fact that the energy scale where \bar{L} -invariance holds and the mass matrices M_D and \mathcal{M}_ν have the forms (14) and (15), respectively, is the seesaw scale. Since our model has *only one* Higgs doublet, the relation between the mass matrix $\mathcal{M}_\nu^{(0)}$ at the seesaw scale and the mass matrix \mathcal{M}_ν at the electroweak scale is simply given by [25]

$$\mathcal{M}_\nu = I \mathcal{M}_\nu^{(0)} I, \quad (32)$$

where I is a diagonal, positive and non-singular matrix, since the charged-lepton mass matrix is diagonal. Now, suppose there is a vector $u^{(0)}$ such that $\mathcal{M}_\nu^{(0)} u^{(0)} = 0$. Then the vector $u \equiv I^{-1} u^{(0)}$ is an eigenvector to \mathcal{M}_ν with eigenvalue zero⁸. Moreover, if one entry of $u^{(0)}$ is zero, then the corresponding entry of u is zero as well, due to I being diagonal. We stress that these observations only hold for eigenvectors with eigenvalue zero.

Applying this to the present model, we find that $m_3 = 0$ together with $\theta_{13} = 0$ are predictions *stable under the renormalization-group evolution* (see also [26]). The matrices \mathcal{M}_ν and $\mathcal{M}_\nu^{(0)}$ are related through $M_D = M_D^{(0)} I$, where $M_D^{(0)}$ is the neutrino Dirac mass matrix at the seesaw scale; again, due to I being *diagonal*, both Dirac mass matrices have the same form (14). Therefore, all our discussions in the previous section hold for the physical quantities at the low (electroweak) scale.

4. Leptogenesis

The model in this paper has very few parameters and only one Higgs doublet. Therefore, it allows clear-cut predictions for leptogenesis—for reviews see, for instance, [27]. It turns out that the computations for this model resemble closely the ones for the \mathbb{Z}_2 model [10], which were performed in a previous paper [12]. We give here only the gist of the argument.

Let the matrix M_R in (13) be diagonalized by the 2×2 unitary matrix

$$V = \begin{pmatrix} c' e^{i\omega} & s' e^{i\sigma} \\ -s' e^{i\tau} & c' e^{i(\sigma+\tau-\omega)} \end{pmatrix} \quad (33)$$

as

$$V^T M_R V = \text{diag}(M_1, M_2), \quad (34)$$

with real, non-negative M_1 and M_2 . We assume $M_1 \ll M_2$. In (33), $c' \equiv \cos \theta'$ and $s' \equiv \sin \theta'$, where θ' is an angle of the first quadrant. Defining the Hermitian matrix

$$H \equiv V^T M_D M_D^\dagger V^*, \quad (35)$$

the relevant quantity for leptogenesis is [27]

$$\mathcal{Q} \equiv \frac{\text{Im}[(H_{12})^2]}{(H_{11})^2}. \quad (36)$$

⁸ This statement would still be true for a non-diagonal matrix I .

One may use as input for leptogenesis the heavy-neutrino masses $M_{1,2}$ together with $m_{1,2}$, θ_{12} and the Majorana phase Δ . One can demonstrate that a and $b = \sqrt{b'^2 + b''^2}$ satisfy

$$a^2 b^2 = m_1 m_2 M_1 M_2 \quad (37)$$

and

$$\begin{aligned} & |s_{12}^2 m_1 + c_{12}^2 m_2 e^{i\Delta}|^2 a^4 + |c_{12}^2 m_1 + s_{12}^2 m_2 e^{i\Delta}|^2 b^4 \\ &= m_1^2 m_2^2 (M_1^2 + M_2^2) - 2m_1 m_2 M_1 M_2 c_{12}^2 s_{12}^2 |m_1 - m_2 e^{i\Delta}|^2, \end{aligned} \quad (38)$$

where $c_{12} \equiv \cos \theta_{12}$ and $s_{12} \equiv \sin \theta_{12}$. By using (37) and (38), one finds the values of a and b from the input, with a two-fold ambiguity only. Then θ' is given by

$$c'^2 - s'^2 = \frac{1}{m_1^2 m_2^2 (M_1^2 - M_2^2)} (|s_{12}^2 m_1 + c_{12}^2 m_2 e^{i\Delta}|^2 a^4 - |c_{12}^2 m_1 + s_{12}^2 m_2 e^{i\Delta}|^2 b^4). \quad (39)$$

With a and b known, \mathcal{Q} is found as a function of the input by use of

$$H_{11} = a^2 c'^2 + b^2 s'^2, \quad (40)$$

$$\text{Im}[(H_{12})^2] = (b^2 - a^2)^2 \frac{M_1 M_2}{M_2^2 - M_1^2} \frac{m_2^2 - m_1^2}{m_1 m_2} c_{12}^2 s_{12}^2 \sin \Delta. \quad (41)$$

Equations (37)–(41) are identical with those of the \mathbb{Z}_2 model, derived in [12]. In order to compute the baryon-to-photon ratio of the universe, η_B , one must [12] multiply \mathcal{Q} by (i) $M_1/(10^{11} \text{ GeV})$, (ii) a numerical factor of order 10^{-9} , (iii) a function of M_2/M_1 and (iv) $(\ln K_1)^{-3/5}$, where $K_1 \propto H_{11}/M_1$. (All these factors are given and explained in [12], together with references to the original papers.) One may then compute η_B as a function of the input.

Most crucial is the behaviour of η_B as a function of m_1 when $m_2^2 - m_1^2 = \Delta m_\odot^2$ is kept fixed. One finds that η_B grows with m_1 , finding a maximum for $m_1 \sim 4 \times 10^{-3} \text{ eV}$, afterwards decreasing rapidly for a larger m_1 . Now, the present model—contrary to what happened in the model treated in [12], wherein m_1 was free—has $m_3 = 0$ and, therefore, $m_1 \simeq \sqrt{\Delta m_{\text{atm}}^2} \sim 0.05 \text{ eV}$. For such a high value of m_1 the baryon-to-photon ratio turns out to be hopelessly small. Thus, in the present model, contrary to what happened in the \mathbb{Z}_2 model [10] worked out in [12], leptogenesis is not a viable option for explaining the baryon asymmetry of the universe.

5. Conclusions

In this paper, we have discussed an extension of the lepton sector of the standard model with two right-handed neutrino singlets and the seesaw mechanism. The model, which was originally proposed in [21], is based on the lepton number $\bar{L} = L_e - L_\mu - L_\tau$. Zeros in the 2×3 neutrino Dirac mass matrix are enforced by \bar{L} invariance, and as a consequence the model features the predictions $\theta_{13} = 0$ and a hierarchical neutrino mass spectrum with $m_3 = 0$.⁹ The lepton number \bar{L} is softly broken in the 2×2 Majorana mass matrix M_R of the right-handed neutrino singlets, by the two entries R and S in (13) which would be zero in the case of exact \bar{L} invariance. One obtains $\Delta m_\odot^2 \neq 0$ and $\theta_{12} \neq 45^\circ$ from that soft breaking. However, $\theta_{12} \sim 33^\circ$ requires the soft breaking to be ‘strong’, which means that R and S are of the same order of magnitude as the element M in M_R which is allowed by \bar{L} invariance. Thus the model discussed here has the property that in M_R there is no trace of \bar{L} invariance, whereas the form of the Dirac mass matrix is completely determined by that invariance.

⁹ These predictions are common with other models based on \bar{L} -invariance [28].

We have argued that, for models with one Higgs doublet like the present one, the configuration $m_3 = 0$ together with $\theta_{13} = 0$ is stable under the renormalization-group evolution.

A further prediction of our model is the range for the effective mass in neutrinoless $\beta\beta$ decay, in particular the lower bound given by (30); the order of magnitude of that effective mass is the square root of the atmospheric mass-squared difference.

Since there is only one CP -violating phase in our model, we have also considered the possibility of thermal leptogenesis; however, it turns out that this mechanism is unable to generate a realistic baryon asymmetry of the universe. This is because in our model the neutrino mass m_1 is too large, due to the inverted mass hierarchy.

In summary, we have shown by way of a very economical example that—contrary to claims in the literature—models based on the lepton number $L_e - L_\mu - L_\tau$ are not necessarily incompatible with the solar mixing angle being significantly smaller than 45° .

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