Nuclear Physics B 381 (1992) 87–108
North-Holland

NUCLEAR
PHYSICS B

Neutrino masses in supersymmetry with spontaneously broken *R*-parity

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Received 27 September 1991
(Revised 20 January 1992)
Accepted for publication 23 March 1992

The neutrino mass spectrum that arises in supersymmetry models with spontaneously broken R-parity is studied in detail. We analyse the attainable values for m_{ν_e}, m_{ν_μ} and m_{ν_τ}, once all observational constraints have been incorporated, including both those that arise from collider experiments such as LEP, as well as weak interaction constraints, such as the non-observation of neutrinoless double-β decay, the limits from neutrino oscillation searches, etc. For natural choices of the parameters neutrino masses arise in a very striking pattern: while ν_e and ν_μ have masses lying in the adequate range for the explanation of the observed deficit in the solar neutrino flux via the MSW effect, the ν_τ mass is large enough to lead to novel signatures associated with the τ-lepton that could be seen both at LEP as well as at a tau factory. These provide an additional tool to probe the parameters characterizing the solar neutrino conversions in conventional accelerator experiments. The related processes include the single-chargino Z-decay branching ratio $\mathrm{BR}(Z \to \tilde{\chi}\tau)$ as large as $\sim 6 \times 10^{-5}$ and single-majoron emission τ-decay branching ratios $\tau \to \mu + J$ as large as $\sim 10^{-4}$ corresponding to solar neutrino oscillation parameters in the non-adiabatic branch favoured by present solar neutrino data. The τ-neutrino is naturally much heavier than ν_μ and decays to it via majoron emission, with a lifetime short enough to obey cosmological limits.

1. Introduction

Most studies of supersymmetric phenomenology have so far assumed the exact conservation of a discrete symmetry called R-parity. Under this symmetry all particles of the standard model (including the Higgs scalars) are R-even while their

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SUSY partners are R-odd. There is no solid theoretical basis for the assumption of R-parity conservation, and many are the motivations to study the implications of theories without this symmetry [1]. Since R-parity is related to the total lepton number [$R_{\mathrm{p}} = (-1)^{3B+L+2S}$ where S denotes spin, B and L denote baryon and total lepton number, respectively] its violation is bound to generate neutrino masses as well as lepton-number and/or lepton-flavour violating processes [2–4].

There are two ways to break R-parity. The first way is explicitly [2]. Indeed, the most general supersymmetric extension of the standard model contains the following superpotential terms

$$W = \lambda^{(1)} u^{\mathrm{c}} u^{\mathrm{c}} d^{\mathrm{c}} + \lambda^{(2)} l l e^{\mathrm{c}} + \lambda^{(3)} Q l d^{\mathrm{c}}, \tag{1}$$

which explicitly violate R-parity but are consistent with both gauge invariance and supersymmetry. These R-parity violating interactions involve a large number of arbitrary constants, since each of the $\lambda^{(i)}$ has three generation indices, which have been omitted. Many of these couplings must be set to zero in order, for example, to avoid having too fast proton decay rates. Restricted structures of the generation indices may, however, be phenomenologically acceptable. These may be obtained by imposing some global and/or discrete symmetry. There are, however, *other* constraints on R-parity violating couplings that may restrict, in fact, *all* of these couplings. These follow from cosmological arguments related to the baryon asymmetry of the universe [5]. The point is that the interactions in eq. (1) mediate $(B-L)$-violating decays of squarks and sleptons of the kind $\tilde{\mathrm{u}} \to \bar{\mathrm{d}}\bar{\mathrm{d}}$, $\tilde{\mathrm{u}} \to \bar{\ell}\mathrm{d}$, $\tilde{\ell} \to \ell\nu$, etc. The rate for these decay processes may be estimated as

$$\Gamma_{\mathrm{D}} \approx \frac{\lambda}{4\pi} \frac{\tilde{m}^2}{\left(\tilde{m}^2 + T^2\right)^{1/2}}, \tag{2}$$

where λ denotes the appropriate coupling constant in eq. (1) and $\tilde{m}$ the corresponding supersymmetric particle mass relevant for the decay process of interest. Now, it has been realized [6] that at temperatures above $\sim m_{\mathrm{W}}/\alpha_{\mathrm{weak}}$, transition that violate B- and L-number will occur rapidly since these quantum numbers are not conserved due to the electroweak anomaly. These transitions may then erase any primordial B-asymmetry, which could have been generated at some grand-unification energy scale.

There is one anomaly-free linear combination of B and L, namely $B-L$. Thus in order that a pre-existing B-asymmetry survive, a $B-L$ excess must have existed at very early times, and this may be easily generated in most GUTs. However, in this case it is crucial that the $B-L$ asymmetry be not eliminated through some other mechanism, such as the interactions present in eq. (1). Requiring that no such interaction comes into equilibrium after the $B-L$ asymmetry is produced, i.e. that Γ_{D} is smaller than the Hubble constant at any $T > \tilde{m}$, leads to [5]

$$\lambda \lesssim 10^{-7}\left(\frac{\tilde{m}}{\mathrm{TeV}}\right)^{1/2} \tag{3}$$

In the absence of additional symmetries that may restrict the flavour structure of the R_{p} violating couplings [8] this bound holds for each of the coupling constants of the operators in eq. (1), and rules out the possibility of observing new signals associated to *explicitly* R-parity violating interactions in collider experiments *.

The alternative possibility that R-parity can be an exact (or nearly so) symmetry of the lagrangian, broken only spontaneously, through the Higgs mechanism, automatically has a stable proton and survives the restrictions arising from the baryogenesis argument, provided the characteristic R-parity violation scale is below a few TeV. From now on we concentrate entirely on this possibility. As a generic feature, these $\mathrm{SU(2)} \otimes \mathrm{U(1)}$ theories lead to the existence of a physical massless Nambu–Goldstone boson – a majoron – denoted J. The minimal way to break R-parity spontaneously is through a nonzero vacuum expectation value (VEV) for the scalar neutrino [4]

$$v_{\mathrm{L}} = \langle \tilde{\nu}_{\mathrm{L}\tau} \rangle . \tag{4}$$

This, however, is not phenomenologically viable, since it leads to a new invisible decay mode for the neutral gauge boson involving the emission of light scalars,

$$\mathrm{Z} \to \rho + \mathrm{J}, \tag{5}$$

now ruled out by LEP measurements of the *invisible* Z-width [9]. The point is that the magnitude of the left-handed sneutrino VEV is restricted by astrophysical considerations related to stellar cooling rates [10] **

$$v_{\mathrm{L}} < \mathrm{O}(30\ \mathrm{keV}). \tag{6}$$

Since this determines the mass of the scalar ρ (related to the majoron) the decay in eq. (5) is unsuppressed.

However there is a viable variant of the broken R-parity SUSY model which avoids in a simple way the LEP constraint on $\Gamma_{\mathrm{Z}}^{\mathrm{inv}}$ [11]. The majoron being an isosinglet does not couple to the Z. The scale characterizing R-parity breaking is now large, i.e.

$$v_{\mathrm{R}} = \mathrm{O}(1\ \mathrm{TeV}) \tag{7}$$

* Strictly speaking, this could be avoided in the presence of a mechanism that could generate a nonzero baryon asymmetry at low energy [7].

** In the present model these involve majoron emission processes such as the Compton-like reaction $\gamma + \mathrm{e} \to \mathrm{e} + \mathrm{J}$. Being weakly coupled, once produced, majorons freely escape, bleeding energy from the star.

while the analogue of eq. (6) is estimated to be [11]

$$v_{\mathrm{L}} < \mathrm{O}(100\ \mathrm{MeV}) \tag{8}$$

and can be obtained without the need of unnatural fine-tuning of the parameters in the Higgs potential [11,12].

The signatures of supersymmetry in these spontaneously broken R-parity models are different from those of the minimal SUSY standard model. For example, SUSY particles may be singly produced and the "photino" is unstable. In ref. [13] it was shown that these R-parity breaking effects may lead to observable effects at LEP1. Similarly, there can be flavour-violating weak-decay processes involving majoron emission that could be seen in the laboratory [14].

In the present paper we focus on the detailed study of the neutrino mass spectrum that arises in these models. We analyse the attainable values for m_{ν_e}, m_{ν_μ} and m_{ν_τ}, once all observational constraints on the model parameters have been incorporated. We find that the ν_e is massless, while for natural choices of these parameters, the ν_μ mass scales as v_{L}^2 and $m_{\nu_\tau} \sim v_{\mathrm{R}}^2$. As a result, m_{ν_e} and m_{ν_μ} lie in the adequate range for the explanation of the deficit in the solar neutrino flux via the MSW effect, while the τ-neutrino, ν_τ, is naturally much heavier than ν_μ and decays to it via majoron emission, with a lifetime short enough to obey cosmological limits. The model leads to novel signatures associated with the τ-lepton that could be *measurable experimentally*, both at LEP as well as at a tau factory. By studying these new processes, facilities such as LEP or a tau factory will be able to independently restrict the neutrino oscillation parameters responsible for the explanation of the solar neutrino deficit. This would provide an indirect consistency check that, indeed, the MSW effect is the one responsible for the observed depletion in the solar neutrino flux.

2. The model

The possibility of generating the spontaneous violation of R-parity and lepton number was illustrated in the model proposed in ref. [11] and further studied in refs. [13,14]. The model is described by the following superpotential terms *:

$$\begin{aligned} &h_{\mathrm{u}} u^{\mathrm{c}} Q H_{\mathrm{u}} + h_{\mathrm{d}} d^{\mathrm{c}} Q H_{\mathrm{d}} + h_{\mathrm{e}} e^{\mathrm{c}} l H_{\mathrm{d}} + \left(h_0 H_{\mathrm{u}} H_{\mathrm{d}} - \epsilon^2\right)\Phi + h_\nu \nu^{\mathrm{c}} l H_{\mathrm{u}} + h \Phi \nu^{\mathrm{c}} S \\ &\quad + \hat{\mu} H_{\mathrm{u}} H_{\mathrm{d}} + M \nu^{\mathrm{c}} S + M_\Phi \Phi\Phi \end{aligned} \tag{9}$$

* Note that we have added some new terms that were not included in refs. [11,13,14]. These are allowed by our symmetries and give more flexibility in obeying all of the experimental constraints (the $\hat{\mu}$-term) and/or justifying our approximate treatment of the 14×14 matrix (e.g. the bare mass term $M\nu^{\mathrm{c}}S$ and the $\Phi\Phi$ mass term).

The superfields (Φ, ν_i^c, S_i) are singlets under $SU_2 \otimes U(1)$ and carry a conserved lepton number assigned as $(0, -1, 1)$, respectively. All couplings h_u, h_d, h_e, h_ν, h as well as the mass M are described by arbitrary matrices in generation space which explicitly break flavour conservation. These additional singlets, e.g. S, may arise in several extensions of the standard model [15], and may lead to interesting phenomenological signatures [16–18].

While the superpotential of eq. (9) conserves *total* lepton number as well as R-parity, the presence of the new singlets can drive the spontaneous violation of R-parity and electroweak symmetries [11,12]. This leads to the existence of a majoron given by the imaginary part of

$$\frac{v_L^2}{Vv^2}(v_u H_u - v_d H_d) + \frac{v_L}{V}\tilde{\nu}_\tau - \frac{v_R}{V}\tilde{\nu}_\tau^c + \frac{v_S}{V}\tilde{S}_\tau . \tag{10}$$

The isosinglet VEVs

$$v_R = \langle \tilde{\nu}_{R\tau} \rangle, \qquad v_S = \langle \tilde{S}_\tau \rangle \tag{11, 12}$$

with $V = \sqrt{v_R^2 + v_S^2}$ set the scale of R-parity or lepton number breaking, while the isodoublet VEVs

$$v_u = \langle H_u \rangle, \qquad v_d = \langle H_d \rangle \tag{13, 14}$$

drive electroweak breaking and the fermion masses. The combination $v^2 = v_u^2 + v_d^2$ is fixed by the W-mass, while the ratio of isodoublet VEVs determines the parameter

$$\tan\beta = \frac{v_u}{v_d} . \tag{15}$$

A necessary ingredient for the consistency of this model is the presence of a small seed of R-parity breaking in the SU(2) doublet sector. Using the results of ref. [10] we estimate that

$$\frac{v_L^2}{v_R m_W} \lesssim 10^{-7} \tag{16}$$

in order to adequately suppress the resulting stellar energy loss to acceptable levels. This may be easily satisfied for $v_R = O(1\ \text{TeV})$ provided $v_L \leqslant O(100\ \text{MeV})$, eq. (8). This constraint on v_L is more than three orders of magnitude less stringent than the one that holds in the model of ref. [4]. Moreover, the smallness of the ratio v_L/v_R may be achieved in a natural way, from the Higgs potential [11]. This follows because $v_L = \langle \tilde{\nu}_{L\tau} \rangle$ is related to the Yukawa coupling h_ν and vanishes as $h_\nu \to 0$. Last but not least, the present model has only the canonical neutrino counting, since the majoron contribution is avoided.

3. Mass matrices

Much of the phenomenology of this model is determined by the structure of the chargino and neutralino mass matrices. The first matrix is 5×5 and its form is common to all $SU(2) \otimes U(1)$ SUSY models with spontaneously broken R-parity, namely [11]

$$\begin{array}{c|ccc} & e_j^+ & \tilde{H}_u^+ & -i\tilde{W}+ \\ \hline e_i & h_{eij} v_d & -h_{\nu ij} v_{Rj} & \sqrt{2} g_2 v_{Li} \\ \tilde{H}_d^- & -h_{eij} v_{Li} & \mu & \sqrt{2} g_2 v_d \\ -i\tilde{W}^- & 0 & \sqrt{2} g_2 v_u & M_2 \end{array} \tag{17}$$

This (non-symmetric) chargino mass matrix is diagonalized by two matrices U and V, i.e.

$$\chi_i^+ = V_{ij} \psi_j^+ , \quad \chi_i^- = U_{ij} \psi_j^- , \tag{18, 19}$$

where the indices i and j run from 1 to 5 and $\psi_j^+ = (e_1^+, e_2^+, e_3^+, \tilde{H}_u^+, -i\tilde{W}^+)$ and $\psi_j^- = (e_1^-, e_2^-, e_3^-, \tilde{H}_d^-, -i\tilde{W}^-)$.

The form of the neutralino mass matrix is more sensitive to the details of the model considered. In our model it is a 14×14 majorana mass matrix (thus symmetric, due to the Pauli exclusion principle). The basis is defined by all 2-component neutral fermions present, $\psi_j^0 = (\nu_i, \tilde{H}_u, \tilde{H}_d, -i\tilde{W}_3, -i\tilde{B}, \nu_i^c, S_i, \Phi)$. This matrix is determined in terms of the VEVs v_R, v_S, $\langle \Phi \rangle$, v_u, v_d, v_L through eq. (9) and the corresponding D-terms that follow from the $SU(2) \otimes U(1)$ assignments of the superfields.

For most purposes one can, under reasonable approximations, treat the neutralino mass matrix as an effective 7×7 mass matrix, after appropriately removing away the heavy isosinglet leptons present. This *effective* matrix takes the following approximate form *:

$$\begin{array}{c|ccccc} & \nu_i & \tilde{H}_u & \tilde{H}_d & -i\tilde{W}_3 & -i\tilde{B} \\ \hline \nu_i & 0 & h_{\nu ij} v_{Rj} & 0 & g_2 v_{Li} & -g_1 v_{Li} \\ \tilde{H}_u & h_{\nu ij} v_{Rj} & 0 & -\mu & -g_2 v_u & g_1 v_u \\ \tilde{H}_d & 0 & -\mu & 0 & g_2 v_d & -g_1 v_d \\ -i\tilde{W}_3 & g_2 v_{Li} & -g_2 v_u & g_2 v_d & M_2 & 0 \\ -i\tilde{B} & -g_1 v_{Li} & g_1 v_u & -g_1 v_d & 0 & M_1 \end{array} \tag{20}$$

* This truncation is not a good approximation when determining the masses of ν_e and ν_μ and the majoron couplings responsible for ν_τ decays. See sect. 5.

In eqs. (17) and (20) $M_{1,2}$ denote the supersymmetry breaking gaugino mass parameters and $g_{1,2}$ are the SU(2) ⊗ U(1) gauge couplings divided by $\sqrt{2}$. We assume the canonical relation $M_1/M_2 = \frac{5}{3}\tan^2\theta_W$. In some models, such as the one in ref. [11], the effective higgsino mixing parameter μ may be given as $h_0\langle\Phi\rangle$, where $\langle\Phi\rangle$ is the VEV of an appropriate singlet scalar. Here we are assuming, for generality, that it could also have a bare contribution, i.e. $\mu = \hat{\mu} + h_0\langle\Phi\rangle$. This mass matrix is diagonalized by a single 7×7 matrix N, i.e.

$$\chi_i^0 = N_{ij}\psi_j^0, \tag{21}$$

where $\psi_j^0 = (\nu_\alpha,\ \tilde{H}_u,\ \tilde{H}_d,\ -i\tilde{W}_3,\ -i\tilde{B})$, with ν_α denoting weak-eigenstate neutrinos. Here the indices i and j run from 1 to 7. For simplicity we assume CP variance, which implies that these two matrices are real and, consequently, in an appropriate phase convention, also the corresponding diagonalizing matrices.

As a result of R-parity breaking, the supersymmetric fermions in eqs. (17) and (20) (partners of gauge and Higgs particles) mix with the weak-eigenstate leptons. This mixing implies that the masses of the physical charged leptons, especially the tau, are determined in terms of the underlying parameters, such as Yukawa couplings, differently than in the standard model. In the neutral case this mixing generates neutrino masses and decays that we consider in sect. 5.

4. The explicit form of the couplings

Using the diagonalizing matrices U, V and N one can write the electroweak currents of mass-eigenstate fermions in terms of effective K and P mixing matrices [21]. The off-diagonal blocks in these matrices will determine the strength of R-parity violation in the gauge interactions of our SU(2) ⊗ U(1) SUSY model. For example, the charged current weak interaction lagrangian may be written as

$$\frac{g}{\sqrt{2}} W_\mu \bar{\chi}_i^- \gamma^\mu (K_{\mathrm{L}ik} P_\mathrm{L} + K_{\mathrm{R}ik} P_\mathrm{R}) \chi_k^0 + \text{h.c.}, \tag{22}$$

where $P_{\mathrm{L,R}}$ are the two chiral projectors and the 5×7 coupling matrices $K_{\mathrm{L,R}}$ may be written as

$$K_{\mathrm{L}ik} = \eta_i \left(-\sqrt{2}\, U_{i5} N_{k6} - U_{i4} N_{k5} - \sum_{m=1}^{3} U_{im} N_{km} \right), \tag{23}$$

$$K_{\mathrm{R}ik} = \epsilon_k \left(-\sqrt{2}\, V_{i5} N_{k6} + V_{i4} N_{k4} \right). \tag{24}$$

The matrix $K_{\mathrm{L}ik}$ is the analogous of the matrix K introduced in ref. [21]. The η_i and ϵ_k factors are sign factors that follow from the diagonalization of the mass matrices. The diagonal blocks in K describe R-parity conserving lepton–neutrino or chargino–neutralino interactions, while the off-diagonal ones describe R-parity violating lepton–neutralino or neutrino–chargino interactions.

Similarly, the neutral current weak interaction lagrangian may be written as

$$\frac{g}{\cos\theta_{\mathrm{W}}} Z_\mu \{ \bar{\chi}_i^- \gamma^\mu (\eta_i \eta_k O'_{\mathrm{L}ik} P_{\mathrm{L}} + O'_{\mathrm{R}ik} P_{\mathrm{R}}) \chi_k^-$$

$$+ \tfrac{1}{2} \bar{\chi}_i^0 \gamma^\mu (\epsilon_i \epsilon_k O''_{\mathrm{L}ik} P_{\mathrm{L}} + O''_{\mathrm{R}ik} P_{\mathrm{R}}) \chi_k^0 \}, \tag{25}$$

where the 7×7 coupling matrices $O'_{\mathrm{L,R}}$ and $O''_{\mathrm{L,R}}$ are given as

$$O'_{\mathrm{L}ik} = \tfrac{1}{2} U_{i4} U_{k4} + U_{i5} U_{k5} + \tfrac{1}{2} \sum_{m=1}^{3} U_{im} U_{km} - \delta_{ik} \sin^2\theta_{\mathrm{W}}, \tag{26}$$

$$O'_{\mathrm{R}ik} = \tfrac{1}{2} V_{i4} V_{k4} + V_{i5} V_{k5} - \delta_{ik} \sin^2\theta_{\mathrm{W}}, \tag{27}$$

$$O''_{\mathrm{L}ik} = \tfrac{1}{2} \left\{ N_{i4} N_{k4} - N_{i5} N_{k5} - \sum_{m=1}^{3} N_{im} N_{km} \right\} = -O''_{\mathrm{R}ik}. \tag{28}$$

Here the diagonal blocks in O' and O'' describe R-parity conserving lepton–lepton, neutrino–neutrino or chargino–chargino or neutralino–neutralino interactions, while the off-diagonal ones describe R-parity violating neutrino–neutralino, or lepton–chargino interactions.

Similarly, the effective lagrangian interaction of the majoron with charged fermions may be given in terms of the chargino diagonalizing matrices U and V introduced earlier. The result is

$$\sqrt{\tfrac{1}{2}}\, i J \bar{\chi}_j \{ \eta_k P_{\mathrm{L}} A_{kj} - \eta_j P_{\mathrm{R}} A_{jk} \} \chi_k, \tag{29}$$

where χ_i^- are negatively charged Dirac spinors composed out of the two 2-component mass eigenstate fermions obtained, for each value of i, from diagonalizing their mass matrix, eq. (17). Using eq. (29) we can derive the decay rates of μ and τ with single-majoron emission. The single-majoron emission in muon decay corresponds to $k \to \mu$ and $j \to \mathrm{e}$ while, for the case of the τ-decays, the possible assignments are $k \to \tau$ and $j \to \mathrm{e}, \mu$. The matrix A may be obtained, in a good approximation, from the matrices U and V as follows:

$$A_{jk} = \sum_{i=1}^{3} h_{\nu i3} U_{ji} V_{k4} \frac{v_{\mathrm{R}}}{V}. \tag{30}$$

On the other hand, the form of the interaction lagrangian describing the majoron–neutralino interactions is given as

$$-\tfrac{1}{2}iJ\bar{\chi}_k^0(O_{\mathrm{L}kj}P_{\mathrm{L}}+O_{\mathrm{R}kj}P_{\mathrm{R}})\chi_j^0, \tag{31}$$

where

$$O_{\mathrm{R}kj}=-O_{\mathrm{L}kj}. \tag{32}$$

These interactions will determine the ν_τ majoron emission decay rate.

We now will concentrate on the consequences of having spontaneously broken R parity for neutrino masses, mixing and decays.

5. Neutrino masses and decays

Spontaneous R-parity violation implies the violation of L-number and provides a natural origin for neutrino masses [1]. They arise at the three level in these models due to mixing of the heavy neutral R-odd fermions with the neutrinos. The resulting pattern is very striking. Here we focus on the general study of the neutrino mass spectrum in the spontaneously broken R-parity model considered in sect. 2.

The heavier neutrino in our model is the ν_τ. Its mass may be easily estimated in the approximation where we neglect v_{L} from the effective matrix in eq. (20) as

$$m_{\nu_\tau}\simeq\frac{\sum_i h_{\nu i\tau}^2M_0v_{\mathrm{R}}^2v_{\mathrm{d}}^2}{\mu(2v_{\mathrm{u}}v_{\mathrm{d}}M_0-\mu M_1M_2)}, \tag{33}$$

where we have set $M_0=g_1^2M_2+g_2^2M_1$ *.

In order to determine the masses of the two lowest mass states relevant for the description of the propagation of solar neutrinos in our model – ν_{e} and ν_μ – we have to consider the full 14×14 neutralino mass matrix, M_{14}. In complete generality, one can show analytically that one of the neutrinos is massless at the tree level, since the determinant of M_{14} vanishes,

$$\det(M_{14})=0. \tag{34}$$

This fact holds in the model of sect. 2 even if we allow the most general pattern of VEVs for all the scalars **. The corresponding massless neutrino state ν_1 is a

* In our numerical study, however, we have used the full expression for m_{ν_τ} corrected for the general case where $v_{\mathrm{L}}\neq0$.

** Indeed, starting from the mass matrix M_{14} and performing elementary manipulations on its rows and columns, one can cast it in a form that clearly identifies the existence of six linearly dependent vectors that only span a five-dimensional subspace. This follows from the particle content and couplings of the model as specified in sect. 2. At the top loop level there are negligibly small radiative corrections, irrelevant for our considerations.

linear combination of ν_e, ν_μ, ν_τ, with some admixture of $\tilde{H}_d$ and $\tilde{S}_i$ ($i = 1, 2, 3$), the SUSY fermions which have the same lepton number and/or hypercharge as the weak-eigenstate neutrinos. For reasonable choices of parameters, the dominant components of ν_1 are along ν_e and ν_μ. For simplicity, in our numerical analysis we assumed R_p breaking VEVs only along the third generation and, moreover, assumed a simplified flavour structure for relevant Yukawa couplings such as h_ν. In addition we use an improved 7×7 effective matrix obtained from the full matrix in the seesaw approximation, which consists in diagonalizing out the heavy $SU(2) \otimes U(1)$ singlet leptons. We checked that this is a very good approximation. In this approximation we obtain

$$\nu_1 \simeq \cos\theta\, \nu_e - \sin\theta\, \nu_\mu, \tag{35}$$

where

$$\tan\theta = \frac{h_{\nu 13}}{h_{\nu 23}}. \tag{36}$$

In this case the other light neutrino is

$$\nu_2 \simeq \alpha \sin\theta\, \nu_e + \alpha \cos\theta\, \nu_\mu + N_{23} \nu_\tau, \tag{37}$$

where

$$\alpha \simeq \frac{-\mu(v_L/v_d) + h_{\nu 33} v_R}{\sqrt{(h_{\nu 13}^2 + h_{\nu 23}^2) v_R^2 + (\mu(v_L/v_d) + h_{\nu 33} v_R)^2}}, \tag{38}$$

$$N_{23} \simeq \frac{\sqrt{h_{\nu 13}^2 + h_{\nu 23}^2}\, v_R}{\sqrt{(h_{\nu 13}^2 + h_{\nu 23}^2) v_R^2 + (\mu(v_L/v_d) + h_{\nu 33} v_R)^2}}, \tag{39}$$

and its mass can be given, in a reasonable approximation, by

$$m_2 \simeq \frac{(h_{\nu 13}^2 + h_{\nu 23}^2) v_u^2 v_L^2 [h_{33}(\mu + h_0 \langle \Phi \rangle) + h_0 M_{33}]^2}{M_\Phi [h_{33} \langle \Phi \rangle + M_{33}]^2 h_{\nu 33}^2 v_d^2}, \tag{40}$$

showing that it vanishes as $v_L \to 0$. For small values of θ the massless neutrino is mostly ν_e so that ν_2 corresponds to ν_μ. In the following we will always refer to ν_2 and ν_μ and to ν_1 as ν_e although this assignment is not valid for large values for the ν_e–ν_μ mixing angle θ.

The striking point to notice is that in this scheme the ν_μ mass is extremely small on the scale of the ν_τ mass. This is illustrated by eqs. (33) and (40). The ν_τ mass is generated through the dominant $SU(2) \otimes U(1)$ singlet R_p breaking VEV v_R while the ν_μ mass only arises as a result of the small isodoublet VEV $v_{\rm L}$. The smallness of the $v_{\rm L}/v_{\rm R}$ ratio follows in a natural way from the Higgs potential [11,12]. Indeed, $v_{\rm L}$ is related to the Yukawa coupling h_ν and vanishes as $h_\nu \to 0$. In other words,

$$h_\nu \to 0 \Rightarrow \frac{v_{\rm L}}{v_{\rm R}} \to 0. \tag{41}$$

The above limit defines an $R_{\rm p}$ conserving theory. Choosing, for example, reasonable parameters such as $v_{\rm R} = 1$ TeV and $v_{\rm L} = 100$ MeV we obtain a ratio of order

$$\frac{m_{\nu_\mu}}{m_{\nu_\tau}} \sim 10^{-8}. \tag{42}$$

Thus for natural ν_τ mass values in the 10 keV–1 MeV range we expect a ν_μ mass in the range 10^{-4}–10^{-2} eV, just the one relevant for the MSW effect. Similarly, the $\nu_{\rm e}$–ν_μ mixing angle given by eq. (36) may easily lie in the desired range where the MSW effect can effectively reduce the solar neutrino flux.

The expected size of the ν_τ mass in this model appears to be in conflict with the cosmological limit [22]

$$\sum_i m_{\nu_i} \lesssim 100 \text{ eV} \tag{43}$$

on the abundance of relic neutrinos. Fortunately, our model also has the solution to this apparent conflict. It relies on the existence of a new ν_τ decay mode involving majoron emission

$$\nu_\tau \to \nu_\mu + \mathrm{J} \tag{44}$$

that naturally realizes the early proposal considered in refs. [23–25]. This follows from the fact that, although light, ν_μ is not strictly massless, *and* from the fact that, unlike the situation [23] there is no cancellation [24] in the decay amplitude. The relevant coupling is obtained from eq. (31) when $k = 2$, $j = 3$. In a reasonable approximation one has

$$O_{\rm L23} \simeq \frac{m_{\nu_\tau} v_{\rm L}}{\sqrt{2}\, V v_{\rm d}} N_{23} N_{35} \tag{45}$$

with N_{23} given by eq. (39). This coupling can be large enough as to cause a cosmologically efficient ν_τ decay rate. One can see that in the limit $h_{\nu 13} \to 0$ and $h_{\nu 23} \to 0$ both the ν_μ mass, eq. (40), and the coupling in eq. (39), become zero. One can also show that the decay $\nu_\tau \to \nu_{\rm e} + \mathrm{J}$ is strictly forbidden and that the mass of $\nu_{\rm e}$ is zero.

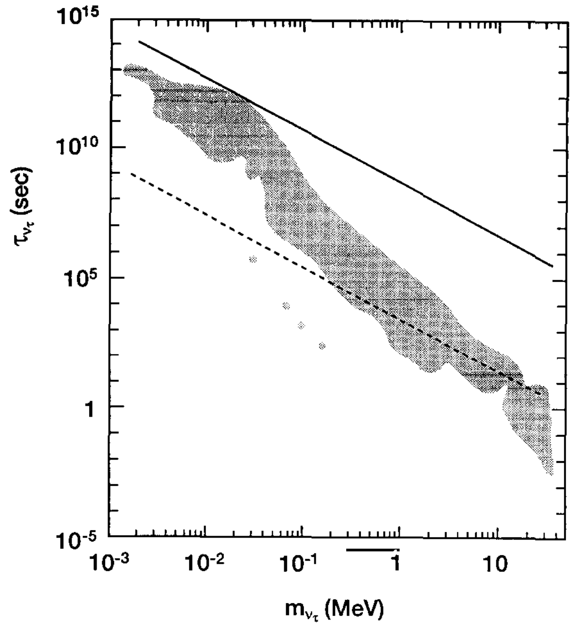

Fig. 1. The shaded area represents the attainable values of the ν_τ lifetime versus its mass, when the model parameters are randomly varied as explained in the text. The curve delimiting the lower part of the shaded contour represents our estimate of the minimum lifetime which is consistent with observational constraints. For comparison we also show the cosmological critical density limit (solid line) and the naive limit that one may derive from galaxy formation (dashed line).

We therefore conclude that the spontaneously broken R-parity model should be added to the list [25] of models with fast decaying majorana neutrinos. The attainable ν_τ lifetimes are given in fig. 1 as a function of the ν_τ mass, and should be compared to the cosmological limit on the ν_τ decay lifetime *.

$$\tau \lesssim 1.5 \times 10^7 \text{ yr}\left(\frac{m_{\nu_\tau}}{\text{keV}}\right)^{-2} \tag{46}$$

that is required in order to efficiently suppress the relic ν_τ contribution. This is shown as the solid line in fig. 1. Clearly the decay lifetimes can be much shorter than required by cosmology for a wide range of values of the parameters. Moreover, since these decays are *invisible*, they are consistent with all astrophysical observations [1].

* This assumes $m_{\nu_\tau} \lesssim$ few MeV. For larger ν_τ masses the limit is weaker, due to the Boltzmann suppression.

If, however, we require that the universe should have become matter-dominated by a redshift of 1000 at the latest (in order that fluctuations have grown by at least a factor of 1000 by today), then the ν_τ lifetime limit becomes [26]

$$\tau \lesssim 2 \times 10^9 \text{ s}(m_\nu/\text{keV})^{-2}. \tag{47}$$

This limit is also shown in fig. 1 (dashed line). Clearly lifetimes below the dashed line are allowed in the present model. However, this lifetime limit is much less reliable than that of eq. (46), since there is not yet an established theory for the formation of structure in the early universe. In our following analysis we have therefore not imposed it.

In short, in this model m_{ν_τ} can be larger than eq. (43), because of the existence of an efficient majoron decay channel, eq. (44). The large m_{ν_τ} values acceptable in this model enable the rare Z- and τ-decays to be correspondingly enhanced. In addition, the large hierarchy between v_L and v_R (and thus between m_{ν_μ} and m_{ν_τ}) present in the model, naturally combines these measurable effects with the MSW oscillations, required to explain the solar neutrino data. This suggests the possibility of having an independent check upon the solar neutrino oscillation parameters as determined by solar neutrino experiments with searches performed in conventional laboratory experiments, as originally suggested in ref. [4]. To analyse this more sharply we first summarize the observational constraints upon the model parameters.

5. Experimental constraints

There are several restrictions on the parameters of any supersymmetric extension of the standard model, such as the spontaneously broken R-parity models. Broadly speaking, these fall into two classes. First, there are constraints that follow from collider experiments, such as the recent LEP data on Z-decays and $\bar{\mathrm{p}}\mathrm{p}$ collider data, e.g. on $\mathrm{W}^\pm$, Z and gluino production. In addition to these constraints, there are important restrictions, characteristic of broken R-parity models, related to weak interactions and neutrino physics. These follow from laboratory, astrophysics and cosmology. This second group of constraints plays a very important role for our present analysis, since they are found to exclude many parameter choices that are allowed by the collider constraints, while the converse is not true. For completeness, we now list the constraints relevant for our present analysis. The first group includes:

(i) The lightest of these charginos, denoted $\tilde{\chi}^\pm$, has not yet been produced in Z-decays at LEP [9], leading to the mass limit

$$m_{\tilde{\chi}^\pm} \gtrsim 45 \text{ GeV}. \tag{48}$$

(ii) The recent measurements of the Z-widths at LEP give [9]

$$\Gamma_{\mathrm{Z}}^{\mathrm{total}} = 2.487 \pm 0.010 \text{ GeV} \Rightarrow \Gamma_{\mathrm{Z}}^{\mathrm{total}} \lesssim 2.504 \text{ GeV (95\% C.L.)}. \tag{49}$$

These measurements restrict the additional decay channels of the Z involving charginos and neutralinos, present in any SUSY model, such as our broken R-parity model.

(iii) LEP limits on the visible Z-width [9]:

$$\Gamma_{\mathrm{Z}}^{\mathrm{inv}} = 496.2 \pm 8.8 \text{ MeV} \Rightarrow \Gamma_{\mathrm{Z}}^{\mathrm{inv}} \lesssim 511 \text{ MeV (95\% C.L.)}. \tag{50}$$

These measurements restrict the additional contributions involving invisibly decaying neutralinos, present in the R-parity broken models.

(iv) The CDF lower limit on the gluino mass $m_{\tilde{\mathrm{g}}}$ [27] restricts the soft supersymmetry breaking electroweak gaugino mass parameter,

$$M_2 > 30 \text{ GeV}. \tag{51}$$

(v) $\mathrm{p\bar{p}}$ collider limits on the ratio $R = \sigma_{\mathrm{W}^{\pm}} \mathrm{BR}(\mathrm{W}^{\pm} \to \mathrm{e}^{\pm}\nu)/\sigma_{\mathrm{Z}} \mathrm{BR}(\mathrm{Z} \to \mathrm{e}^{+}\mathrm{e}^{-})$ imply [28]

$$0.825 \leqslant \frac{R}{R_{\mathrm{SM}}} \leqslant 1.091 \tag{52}$$

while in SUSY models there are additional possible contributions involving charginos and neutralinos, produced in virtual W- or Z-decays.

(vi) LEP constraints on the hadronic peak cross section [9]

$$\sigma_0^{\mathrm{had,T}} \simeq 3.88 \times 10^5 \frac{12\pi}{M_{\mathrm{Z}}^2} \Gamma_{\mathrm{ee}} \frac{\Gamma_{\mathrm{had}}}{(\Gamma^{\mathrm{total}})^2} (\mathrm{nb}) \tag{53}$$

versus total Z-width are given as an allowed ellipsis (σ_0^{had}, $\Gamma_{\mathrm{Z}}^{\mathrm{total}}$). In a SUSY model, such ours, the Z hadronic width in principle receives contributions from hadronically decaying neutralinos.

The constraints in the second group relevant for our analysis are given below:

(i) The non-observation of neutrinoless double β-decay implies a very stringent constraint, i.e.

$$\langle m \rangle = \sum_i K_{\mathrm{Le}i}^2 m_i \leqslant 3 \text{ eV}, \tag{54}$$

where we neglect the induced right-handed current interaction arising from R-parity breaking (see eq. (22)). In the spontaneously broken R-parity model, L is necessarily violated. Even though the seed of this violation lies in the τ-sector, i.e. mostly L_τ is violated, through lepton mixing effects this violation is fed also to the

L_e sector, where it is very well tested. In practice in our model the sum in eq. (54) has really just one dominant term, namely the heavy τ-neutrino contribution. For large masses of the ν_τ we have modified eq. (54) in order to take into account the effects of the two nucleon correlation function, as in ref. [29].

(ii) Neutrino oscillation data and direct searches for anomalous peaks on the energy distribution of the electrons and muons coming from the decays such as π, $K \to e\nu$ and π, $K \to \mu\nu$ lead to constraints on the mixing matrix elements K_{Lei} and $K_{L\mu i}$ $(i = 3)$ [19].

(iii) The ARGUS limit on the τ-neutrino mass,

$$m_{\nu_\tau} \leqslant 35 \text{ MeV}, \tag{55}$$

is a restrictive constraint on spontaneously broken R-parity models since, as seen above, in these models the τ-neutrino may acquire a large mass from the mixing with the heavy neutralinos. Moreover, if it were stable, the ν_τ would easily violate the cosmological limit in eq. (43).

(iv) The cosmological limit on the ν_τ decay lifetime of eq. (46) is imposed. It easily satisfied in this model due to the invisible decay mode $\nu_\tau \to \nu_\mu + J$ where J is the majoron [25].

(v) The limits on the muon and tau lifetimes impose severe restrictions on the parameters of these models.

(vi) The existence of fermion states that cannot be kinematically produced in low-energy weak decays changes the relative rates of various such processes, for instance β- or μ-decays, leading to universality violations. The resulting constraints were discussed in ref. [1] and have been implemented.

(vii) The limits on lepton flavour violating processes such as $\mu \to e + \gamma$ and $\mu \to 3e$ and the corresponding tau decay processes [19]. In this model the $\mu \to e + \gamma$ decay process occurs only at one-loop level, while the three lepton decays can arise at the tree level due to the existence of flavour changing couplings of the Z to charged leptons.

(viii) The limits on lepton flavour violating decays with single-majoron emission. The present experimental limit from TRIUMF on $\mu \to e + J$ is $BR(\mu \to e + J) \leqslant 2.6 \times 10^{-6}$, while the best τ decay limits now available from the ARGUS collaboration are $BR(\tau \to e + J) \leqslant 3.2 \times 10^{-3}$ and $BR(\tau \to \mu + J) \leqslant 5.8 \times 10^{-3}$ [20].

Note that in phenomenological studies of spontaneously broken R-parity models a specially important role is played by constraints related to flavour and/or total-lepton-number-violating processes such as those arising from the non-observation of neutrino oscillations and neutrinoless double β-decay [14].

6. Study of the MSW effect

We now give a summary of our study of the phenomenological implications of our spontanesouly broken R-parity model for the propagation of solar neutrinos.

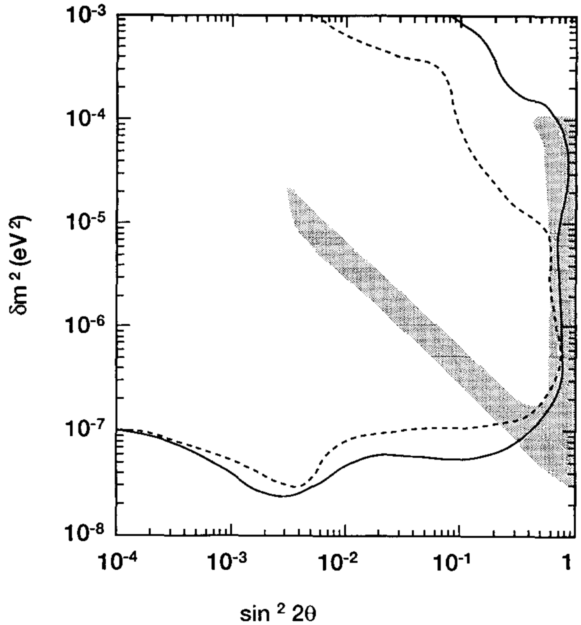

Fig. 2. Contour plots for the $\mathrm{BR}(Z \to \tilde{\chi} + \tau)$. The solid line corresponds to $\mathrm{BR} \geqslant 10^{-6}$, while the dashed line corresponds to $\mathrm{BR} \geqslant 10^{-5}$. Also shown is the region allowed by present solar neutrino data.

In our model solar neutrino oscillations are expected to occur in the channel ν_e to ν_μ, since, as discussed, ν_e remains massless while ν_μ acquires a mass $\sim v_L^2$, much smaller than that of the heavy ν_τ which scales as $\sim v_R^2$. The ν_τ is therefore decoupled from the solar neutrino oscillations.

For definiteness we have fixed in our analysis characteristic values for the following parameters

$$\tan\beta = 10, \quad v_R = 1\ \mathrm{TeV}, \quad v_L = 100\ \mathrm{MeV}, \quad m_\Phi = 10\ \mathrm{TeV}, \quad M_{33} = 1\ \mathrm{TeV}, \tag{56}$$

and assumed, for simplicity, that $v_R = v_S$.

The allowed region of oscillation parameters has been determined as a function of the relevant SUSY parameters μ and M_2. These have been randomly varied in the range *

$$30 \leqslant \frac{M_2}{\mathrm{GeV}} \leqslant 250, \qquad -250 \leqslant \frac{\mu}{\mathrm{GeV}} \leqslant 250. \tag{57}$$

* It is always possible, if CP is conserved in this sector, to choose $M_2 > 0$, while μ may have either sign.

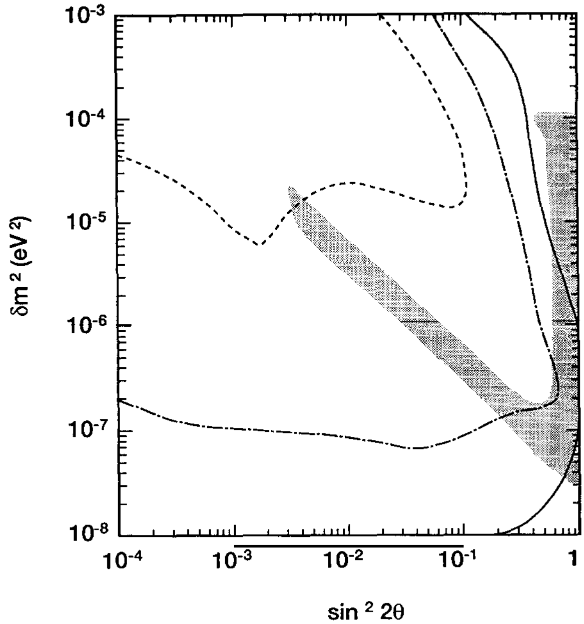

Fig. 3. Contour plots for the BR($\tau \to \mu + J$). The solid line corresponds to BR $\geqslant 10^{-6}$, while the dot-dashed and the dashed line correspond to BR $\geqslant 10^{-5}$ and BR $\geqslant 10^{-4}$, respectively. Also shown is the region allowed by present solar neutrino data.

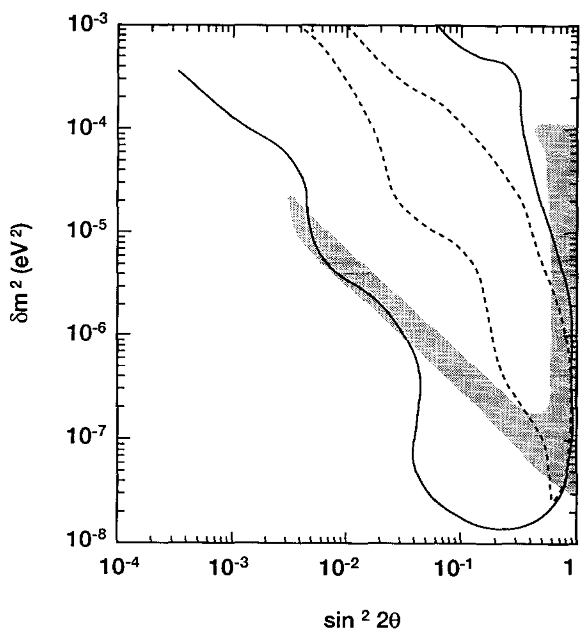

Fig. 4. Contour plots for the BR($\tau \to e + J$). The solid line corresponds to BR $\geqslant 10^{-7}$, while the dashed line corresponds to BR $\geqslant 10^{-6}$. Also shown is the region allowed by present solar neutrino data.

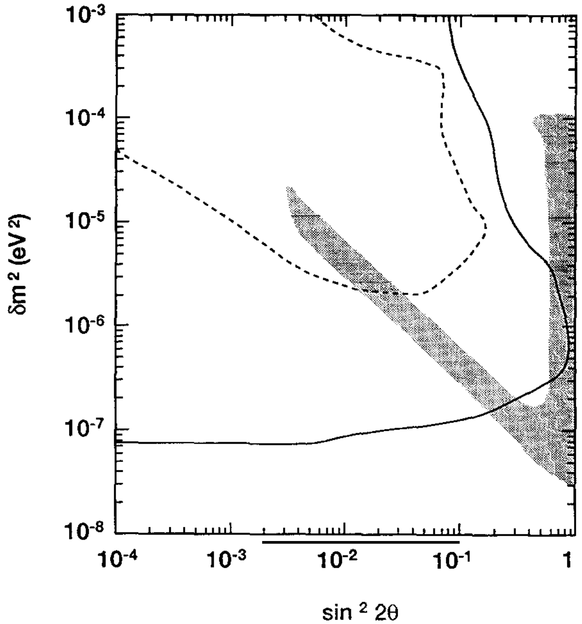

Fig. 5. Contour plots for the BR($\tau \to \mu + J$) and BR($Z \to \tilde{\chi} + \tau$). The solid line corresponds to both BR $\geqslant 10^{-6}$, while the dashed line corresponds to both BR $\geqslant 10^{-5}$. Also shown is the region allowed by present solar neutrino data.

The ν_τ and ν_μ masses are controlled by the parameters $h_{\nu_{ij}}$ as can be seen from eqs. (33) and (40). These were varied randomly in the interesting range given by

$$10^{-10} \leqslant h_{\nu 13}, h_{\nu 23} \leqslant 10^{-1}, \qquad 10^{-4} \leqslant h_{\nu 33} \leqslant 10^{-1}. \tag{58}$$

We have performed a careful sampling of the points in our parameter space that are allowed by all the constraints discussed in sect. 5 in order to evaluate the attainable magnitudes of m_{ν_μ} and the corresponding mixing angle θ. These will determine the ν_e–ν_μ conversion probabilities and thus the solar neutrino predictions. We have verified that, with all the constraints discussed in the previous section, the regions of neutrino mass and mixing parameters characterizing ν_e–ν_μ oscillations presently allowed in our model covers the region where the resonant effects can play an important role in the oscillations of solar neutrinos. On the other hand, the constraints that follow from present solar neutrino observations, both from the Homestake experiment, as well as from Kamiokande, are indicated by the shaded region shown in figs. 2–5, adapted from the first paper in ref. [31]. We also note that the preliminary results of the SAGE collaboration point towards a very drastic reduction of the low-energy pp neutrino flux, as possible at the lower tip of the shaded region. As figs. 2–5 show, in the spontaneously broken R-parity model the solar neutrino oscillation parameters cover the region of interest where the MSW oscillations between ν_e and ν_μ provide a successful explanation of the reduced solar neutrino flux. In addition, since we have a model where these MSW oscillations naturally coexist with a very massive ν_τ, we also expect to have other exotic effects that may be measurable in the laboratory, and *independently* check the underlying physics of the neutrino mass generation mechanism. We now turn to these signatures.

7. Signatures

In the spontaneously broken R-parity model there are observable effects in Z as well as in τ-decays that could be observable both at LEP [13] as well as at a tau factory [14]. Here we investigate whether their observability is consistent with the understanding of present solar neutrino observations in terms of matter-enhanced neutrino oscillations. The smallness of neutrino masses needed in the MSW effect makes this possibility hard to achieve in practice. In the present model, however, the ν_τ mass is expected to be large, thus opening the way to this interesting complementarity between laboratory and astrophysical observations. The latter also involve considerations related to stellar cooling rates inasmuch as these are affected by majoron emission processes [10].

We now show that, indeed, when the neutrino oscillation parameters lie in the required domain indicated by solar neutrino observations, there are related processes that could be observable at the laboratory. We consider two examples. In our analysis we have fixed some of the parameters as in eq. (56), while the others have been randomly varied in the ranges given in eqs. (57) and (58).

7.1. SINGLE-CHARGINO PRODUCTION AT LEP1

If R-parity does not hold exactly then SUSY particles may be singly produced. In ref. [13] a study was made of the *single*-chargino decay mode

$$Z \to \tilde{\chi}\tau \tag{59}$$

and shown that it may be observable at LEP1 *.

Here we have performed a systematic analysis of the attainable values of this branching ratio once constraints that follow from all existing observations, including those from laboratory, cosmology and astrophysics are taken into account. The latter include the requirement that the solar neutrino oscillation parameters lie in

* The same situation holds in the case where R-parity is broken spontaneously and the majoron is absorbed by an additional gauge boson. See e.g. fig. 7 in ref. [30].

the desired range allowed by present solar neutrino observations. Our results are summarized in fig. 2. They show that the corresponding $Z \to \tilde{\chi}\tau$ branching ratio could possibly be measured at LEP1, even when the solar neutrino oscillation parameters lie in the non-adiabatic region favoured by present observations.

7.2. MAJORON EMISSION IN μ- AND τ-DECAYS

In the spontaneously broken R-parity model the spectra of the decay leptons produced in μ- and τ-decays could be significantly different from the standard model predictions, as a result of majoron emission processes in these weak decays [4,14]

$$\mu \to e + J, \qquad \tau \to e + J, \qquad \tau \to \mu + J. \tag{60}$$

Since the majoron is very weakly interacting, it will lead to an observable signal only insofar as it will affect the energy spectra of the decay-produced leptons. The decays in eq. (60) would lead to bumps in the final lepton energy spectrum, at half of the parent mass, in its rest frame. These have been searched for experimentally, but the limits on their possible existence are still rather poor, especially for the case of taus [20].

A detailed study of the attainable sizes of these signals in the present model was made in ref. [14], as a function of the tau neutrino mass. Our results showed that these flavour-violating τ-decay processes with single-majoron emission could lead to observable effects for a wide range of the allowed parameters, especially for the $\tau \to \mu + J$ decay mode. They all become larger, the larger the value of the parameter $\tan\beta$, that we have fixed at 10 but which could well be as large as m_t/m_b, the top–bottom quark mass ratio. As seen from figs. 3 and 4 these rates are well within the expected sensitivities of a tau factory and even LEP, with the data that are expected to be accumulated by the end of 1992. In contrast, the single-majoron emission muon decay branching ratio is always below 10^{-8} when the neutrino oscillation parameters lie in the region favored by present solar neutrino experiments. Note that this is not so if the MSW restrictions are not applied, as shown in fig. 2 of ref. [14].

In addition, we have shown explicitly that *both* the Z and the $\tau \to \mu + J$ decay branching ratios can be simultaneously larger than 10^{-5}. Such large branching can certainly be probed experimentally both at LEP1 as well as at a tau factory of the type now under consideration [32]. It is also worth stressing the fact that these rates are completely consistent with all existing observations and, in addition, consistent with having the ν_e–ν_μ oscillation parameters in the desired range where MSW oscillations explain present solar neutrino observations. This is shown in fig. 5.

8. Discussion

We conclude that the MSW oscillations responsible for the explanation of the reduced solar neutrino flux may well be accompanied by effects that can be observable in the laboratory and therefore these can be used as an additional tool to restrict the underlying physics. In principle, the non-observation of the new effects would place independent restriction upon solar neutrino oscillation parameters. This is the case for the present spontaneously broken R-parity model. We showed that the peculiar nature of the neutrino mass spectrum present in this model, that combines a massless ν_e, an ultralight ν_μ and a heavy majorana ν_τ, naturally allows for the interesting possibility that the ν_e–ν_μ oscillations that can successfully explain present solar neutrino observations are accompanied by effects that can be probed in conventional particle physics accelerator experiments. Two examples were given of these. First, the Z could reveal unconventional R-parity violating decay modes, such as eq. (59) with branching ratio as large as $\mathrm{BR}(Z \to \tilde{\chi}\tau) \sim 6 \times 10^{-5}$, which can be studied at LEP1. In addition, our model allows for substantial modifications in the τ-lepton decay properties, with respect to the standard model predictions. These arise from single-majoron emission τ decay modes such as $\tau \to \mu + J$ which can occur with branching ratios as large as $\sim 10^{-4}$ which could be observable at LEP and/or at a tau factory. Therefore these effects might be used as an independent tool to probe the physics underlying the explanation of the solar neutrino deficit. This possibility does not contradict any laboratory, cosmological or astrophysical observation.

This work was supported by Accion Integrada Hispano–Portuguesa N. 59 and by CICYT under grant N. AEN-90-0040.

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