Problems in Quantum Field Theory

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Abstract

We collect here problems for the four Lectures I gave in Quantum Field Theory at the 2nd IDPASC School held at Udine, January 23rd to February 3rd, 2012.

1 Problems Quantum Mechanics: Lecture 1

1.1 Consider the system of units used in high energy physics, that is, where we define $\hbar = 1$, $c = 1$. In this system all the physical quantities can be expressed in units of the energy or powers of the energy.

a) Express 1 s, 1 Kg and 1 m in MeV.

b) Write you weight, height and age in MeV.

1.2 The lifetime τ of an unstable particle is defined as the time needed for the initial number of particles to reduced to $1/e$ of its value, that is,

$$N(t) = N_0 \, e^{-\frac{t}{\tau}}$$

where N_0 is the number of particles at $t = 0$. Knowing that the charged pions have, in their rest frame, $\tau_\pi = 2.6 \times 10^{-8}$ s and $m_\pi = 140$ MeV evaluate:

a) The γ factor for a beam of 200 GeV pions.

b) The lifetime in the laboratory frame.

c) The percentage of pions that have decayed after travelling 300 m in the laboratory. If there was no time dilation, what would have been the percentage?

1.3 Consider the decay $\pi^- \to \mu^- + \overline{\nu}$, where $m_\pi = 139.6$ MeV, $m_\mu = 105.7$ MeV and $m_\nu = 0$. Determine:

a) The linear momenta of the μ^- and of the $\overline{\nu}$ in the center of mass frame, that is, where the π^- is at rest.

b) The linear momenta of the μ^- and of the $\overline{\nu}$ in the laboratory frame, assuming that the $\overline{\nu}$ is emitted in the same direction of the π^-.

c) Repeat b) assuming now that it was the μ^- that was emitted in the direction of the π^-.

1.4 A photon can be described as a particle of zero mass and 4-momenta $k^\alpha = (\omega, \vec{k})$ where $\omega = 2\pi\nu = 2\pi/\lambda$ and $|\vec{k}| = \omega$ ($\hbar = c = 1$). If a photon collides with an electron with mass m_e at rest, it will be scattered with an angle θ and with energy ω' (Compton scattering). Show that

$$\lambda' - \lambda = 2\lambda_c \sin^2 \frac{\theta}{2} \qquad \text{onde} \qquad \lambda_c = \frac{2\pi}{m}$$

1.5 Consider the electromagnetic field tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. From this we can define the *dual* tensor

$$\mathcal{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \ .$$

a) Show that Maxwell equations with sources (Gauss's and Ampère's Laws) can be written

$$\partial_\mu F^{\mu\nu} = J^\nu$$

b) Show that we have

$$\partial_\mu \mathcal{F}^{\mu\nu} = 0$$

Verify that this equation contains the so-called homogeneous Maxwell equations, $\vec{\nabla} \cdot \vec{B} = 0$, and $\vec{\nabla} \times \vec{E} = -\partial \vec{B}/\partial t$. Verify that the above relation is equivalent to the more usual form (Bianchi identity)

$$\partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0$$

c) Express the invariants $F_{\mu\nu}F^{\mu\nu}$, $F_{\mu\nu}\mathcal{F}^{\mu\nu}$ and $\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$ in terms of the fields $\vec{E}$ and $\vec{B}$.

d) Show that if $\vec{E}$ and $\vec{B}$ are orthogonal in a reference frame they will remain orthogonal in all reference frames

e) Consider a reference frame where $\vec{E} \neq 0$ and $\vec{B} = 0$. Can we find a reference frame where $\vec{E} = 0$ e $\vec{B} \neq 0$? Justify your answer.

1.6 Use the relations

$$a^\mu{}_\alpha g_{\mu\nu} a^\nu{}_\beta = g_{\alpha\beta} \qquad \text{or in matrix form} \qquad a^T g \, a = g$$

to show that for infinitesimal transformations

$$a^\nu{}_\mu = g^\nu{}_\mu + \omega^\nu{}_\mu + \cdots$$

we have

$$\omega^{\mu\nu} = -\omega^{\nu\mu}$$

1.7 Use the explicit expressions

$$\begin{aligned} S_R &= \cos\frac{\theta}{2} + i\hat{\theta}\cdot\vec{\Sigma}\sin\frac{\theta}{2} \\ S_L &= \cosh\frac{\omega}{2} - \hat{\omega}\cdot\vec{\alpha}\sinh\frac{\omega}{2} \end{aligned}$$

to verify that for finite transformations we also have

$$S^{-1}\gamma^\mu S = a^\mu{}_\nu\gamma^\nu$$

1.8 Show the following relations

$$(\Gamma^a)^2 = \pm 1$$

$$\mathrm{Tr}\,(\Gamma^a) = 0\ ,\forall a \neq s$$

$$\gamma^\mu\gamma_\mu = 4\ ;\ \gamma^\mu\gamma^\nu\gamma_\mu = -2\gamma^\nu\ ;\ \gamma^\mu\gamma^\nu\gamma^\rho\gamma_\mu = 4g^{\nu\rho}$$

$$\gamma^\mu\gamma^\nu\gamma^\rho = g^{\mu\nu}\gamma^\rho - g^{\mu\rho}\gamma^\nu + g^{\nu\rho}\gamma^\mu + i\varepsilon^{\mu\nu\rho\alpha}\ \gamma_\alpha\gamma_5$$

1.9 Show that the spinors $w^r(\vec{p})$ obey the relations

$$(\not{p} - \varepsilon_r m)w^r(\vec{p}) = 0\ ;\ \overline{w}^r(\vec{p})\ (\not{p} - \varepsilon_r m) = 0$$

$$\overline{w}^r(\vec{p})w^{r'}(\vec{p}) = 2m\ \delta_{rr'}\varepsilon_r$$

$$\sum_{r=1}^{4}\varepsilon_r w^r_\alpha(\vec{p})\overline{w}^r_\beta(\vec{p}) = 2m\ \delta_{\alpha\beta}$$

$$w^{r\dagger}(\varepsilon_r\vec{p})w^{r'}(\varepsilon_{r'}\vec{p}) = 2E\ \delta_{rr'}$$

1.10 Show that for the Dirac equation the eigenvalue of W^2 is

$$W^2 = -\frac{3}{4}\ m^2$$

1.11 Show that

$$(\vec{\sigma}\cdot\vec{\pi})(\vec{\sigma}\cdot\vec{\pi}) = \vec{\pi}\cdot\vec{\pi} - e\vec{\sigma}\cdot\vec{B}$$

where

$$\left\{ \begin{aligned} \vec{\pi} &= -i\vec{\nabla} - e\vec{A} \\ \vec{B} &= \vec{\nabla}\times\vec{A} \end{aligned} \right.$$

1.12 Verify the following relations

$$\overline{u}(p,s)u(p,s') = 2m\ \delta_{ss'}$$

$$\overline{v}(p,s)v(p,s') = -2m\ \delta_{ss'}$$

$$u^\dagger(p,s)u(p,s') = 2E_p\ \delta_{ss'}$$

$$v^\dagger(p,s)v(p,s') = 2E_p\ \delta_{ss'}$$

$$\overline{v}(p,s)u(p,s') = 0$$

$$v^\dagger(p,s)u(-p,s') = 0$$

$$\sum_s [u_\alpha(p,s)\overline{u}_\beta(p,s)] = (\not{p} + m)_{\alpha\beta}$$

$$\sum_s [v_\alpha(p,s)\overline{v}_\beta(p,s)] = -(-\not{p} + m)_{\alpha\beta}$$

$$\sum_s [u_\alpha(p,s)\overline{u}_\beta(p,s) - v_\alpha(p,s)\overline{v}_\beta(p,s)] = 2m\ \delta_{\alpha\beta}$$

2 Problems Field Theory: Lecture 2

2.1 Derive the following results

1. The trace of an odd number of γ matrices is zero.
2. The following recurrence form holds for n even.

$$\begin{aligned}\mathrm{Tr}\left[\not{a}_1 \cdots \not{a}_n\right] =& a_1 \cdot a_2 \ \mathrm{Tr}\left[\not{a}_3 \cdots \not{a}_n\right] - a_1 \cdot a_3 \ \mathrm{Tr}\left[\not{a}_2 \not{a}_4 \cdots \not{a}_n\right] \\ &+ a_1 \cdot a_n \ \mathrm{Tr}\left[\not{a}_2 \cdots \not{a}_{n-1}\right]\end{aligned}$$

3. Evaluate the trace of 4 γ matrices

$$\begin{aligned}\mathrm{Tr}\left[\not{a}_1 \not{a}_2 \not{a}_3 \not{a}_4\right] =& a_1 \cdot a_2 \ \mathrm{Tr}\left[\not{a}_3 \not{a}_4\right] - a_1 \cdot a_3 \ \mathrm{Tr}\left[\not{a}_2 \not{a}_4\right] + a_1 \cdot a_4 \ \mathrm{Tr}\left[\not{a}_2 \not{a}_3\right] \\ =& 4\left[a_1 \cdot a_2 \ a_3 \cdot a_4 - a_1 \cdot a_3 \ a_2 \cdot a_4 + a_1 \cdot a_4 \ a_2 \cdot a_3\right]\end{aligned}$$

4. Derive the following results

$$\begin{aligned}&\mathrm{Tr}\left[\gamma_5\right] = 0 \\ &\mathrm{Tr}\left[\gamma_5 \not{a} \not{b}\right] = 0 \\ &\mathrm{Tr}\left[\gamma_5 \not{a} \not{b} \not{c} \not{d}\right] = -4i\varepsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma\end{aligned}$$

5. Derive the following identities

$$\begin{aligned}&\gamma_\mu \gamma^\mu = 4 \\ &\gamma_\mu \not{a} \gamma^\mu = -2\not{a} \\ &\gamma_\mu \not{a} \not{b} \gamma^\mu = 4a.b \\ &\gamma_\mu \not{a} \not{b} \not{c} \gamma^\mu = -2\not{c} \not{b} \not{a} \\ &\gamma_\mu \not{a} \not{b} \not{c} \not{d} \gamma^\mu = 2\left[\not{d} \not{a} \not{b} \not{c} + \not{c} \not{b} \not{a} \not{d}\right]\end{aligned}$$

2.2 Consider the matrix Γ defined by

$$\Gamma = \gamma^\mu (g_V - g_A \gamma_5) \tag{1}$$

where g_V and g_A are constants. Show that

$$\overline{\Gamma} = \Gamma \tag{2}$$

where $\overline{\Gamma} = \gamma^0 \Gamma^\dagger \gamma^0$.

2.3 Consider the decay of an unstable particle of mass M and 4-momentum P in n fragments ($n \geq 2$) with 4-momenta q_i. Show that the expression for the decay width, defined as the rate of transition per unit time, per unit volume and per unit particle that decays is given by

$$d\Gamma = \frac{1}{2M} \overline{|M_{fi}|^2} \, (2\pi)^4 \delta^4 \left(P - \sum_i^n q_i \right) \prod_i^n \frac{d^3 q_i}{(2\pi)^3 2 q_i^0} \tag{3}$$

2.4 Write the relation between the lifetime expressed in seconds, $\tau(seg)$, and the decay with expressed in MeV, $\Gamma(MeV)$.

2.5 Show that the differential cross section for the process $p_1 + p_2 \to p_3 + p_4$ can be written in the center of mass frame as

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{|\vec{p}_{3cm}|}{|\vec{p}_{1cm}|} \overline{|M_{fi}|^2} \tag{4}$$

where $|\vec{p}_{1cm}|$ and $|\vec{p}_{3cm}|$ are the momenta of particles 1 and 3 in the center of mass frame. Consider then the particular case when the incident particles are massless.

2.6 Show that for the decay $P \to q_1 + q_2$, the width, in the rest frame of the decaying particle, can be written as

$$\frac{d\Gamma}{d\Omega} = \frac{1}{32\pi^2} \frac{|\vec{q}_{1cm}|}{M^2} \overline{|M_{fi}|^2} \tag{5}$$

where $P^2 = M^2$.

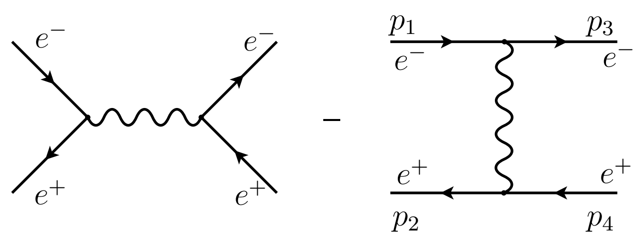
2.7 Consider in QED the process $\gamma\gamma \to e^+ e^-$.

a) Write the amplitude $M = \epsilon^\mu(k_1)\epsilon^\nu(k_2) M_{\mu\nu}$ for the process, where k_1, k_2 are the 4-momenta of the photons.

b) Show that the amplitude is gauge invariant, that is

$$k_1^\mu M_{\mu\nu} = k_2^\nu M_{\mu\nu} = 0$$

2.8 Consider the process $e^-e^+ \to e^-e^+$, known as *Bhabha scattering*. In QED there are two diagrams contributing to the process



and there is a relative minus sign between them. Show that in the high energy limit, where $\sqrt{s} \gg m$, and $\sqrt{s}$ is the total center of mass energy, we get

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left[\frac{1+\cos^4(\theta/2)}{\sin^4(\theta/2)} - \frac{2\cos^4(\theta/2)}{\sin^2(\theta/2)} + \frac{1+\cos^2\theta}{2}\right] \tag{6}$$

where θ is the electron scattering angle in the center of mass frame.

3 Problems Field Theory: Lecture 3

3.1 Show that the Lagrangian

$$\mathcal{L}_{YM} = -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

is invariant under the transformations

$$\delta A^a_\mu = -f^{bca}\ \varepsilon^b\ A^c_\mu - \frac{1}{g}\ \partial_\mu \varepsilon^a$$

3.2 Show that

$$P^{\mu\nu}(k) \equiv \sum_\lambda \varepsilon^\mu(k,\lambda)\varepsilon^\nu(k,\lambda) = -g^{\mu\nu} + \frac{k^\mu \eta^\nu + k^\nu \eta^\mu}{k \cdot \eta}$$

where $k^\mu, \varepsilon^\nu(k,1), \varepsilon^\rho(k,2)$ are η^σ four independent 4-vectors satisfying

$$\begin{aligned}
&\eta \cdot \varepsilon(k,\sigma) = 0 \qquad \sigma = 1,2 \\
&\varepsilon(k,1) \cdot \varepsilon(k,2) = 0 \\
&k \cdot \varepsilon(k,\sigma) = 0 \qquad \sigma = 1,2 \\
&k^2 = 0 \\
&\eta^2 = 0 \qquad \text{(convenient choice)} \\
&\varepsilon^2(k,\sigma) = -1 \qquad \sigma = 1,2
\end{aligned} \tag{7}$$

Hint: The most general expression for $P^{\mu\nu}$ is

$$P^{\mu\nu} = a g^{\mu\nu} + b k^\mu k^\nu + c \eta^\mu \eta^\nu + d(k^\mu \eta^\nu + k^\nu \eta^\mu)\ .$$

Use the above relations to determine a, b, c, d.

3.3 Show that the Yang-Mills tensor $F^a_{\mu\nu}$ satisfies the Bianchi identities

$$D_\mu^{ab} F^b_{\rho\sigma} + D_\rho^{ab} F^b_{\sigma\mu} + D_\sigma^{ab} F^b_{\mu\rho} = 0$$

or

$$D_\mu^{ab\,*} F^{\mu\nu\ b} = 0$$

where

$${}^*F^{\mu\nu\ a} = \frac{1}{2}\ \varepsilon^{\mu\nu\rho\sigma} F^a_{\rho\sigma}$$

3.4 Consider the pure Yang-Mills theory. Show that the field equations can be written as

$$\begin{cases} \vec{\nabla}\cdot\vec{E}^a = \rho^a \\ \vec{\nabla}\cdot\vec{B}^a = {}^*\rho^a \\ \vec{\nabla}\times\vec{E}^a = -\dfrac{\partial\vec{B}^a}{\partial t} + \vec{J}^a \\ \vec{\nabla}\times\vec{B}^a = -\dfrac{\partial\vec{E}^a}{\partial t} + {}^*\vec{J}^a \end{cases}$$

and evaluate ρ^a, ${}^*\rho^a$, $\vec{J}^a$ and ${}^*\vec{J}^a$.

3.5 Consider the Wu-Yang *Ansatze* for static solutions of pure SU(2) Yang-Mills,

$$A^{0a} = x^a\ \frac{G(r)}{r^2} \qquad\qquad A^{ia} = \varepsilon^{aij}\ x^j\frac{F(r)}{r^2}$$

a) Derive the equations of motion for F and G
b) Show that they are satisfied for $F = -1/g$ and G = constant. Show that these solutions correspond to $\rho^a = {}^*\rho^a = 0$ and $\vec{J}^a = {}^*\vec{J}^a = 0$ where $\rho^a \ldots$ are define in problem 3.4.

4 Problems Standard Model: Lecture 4

4.1 Consider the two decays of the Z^0

$$Z^0 \to \nu\overline{\nu}, \quad Z^0 \to e^- e^+ \ .$$

Show that

$$\frac{\Gamma(Z^0 \to \nu\overline{\nu})}{\Gamma(Z^0 \to e^- e^)} \simeq 2 \ .$$

4.2 Evaluate the trace

$$\begin{aligned} T_1 =& \text{Tr}\left[(\not{q}_1 + m_f)\gamma_\mu \left(g_V^f - g_A^f\gamma_5\right)(\not{q}_2 - m_f)\gamma_\nu \left(g_V^f - g_A^f\gamma_5\right)\right] \\ =& 4\left[\left(g_V^{f\,2} + g_A^{f\,2}\right)(q_{1\mu}q_{2\nu} + q_{1\nu}q_{2\mu} - g_{\mu\nu}\ q_1 \cdot q_2) - g_{\mu\nu}\ m_f^2 \left(g_V^{f\,2} - g_A^{f\,2}\right)\right. \\ & \left. -2i\epsilon^{\alpha\beta}{}_{\mu\nu}q_{1\alpha}q_{2\beta}\ g_V^f g_A^f\right] \end{aligned}$$

4.3 Neglecting the fermions masses show that

$$BR(Z^0 \to e^-\ e^+) \equiv \frac{\Gamma(Z^0 \to e^-\ e^+)}{\Gamma_Z} \simeq 3.4\%$$

where $\Gamma_Z \equiv \Gamma(Z^0 \to \text{all})$.

4.4 Consider the process $e^+e^- \to \nu_e\overline{\nu}_e$.

a) What are the diagrams that contribute?

b) Write the amplitude corresponding to the dominant diagram for $\sqrt{s} \simeq M_z$.

c) Show that for $\sqrt{s} \simeq M_Z$ we have

$$\frac{\sigma(e^+e^- \to \nu_e\overline{\nu}_e)}{\sigma(e^+e^- \to e^+e^-)} \simeq 2$$

4.5 Consider the decay $W^- \to e^-\ \overline{\nu}_e$.

a) Calculate the speed of the electron in the frame where the W is at rest.

b) Write the amplitude for the process.

c) Neglecting the electron mass calculate the decay width.

4.6 Evaluate the **Branching Ratio**, $BR(W^- \to e^- \nu)$, defined by

$$BR(W^- \to e^- \nu) \equiv \frac{\Gamma(W^- \to e^- \nu)}{\Gamma(W^- \to \text{all})}$$

where $\Gamma(W^- \to \text{all}) = \Gamma_W \simeq 2.0$ GeV.

4.7 Consider the process $Z^0 \to e^- e^+ \gamma$.

a) Draw the diagrams in lowest order.

b) Write the amplitude and verify gauge invariance, that is, if

$$\mathcal{M} = \varepsilon^\mu(k) V_\mu$$

then

$$k^\mu V_\mu = 0$$

where k^μ is the 4-momentum of the photon.

4.8 When we neglect the lepton masses and consider that the energy in the `CM`, $\sqrt{s}$, is much less than the W and Z masses, then the cross section for the processes in the table

Process	λ_i
$\nu_\mu + e^- \to \mu^- + \nu_e$	1
$\overline{\nu}_e + e^- \to \mu^- + \overline{\nu}_\mu$	$\frac{1}{3}$
$\nu_\mu + e^- \to \nu_\mu + e^-$	$\sigma = \frac{32}{3} \left[\left({g_V^\nu}^2 + {g_A^\nu}^2\right)\left({g_V^e}^2 + {g_A^e}^2\right) + 2 g_V^\nu g_A^\nu g_V^e g_A^e\right]$
$\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-$	
$\mu^- + e^+ \to \nu_\mu + \overline{\nu}_e$	
$\nu_e + e^- \to \nu_e + e^-$	

can be written as

$$\sigma_i = \frac{\lambda_i}{\pi} G_F^2 \, s$$

a) Show this and fill the entries

b) Show that

$$\frac{\sigma(\nu_\mu + e^- \to \nu_\mu + e^-)}{\sigma(\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-)} = \frac{3L_e^2 + R_e^2}{L_e^2 + 3R_e^2}$$

where

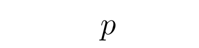
$$L_e = g_V^e + g_A^e, \qquad R_e = g_V^e - g_A^e$$

c) Define $R(x) = \sigma(\nu_\mu e^- \to \nu_\mu e^-)/\sigma(\overline{\nu}_\mu + e^- \to \overline{\nu}_\mu + e^-)$ where $x = \sin^2\theta_W$. Verify that $R(0.25) = 1$.

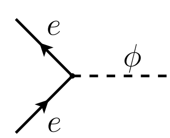
4.9 Consider the process $e^+ + e^- \to \phi + \gamma$ in the theory described by the following Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm QED} + \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m_\phi^2\phi^2 - \beta\,\overline{\psi}\gamma_5\psi\,\phi$$

where ϕ is a neutral spin 0 scalar field and ψ is the electron Besides the propagators and vertex of QED we have



$$\frac{i}{p^2 - m_\phi^2}$$



$$-i\beta\,\gamma_5$$

a) Draw the diagram(s) that contribute in lowest order to the process.

b) Write the amplitude for the process.

c) Show that the amplitude is gauge invariant, that is if $\mathcal{M} \equiv \epsilon^\mu(k)\,\mathcal{M}_\mu$ where k is the 4-momentum of the photon, then we have $k^\mu\mathcal{M}_\mu = 0$.

4.10 Consider the process $\phi \to e^+ + e^-$ in the theory described in problem 4.9.

a) Write the amplitude for the process

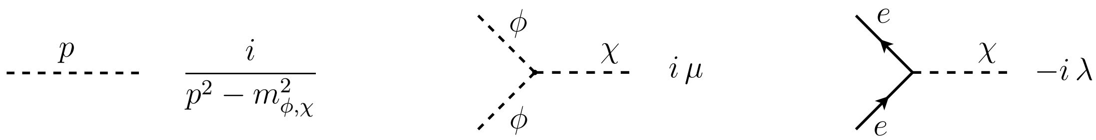
b) Evaluate the decay width $\Gamma(\phi \to e^+ + e^-)$ as a function of the parameters of the model.

c) Assume that you measure $m_\phi = 1.8$ GeV and a lifetime $\tau_\phi = 8.5 \times 10^{-23}$ s. What is the value of β? ($m_e = 0.511$ MeV)

4.11 Consider the process $e^+ + e^- \to \phi + \phi + \gamma$ in the theory described by the following Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{QED}} + \frac{1}{2}\partial_\mu \chi \, \partial^\mu \chi + \frac{1}{2}\partial_\mu \phi \, \partial^\mu \phi \; - \frac{1}{2} m_\chi^2 \, \chi^2 - \frac{1}{2} m_\phi^2 \, \phi^2 + \frac{\mu}{2} \, \phi^2 \, \chi - \lambda \, \overline{\psi} \psi \, \chi$$

where χ and ϕ are real neutral scalar fields (spin 0) and ψ is the electron. The constant μ has the dimension of a mass (in the system $\hbar = c = 1$). The new propagators and vertices are



a) Draw the diagram(s) that contribute to the process in lowest order.

b) Write the amplitude for the process.

c) Show that the amplitude is gauge invariant, that is if $\mathcal{M} \equiv \epsilon^\mu(k) \, \mathcal{M}_\mu$ where k is the 4-momentum of the photon, then we have $k^\mu \mathcal{M}_\mu = 0$.

4.12 Consider the decay $\chi \to e^+ + e^-$ in the model described in problem 4.11.

a) Write the amplitude in lowest order.

b) Evaluate the decay width $\Gamma(\chi \to e^+ + e^-)$.

c) Assume that you measure $m_\chi = 1.8$ GeV and a lifetime $\tau_\chi = 1.3 \times 10^{-25}$ s. What is the value of λ? ($m_e = 0.511$ MeV)

4.13 Consider the decay of the top quark, $t \to b + W^+$, in the Standard Model. In this problem neglect the mass of the bottom quark b.

a) Write the amplitude for the process

b) What is the speed of the W in the rest frame of the top.

c) Evaluate the decay width $\Gamma(t \to b + W^+)$ as a function of the model parameters.

d) Knowing that the polarization vector of the W^+ in the frame where it moves with velocity $\vec{\beta}$ is $\varepsilon_L^\mu = (\gamma\beta, \gamma\vec{\beta}/\beta)$, show that the fraction of the decays where the W^+ is polarized longitudinally is

$$F_L = \frac{m_t^2}{m_t^2 + 2M_W^2}$$