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Nuclear Physics B 524 (1998) 23–40

NUCLEAR PHYSICS B

Minimal supergravity with R-parity breaking

Marco A. Díaz [a], Jorge C. Romão [b], José W.F. Valle [a]

[a] *Departamento de Física Teórica, Universidad de Valencia, Burjassot, Valencia 46100, Spain*
[b] *Departamento de Física, Instituto Superior Técnico, A. Rovisco Pais, 1096 Lisbon Codex, Portugal*

Received 9 June 1997; revised 13 February 1998; accepted 16 March 1998

Abstract

We show that the minimal R-parity breaking model characterized by an effective bilinear violation of R-parity in the superpotential is consistent with minimal $N = 1$ supergravity unification with radiative breaking of the electroweak symmetry and universal scalar and gaugino masses. This one-parameter extension of the MSSM-SUGRA model provides therefore the simplest reference model for the breaking of R-parity and constitutes a consistent truncation of the complete dynamical models with spontaneous R-parity breaking proposed previously. We comment on the lowest-lying CP-even Higgs boson mass and discuss its minimal $N = 1$ supergravity limit, determine the ranges of $\tan\beta$ and bottom quark Yukawa couplings allowed in the model, as well as the relation between the tau neutrino mass and the bilinear R-parity violating parameter. © 1998 Elsevier Science B.V.

1. Introduction

Supersymmetry apart from being attractive from the point of view of providing a solution to the hierarchy problem and the unification of the gauge couplings, provides an elegant mechanism for the breaking of the electroweak symmetry via radiative corrections [1]. So far most attention to the study of supersymmetric phenomenology has been made in the framework of the Minimal Supersymmetric Standard Model (MSSM) [2] with conserved R-parity [3]. R-parity is a discrete symmetry assigned as $R_p = (-1)^{(3B+L+2S)}$, where L is the lepton number, B is the baryon number and S is the spin of the state. If R-parity is conserved all supersymmetric particles must always be pair-produced, while the lightest of them must be stable. Whether or not supersymmetry is realized with a conserved R-parity is an open dynamical question, sensitive to physics at a more fundamental scale.

The study of alternative supersymmetric scenarios where the effective low energy theory violates R-parity [4] has received recently a lot of attention [5–7]. As is well

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PII S0550-3213(98)00234-X

known, the simplest supersymmetric extension of the Standard Model violates R-parity through a set of cubic superpotential terms involving a very large number of arbitrary Yukawa couplings. Although highly constrained by proton stability, one cannot exclude that a large number of such scenarios could be viable. Nevertheless their systematic study at a phenomenological level is hardly possible, due to the large number of parameters (almost fifty) characterizing these models, in addition to those of the MSSM.

As other fundamental symmetries, it could well be that R-parity is a symmetry at the Lagrangian level but is broken by the ground state. Such scenarios provide a very *systematic* way to include R-parity violating effects, automatically consistent with low energy *baryon number conservation*. They have many added virtues, such as the possibility of having a dynamical origin for the breaking of R-parity, through radiative corrections, similar to the electroweak symmetry [8].

In this paper we focus on the simplest truncated version of such a model, in which the violation of R-parity is effectively parametrized by a bilinear superpotential term $\epsilon_i \widehat{L}_i^a \widehat{H}_2^b$. In this effective truncated model the superfield content is exactly the standard one of the MSSM. In this case there is no physical Goldstone boson, the Majoron, associated to the spontaneous breaking of R-parity. Formulated at the weak scale, the model contains only two new parameters in addition to those of the MSSM. Alternatively, the unified version of the model, contains exactly a single additional parameter when compared to the unified version of the MSSM, which we will from now on call MSSM-SUGRA. Therefore our model is the simplest way to break R-parity and can thus be regarded as a reference model for R-parity breaking. In contrast to models with trilinear R-parity breaking couplings, it leads to a very restrictive and systematic pattern of R-parity violating interactions.

Here we show that this simplest truncated version of the R-parity breaking model of Ref. [8], characterized by a bilinear violation of R-parity in the superpotential, is consistent with minimal $N = 1$ supergravity models with radiative electroweak symmetry breaking and universal scalar and gaugino masses at the unification scale. In particular, we perform a thorough study of the minimization of the scalar boson potential using the tadpole method needed for an accurate determination of the Higgs boson mass spectrum. We comment on the lowest-lying CP-even Higgs boson mass and discuss its minimal $N = 1$ supergravity limit, determining also the ranges of $\tan\beta$ and bottom quark Yukawa couplings allowed at unification, as well as the relation between the tau neutrino mass and the effective bilinear R-parity violating parameter. Our results encourage further theoretical work on this and on more complete versions of the model, like that of Ref. [8], as well as phenomenological studies of the related signals.

2. The model

The supersymmetric Lagrangian is specified by the superpotential W given by [1]

[1] We are using here the notation of Refs. [2,9].

$$W = \varepsilon_{ab}\left[h_U^{ij}\widehat{Q}_i^a\widehat{U}_j\widehat{H}_2^b + h_D^{ij}\widehat{Q}_i^b\widehat{D}_j\widehat{H}_1^a + h_E^{ij}\widehat{L}_i^b\widehat{R}_j\widehat{H}_1^a - \mu\widehat{H}_1^a\widehat{H}_2^b + \epsilon_i\widehat{L}_i^a\widehat{H}_2^b\right], \quad (1)$$

where $i,j = 1,2,3$ are generation indices, $a,b = 1,2$ are $SU(2)$ indices, and ε is a completely antisymmetric 2×2 matrix, with $\varepsilon_{12} = 1$. The symbol "hat" over each letter indicates a superfield, with $\widehat{Q}_i$, $\widehat{L}_i$, $\widehat{H}_1$, and $\widehat{H}_2$ being $SU(2)$ doublets with hyper-charges $\frac{1}{3}$, -1, -1, and 1 respectively, and $\widehat{U}$, $\widehat{D}$, and $\widehat{R}$ being $SU(2)$ singlets with hyper-charges $-\frac{4}{3}$, $\frac{2}{3}$, and 2 respectively. The couplings h_U, h_D and h_E are 3×3 Yukawa matrices, and μ and ϵ_i are parameters with units of mass. The first four terms in the superpotential are common to the MSSM, and the last one is the only R-parity violating term. From now on, we work only with the third generation of quarks and leptons.

Experimental evidence indicate that supersymmetry must be broken. The actual supergravity mechanism is unknown, but can be parametrized with a set of soft supersymmetry breaking terms which do not introduce quadratic divergences to the unrenormalized theory [10]

$$\begin{aligned}V_{\rm soft} = {} & M_Q^2\widetilde{Q}_3^{a*}\widetilde{Q}_3^a + M_U^2\widetilde{U}_3^*\widetilde{U}_3 + M_D^2\widetilde{D}_3^*\widetilde{D}_3 + M_L^2\widetilde{L}_3^{a*}\widetilde{L}_3^a + M_R^2\widetilde{R}_3^*\widetilde{R}_3 \\ & + m_{H_1}^2 H_1^{a*}H_1^a + m_{H_2}^2 H_2^{a*}H_2^a - \left[\tfrac{1}{2}M_3\lambda_3\lambda_3 + \tfrac{1}{2}M_2\lambda_2\lambda_2 + \tfrac{1}{2}M_1\lambda_1\lambda_1 + \text{h.c.}\right] \\ & + \varepsilon_{ab}\Big[A_t h_t\widetilde{Q}_3^a\widetilde{U}_3 H_2^b + A_b h_b\widetilde{Q}_3^b\widetilde{D}_3 H_1^a + A_\tau h_\tau\widetilde{L}_3^b\widetilde{R}_3 H_1^a \\ & - B\mu H_1^a H_2^b + B_2\epsilon_3\widetilde{L}_3^a H_2^b\Big], \end{aligned} \quad (2)$$

where we are already using a one-generation notation.

Note that in the effective low-energy supergravity model the bilinear R-parity violating term *cannot* be eliminated by superfield redefinition even though it appears to be so at high scales, before electroweak and supersymmetry breaking take place [4]. The reason is that the bottom Yukawa coupling, usually neglected in the renormalization group evolution, plays a crucial role in splitting the soft-breaking parameters B and B_2 as well as the scalar masses $m_{H_1}^2$ and M_L^2, assumed to be equal at the unification scale. This can be seen explicitly from Eqs. (A.17) and (A.18) as well as Eqs. (A.10) and (A.13) in Appendix A. This ensures that R-parity violating effects can not be rotated away by going to a new basis [2] [11,12], even if the starting RGE boundary conditions for the soft-breaking terms are universal.

It goes without saying that, in a supergravity model where soft-breaking terms are not universal at the GUT scale, such as string models, the bilinear violation of R-parity is also not removable. However, in this case its effects are not calculable, in contrast to our case. The same is true for the case of the most general low-energy supersymmetric model [13].

The electroweak symmetry is broken when the two Higgs doublets H_1 and H_2, and the tau-sneutrino acquire vacuum expectation values (VEVS):

[2] Obviously physics does not depend on the choice of basis [11]. In this paper we choose to work with the unrotated fields.

$$H_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}[\chi_1^0 + v_1 + i\varphi_1^0] \\ H_1^- \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}[\chi_2^0 + v_2 + i\varphi_2^0] \end{pmatrix},$$
$$\widetilde{L}_3 = \begin{pmatrix} \frac{1}{\sqrt{2}}[\tilde{\nu}_\tau^R + v_3 + i\tilde{\nu}_\tau^I] \\ \tilde{\tau}^- \end{pmatrix}. \tag{3}$$

Note that the gauge bosons W and Z acquire masses given by $m_W^2 = \frac{1}{4}g^2v^2$ and $m_Z^2 = \frac{1}{4}(g^2 + g'^2)v^2$, where $v^2 \equiv v_1^2 + v_2^2 + v_3^2 = (246\ \text{GeV})^2$. We introduce the following notation in spherical coordinates:

$$v_1 = v\sin\theta\cos\beta, \qquad v_2 = v\sin\theta\sin\beta, \qquad v_3 = v\cos\theta, \tag{4}$$

which preserves the MSSM definition $\tan\beta = v_2/v_1$. The angle θ is equal to $\pi/2$ in the MSSM limit.

The full scalar potential may be written as

$$V_{\text{total}} = \sum_i \left|\frac{\partial W}{\partial z_i}\right|^2 + V_D + V_{\text{soft}} + V_{\text{RC}}, \tag{5}$$

where z_i denotes any one of the scalar fields in the theory, V_D are the usual D-terms, V_{soft} the SUSY soft breaking terms given in Eq. (2), and V_{RC} are the one-loop radiative corrections. It is popular to treat radiative corrections with the effective potential. In this case, V_{RC} corresponds to the one-loop contributions to the effective potential. Here we prefer to use the diagrammatic method and find the minimization conditions by correcting to one loop the tadpole equations. At the level of finding the minima, the two methods are equivalent [14]. Nevertheless, the diagrammatic (tadpole) method has advantages with respect to the effective potential when we calculate the one-loop corrected scalar masses [15].

The scalar potential contains linear terms

$$V_{\text{linear}} = t_1^0\chi_1^0 + t_2^0\chi_2^0 + t_3^0\tilde{\nu}_\tau^R, \tag{6}$$

where

$$\begin{aligned} t_1^0 &= (m_{H_1}^2 + \mu^2)v_1 - B\mu v_2 - \mu\epsilon_3 v_3 + \tfrac{1}{8}(g^2 + g'^2)v_1(v_1^2 - v_2^2 + v_3^2), \\ t_2^0 &= (m_{H_2}^2 + \mu^2 + \epsilon_3^2)v_2 - B\mu v_1 + B_2\epsilon_3 v_3 - \tfrac{1}{8}(g^2 + g'^2)v_2(v_1^2 - v_2^2 + v_3^2), \\ t_3^0 &= (m_{L_3}^2 + \epsilon_3^2)v_3 - \mu\epsilon_3 v_1 + B_2\epsilon_3 v_2 + \tfrac{1}{8}(g^2 + g'^2)v_3(v_1^2 - v_2^2 + v_3^2). \end{aligned} \tag{7}$$

These $t_i^0, i = 1, 2, 3$ are the tree-level tadpoles, and are equal to zero at the minimum of the potential.

3. Squark sector and radiative corrections

To find the correct electroweak symmetry breaking radiatively, we need to relate parameters at the GUT scale with parameters at the weak scale. This means that we are promoting the parameters in the tree-level tadpoles in Eq. (7) to running parameters. Therefore, in order to find the correct minima of the scalar potential it is essential to include the one-loop contributions to the tadpoles, otherwise our tadpoles would be extremely scale dependent, i.e. unphysical.

The main one-loop contributions to the tadpoles come from loops involving top and bottom quarks and squarks. Therefore, we need to study the scalar quark sector, and in particular, the spectrum and couplings to CP-even neutral scalars.

The term $\epsilon_3 \widehat{L}_3^a \widehat{H}_2^b$ in the superpotential induce F-terms in the scalar potential, leading to squark mass terms of the form $\tilde{t}_L \tilde{t}_R^*$ proportional to ϵ_3. In addition, the non-zero value of the vacuum expectation value of the tau-sneutrino generates, from the D-terms, squark mass terms of the form $\tilde{t}_i \tilde{t}_i^*$ and $\tilde{b}_i \tilde{b}_i^*$, $i = L, R$. The new squark mass matrices are

$$M_{\tilde{t}}^2 = \begin{bmatrix} M_Q^2 + m_t^2 + \frac{1}{8}(g^2 - \frac{1}{3}g'^2)(v_1^2 - v_2^2 + v_3^2) & m_t(A_t - \mu v_1/v_2 + \epsilon_3 v_3/v_2) \\ m_t(A_t - \mu v_1/v_2 + \epsilon_3 v_3/v_2) & M_U^2 + m_t^2 + \frac{1}{6}g'^2(v_1^2 - v_2^2 + v_3^2) \end{bmatrix} \tag{8}$$

for the top squarks, and

$$M_{\tilde{b}}^2 = \begin{bmatrix} M_Q^2 + m_b^2 - \frac{1}{8}(g^2 + \frac{1}{3}g'^2)(v_1^2 - v_2^2 + v_3^2) & m_b(A_b - \mu v_2/v_1) \\ m_b(A_b - \mu v_2/v_1) & M_D^2 + m_b^2 - \frac{1}{12}g'^2(v_1^2 - v_2^2 + v_3^2) \end{bmatrix} \tag{9}$$

for the bottom squarks. The reader can recover the MSSM squark mass matrices by taking $\epsilon_3 = v_3 = 0$ in the above two equations. The quark masses are related to the quark Yukawa couplings in the same way as in the MSSM: $m_t = h_t v_2/\sqrt{2}$ and $m_b = h_b v_1/\sqrt{2}$. Nevertheless, the numerical value of the quark Yukawas is higher in comparison with the MSSM to compensate with smaller vacuum expectation values

$$h_t = \frac{g m_t}{\sqrt{2} m_W s_\beta s_\theta}, \qquad h_b = \frac{g m_b}{\sqrt{2} m_W c_\beta s_\theta}, \tag{10}$$

and this is represented by the term $\sin\theta \equiv s_\theta$ in the denominators in the above equations.

Squark mass matrices $M_{\tilde{t}}^2$ and $M_{\tilde{b}}^2$ are diagonalized by two rotation matrices such that:

$$R_{\tilde{t}} M_{\tilde{t}}^2 R_{\tilde{t}}^T = \begin{bmatrix} m_{\tilde{t}_1} & 0 \\ 0 & m_{\tilde{t}_2} \end{bmatrix}, \qquad R_{\tilde{b}} M_{\tilde{b}}^2 R_{\tilde{b}}^T = \begin{bmatrix} m_{\tilde{b}_1} & 0 \\ 0 & m_{\tilde{b}_2} \end{bmatrix}, \tag{11}$$

where $m_{\tilde{q}_1} < m_{\tilde{q}_2}$ by convention. These rotation matrices play an important role in the determination of the scalar couplings to a pair of squarks.

We introduce the notation for the CP-even neutral scalars $S_i^0 = \chi_1^0, \chi_2^0, \tilde{\nu}_\tau^R$ for $i = 1, 2, 3$ respectively. In this way, the Feynman rules of the type $S_i^0 q\overline{q}$ are

$$\chi_1^0 b\overline{b} \longrightarrow -i\frac{1}{\sqrt{2}}h_b, \qquad \chi_2^0 t\overline{t} \longrightarrow -i\frac{1}{\sqrt{2}}h_t. \tag{12}$$

as in the MSSM, but with the quark Yukawa couplings given by Eq. (10). Feynman rules of the type $S_i^0 q\overline{q}$ not listed in Eq. (12) vanish.

In a similar way, we find Feynman rules of the type $S_i^0\tilde{q}\tilde{q}^*$, i.e. CP-even neutral scalars couplings to a pair of squarks. We start with χ_1^0 couplings to top squarks:

$$\chi_1^0\tilde{t}\tilde{t}^* \longrightarrow i\boldsymbol{M}_{\chi_1^0\tilde{t}\tilde{t}}, \qquad \boldsymbol{M}_{\chi_1^0\tilde{t}\tilde{t}} = \boldsymbol{R}_{\tilde{t}}\boldsymbol{M}'_{\chi_1^0\tilde{t}\tilde{t}}\boldsymbol{R}_{\tilde{t}}^T,$$
$$\boldsymbol{M}'_{\chi_1^0\tilde{t}\tilde{t}} = \begin{bmatrix} -\frac{1}{4}(g^2 - \frac{1}{3}g'^2)v_1 & \frac{1}{\sqrt{2}}h_t\mu \\ \frac{1}{\sqrt{2}}h_t\mu & -\frac{1}{3}g'^2v_1 \end{bmatrix} \tag{13}$$

and to bottom squarks:

$$\chi_1^0\tilde{b}\tilde{b}^* \longrightarrow i\boldsymbol{M}_{\chi_1^0\tilde{b}\tilde{b}}, \qquad \boldsymbol{M}_{\chi_1^0\tilde{b}\tilde{b}} = \boldsymbol{R}_{\tilde{b}}\boldsymbol{M}'_{\chi_1^0\tilde{b}\tilde{b}}\boldsymbol{R}_{\tilde{b}}^T,$$
$$\boldsymbol{M}'_{\chi_1^0\tilde{b}\tilde{b}} = \begin{bmatrix} -h_b^2v_1 + \frac{1}{4}(g^2 + \frac{1}{3}g'^2)v_1 & -\frac{1}{\sqrt{2}}h_bA_b \\ -\frac{1}{\sqrt{2}}h_bA_b & -h_b^2v_1 + \frac{1}{6}g'^2v_1 \end{bmatrix}. \tag{14}$$

These couplings have the same form in the MSSM but, as it was said before, the Yukawa couplings are different and given by Eq. (10). In addition, vacuum expectation values v_1 and v_2 are different with respect to the MSSM and given by $v_1 = 2m_Wc_\beta s_\theta/g$ and $v_2 = 2m_Ws_\beta s_\theta/g$ and again, the deviation from the MSSM is parametrized by the angle θ.

Now we turn to the neutral CP-even Higgs χ_2^0 that comes from the second Higgs doublet. Its couplings to top squarks are

$$\chi_2^0\tilde{t}\tilde{t}^* \longrightarrow i\boldsymbol{M}_{\chi_2^0\tilde{t}\tilde{t}}, \qquad \boldsymbol{M}_{\chi_2^0\tilde{t}\tilde{t}} = \boldsymbol{R}_{\tilde{t}}\boldsymbol{M}'_{\chi_2^0\tilde{t}\tilde{t}}\boldsymbol{R}_{\tilde{t}}^T,$$
$$\boldsymbol{M}'_{\chi_2^0\tilde{t}\tilde{t}} = \begin{bmatrix} -h_t^2v_2 + \frac{1}{4}(g^2 - \frac{1}{3}g'^2)v_2 & -\frac{1}{\sqrt{2}}h_tA_t \\ -\frac{1}{\sqrt{2}}h_tA_t & -h_t^2v_2 + \frac{1}{3}g'^2v_2 \end{bmatrix} \tag{15}$$

and to bottom squarks

$$\chi_2^0\tilde{b}\tilde{b}^* \longrightarrow i\boldsymbol{M}_{\chi_2^0\tilde{b}\tilde{b}}, \qquad \boldsymbol{M}_{\chi_2^0\tilde{b}\tilde{b}} = \boldsymbol{R}_{\tilde{b}}\boldsymbol{M}'_{\chi_2^0\tilde{b}\tilde{b}}\boldsymbol{R}_{\tilde{b}}^T,$$
$$\boldsymbol{M}'_{\chi_2^0\tilde{b}\tilde{b}} = \begin{bmatrix} -\frac{1}{4}(g^2 + \frac{1}{3}g'^2)v_2 & \frac{1}{\sqrt{2}}h_b\mu \\ \frac{1}{\sqrt{2}}h_b\mu & -\frac{1}{6}g'^2v_2 \end{bmatrix}. \tag{16}$$

Finally, we turn to the real part of the tau-sneutrino field, which mixes with χ_1^0 and χ_2^0. Its couplings to top squarks are

$$\tilde{\nu}_\tau^R\tilde{t}\tilde{t}^* \longrightarrow i\boldsymbol{M}_{\tilde{\nu}_\tau^R\tilde{t}\tilde{t}}, \qquad \boldsymbol{M}_{\tilde{\nu}_\tau^R\tilde{t}\tilde{t}} = \boldsymbol{R}_{\tilde{t}}\boldsymbol{M}'_{\tilde{\nu}_\tau^R\tilde{t}\tilde{t}}\boldsymbol{R}_{\tilde{t}}^T,$$
$$\boldsymbol{M}'_{\tilde{\nu}_\tau^R\tilde{t}\tilde{t}} = \begin{bmatrix} -\frac{1}{4}(g^2 - \frac{1}{3}g'^2)v_3 & -\frac{1}{\sqrt{2}}h_t\epsilon_3 \\ -\frac{1}{\sqrt{2}}h_t\epsilon_3 & -\frac{1}{3}g'^2v_3 \end{bmatrix} \tag{17}$$

and to bottom squarks

$$\tilde{\nu}_\tau^R\tilde{b}\tilde{b}^* \longrightarrow i\boldsymbol{M}_{\tilde{\nu}_\tau^R\tilde{b}\tilde{b}}, \qquad \boldsymbol{M}_{\tilde{\nu}_\tau^R\tilde{b}\tilde{b}} = \boldsymbol{R}_{\tilde{b}}\boldsymbol{M}'_{\tilde{\nu}_\tau^R\tilde{b}\tilde{b}}\boldsymbol{R}_{\tilde{b}}^T,$$
$$\boldsymbol{M}'_{\tilde{\nu}_\tau^R\tilde{b}\tilde{b}} = \begin{bmatrix} \frac{1}{4}(g^2 + \frac{1}{3}g'^2)v_3 & 0 \\ 0 & \frac{1}{6}g'^2v_3 \end{bmatrix}. \tag{18}$$

These couplings $\tilde{\nu}_\tau^R\tilde{q}\tilde{q}^*$ vanish in the MSSM limit $v_3 = \epsilon_3 = 0$, as it should.

We are now ready to include the effect of the one-loop tadpoles in Eq. (7). The first step towards the calculation of radiative corrections is the introduction of counter-terms. All parameters in the Lagrangian are shifted from bare parameters to renormalized parameters minus a counter-term:

$$\begin{aligned}
\lambda &\longrightarrow \lambda - \delta\lambda, & \lambda &= g, g', h_t, h_b, h_\tau, \\
m^2 &\longrightarrow m^2 - \delta m^2, & m^2 &= m_{H_1}^2, m_{H_2}^2, m_{L_3}^2, m_{R_3}^2, \mu, \epsilon_3, \\
v_i &\longrightarrow v_i - \delta v_i, & i &= 1,2,3, \qquad (19) \\
A &\longrightarrow A - \delta A, & A &= A_t, A_b, A_\tau, \\
B &\longrightarrow B - \delta B, & B &= B, B_2, \qquad (20)
\end{aligned}$$

for couplings, masses, vacuum expectation values, trilinear soft parameters, and bilinear soft parameters respectively. If we make this shift in the tadpole equations given in Eq. (7), the tadpole themselves get a counter-term δt_i for $i = 1,2,3$. Therefore, the one-loop tadpole equations are

$$t_i = t_i^0 - \delta t_i + T_i(Q)\,, \qquad i = 1,2,3, \tag{21}$$

where t_i are the one-loop renormalized tadpoles and $T_i(Q)$ are the one-loop contributions to the tadpoles, with Q being the arbitrary mass scale introduced by dimensional reduction.

The renormalization scheme we choose to work with is the $\overline{\rm MS}$ scheme, where by definition the tadpole counter-terms are taken such that they cancel the divergent pieces of $T_i(Q)$ proportional to Δ:

$$\Delta = \frac{2}{4-n} + \ln 4\pi - \gamma_E, \tag{22}$$

where Δ is the regulator of dimensional regularization, n is the number of space-time dimensions, and γ_E is the Euler's constant. The $\overline{\rm MS}$ counter-terms chosen in this way make the tadpoles finite. We introduce the notation

$$\widetilde{T}_i^{\overline{\rm MS}}(Q) = -\delta t_i^{\overline{\rm MS}} + T_i(Q), \tag{23}$$

for the finite one-loop contribution to the tadpoles. These finite one-loop tadpoles depend explicitly on the arbitrary scale Q.

The one-loop tadpoles t_i must be scale independent (at least in the one-loop approximation), therefore the renormalized parameters are promoted to running parameters, i.e. they evolve with the scale Q according to their renormalization group equations (RGE). The explicit Q dependence on $\widetilde{T}_i^{\overline{\rm MS}}(Q)$ is cancelled at one-loop by the implicit Q dependence on the parameters of the tree-level tadpoles. Renormalized tadpoles must be zero at the minimum of the potential $t_i = 0$, thus the generalization of the tadpole equations is

$$\begin{aligned}
&\left[(m_{H_1}^2 + \mu^2)v_1 - B\mu v_2 - \mu\epsilon_3 v_3 + \tfrac{1}{8}(g^2 + g'^2)v_1(v_1^2 - v_2^2 + v_3^2)\right](Q) \\
&\quad + \widetilde{T}_1^{\overline{\rm MS}}(Q) = 0\,,
\end{aligned}$$

$$
\begin{aligned}
&\left[(m_{H_2}^2+\mu^2+\epsilon_3^2)v_2-B\mu v_1+B_2\epsilon_3 v_3-\tfrac{1}{8}(g^2+g'^2)v_2(v_1^2-v_2^2+v_3^2)\right](Q)\\
&\quad+\widetilde{T}_2^{\overline{\rm MS}}(Q)=0\,,\\
&\left[(m_{L_3}^2+\epsilon_3^2)v_3-\mu\epsilon_3 v_1+B_2\epsilon_3 v_2+\tfrac{1}{8}(g^2+g'^2)v_3(v_1^2-v_2^2+v_3^2)\right](Q)\\
&\quad+\widetilde{T}_3^{\overline{\rm MS}}(Q)=0\,,
\end{aligned}
\tag{24}
$$

and these are the minimization condition we impose.[3] We choose to work at the scale $Q=m_Z$. The RGE's for each parameter are given in Appendix A, and the boundary condition at the GUT scale are described later.

Now we find the one-loop contributions to the tadpoles. Quarks contribute to χ_1^0 and χ_2^0 one-loop tadpoles only. On the contrary, squarks contribute to all three tadpoles. Using the notation for the Feynman rules introduced in the previous section, the quark and squark one-loop contribution to the tadpoles can be written as

$$
\begin{aligned}
\left[T_{\chi_1^0}\right]^{tb\tilde{t}\tilde{b}}&=\frac{N_c}{16\pi^2}\sum_{i=1}^{2}\left[M^{ii}_{\chi_1^0\tilde{t}\tilde{t}}A_0(m_{\tilde{t}_i}^2)+M^{ii}_{\chi_1^0\tilde{b}\tilde{b}}A_0(m_{\tilde{b}_i}^2)\right]+\frac{N_c g m_b^2}{8\pi^2 m_W c_\beta s_\theta}A_0(m_b^2),\\
\left[T_{\chi_2^0}\right]^{tb\tilde{t}\tilde{b}}&=\frac{N_c}{16\pi^2}\sum_{i=1}^{2}\left[M^{ii}_{\chi_2^0\tilde{t}\tilde{t}}A_0(m_{\tilde{t}_i}^2)+M^{ii}_{\chi_2^0\tilde{b}\tilde{b}}A_0(m_{\tilde{b}_i}^2)\right]+\frac{N_c g m_t^2}{8\pi^2 m_W s_\beta s_\theta}A_0(m_t^2),\\
\left[T_{\tilde{\nu}_\tau^R}\right]^{tb\tilde{t}\tilde{b}}&=\frac{N_c}{16\pi^2}\sum_{i=1}^{2}\left[M^{ii}_{\tilde{\nu}_\tau^R\tilde{t}\tilde{t}}A_0(m_{\tilde{t}_i}^2)+M^{ii}_{\tilde{\nu}_\tau^R\tilde{b}\tilde{b}}A_0(m_{\tilde{b}_i}^2)\right],
\end{aligned}
\tag{25}
$$

where A_0 is the first Veltman's function defined by

$$
A_0(m^2)=m^2\left(\Delta-\ln\frac{m^2}{Q^2}+1\right). \tag{26}
$$

The finite tadpoles $\widetilde{T}_i^{\overline{\rm MS}}(Q)$ are found simply by setting $\Delta=0$ in the previous expressions.

4. Unification

We now discuss the corresponding boundary conditions at unification. We assume that at the unification scale the model is characterized by one universal soft supersymmetry-breaking mass m_0 for all the scalars (the gravitino mass), and a universal gaugino mass $M_{1/2}$. Moreover we assume that there is a single trilinear soft breaking scalar mass parameter A and that the bilinear soft breaking parameter B is related to A through $B=A-1$. In other words, we make the standard minimal supergravity assumptions:

$$
A_t=A_b=A_\tau\equiv A, \tag{27}
$$

[3] To see the effect one-loop tadpoles have on the determination of MSSM-SUGRA parameters, see Ref. [16].

$$B = B_2 = A - 1, \tag{28}$$

$$m^2_{H_1} = m^2_{H_2} = M^2_L = M^2_R = m^2_0, \tag{29}$$

$$M^2_Q = M^2_U = M^2_D = m^2_0, \tag{30}$$

$$M_3 = M_2 = M_1 = M_{1/2} \tag{31}$$

at $Q = M_{\rm GUT}$. At energies below $M_{\rm GUT}$ these conditions do not hold, due to the renormalization group evolution from the unification scale down to the relevant scale.

In order to determine the values of the Yukawa couplings and of the soft breaking scalar masses at low energies we first run the RGE's from the unification scale $M_{\rm GUT} \sim 10^{16}$ GeV down to the weak scale. In doing this we randomly give values at the unification scale for the parameters of the theory. The range of variation of the MSSM-SUGRA parameters at the unification scale is as follows:

$$\begin{array}{rcccl} 10^{-2} & \leqslant & h^2_{t\,\rm GUT}/4\pi & \leqslant & 1, \\ 10^{-5} & \leqslant & h^2_{b\rm GUT}/4\pi & \leqslant & 1, \\ -3 & \leqslant & A/m_0 & \leqslant & 3, \\ 0 & \leqslant & \mu^2_{\rm GUT}/m^2_0 & \leqslant & 10, \\ 0 & \leqslant & M_{1/2}/m_0 & \leqslant & 5, \end{array} \tag{32}$$

while the range of variation of ϵ_3 is

$$10^{-2} \leqslant \epsilon^2_{3\rm GUT}/m^2_0 \leqslant 10 \tag{33}$$

and the value of $h^2_{\tau\rm GUT}/4\pi$ is defined in such a way that we get the τ mass correctly. After running the RGE we have a complete set of parameters, Yukawa couplings and soft-breaking masses m^2_i(RGE) to study the minimization.

Similar to what happens in the MSSM-SUGRA (see Appendix B) the number of independent parameters of this model is actually less than given above, as one must take into account the W mass constraint as well as the minimization conditions. In the end there is a single new parameter characterizing our model, namely ϵ_3.

5. Results and phenomenology

The main parameters characterizing electroweak breaking are the SU(2) doublet VEVs v_1, v_2 and v_3. In our model these are obtained as explained in Appendix B. Basically we assign random values for the top and bottom quark Yukawa couplings h_t and h_b at the GUT scale and evolve them down to the weak scale through the renormalization group equations, given in Appendix A. Using the measured top and bottom quark masses we determine the corresponding running masses at the weak scale. Combining this with the values of h_t and h_b at the weak scale, obtained through the use of the RGE's, we calculate the standard MSSM VEVS v_1 and v_2. The third VEV v_3, which breaks R-parity, is determined through the W mass formula. The resulting VEVs may not be consistent with the minimization conditions. In Appendix B we present a

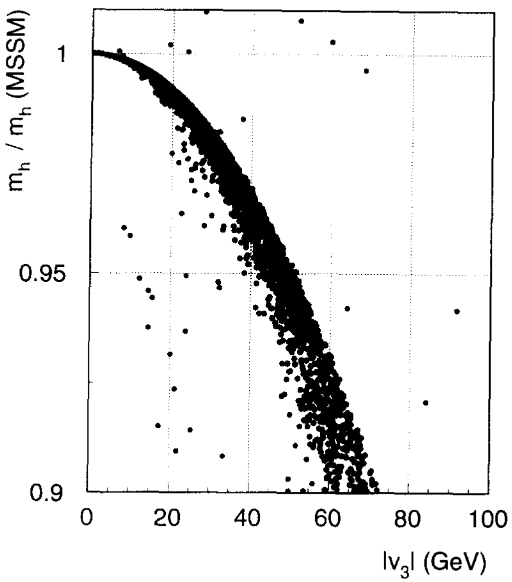

Fig. 1. Lightest CP-even Higgs boson mass m_h as a function of v_3 in our model.

procedure to ensure a consistent solution. Note that due to the contribution of v_3 to the intermediate gauge boson masses, $v_1^2 + v_2^2$ is smaller than in the MSSM. The first check of we can do to verify the consistency of the model is to study the allowed values of the lightest CP-even Higgs boson mass m_h as a function of the third VEV v_3. This is displayed in Fig. 1 The unrotated neutral CP-even Higgs bosons χ_1^0 and χ_2^0 mix with the real part of the tau sneutrino $\tilde{\nu}_\tau^R$. These are the CP-even scalars S_i^0, $i = 1,2,3$, introduced in Section 3. The mass matrix can be written as

$$M_{S^0}^2 = M_{S^0,\mathrm{MSSM}}^2 + M_{S^0,\epsilon_3}^2 + M_{S^0,\mathrm{RC}}^2 \tag{34}$$

where $M_{S^0,\mathrm{MSSM}}^2$ is the MSSM mass matrix given by

$$M_{S^0,\mathrm{MSSM}}^2 = \begin{bmatrix} B\mu\frac{v_2}{v_1} + \frac{1}{4}g_Z^2 v_1^2 & -B\mu - \frac{1}{4}g_Z^2 v_1 v_2 & 0 \\ -B\mu - \frac{1}{4}g_Z^2 v_1 v_2 & B\mu\frac{v_1}{v_2} + \frac{1}{4}g_Z^2 v_2^2 & 0 \\ 0 & 0 & m_{L_3}^2 + \frac{1}{8}g_Z^2(v_1^2 - v_2^2) \end{bmatrix}, \tag{35}$$

where we have defined $g_Z^2 \equiv g^2 + g'^2$. As expected, this mass matrix has no mixing between the Higgs and stau sectors. The extra terms that appear in our ϵ_3-model are

$$M_{S^0,\epsilon_3}{}^2 = \begin{bmatrix} \mu\epsilon_3\frac{v_3}{v_1} & 0 & -\mu\epsilon_3 + \frac{1}{4}g_Z^2 v_1 v_3 \\ 0 & -B_2\epsilon_3\frac{v_3}{v_2} & B_2\epsilon_3 - \frac{1}{4}g_Z^2 v_2 v_3 \\ -\mu\epsilon_3 + \frac{1}{4}g_Z^2 v_1 v_3 & B_2\epsilon_3 - \frac{1}{4}g_Z^2 v_2 v_3 & \epsilon_3^2 + \frac{3}{8}g_Z^2 v_3^2 \end{bmatrix}, \tag{36}$$

which introduce a Higgs–Stau mixing. Finally, in $M_{S^0,\mathrm{RC}}^2$ we introduce the largest term in the one-loop radiative corrections, i.e. the term proportional to m_t^4:

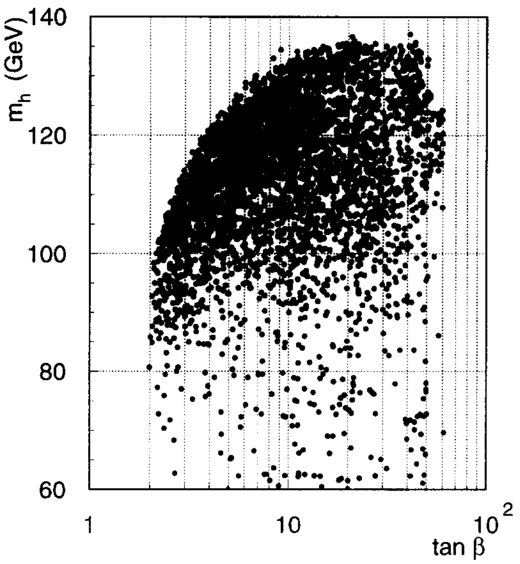

Fig. 2. Lightest CP-even Higgs boson mass m_h versus $\tan\beta$

$$M^2_{S^0,\mathrm{RC}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta_t & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \Delta_t = \frac{3g^2 m_t^4}{16\pi^2 m_W^2 s_\beta^2 s_\theta^2} \ln \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4}. \tag{37}$$

This formula gives good results in first approximation, nevertheless, already in the MSSM can give wrong results in certain regions of parameter space [17], and should be improved.

As one can see in Fig. 1, in the limit $v_3 \to 0$ our model reproduces exactly the expected minimal SUGRA limit for the lightest CP-even Higgs boson mass. Another view of the Higgs boson mass spectrum allowed in our model is obtained by plotting m_h as a function of $\tan\beta$, as illustrated in Fig. 2. One sees that all values of $\tan\beta$ in the range 2 to 60 or so are possible in our model. As in the MSSM-SUGRA, $\tan\beta$ smaller than 2 is not possible because the top Yukawa coupling diverges as we approach the unification scale. This is related to the fact that in that region we are close to the infrared quasi-fixed point. Note that the range of $\tan\beta$ values obtained in our model is consistent with the unification hypothesis for a large range of the bottom quark Yukawa coupling at unification, as illustrated in Fig. 3.

Another important feature of our broken R-parity model is that the tau neutrino ν_τ acquires a mass, due to the fact that ϵ_3 and v_3 are non-zero. Consider the basis $(\Psi^0)^T = (-i\lambda_1, -i\lambda_2^3, \widetilde{H}_1^1, \widetilde{H}_2^2, \nu_\tau)$, where λ_1 is the $U(1)$ gaugino introduced in Eq. (2), λ_2^3 is the neutral $SU(2)$ gaugino, $\widetilde{H}_i^i$, $i = 1, 2$ are the neutral Higgsinos, and ν_τ is the SM tau neutrino. In this base, the mass terms in the Lagrangian for the neutralino–neutrino sector are

$$\mathcal{L}_m = -\tfrac{1}{2}(\Psi^0)^T M_N \Psi^0 + \text{h.c.}, \tag{38}$$

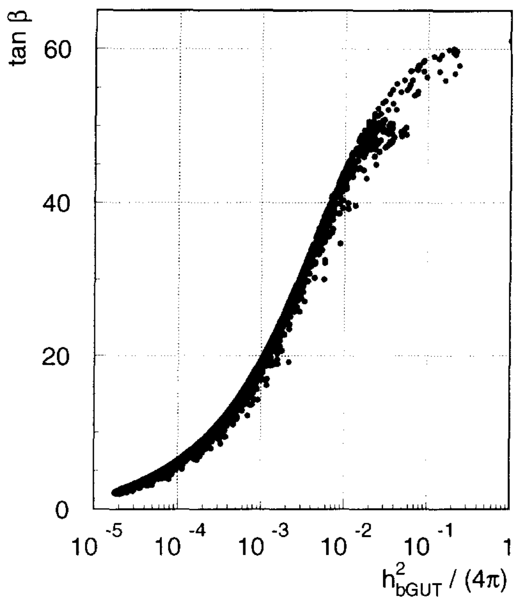

Fig. 3. $\tan\beta$ versus bottom quark Yukawa coupling at unification

where the mass matrix is[4]

$$M_N = \begin{bmatrix} M_1 & 0 & -\frac{1}{2}g'v_1 & \frac{1}{2}g'v_2 & -\frac{1}{2}g'v_3 \\ 0 & M_2 & \frac{1}{2}gv_1 & -\frac{1}{2}gv_2 & \frac{1}{2}gv_3 \\ -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & -\mu & 0 \\ \frac{1}{2}g'v_2 & -\frac{1}{2}gv_2 & -\mu & 0 & \epsilon_3 \\ -\frac{1}{2}g'v_3 & \frac{1}{2}gv_3 & 0 & \epsilon_3 & 0 \end{bmatrix}. \quad (39)$$

The only new terms appear in the mixing between neutralinos and the tau neutrino, determined by the parameters ϵ_3 and v_3. If we stick to the simplest unified supergravity version of the model with bilinear breaking of R-parity and universal boundary conditions for the soft breaking parameters, then the *effective* neutralino mixing parameter $\xi \equiv (\epsilon_3 v_1 + \mu v_3)^2$ characterizing the violation of R-parity, either through v_3 or ϵ_3 will be small since contributions arising from *gaugino* mixing will cancel, to a large extent, those from *Higgsino* mixing. This cancellation will happen automatically in these models if the soft breaking parameters are universal [11] so that in this case m_{ν_τ} will be naturally small and radiatively calculable in terms of the bottom Yukawa coupling h_b. This will explain naturally the smallness of the neutrino mass in this model. The above scenario is therefore a *hybrid* of the see-saw and radiative schemes of neutrino mass generation. The rôle of the right-handed mass which appears in the see-saw model is played by the neutralino mass (which lies at the SUSY scale) while the rôle of the seesaw-scheme Dirac mass is played by the *effective* neutralino mixing ξ which is induced radiatively. The ν_τ mass induced this way is directly correlated with the magnitude of the effective

[4] More complete forms of this matrix have been given in many places. See, e.g., Ref. [18].

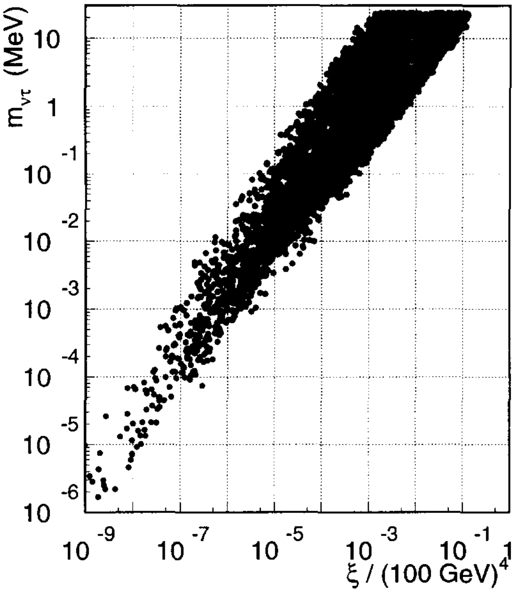

Fig. 4. Tau neutrino mass versus ϵ_3.

parameter ξ so that R-parity violation acts as the origin for neutrino mass. In Fig. 4 we display the allowed values of m_{ν_τ}. Notice that m_{ν_τ} values can cover a very wide range, from eV to values in the MeV range, comparable to the present LEP limit [19]. Note that the individual values of v_3 and ϵ_3 can be rather large (see, for example, Fig. 1).

Now a word about cosmology. Clearly our model leads to a tau neutrino which can be much heavier than the limits that follow from the cosmological critical density as well as primordial nucleosynthesis would allow [20]. However, in this model the ν_τ is unstable and decays via neutral current into three lighter neutrinos [21]. In order for this mode to be efficient we estimate that m_{ν_τ} must exceed 100 keV or so. On the other hand, in order to avoid problems with primordial nucleosynthesis it is safer to consider m_{ν_τ} masses below 1 MeV or so. In order to sharpen these estimates (which are not strict bounds) a detailed investigation is required.

One should bear in mind, however, that m_{ν_τ} can be as large as the present laboratory bound [19] in the more complete versions of the model in which R-parity is broken spontaneously due to sneutrino expectation values [5,6]. This is so because such models contain a majoron, denoted as J, which opens new decay channels $\nu_\tau \rightarrow \nu + J$ where ν is a lighter neutrino [22] as well as new annihilation channels $\nu_\tau\ \nu_\tau \rightarrow J + J$. It has been shown explicitly that the lifetimes that can be achieved in the spontaneous broken R-parity versions of the model can be sufficiently short to obey the critical density limit [18]. Moreover, it has been shown that the annihilation channel is efficient enough in order to comply the primordial nucleosynthesis bound [23], while decays may also play an important rôle [24].

6. Discussion and conclusions

Here we have shown that this simplest truncated version of the R-parity breaking model of Ref. [5], characterized by a bilinear violation of R-parity in the superpotential, is consistent with minimal $N = 1$ supergravity models with radiative electroweak symmetry breaking and universal scalar and gaugino masses at the unification scale. We have performed a thorough study of the minimization of the scalar boson potential of the model. We have determined the lowest-lying CP-even Higgs boson mass spectrum. We have discussed how the minimal $N = 1$ supergravity limit of this theory is obtained and verified that it works as expected. We have determined also the ranges of $\tan\beta$ and bottom quark Yukawa couplings allowed at unification, as well as the relation between the tau neutrino mass and the effective bilinear R-parity violating parameter. We showed that in the case of universal soft breaking terms this model leads to radiatively induced R-parity violation and, as a result, to a model with naturally small neutrino mass determined by the bottom quark Yukawa coupling.

Our results should encourage further theoretical work on this model, as well as more complete versions of the model, like that of Ref. [8]. Phenomenological studies of the related signals should also be desirable, given the fact that the production and decay patterns of Higgs bosons and supersymmetric particles in this model are substantially different than expected in the MSSM or MSSM-SUGRA. For example, Higgs bosons may have sizeable R-parity violating decays [13]. Similarly, sneutrinos and staus can be the LSP and can have unsuppressed decays into standard model states, thus violating R-parity. Finally, chargino and neutralino production can lead to totally different signals as, for example, the lightest neutralino can decay [25]. These features could play an important role in designing strategies for searching for supersymmetric particles at future accelerators. One should bear in mind that (i) the effects of R-parity violation can be large even in the case where the neutrino mass is naturally small. This applies for example to branching ratios into R-parity breaking modes and (ii) there are striking effects of R-parity violation which do not require it to have a large strength. The obvious example is the fact that, unless the violation is really tiny as in [26], the lightest neutralino decay will typically decay inside the detector. This would, for example, lead to high multiplicity lepton events at the LHC from gluino and squark cascade decays [27].

Acknowledgements

This work was supported by DGICYT under grants PB95-1077 and Acción Integrada HP96-055, and by the TMR network grant ERBFMRXCT960090 of the European Union. M.A.D. was supported by a DGICYT postdoctoral grant.

Appendix A. The renormalization group equations

Here we give the renormalization group equations for the model described by the superpotential in Eq. (1), but including only the third generation, and by the soft supersymmetry breaking terms given in Eq. (2). First we write the equations for the Yukawa couplings of the trilinear terms,

$$16\pi^2\frac{dh_U}{dt} = h_U\left(6h_U^2 + h_D^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{9}g_1^2\right), \tag{A.1}$$

$$16\pi^2\frac{dh_D}{dt} = h_D\left(6h_D^2 + h_U^2 + h_\tau^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{9}g_1^2\right), \tag{A.2}$$

$$16\pi^2\frac{dh_\tau}{dt} = h_\tau\left(4h_\tau^2 + 3h_D^2 - 3g_2^2 - 3g_1^2\right) \tag{A.3}$$

now the corresponding cubic soft supersymmetry breaking parameters

$$8\pi^2\frac{dA_U}{dt} = 6h_U^2A_U + h_D^2A_D + \frac{16}{3}g_3^2M_3 + 3g_2^2M_2 + \frac{13}{9}g_1^2M_1, \tag{A.4}$$

$$8\pi^2\frac{dA_D}{dt} = 6h_D^2A_D + h_U^2A_U + h_\tau^2A_\tau + \frac{16}{3}g_3^2M_3 + 3g_2^2M_2 + \frac{7}{9}g_1^2M_1, \tag{A.5}$$

$$8\pi^2\frac{dA_\tau}{dt} = 4h_\tau^2A_\tau + 3h_D^2A_D + 3g_2^2M_2 + 3g_1^2M_1. \tag{A.6}$$

For the soft supersymmetry breaking mass parameters we have

$$8\pi^2\frac{dM_Q^2}{dt} = h_U^2(m_{H_2}^2 + M_Q^2 + M_U^2 + A_U^2) + h_D^2(m_{H_1}^2 + M_Q^2 + M_D^2 + A_D^2)$$
$$-\frac{16}{3}g_3^2M_3^2 - 3g_2^2M_2^2 - \frac{1}{9}g_1^2M_1^2 + \frac{1}{6}g_1^2\mathcal{S}, \tag{A.7}$$

$$8\pi^2\frac{dM_U^2}{dt} = 2h_U^2(m_{H_2}^2 + M_Q^2 + M_U^2 + A_U^2) - \frac{16}{3}g_3^2M_3^2 - \frac{16}{9}g_1^2M_1^2 - \frac{2}{3}g_1^2\mathcal{S}, \tag{A.8}$$

$$8\pi^2\frac{dM_D^2}{dt} = 2h_D^2(m_{H_1}^2 + M_Q^2 + M_D^2 + A_D^2) - \frac{16}{3}g_3^2M_3^2 - \frac{4}{9}g_1^2M_1^2 + \frac{1}{3}g_1^2\mathcal{S}, \tag{A.9}$$

$$8\pi^2\frac{dM_L^2}{dt} = h_\tau^2(m_{H_1}^2 + M_L^2 + M_R^2 + A_\tau^2) - 3g_2^2M_2^2 - g_1^2M_1^2 - \frac{1}{2}g_1^2\mathcal{S}, \tag{A.10}$$

$$8\pi^2\frac{dM_R^2}{dt} = 2h_\tau^2(m_{H_1}^2 + M_L^2 + M_R^2 + A_\tau^2) - 4g_1^2M_1^2 + g_1^2\mathcal{S}, \tag{A.11}$$

$$8\pi^2\frac{dm_{H_2}^2}{dt} = 3h_U^2(m_{H_2}^2 + M_Q^2 + M_U^2 + A_U^2) - 3g_2^2M_2^2 - g_1^2M_1^2 + \frac{1}{2}g_1^2\mathcal{S}, \tag{A.12}$$

$$8\pi^2\frac{dm_{H_1}^2}{dt} = 3h_D^2(m_{H_1}^2 + M_Q^2 + M_D^2 + A_D^2) + h_\tau^2(m_{H_1}^2 + M_L^2 + M_R^2 + A_\tau^2)$$
$$-3g_2^2M_2^2 - g_1^2M_1^2 - \frac{1}{2}g_1^2\mathcal{S}, \tag{A.13}$$

where

$$\mathcal{S} = m_{H_2}^2 - m_{H_1}^2 + M_Q^2 - 2M_U^2 + M_D^2 - M_L^2 + M_R^2. \tag{A.14}$$

For the bilinear terms in the superpotential we get

$$16\pi^2\frac{d\mu}{dt} = \mu\left(3h_U^2 + 3h_D^2 + h_\tau^2 - 3g_2^2 - g_1^2\right), \tag{A.15}$$

$$16\pi^2 \frac{d\epsilon_3}{dt} = \epsilon_3 \left(3h_U^2 + h_\tau^2 - 3g_2^2 - g_1^2\right) \tag{A.16}$$

and for the corresponding soft breaking terms

$$8\pi^2 \frac{dB}{dt} = 3h_U^2 A_U + 3h_D^2 A_D + h_\tau^2 A_\tau + 3g_2^2 M_2 + g_1^2 M_1, \tag{A.17}$$

$$8\pi^2 \frac{dB_2}{dt} = 3h_U^2 A_U + h_\tau^2 A_\tau + 3g_2^2 M_2 + g_1^2 M_1. \tag{A.18}$$

The g_i are the $SU(3) \times SU(2) \times U(1)$ gauge couplings and the M_i are the corresponding the soft breaking gaugino masses.

Appendix B. Minimization procedure

To minimize the scalar potential we use the procedure developed in Refs. [8,28]. We solve the tadpole equations, Eq. (24), for the soft mass-squared parameters in terms of the VEVs and of the other parameters at the weak scale. This is particularly simple because those equations are linear in the soft masses squared. To do this we need to know the values for the VEVs. These are obtained in the following way.

(1) We start with random values for h_t and h_b at $M_{\rm GUT}$ in the range given in Eq. (32). The value of h_τ at $M_{\rm GUT}$ is fixed in order to get the correct τ mass.

(2) The value of v_1 is determined from $m_b = h_b v_1/\sqrt{2}$ for $m_b = 3$ GeV (running b mass at m_Z).

(3) The value of v_2 is determined from $m_t = h_t v_2/\sqrt{2}$ for $m_t = 176 \pm 5$ GeV. If

$$v_1^2 + v_2^2 > v^2 = \frac{4}{g^2} m_W^2 = (246 \text{ GeV})^2 \tag{B.1}$$

we go back and choose another starting point.

(4) The value of v_3 is then obtained from $v_3 = \pm\sqrt{(4/g^2)m_W^2 - v_1^2 - v_2^2}$.

We see that the freedom in h_t and h_b at $M_{\rm GUT}$ can be translated into the freedom in the mixing angles β and θ. Comparing, at this point, with the MSSM we have one extra parameter θ. We will discuss this in more detail below. In the MSSM we would have $\theta = \pi/2$.

After doing this, for each point in parameter space, we solve the extremum equations, Eq. (24), for the soft breaking masses, which we now call m_i^2 ($i = H_1, H_2, L$). Then we calculate numerically the eigenvalues for the real and imaginary part of the neutral scalar mass-squared matrix. If they are all positive, except for the Goldstone boson, the point is a good one. If not, we go back to the next random value. After doing this we end up with a set of solutions for which the following holds.

(1) The Yukawa couplings are determined by the procedure described above.

(2) The other parameters are given by the RGE evolution once the values at $M_{\rm GUT}$ are fixed. Notice, however, that these parameters may not satisfy the tadpole equations. We will come back to this later.

Table B.1
Counting of free parameters in minimal $N = 1$ supergravity

Parameters	Conditions	Free parameters
h_t, h_b, h_τ, v_1, v_2	m_W, m_t, m_b, m_τ	$\tan\beta$
A, m_0, $M_{1/2}$, μ	$t_i = 0$, $i = 1, 2$	Two extra free parameters
Total = 9	Total = 6	Total = 3

Table B.2
Counting of free parameters in our model

Parameters	Conditions	Free parameters
h_t, h_b, h_τ, v_1, v_2, v_3	m_W, m_t, m_b, m_τ	$\tan\beta$, $\cos\theta$
A, m_0, $M_{1/2}$, μ, ϵ_3	$t_i = 0$, $i = 1, 2, 3$	Two extra free parameters
Total = 11	Total = 7	Total = 4

(3) For a given set of m_i^2 $(i = H_1, H_2, L)$ each point is also a solution of the minimization of the potential.

(4) However, the m_i^2 obtained from the minimization of the potential differ from those obtained from the RGE, which we call $m_i^2(\text{RGE})$.

Our next goal is to find which solutions, for the m_i^2 that minimize the effective low-energy potential, have the property that they coincide with the $m_i^2(\text{RGE})$ obtained, for a given unified theory, from the RGE, namely

$$m_i^2 = m_i^2(\text{RGE}), \quad i = H_1, H_2, L. \tag{B.2}$$

Following Ref. [8] we define a function

$$\eta = \max\left(\frac{m_i^2}{m_i^2(\text{RGE})}, \frac{m_i^2(\text{RGE})}{m_i^2}\right) \quad \forall i. \tag{B.3}$$

Defined in this way it is easy to see that we always have $\eta \geqslant 1$, the equality being what we are looking for.

We are then all set for a minimization procedure. We want, by varying the parameters at the GUT scale, to get η as close to 1 as possible. With these conditions we used the `MINUIT` package in order to find the minimum of η. We considered a point in parameter space to be a good solution if $\eta < 1.001$.

Before we end this appendix, let us discuss the counting of free parameters in this model and in the minimal $N = 1$ supergravity unified version of the MSSM. As we explained above, after requiring the correct masses for the W, t, b and τ we get one free parameter in the MSSM, $\tan\beta$, and two in our model, $\tan\beta$ and $\cos\theta$ or, equivalently, v_3. As for the other parameters we have at the GUT scale one extra parameter, ϵ_3. But we also have an extra equation for the tadpoles. So in the end our model has just one more free parameter. This has been summarized in Tables 1 and 2.

Finally, we note that in either case the sign of the mixing parameter μ is physical and has to be taken into account.

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