The Minimal Supersymmetric Standard Model

Version 1.9.175

June 24, 2011

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Abstract

We write down the mass matrices and couplings needed for Minimal Supersymmetric Standard Model (MSSM) calculations. These include the mass matrices for **all** the particles in the model, the charged and neutral current couplings of the spin $\frac{1}{2}$ particles with the gauge bosons, and the couplings of the charged and neutral scalars with the spin $\frac{1}{2}$ particles. To fix our notation, we have also included the couplings of the gauge bosons with themselves, although these are the same as in the SM.

Contents

1 Introduction and Motivation

In recent years it has been established [1] with great precision (in some cases better than 0.1%) that the interactions of the gauge bosons with the fermions are described by the Standard Model (SM) [2]. However other sectors of the SM have been tested to a much lesser degree. In fact only now we are beginning to probe the self–interactions of the gauge bosons through their pair production at the Tevatron [3] and LEP [4] and the Higgs sector, responsible for the symmetry breaking has not yet been tested.
Despite all its successes, the SM still has many unanswered questions. Among the various candidates to Physics Beyond the Standard Model, supersymmetric theories play a special role. Although there is not yet direct experimental evidence for supersymmetry (SUSY), there are many theoretical arguments indicating that SUSY might be of relevance for physics below the 1 TeV scale.

The most commonly invoked theoretical arguments for SUSY are:

i. Interrelates matter fields (leptons and quarks) with force fields (gauge and/or Higgs bosons).

ii. As local SUSY implies gravity (supergravity)it could provide a way to unify gravity with the other interactions.

iii. As SUSY and supergravity have fewer divergences than conventional filed theories, the hope is that it could provide a consistent (finite) quantum gravity theory.

iv. SUSY can help to understand the mass problem, in particular solve the naturalness problem (and in some models even the hierarchy problem) if SUSY particles have masses $\leq \mathcal{O}(1\text{TeV})$.

As it is the last argument that makes SUSY particularly attractive for the experiments being done or proposed for the next decade, let us explain the idea in more detail. As the SM is not asymptotically free, at some energy scale Λ, the interactions must become strong indicating the existence of new physics. Candidates for this scale are, for instance, $M_X \simeq \mathcal{O}(10^{16}$ GeV) in GUT's or more fundamentally the Planck scale $M_P \simeq \mathcal{O}(10^{19}\text{GeV})$. This alone does not indicate that the new physics should be related to SUSY, but the so–called mass problem does. The only consistent way to give masses to the gauge bosons and fermions is through the Higgs mechanism involving at least one spin zero Higgs boson. Although the Higgs boson mass is not fixed by the theory, a value much bigger than $< H^0 >\simeq G_F^{-1/2} \simeq 250$ GeV would imply that the Higgs sector would be strongly coupled making it difficult to understand why we are seeing an apparently successful perturbation theory at low energies. Now the one loop radiative corrections to the Higgs boson mass would give

$$\delta m_H^2 = \mathcal{O}\left(\frac{\alpha}{4\pi}\right)\ \Lambda^2 \tag{1}$$

which would be too large if Λ is identified with Λ_{GUT} or Λ_{Planck}. SUSY cures this problem in the following way. If SUSY were exact, radiative corrections to the scalar masses squared would be absent because the contribution of fermion loops exactly cancels against the boson loops. Therefore if SUSY is broken, as it must, we should have

$$\delta m_H^2 = \mathcal{O}\left(\frac{\alpha}{4\pi}\right)\ |m_B^2 - m_F^2| \tag{2}$$

We conclude that SUSY provides a solution for the the naturalness problem if the masses of the superpartners are below $\mathcal{O}(1\ \text{TeV})$. This is the main reason behind all the phenomenological interest in SUSY.
In the following we will give a brief review of the main aspects of the SUSY extension of the SM, the so–called Minimal Supersymmetric Standard Model (MSSM). Almost all the material is covered in many excellent reviews that exist in the literature [5, 6].

2 SUSY Algebra, Representations and Particle Content

2.1 SUSY Algebra

The SUSY generators obey the following algebra

$$\{Q_\alpha, Q_\beta\} = 0 \tag{3}$$

$$\{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} = 0 \tag{4}$$

$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2\,(\sigma^\mu)_{\alpha\dot\beta}\ P_\mu \tag{5}$$

where

$$\sigma^\mu \equiv (1, \sigma^i) \quad ; \quad \overline{\sigma}^\mu \equiv (1 - \sigma^i) \tag{6}$$

and $\alpha, \beta, \dot\alpha, \dot\beta = 1, 2$ (Weyl 2–component spinor notation). The commutation relations with the generators of the Poincaré group are

$$\begin{aligned} [P^\mu, Q_\alpha] &= 0 \\ [M^{\mu\nu}, Q_\alpha] &= -i\,(\sigma^{\mu\nu})_\alpha{}^\beta\, Q_\beta \end{aligned}$$

From these relations one can easily derive that the two invariants of the Poincaré group,

$$\begin{aligned} P^2 &= P_\alpha P^\alpha \\ W^2 &= W_\alpha W^\alpha \end{aligned} \tag{7}$$

where W^μ is the Pauli–Lubanski vector operator

$$W_\mu = -\frac{i}{2}\epsilon_{\mu\nu\rho\sigma}M^{\nu\rho}P^\sigma \tag{8}$$

are no longer invariants of the Super Poincaré group. In fact

$$\begin{aligned}[Q_\alpha, P^2] &= 0 \\ [Q_\alpha, W^2] &\neq 0 \end{aligned} \tag{9}$$

showing that the irreducible multiplets will have particles of the same mass but different spin.

2.2 Simple Results from the Algebra

From the supersymmetric algebra one can derive two important results:

A. Number of Bosons = Number of Fermions

We have

$$\begin{aligned} Q_\alpha|B>=|F> \quad &; \quad (-1)^{N_F}|B>=|B> \\ Q_\alpha|F>=|B> \quad &; \quad (-1)^{N_F}|F>=-|F> \end{aligned} \tag{10}$$

where $(-1)^{N_F}$ is the fermion number of a given state. Then we obtain

$$Q_\alpha(-1)^{N_F} = -(-1)^{N_F}Q_\alpha \tag{11}$$

Using this relation we can show that

$$\begin{aligned} Tr\left[(-1)^{N_F}\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\}\right] &= Tr\left[(-1)^{N_F}Q_\alpha\overline{Q}_{\dot\alpha} + (-1)^{N_F}\overline{Q}_{\dot\alpha}Q_\alpha\right] \\ &= Tr\left[-Q_\alpha(-1)^{N_F}\overline{Q}_{\dot\alpha} + Q_\alpha(-1)^{N_F}\overline{Q}_{\dot\alpha}\right] \\ &= 0 \end{aligned}$$

But using Eq. 5 we also have

$$Tr\left[(-1)^{N_F}\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\}\right] = Tr\left[(-1)^{N_F}2\sigma^\mu_{\alpha\dot\alpha}P_\mu\right] \tag{12}$$

This in turn implies

$$\boxed{Tr(-1)_{N_F} = \#\text{Bosons} - \#\text{Fermions} = 0}$$

showing that in a given representation the number of degrees of freedom of the bosons equals those of the fermions.

B. $\langle 0|H|0\rangle \geq 0$

From the algebra we get

$$\begin{aligned}\left\{Q_1,\overline{Q}_{\dot{1}}\right\}+\left\{Q_2,\overline{Q}_{\dot{2}}\right\} &= 2Tr\left(\sigma^{\mu}\right)P_{\mu}\\ &= 4H\end{aligned} \tag{13}$$

Then

$$H=\frac{1}{4}\left(Q_1\overline{Q}_{\dot{1}}+Q_2\overline{Q}_{\dot{2}}+\overline{Q}_{\dot{1}}Q_1+\overline{Q}_{\dot{2}}Q_2\right) \tag{14}$$

and

$$\begin{aligned}\langle 0|H|0\rangle &= \left(||Q_1\left|0\right\rangle||^2+||Q_1\left|0\right\rangle||^2+||\overline{Q}_{\dot{1}}\left|0\right\rangle||^2+||\overline{Q}_{\dot{2}}\left|0\right\rangle||^2\right)\\ &\geq 0\end{aligned} \tag{15}$$

showing that the energy of the vacuum state is always positive definite.

2.3 SUSY Representations

We consider separately the massive and the massless case.

A. Massive case

In the rest frame

$$\left\{Q_{\alpha},\overline{Q}_{\dot{\alpha}}\right\}=2\,m\;\delta_{\alpha\dot{\alpha}} \tag{16}$$

This algebra is similar to the algebra of the spin 1/2 creation and annihilation operators. Choose $\left|\Omega\right\rangle$ such that

$$Q_1\left|\Omega\right\rangle=Q_2\left|\Omega\right\rangle=0 \tag{17}$$

Then we have 4 states

$$\left|\Omega\right\rangle\ ;\ \overline{Q}_1\left|\Omega\right\rangle\ ;\ \overline{Q}_2\left|\Omega\right\rangle\ ;\ \overline{Q}_1\overline{Q}_2\left|\Omega\right\rangle \tag{18}$$

If $J_3\left|\Omega\right\rangle=j_3\left|\Omega\right\rangle$ we show in Table 1 the values of J_3 for the 4 states. We notice

State	J_3 Eigenvalue
$\left\|\Omega\right\rangle$	j_3
$\overline{Q}_1\left\|\Omega\right\rangle$	$j_3+\frac{1}{2}$
$\overline{Q}_2\left\|\Omega\right\rangle$	$j_3-\frac{1}{2}$
$\overline{Q}_1\overline{Q}_2\left\|\Omega\right\rangle$	j_3

Table 1: Massive states

that there two bosons and two fermions and that the states are separated by one half unit of spin.

B. Massless case

If $m = 0$ then we can choose $P^\mu = (E, 0, 0, E)$. In this frame

$$\left\{Q_\alpha, \overline{Q}_{\dot\alpha}\right\} = M_{\alpha\dot\alpha} \tag{19}$$

where the matrix M takes the form

$$M = \begin{pmatrix} 0 & 0 \\ 0 & 4E \end{pmatrix} \tag{20}$$

Then

$$\left\{Q_2, \overline{Q}_2\right\} = 4E \tag{21}$$

all others vanish. We have then just **two** states

$$|\Omega\rangle \;\; ; \;\; \overline{Q}_2 \, |\Omega\rangle \tag{22}$$

If $J_3 \, |\Omega\rangle = \lambda \, |\Omega\rangle$ we have the states shown in Table 2,

State	J_3 Eigenvalue
$\lvert\Omega\rangle$	λ
$\overline{Q}_2 \, \lvert\Omega\rangle$	$\lambda - \frac{1}{2}$

Table 2: Massless states

3 How to Build a SUSY Model

To construct supersymmetric Lagrangians one normally uses superfield methods (see for instance [5, 6]). Here we do not go into the details of that construction. We will take a more pragmatic view and give the results in the form of a *recipe*. To simplify matters even further we just consider one gauge group G. Then the gauge bosons W^a_μ are in the adjoint representation of G and are described by the massless gauge supermultiplet

$$V^a \equiv (\lambda^a, W^a_\mu) \tag{23}$$

where λ^a are the superpartners of the gauge bosons, the so–called *gauginos*. We also consider only one matter chiral superfield

$$\Phi_i \equiv (A_i, \psi_i) \qquad ; \qquad (i = 1, \ldots, N) \tag{24}$$

belonging to some N dimensional representation of G. We will give the rules for the different parts of the Lagrangian for these superfields. The generalization to the case where we have more complicated gauge groups and more matter supermultiplets, like in the MSSM, is straightforward.

3.1 Kinetic Terms

Like in any gauge theory we have

$$\mathcal{L}_{kin} = -\tfrac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \frac{i}{2}\, \overline{\lambda^a} \gamma^\mu D_\mu \lambda^a + (D_\mu A)^\dagger\, D^\mu A + i\overline{\psi}\gamma^\mu D_\mu P_L \psi \tag{25}$$

where the field strength $F^a_{\mu\nu}$ is given by

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g f^{abc}\, W^b_\mu W^c_\nu \tag{26}$$

and f^{abc} are the structure constants of the gauge group G. The covariant derivative is

$$D_\mu = \partial_\mu + igW^a_\mu T^a \tag{27}$$

In Eq. 25 one should note that ψ is left handed and that λ is a Majorana spinor.

3.2 Self Interactions of the Gauge Multiplet

For a non Abelian gauge group G we have the usual self–interactions (cubic and quartic) of the gauge bosons with themselves. These are well known and we do write them here again. But we have a new interaction of the gauge bosons with the gauginos. In two component spinor notation it reads [5, 6]

$$\mathcal{L}_{\lambda\lambda W} = ig f_{abc}\, \lambda^a \sigma^\mu \overline{\lambda}^b\, W^c_\mu + h.c. \tag{28}$$

where f_{abc} are the structure constants of the gauge group G and the matrices σ^μ were introduced in Eq. 6.

3.3 Interactions of the Gauge and Matter Multiplets

In the usual non Abelian gauge theories we have the interactions of the gauge bosons with the fermions and scalars of the theory. In the supersymmetric case we also have interactions of the gauginos with the fermions and scalars of the chiral matter multiplet. The general form, in two component spinor notation, is [5, 6],

$$\begin{aligned} \mathcal{L}_{\Phi W} &= -gT^a_{ij} W^a_\mu \left(\overline{\psi}_i \overline{\sigma}^\mu \psi_j + iA^*_i \overleftrightarrow{\partial}_\mu A_j \right) + g^2 \left(T^a T^b \right)_{ij} W^a_\mu W^{\mu b}\, A^*_i A_j \\ &\quad + ig\sqrt{2}\, T^a_{ij} \left(\lambda^a \psi_j A^*_i - \overline{\lambda}^a \overline{\psi}_i A_j \right) \end{aligned} \tag{29}$$

where the new interactions of the gauginos with the fermions and scalars are given in the last term.

3.4 Self Interactions of the Matter Multiplet

These correspond in non supersymmetric gauge theories both to the Yukawa interactions and to the scalar potential. In supersymmetric gauge theories we have less freedom to construct these terms. The first step is to construct the superpotential W. This must be a gauge invariant polynomial function of the *scalar* components of the chiral multiplet Φ_i, that is A_i. It *does not* depend on A_i^*. In order to have renormalizable theories the degree of the polynomial must be at most three. This is in contrast with non supersymmetric gauge theories where we can construct the scalar potential with a polynomial up to the fourth degree.

Once we have the superpotential W, then the theory is defined and the Yukawa interactions are

$$\mathcal{L}_{Yukawa} = -\tfrac{1}{2}\left[\frac{\partial^2 W}{\partial A_i \partial A_j}\ \psi_i\psi_j + \left(\frac{\partial^2 W}{\partial A_i \partial A_j}\right)^*\ \overline{\psi}_i\overline{\psi}_j\right] \tag{30}$$

and the scalar potential is

$$V_{scalar} = \tfrac{1}{2} D^a D^a + F_i F_i^* \tag{31}$$

where

$$\begin{aligned} F_i &= \frac{\partial W}{\partial A_i} \\ D^a &= g\ A_i^* T_{ij}^a A_j \end{aligned} \tag{32}$$

We see easily from these equations that, if the polynomial degree of W were higher than three, then the scalar potential would be a polynomial of degree higher than four and hence non renormalizable.

3.5 Supersymmetry Breaking Lagrangian

As we have not discovered superpartners of the known particles with the same mass, we conclude that SUSY has to be broken. How this done is the least understood sector of the theory. In fact, as we shall see, the majority of the unknown parameters come from this sector. As we do not want to spoil the good features of SUSY, the form of these SUSY breaking terms has to obey some restrictions. It has been shown that the added terms can only be mass terms, or have the same form of the superpotential, with arbitrary coefficients. These are called *soft terms*. Therefore, for the model that we are considering, the general form would be[1]

$$\mathcal{L}_{SB} = m_1^2\, Re(A^2) + m_2^2\, Im(A^2) - m_3\left(\lambda^a\lambda^a + \overline{\lambda}^a\overline{\lambda}^a\right) + m_4\,(A^3 + h.c.) \tag{33}$$

[1] We do not consider a term linear in A because we are assuming that Φ, and hence A, are not gauge singlets.

where A^2 and A^3 are gauge invariant combinations of the scalar fields. From its form, we see that it only affects the scalar potential and the masses of the gauginos. The parameters m_i have the dimension of a mass and are in general arbitrary.

3.6 R–Parity

In many models there is a multiplicatively conserved quantum number the called *R–parity.* It is defined as

$$R = (-1)^{2J+3B+L} \tag{34}$$

With this definition it has the value +1 for the known particles and −1 for their superpartners. The MSSM it is a model where R–parity is conserved. The conservation of R–parity has three important consequences: *i)* SUSY particles are pair produced, *ii)* SUSY particles decay into SUSY particles and *iii)* The lightest SUSY particle is stable (LSP).

4 The Minimal Supersymmetric Standard Model

4.1 The Gauge Group and Particle Content

We want to describe the supersymmetric version of the SM. Therefore the gauge group is considered to be that of the SM, that is

$$G = SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \tag{35}$$

We will now describe the minimal particle content.

- **Gauge Fields**

  We want to have gauge fields for the gauge group $G = SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$. Therefore we will need three vector superfields (or vector supermultiplets) $\widehat{V}_i$ with the following components:

$$\begin{aligned} \widehat{V}_1 &\equiv (\lambda', W_1^{\mu}) && \rightarrow && U_Y(1) \\ \widehat{V}_2 &\equiv (\lambda^a, W_2^{\mu a}) && \rightarrow && SU_L(2) \quad , \quad a = 1,2,3 \\ \widehat{V}_3 &\equiv (\widetilde{g}^b, W_3^{\mu b}) && \rightarrow && SU_c(3) \quad , \quad b = 1,\ldots,8 \end{aligned} \tag{36}$$

  where W_i^{μ} are the gauge fields and λ', λ and $\widetilde{g}$ are the $U_Y(1)$ and $SU_L(2)$ gauginos and the gluino, respectively.

- **Leptons**

  The leptons are described by chiral supermultiplets. As each chiral multiplet only describes one helicity state, we will need two chiral multiplets for each charged lepton[2]. The multiplets are given in Table 3, where the $U_Y(1)$ hypercharge is defined

Supermultiplet	$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers
$\widehat{L}_i \equiv (\widetilde{L}, L)_i$	$(1, 2, -\frac{1}{2})$
$\widehat{R}_i \equiv (\widetilde{\ell}_R, \ell^c_L)_i$	$(1, 1, 1)$

Table 3: Lepton Supermultiplets

through $Q = T_3 + Y$. Notice that each helicity state corresponds to a complex scalar and that $\hat{L}_i$ is a doublet of $SU_L(2)$, that is

$$\widetilde{L}_i = \begin{pmatrix} \widetilde{\nu}_{Li} \\ \widetilde{\ell}_{Li} \end{pmatrix} \qquad ; \qquad L_i = \begin{pmatrix} \nu_{Li} \\ \ell_{Li} \end{pmatrix} \tag{37}$$

- **Quarks**

  The quark supermultiplets are given in Table 4. The supermultiplet $\widehat{Q}_i$ is also a doublet of $SU_L(2)$, that is

Supermultiplet	$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers
$\widehat{Q}_i \equiv (\widetilde{Q}, Q)_i$	$(3, 2, \frac{1}{6})$
$\widehat{D}_i \equiv (\widetilde{d}_R, d^c_L)_i$	$(3, 1, \frac{1}{3})$
$\widehat{U}_i \equiv (\widetilde{u}_R, u^c_L)_i$	$(3, 1, -\frac{2}{3})$

Table 4: Quark Supermultiplets

$$\widetilde{Q}_i = \begin{pmatrix} \widetilde{u}_{Li} \\ \widetilde{d}_{Li} \end{pmatrix} \qquad ; \qquad Q_i = \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix} \tag{38}$$

- **Higgs Bosons**

  Finally the Higgs sector. In the MSSM we need at least two Higgs doublets. This is in contrast with the SM where only one Higgs doublet is enough to give masses to all the particles. The reason can be explained in two ways. Either the need to cancel the anomalies, or the fact that, due to the analyticity of the superpotential, we have to have two Higgs doublets of opposite hypercharges to give masses to the up and down type of quarks. The two supermultiplets, with their quantum numbers, are given in Table 5.

[2]We will assume that the neutrinos do not have mass.

Supermultiplet	$SU_c(3) \otimes SU_L(2) \otimes U_Y(1)$ Quantum Numbers
$\widehat{H}_1 \equiv (H_1, \widetilde{H}_1)$	$(1, 2, -\frac{1}{2})$
$\widehat{H}_2 \equiv (H_2, \widetilde{H}_2)$	$(1, 2, +\frac{1}{2})$

Table 5: Higgs Supermultiplets

4.2 Field Strengths and Covariant Derivatives

For the derivation of the Feynman rules for the interactions it is important to fix the notation for the the field strenghts and covariant derivatives. Here we just write the $SU_L(2) \otimes U_Y(1)$ part. We have

$$\mathcal{L}_{Gauge} = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} - B_{\mu\nu} B^{\mu\nu} \tag{39}$$

where

$$F^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g \epsilon^{abc} W^b_\mu W^c_\nu \tag{40}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{41}$$

where $\epsilon^{123} = 1$. The covariant derivative reads

$$D_\mu = \partial_\mu + i\, g\, \frac{\tau^a}{2} W^a_\mu + i\, g'\, \frac{Y}{2} B_\mu \tag{42}$$

where our normalization for the hypercharge is

$$Q = T_3 + Y \tag{43}$$

After symmetry breaking (see below) we have

$$\begin{aligned} W^3_\mu &= \sin\theta_W A_\mu + \cos\theta_W Z_\mu \\ B^3_\mu &= \cos\theta_W A_\mu - \sin\theta_W Z_\mu \end{aligned} \tag{44}$$

with

$$e = g\ \sin\theta_W = g' \cos\theta_W; \quad ; \quad \frac{g'}{g} = \tan\theta_W \tag{45}$$

We can then write the covariant derivative in the more useful form

$$\begin{aligned} D_\mu &= \partial_\mu + i \left[\frac{g}{\sqrt{2}} \begin{pmatrix} 0 & W^+_\mu \\ 0 & 0 \end{pmatrix} + \frac{g}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ W^-_\mu & 0 \end{pmatrix} \right. \\ &\qquad \left. + \frac{g}{\cos\theta_W} \left(T_3 - \sin^2\theta_W Q \right)\ Z_\mu + e\, Q\, A_\mu \right] \end{aligned} \tag{46}$$

4.3 The Superpotential and SUSY Breaking Lagrangian

The MSSM Lagrangian is specified by the R–parity conserving superpotential W

$$W = \varepsilon_{ab}\left[h_U^{ij}\widehat{Q}_i^a\widehat{U}_j\widehat{H}_2^b + h_D^{ij}\widehat{Q}_i^b\widehat{D}_j\widehat{H}_1^a + h_E^{ij}\widehat{L}_i^b\widehat{R}_j\widehat{H}_1^a - \mu\widehat{H}_1^a\widehat{H}_2^b\right] \tag{47}$$

where $i, j = 1, 2, 3$ are generation indices, $a, b = 1, 2$ are $SU(2)$ indices, and ε is a completely antisymmetric 2×2 matrix, with $\varepsilon_{12} = 1$. The coupling matrices h_U, h_D and h_E will give rise to the usual Yukawa interactions needed to give masses to the leptons and quarks. If it were not for the need to break SUSY, the number of parameters involved would be less than in the SM. This can be seen in Table 6.

The most general SUSY soft breaking is

$$\begin{aligned}
-\mathcal{L}_{SB} = {} & M_Q^{ij2}\widetilde{Q}_i^{a*}\widetilde{Q}_j^a + M_U^{ij2}\widetilde{U}_i\widetilde{U}_j^* + M_D^{ij2}\widetilde{D}_i\widetilde{D}_j^* + M_L^{ij2}\widetilde{L}_i^{a*}\widetilde{L}_j^a + M_R^{ij2}\widetilde{R}_i\widetilde{R}_j^* + m_{H_1}^2 H_1^{a*}H_1^a \\
& + m_{H_2}^2 H_2^{a*}H_2^a - \left[\tfrac{1}{2}M_s\lambda_s\lambda_s + \tfrac{1}{2}M\lambda\lambda + \tfrac{1}{2}M'\lambda'\lambda' + h.c.\right] \\
& + \varepsilon_{ab}\left[A_U^{ij}h_U^{ij}\widetilde{Q}_i^a\widetilde{U}_j H_2^b + A_D^{ij}h_D^{ij}\widetilde{Q}_i^b\widetilde{D}_j H_1^a + A_E^{ij}h_E^{ij}\widetilde{L}_i^b\widetilde{R}_j H_1^a - B\mu H_1^a H_2^b\right]
\end{aligned} \tag{48}$$

Theory	Gauge Sector	Fermion Sector	Higgs Sector
SM	e, g, α_s	h_U, h_D, h_E	μ^2, λ
MSSM	e, g, α_s	h_U, h_D, h_E	μ
Broken MSSM	e, g, α_s	h_U, h_D, h_E	$\mu, M_1, M_2, M_3, A_U, A_D, A_E, B$ $m_{H_2}^2, m_{H_1}^2, m_Q^2, m_U^2, m_D^2, m_L^2, m_R^2$

Table 6: Comparative counting of parameters

4.4 Symmetry Breaking

The electroweak symmetry is broken when the two Higgs doublets H_1 and H_2 acquire VEVs

$$H_1 = \begin{pmatrix} \frac{1}{\sqrt{2}}[\sigma_1^0 + v_1 + i\varphi_1^0] \\ H_1^- \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}[\sigma_2^0 + v_2 + i\varphi_2^0] \end{pmatrix} \tag{49}$$

with $m_W^2 = \frac{1}{4}g^2v^2$ and $v^2 \equiv v_1^2 + v_2^2 = (246\text{ GeV})^2$. The full scalar potential at tree level is

$$V_{total} = \sum_i \left|\frac{\partial W}{\partial z_i}\right|^2 + V_D + V_{soft} \tag{50}$$

The scalar potential contains linear terms

$$V_{linear} = t_1^0\sigma_1^0 + t_2^0\sigma_2^0 \tag{51}$$

where

$$
\begin{aligned}
t_1 &= (m_{H_1}^2 + \mu^2)v_1 - B\mu v_2 + \tfrac{1}{8}(g^2 + g'^2)v_1(v_1^2 - v_2^2)\,, \\
t_2 &= (m_{H_2}^2 + \mu^2)v_2 - B\mu v_1 - \tfrac{1}{8}(g^2 + g'^2)v_2(v_1^2 - v_2^2)
\end{aligned}
\tag{52}
$$

One can determine in the tree-level approximation the minimum of the scalar potential by imposing the condition of vanishing tadpoles in Eq. 52. One–loop corrections change these equations to

$$
t_i = t_i^0 - \delta t_i + T_i(Q) \tag{53}
$$

where t_i, with $i = 1, 2$, are the renormalized tadpoles, t_i^0 are the tree level tadpoles given in Eq. 52, δt_i are the tadpole counter-terms, and $T_i(Q)$ are the sum of all one–loop contributions to the corresponding one–point functions with zero external momentum. The contribution from quarks and squarks to these tadpoles in our model can be found in ref. [8]. In an on shell scheme we identify the tree level tadpoles with the renormalized ones. Therefore, to find the correct minima we use Eq. 52 unchanged, where now all the parameters are understood to be renormalized quantities. Eqs. (52) can be solved for B and μ up to a sign. We have

$$
\begin{aligned}
\mu^2 &= -\tfrac{1}{2}m_Z^2 + \frac{m_{H_2}^2 \sin^2\beta - m_{H_1}^2 \cos^2\beta}{\cos 2\beta} \\
B\mu &= \tfrac{1}{2}\sin 2\beta \left(m_{H_1}^2 + m_{H_2}^2 + 2\mu^2\right)
\end{aligned}
\tag{54}
$$

It can be shown that necessary conditions for the existence of a stable minimum are

$$
\begin{aligned}
&(B\mu)^2 > (m_{H_1}^2 + \mu^2)(m_{H_2}^2 + \mu^2) \\
&m_{H_1}^2 + m_{H_2}^2 + 2\mu^2 > |B\mu|
\end{aligned}
\tag{55}
$$

Notice that Eq. 54 only makes sense if $\mu^2 > 0$ and, as we shall see in Section 5.3, also $B\mu > 0$ because is related with the mass squared, m_A^2, of the physical CP–odd state.

4.5 The Constrained Minimal Supersymmetric Standard Model

We have seen in the previous section that the parameters of the MSSM can be considered arbitrary at the weak scale. This is completely consistent. However the number of independent parameters in Table 6 can be reduced if we impose some further constraints. That is usually done by embedding the MSSM in a grand unified scenario. Different schemes are possible but in all of them some kind of unification is imposed at the GUT scale. Then we run the Renormalization Group (RG) equations down to the weak scale to get the values of the parameters at that scale. This is sometimes called the constrained MSSM model.

Among the possible scenarios, the most popular is the MSSM coupled to $N = 1$ Supergravity (SUGRA). Here at M_{GUT} one usually takes the conditions:

$$\begin{aligned}
&A_t = A_b = A_\tau \equiv A\,, B = A - 1\,,\\
&m_{H_1}^2 = m_{H_2}^2 = M_L^2 = M_R^2 = m_0^2\,, M_Q^2 = M_U^2 = M_D^2 = m_0^2\,,\\
&M_3 = M_2 = M_1 = M_{1/2}
\end{aligned} \tag{56}$$

The counting of free parameters[3] is done in Table 7

Parameters	Conditions	Free Parameters
$h_t,\, h_b,\, h_\tau,\, v_1,\, v_2$	$m_W,\, m_t,\, m_b,\, m_\tau$	$\tan\beta$
$A,\, m_0,\, M_{1/2},\, \mu$	$t_i = 0,\ i = 1, 2$	2 Extra free parameters
Total = 9	Total = 6	Total = 3

Table 7: Counting of free parameters in the MSSM coupled to N=1 SUGRA

where we introduced the usual notation

$$\tan\beta = \frac{v_2}{v_1} \tag{57}$$

It is remarkable that with so few parameters we can get the correct values for the parameters, in particular $m_{H_2}^2 < 0$. For this to happen the top Yukawa coupling has to be large which we know to be true.

5 Mass Matrices in the MSSM

5.1 The Chargino Mass Matrices

The charged gauginos mix with the charged higgsinos giving the so–called charginos. In a basis where $\psi^{+T} = (-i\lambda^+, \widetilde{H}_2^+)$ and $\psi^{-T} = (-i\lambda^-, \widetilde{H}_1^-)$, the chargino mass terms in the Lagrangian are

$$\mathcal{L}_m = -\frac{1}{2}(\psi^{+T}, \psi^{-T}) \begin{pmatrix} \mathbf{0} & \boldsymbol{M}_C^T \\ \boldsymbol{M}_C & \mathbf{0} \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + h.c. \tag{58}$$

where the chargino mass matrix is given by

$$\boldsymbol{M}_C = \begin{bmatrix} M_2 & \frac{1}{\sqrt{2}} g v_2 \\ \frac{1}{\sqrt{2}} g v_1 & \mu \end{bmatrix} \tag{59}$$

and M_2 is the $SU(2)$ gaugino soft mass. It is sometimes more useful to write this mass matrix in terms of the physical masses, that is

$$\boldsymbol{M}_C = \begin{bmatrix} M_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & \mu \end{bmatrix} \tag{60}$$

[3]For one family and without counting the gauge couplings.

The chargino mass matrix is diagonalized by two rotation matrices $\boldsymbol{U}$ and $\boldsymbol{V}$ defined by

$$F_i^- = \boldsymbol{U}_{ij}\,\psi_j^- \quad ; \quad F_i^+ = \boldsymbol{V}_{ij}\,\psi_j^+ \tag{61}$$

Then

$$\boldsymbol{U}^* \boldsymbol{M_C} \boldsymbol{V}^{-1} = \boldsymbol{M_C^{\mathrm{diag}}} \tag{62}$$

where $\boldsymbol{M_C^{\mathrm{diag}}}$ is the diagonal chargino mass matrix. To determine $\boldsymbol{U}$ and $\boldsymbol{V}$ we note that

$$\left(\boldsymbol{M_C^{\mathrm{diag}}}\right)^2 = \boldsymbol{V}\boldsymbol{M_C^\dagger}\boldsymbol{M_C}\boldsymbol{V}^{-1} = \boldsymbol{U^*}\boldsymbol{M_C}\boldsymbol{M_C^\dagger}(\boldsymbol{U^*})^{-1} \tag{63}$$

implying that $\boldsymbol{V}$ diagonalizes $\boldsymbol{M_C^\dagger M_C}$ and $\boldsymbol{U^*}$ diagonalizes $\boldsymbol{M_C M_C^\dagger}$. In the previous expressions the $F_i^\pm$ are two component spinors. We construct the four component Dirac spinors out of the two component spinors with the conventions[4],

$$\chi_i^- = \begin{pmatrix} F_i^- \\ \overline{F_i^+} \end{pmatrix} \tag{64}$$

5.2 The Neutralino Mass Matrices

In the basis $\psi^{0T} = (-i\lambda', -i\lambda^3, \widetilde{H}_1^1, \widetilde{H}_2^2)$ the neutral fermions mass terms in the Lagrangian are given by

$$\mathcal{L}_m = -\frac{1}{2}(\psi^0)^T \boldsymbol{M}_N \psi^0 + h.c. \tag{65}$$

where the neutralino mass matrix is

$$\boldsymbol{M}_N = \begin{bmatrix} M_1 & 0 & -\frac{1}{2}g'v_1 & \frac{1}{2}g'v_2 \\ 0 & M_2 & \frac{1}{2}gv_1 & -\frac{1}{2}gv_2 \\ -\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & -\mu \\ \frac{1}{2}g'v_2 & -\frac{1}{2}gv_2 & -\mu & 0 \end{bmatrix} \tag{66}$$

and M_1 is the $U(1)$ gaugino soft mass. The neutralino mass matrix can be written in terms of the physical masses

$$\boldsymbol{M}_N = \begin{bmatrix} M_1 & 0 & -m_Z \sin\theta_W \cos\beta & m_Z \sin\theta_W \sin\beta \\ 0 & M_2 & m_Z \cos\theta_W \cos\beta & -m_Z \cos\theta_W \sin\beta \\ -m_Z \sin\theta_W \cos\beta & m_Z \cos\theta_W \cos\beta & 0 & -\mu \\ m_Z \sin\theta_W \sin\beta & -m_Z \cos\theta_W \sin\beta & -\mu & 0 \end{bmatrix} \tag{67}$$

This neutralino mass matrix is diagonalized by a 4×4 rotation matrix $\boldsymbol{N}$ such that

$$\boldsymbol{N}^* \boldsymbol{M}_N \boldsymbol{N}^{-1} = \mathrm{diag}(m_{F_1^0}, m_{F_2^0}, m_{F_3^0}, m_{F_4^0}) \tag{68}$$

[4]Here we depart from the conventions of ref. [5, 6] because we want the χ^- to be the particle and not the anti–particle.

and

$$F_k^0 = \boldsymbol{N}_{kj}\,\psi_j^0 \tag{69}$$

The four component Majorana neutral fermions are obtained from the two component via the relation

$$\chi_i^0 = \begin{pmatrix} F_i^0 \\ \overline{F_i^0} \end{pmatrix} \tag{70}$$

5.3 Neutral Higgs Mass Matrices

The quadratic scalar potential includes

$$V_{quadratic} = \tfrac{1}{2}[\varphi_1^0, \varphi_2^0]\ \boldsymbol{M}^2_{P^0} \begin{bmatrix} \varphi_1^0 \\ \varphi_2^0 \end{bmatrix} + \tfrac{1}{2}[\sigma_1^0, \sigma_2^0]\ \boldsymbol{M}^2_{S^0} \begin{bmatrix} \sigma_1^0 \\ \sigma_2^0 \end{bmatrix} + \cdots \tag{71}$$

where the CP-odd neutral scalar mass matrix is

$$\boldsymbol{M}^2_{P^0} = \begin{bmatrix} B\mu \tan\beta + \frac{t_1}{v_1} & B\mu \\ B\mu & B\cot\beta + \frac{t_2}{v_2} \end{bmatrix} \tag{72}$$

This matrix also has a vanishing determinant after the tadpoles are set to zero, and the zero eigenvalue corresponds to the mass of the neutral Goldstone boson. The mass of the CP-odd state, usually called A, is

$$m_A^2 = \frac{2B\mu}{\sin 2\beta} \tag{73}$$

The relation between the lagrangian states and the mass eigenstates is, as can be easily verified,

$$\begin{bmatrix} G^0 \\ A^0 \end{bmatrix} = \begin{bmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} \varphi_1^0 \\ \varphi_2^0 \end{bmatrix} \tag{74}$$

that can be written in matrix form as

$$P^0 = \boldsymbol{R}^{\boldsymbol{P}} P'^0 \tag{75}$$

where, $P^{0T} = (G^0, A^0)$, $P'^{0T} = (\varphi_1^0, \varphi_2^0)$ and the rotation matrix is

$$\boldsymbol{R}^{\boldsymbol{P}} = \begin{bmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \tag{76}$$

for future reference we note that

$$\varphi_1^0 = \boldsymbol{R}^{\boldsymbol{P}*}_{\ 11} G^0 + \boldsymbol{R}^{\boldsymbol{P}*}_{\ 21} A^0$$

$$\varphi_2^0 = \boldsymbol{R}^{\boldsymbol{P}*}_{\ 12} G^0 + \boldsymbol{R}^{\boldsymbol{P}*}_{\ 22} A^0 \tag{77}$$

The neutral CP-even scalar sector mass matrix in Eq. 71 is given by

$$\boldsymbol{M}^2_{S^0} = \begin{bmatrix} B\mu\tan\beta + m_Z^2\cos^2\beta + \frac{t_1}{v_1} & -B\mu - m_Z^2\sin\beta\cos\beta \\ -B\mu - m_Z^2\sin\beta\cos\beta & B\mu\cot\beta + m_Z^2\sin^2\beta + \frac{t_2}{v_2} \end{bmatrix} \tag{78}$$

As before we define the rotation matrix as

$$S^0 = \boldsymbol{R^S} S'^0 \tag{79}$$

where $S^{0T} = (h^0, H^0)$, $S'^{0T} = (\sigma_1^0, \sigma_2^0)$. For future reference we note that

$$\sigma_1^0 = \boldsymbol{R^S}^*_{11} h^0 + \boldsymbol{R^S}^*_{21} H^0$$

$$\sigma_2^0 = \boldsymbol{R^S}^*_{12} h^0 + \boldsymbol{R^S}^*_{22} H^0 \tag{80}$$

For completeness we note that this (orthogonal) matrix is normally parameterized by an angle α in the following way

$$\boldsymbol{R^S} = \begin{bmatrix} -\sin\alpha & \cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} . \tag{81}$$

5.4 Charged Higgs Mass Matrices

The mass matrix of the charged Higgs bosons follows from the quadratic terms in the scalar potential

$$V_{quadratic} = [H_1^-, H_2^-,]\ \boldsymbol{M^2_{H^\pm}} \begin{bmatrix} H_1^+ \\ H_2^+ \end{bmatrix} \tag{82}$$

where the charged Higgs mass matrix is

$$\boldsymbol{M}^2_{H^\pm} = \begin{bmatrix} B\mu\tan\beta + m_W^2\sin^2\beta + \frac{t_1}{v_1} & B\mu + m_W^2\sin\beta\cos\beta \\ B\mu + m_W^2\sin\beta\cos\beta & B\mu\cot\beta + m_W^2\cos^2\beta + \frac{t_2}{v_2} \end{bmatrix} \tag{83}$$

The relation between the lagrangian states and the mass eigenstates is, again as in Eq. (74)

$$\begin{bmatrix} G^\pm \\ H^\pm \end{bmatrix} = \begin{bmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \begin{bmatrix} H_1^\pm \\ H_2^\pm \end{bmatrix} \tag{84}$$

that can be written in matrix form as

$$S^\pm = \boldsymbol{R^{S^\pm}} S'^\pm \tag{85}$$

where, $S^{\pm T} = (G^\pm, H^\pm)$, $S'^{\pm T} = (H_1^\pm, H_2^\pm)$ and the rotation matrix is

$$\boldsymbol{R^{S^\pm}} = \begin{bmatrix} -\cos\beta & \sin\beta \\ \sin\beta & \cos\beta \end{bmatrix} \tag{86}$$

for future reference we note that

$$H_1^\pm = \boldsymbol{R^S}^*_{11} G^\pm + \boldsymbol{R^{S^\pm}}^*_{21} H^\pm$$

$$H_2^\pm = \boldsymbol{R^{S^\pm}}^*_{12} G^\pm + \boldsymbol{R^{S^\pm}}^*_{22} H^\pm \tag{87}$$

5.5 Lepton Mass Matrices

In the superpotential in Eq. 47 the Yukawa couplings are matrices in generation space. For the case of the leptons it is possible to start with the matrix h_E already diagonalized. However, for some applications, could be useful to have a general matrix. Here we consider an arbitrary matrix. In 2–component spinor
notation the relevant mass terms in the Lagrangian are

$$\mathcal{L}_M = -\frac{v_1}{\sqrt{2}}\, (h_E)_{ij}\, \ell'_{Li}\, \ell'^c_{Lj} - \frac{v_1}{\sqrt{2}}\, (h^*_E)_{ij}\, \overline{\ell'_{Li}}\, \overline{\ell'^c_{Lj}} \tag{88}$$

where ℓ' are the interaction eigenstates. The 4–component spinors are

$$\ell' = \begin{pmatrix} \ell'_L \\ \overline{\ell'^c_L} \end{pmatrix} \tag{89}$$

and therefore we have

$$\begin{aligned}\mathcal{L}_M &= -\overline{\ell'}\, \boldsymbol{M_E} P_R \ell' - \overline{\ell'}\, \boldsymbol{M_E}^\dagger P_L \ell' \\ &= -\overline{\ell'_L}\, \boldsymbol{M_E} \ell'_R - \overline{\ell'_R}\, \boldsymbol{M_E}^\dagger \ell'_L \end{aligned} \tag{90}$$

where

$$(\boldsymbol{M_E})_{ij} = \frac{v_1}{\sqrt{2}}\, (h^*_E)_{ij} \tag{91}$$

To diagonalize the mass matrix $\boldsymbol{M_E}$ we need different rotations for the left handed and right handed components. We introduce [5]

$$\ell_R = \boldsymbol{R^\ell_R}\, \ell'_R \qquad ; \qquad \ell_L = \boldsymbol{R^\ell_L}\, \ell'_L \tag{92}$$

Then

$$\boldsymbol{R^\ell_L}\, \boldsymbol{M_E} \boldsymbol{R^\ell_R}^\dagger = \boldsymbol{M^{diag}_E} \qquad \text{or} \qquad \boldsymbol{R^\ell_L}^\dagger\, \boldsymbol{M^{diag}_E} \boldsymbol{R^\ell_R} = \boldsymbol{M_E} \tag{93}$$

where $\boldsymbol{M^{diag}_E}$ is a diagonal matrix. The rotation matrices are obtained by noticing that

$$\boldsymbol{M_E}\boldsymbol{M_E}^\dagger = \boldsymbol{R^\ell_L}^\dagger \left(\boldsymbol{M^{diag}_E}\right)^2 \boldsymbol{R^\ell_L} \qquad \text{and} \qquad \boldsymbol{M_E}^\dagger\boldsymbol{M_E} = \boldsymbol{R^\ell_R}^\dagger \left(\boldsymbol{M^{diag}_E}\right)^2 \boldsymbol{R^\ell_R} \tag{94}$$

that tell us that $\boldsymbol{R^\ell_L}$ diagonalizes $\boldsymbol{M_E}\boldsymbol{M_E}^\dagger$ and $\boldsymbol{R^\ell_R}$ diagonalizes $\boldsymbol{M_E}^\dagger\boldsymbol{M_E}$. For future reference we write the relations between the mass and the interaction eigenstates. We have

$$\begin{aligned} \ell'_{Ri} &= \left(\boldsymbol{R^\ell_R}\right)^*_{ji}\, \ell_{Rj} \qquad ; \qquad \overline{\ell'_{Ri}} = \overline{\ell_{Rj}}\, \left(\boldsymbol{R^\ell_R}\right)_{ji} \\ \ell'_{Li} &= \left(\boldsymbol{R^\ell_L}\right)^*_{ji}\, \ell_{Lj} \qquad ; \qquad \overline{\ell'_{Li}} = \overline{\ell_{Lj}}\, \left(\boldsymbol{R^\ell_L}\right)_{ji} \end{aligned} \tag{95}$$

[5] I changed here the convention in comparison with previous versions. Now the convention is uniform for all fields and it is consistent with the convention used in `SPheno` [9].

5.6 Quark Mass Matrices

In 2–component spinor notation the relevant terms in the Lagrangian are

$$\mathcal{L}_M = -\frac{v_1}{\sqrt{2}}\,(h_D)_{ij}\; d'_{Li}\, d'^c_{Lj} - \frac{v_2}{\sqrt{2}}\,(h_U)_{ij}\; u'_{Li}\, u'^c_{Lj} + h.c. \tag{96}$$

where the primed states are again the interaction eigenstates. In 4–component spinor notation with the definitions

$$d' = \begin{pmatrix} d'_L \\ \\ \overline{d'^c_L} \end{pmatrix} \qquad ; \qquad u' = \begin{pmatrix} u'_L \\ \\ \overline{u'^c_L} \end{pmatrix} \tag{97}$$

we get

$$\mathcal{L}_M = -\overline{d'_L}\,\boldsymbol{M_D} d'_R - \overline{u'_L}\,\boldsymbol{M_U} u'_R + h.c. \tag{98}$$

where

$$(\boldsymbol{M_D})_{ij} = \frac{v_1}{\sqrt{2}}\,(h^*_D)_{ij} \qquad ; \qquad (\boldsymbol{M_U})_{ij} = \frac{v_2}{\sqrt{2}}\,(h^*_U)_{ij} \tag{99}$$

To obtain the eigenstates of the mass we rotate the quark fields through

$$d_R = \boldsymbol{R^d_R}\, d'_R \qquad ; \qquad d_L = \boldsymbol{R^d_L}\, d'_L \qquad ; \qquad u_R = \boldsymbol{R^u_R}\, u'_R \qquad ; \qquad u_L = \boldsymbol{R^u_L}\, u'_L \tag{100}$$

For future reference we write the relations between the mass and the interaction eigenstates. We have

$$\begin{aligned} q'_{Ri} &= (\boldsymbol{R^q_R})^*_{ji}\; q_{Rj} \qquad ; \qquad \overline{q'_{Ri}} = \overline{q_{Rj}}\; (\boldsymbol{R^q_R})_{ji} \\ & \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad q = (d, u) \\ q'_{Li} &= (\boldsymbol{R^q_L})^*_{ji}\; q_{Lj} \qquad ; \qquad \overline{q'_{Li}} = \overline{q_{Lj}}\; (\boldsymbol{R^q_L})_{ji} \end{aligned} \tag{101}$$

Then

$$\boldsymbol{R^d_L}\, \boldsymbol{M_D} \boldsymbol{R^{d\,\dagger}_R} = \boldsymbol{M^{\text{diag}}_D} \qquad \text{and} \qquad \boldsymbol{R^u_L}\, \boldsymbol{M_U} \boldsymbol{R^{u\,\dagger}_R} = \boldsymbol{M^{\text{diag}}_U} \tag{102}$$

where $\boldsymbol{M^{\text{diag}}_{D(U)}}$ are a diagonal matrices. These are diagonalized by noticing that

$$\begin{aligned} \boldsymbol{R^d_L}\, \boldsymbol{M_D} \boldsymbol{M_D}^\dagger\, \boldsymbol{R^d_L}^\dagger &= \left(\boldsymbol{M^{\text{diag}}_D}\right)^2 \qquad ; \qquad \boldsymbol{R^d_R}\, \boldsymbol{M_D}^\dagger \boldsymbol{M_D}\, \boldsymbol{R^d_R}^\dagger = \left(\boldsymbol{M^{\text{diag}}_D}\right)^2 \\ \boldsymbol{R^u_L}\, \boldsymbol{M_U} \boldsymbol{M_U}^\dagger\, \boldsymbol{R^u_L}^\dagger &= \left(\boldsymbol{M^{\text{diag}}_U}\right)^2 \qquad ; \qquad \boldsymbol{R^u_R}\, \boldsymbol{M_U}^\dagger \boldsymbol{M_U}\, \boldsymbol{R^u_R}^\dagger = \left(\boldsymbol{M^{\text{diag}}_U}\right)^2 \end{aligned} \tag{103}$$

Before we close this section let us write down the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix with our conventions. The couplings of the $W^\pm$ with the quarks are

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\, W^-_\mu \overline{d'_{Li}}\, \gamma^\mu\, u'_{Li} - \frac{g}{\sqrt{2}}\, W^+_\mu \overline{u'_{Li}}\, \gamma^\mu\, d'_{Li} \tag{104}$$

Then in terms of the mass eigenstates the charged current Lagrangian reads

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}\, \boldsymbol{V}^{\text{CKM}*}_{ij}\, W^-_\mu\, \overline{d_{Lj}}\, \gamma^\mu\, u_{Li} - \frac{g}{\sqrt{2}}\, \boldsymbol{V}^{\text{CKM}}_{ij}\, W^+_\mu\, \overline{u_{Li}}\, \gamma^\mu\, d_{Lj} \tag{105}$$

where the CKM matrix $\boldsymbol{V}^{\text{CKM}}$ is defined through

$$\boldsymbol{V}^{\text{CKM}} = \boldsymbol{R^u_L}\, \boldsymbol{R^d_L}^\dagger \tag{106}$$

5.7 Slepton Mass Matrices

In the unrotated basis $\widetilde{\ell'}_i = (\widetilde{e}_L, \widetilde{\mu}_L, \widetilde{\tau}_L, \widetilde{e}^*_R, \widetilde{\mu}^*_R, \widetilde{\tau}^*_R)$ we get

$$\mathcal{L}_M = -\tfrac{1}{2} \widetilde{\ell'}^\dagger \, \boldsymbol{M_{\widetilde{\ell}}}^2 \, \widetilde{\ell'} \tag{107}$$

where

$$\boldsymbol{M_{\widetilde{\ell}}}^2 = \begin{pmatrix} \boldsymbol{M^2_{LL}} & \boldsymbol{M^2_{LR}} \\ \boldsymbol{M^2_{RL}} & \boldsymbol{M^2_{RR}} \end{pmatrix} \tag{108}$$

and

$$\begin{aligned} \boldsymbol{M^2_{LL}} &= \tfrac{1}{2} v_1^2 \, h_E^* h_E^T + M_L^2 - \tfrac{1}{2} \, (2m_W^2 - m_Z^2) \cos 2\beta \\ \boldsymbol{M^2_{RR}} &= \tfrac{1}{2} v_1^2 \, h_E^T h_E^* + M_R^2 - (m_Z^2 - m_W^2) \cos 2\beta \\ \boldsymbol{M^2_{LR}} &= \frac{v_1}{\sqrt{2}} \, A_E^* - \mu \frac{v_2}{\sqrt{2}} \, h_E^* \\ \boldsymbol{M^2_{RL}} &= \boldsymbol{M^2_{LR}}^\dagger \end{aligned} \tag{109}$$

We define the mass eigenstates

$$\widetilde{\ell} = \boldsymbol{R^{\widetilde{\ell}}} \, \widetilde{\ell'} \tag{110}$$

which implies

$$\widetilde{\ell'}_i = \boldsymbol{R^{\widetilde{\ell}}}^*_{ji} \, \widetilde{\ell}_j \tag{111}$$

The rotation matrices are obtained from

$$\boldsymbol{R^{\widetilde{\ell}}}^\dagger \left(\boldsymbol{M^{\mathrm{diag}}_{\widetilde{\ell}}} \right)^2 \boldsymbol{R^{\widetilde{\ell}}} = \boldsymbol{M_{\widetilde{\ell}}}^2 \tag{112}$$

In most applications the matrices in Eq. 108 are real and therefore the rotation matrices $\boldsymbol{R^{\widetilde{\ell}}}$ are orthogonal matrices.

5.8 Sneutrino Mass Matrices

In the unrotated basis $\widetilde{\nu}'_i = \widetilde{\nu}_{iL}$ we have

$$\mathcal{L}_M = -\tfrac{1}{2} \, \widetilde{\nu}'^\dagger \, \boldsymbol{M_{\widetilde{\nu}}}^2 \, \widetilde{\nu}' \tag{113}$$

where

$$\boldsymbol{M_{\widetilde{\nu}}}^2 = M_L^2 + \tfrac{1}{2} \, m_Z^2 \cos 2\beta \tag{114}$$

We define the mass eigenstates

$$\widetilde{\nu} = \boldsymbol{R^{\widetilde{\nu}}} \, \widetilde{\nu}' \tag{115}$$

which implies

$$\widetilde{\nu}'_i = \boldsymbol{R^{\widetilde{\nu}}}^*_{ji} \, \widetilde{\nu}_j \tag{116}$$

The rotation matrices are obtained from

$$\boldsymbol{R}^{\tilde{\nu}\,\dagger}\left(\boldsymbol{M}_{\tilde{\nu}}^{\mathbf{diag}}\right)^2 \boldsymbol{R}^{\tilde{\nu}} = \boldsymbol{M}_{\tilde{\nu}}{}^2 \tag{117}$$

In most applications the matrix in Eq. 114 is real and therefore the rotation matrices $\boldsymbol{R}^{\tilde{\ell}}$ are orthogonal matrices.

5.9 Squark Mass Matrices

In the unrotated basis $\tilde{u}'_i = (\tilde{u}_{Li}, \tilde{u}^*_{Ri})$ and $\tilde{d}'_i = (\tilde{d}_{Li}, \tilde{d}^*_{Ri})$ we get

$$\mathcal{L}_M = -\tfrac{1}{2}\tilde{u}'^{\dagger}\, \boldsymbol{M}_{\tilde{u}}{}^2\, \tilde{u}' - \tfrac{1}{2}\tilde{d}'^{\dagger}\, \boldsymbol{M}_{\tilde{d}}{}^2\, \tilde{d}' \tag{118}$$

where

$$\boldsymbol{M}_{\tilde{q}}{}^2 = \begin{pmatrix} \boldsymbol{M}^2_{\tilde{q}LL} & \boldsymbol{M}^2_{\tilde{q}LR} \\ \boldsymbol{M}^2_{\tilde{q}RL} & \boldsymbol{M}^2_{\tilde{q}RR} \end{pmatrix} \tag{119}$$

with $\tilde{q} = (\tilde{u}, \tilde{d})$. The blocks are different for up and down type squarks. We have

$$\begin{aligned} \boldsymbol{M}^2_{\tilde{u}LL} &= \tfrac{1}{2}\, v_2^2\, h_U^* h_U^T + M_Q^2 + \tfrac{1}{6}\,(4m_W^2 - m_Z^2)\cos 2\beta \\ \boldsymbol{M}^2_{\tilde{u}RR} &= \tfrac{1}{2}\, v_2^2\, h_U^T h_U^* + M_U^2 + \tfrac{2}{3}(m_Z^2 - m_W^2)\cos 2\beta \\ \boldsymbol{M}^2_{\tilde{u}LR} &= \frac{v_2}{\sqrt{2}}\, A_U^* - \mu \frac{v_1}{\sqrt{2}}\, h_U^* \\ \boldsymbol{M}^2_{\tilde{u}RL} &= \boldsymbol{M}^2_{\tilde{u}LR}{}^{\dagger} \end{aligned} \tag{120}$$

and

$$\begin{aligned} \boldsymbol{M}^2_{\tilde{d}LL} &= \frac{1}{2}\, v_1^2\, h_D^* h_D^T + M_Q^2 - \tfrac{1}{6}\,(2m_W^2 + m_Z^2)\cos 2\beta \\ \boldsymbol{M}^2_{\tilde{d}RR} &= \tfrac{1}{2}\, v_1^2\, h_D^T h_D^* + M_D^2 - \tfrac{1}{3}(m_Z^2 - m_W^2)\cos 2\beta \\ \boldsymbol{M}^2_{\tilde{d}LR} &= \frac{v_1}{\sqrt{2}}\, A_D^* - \mu \frac{v_2}{\sqrt{2}}\, h_D^* \\ \boldsymbol{M}^2_{\tilde{d}RL} &= \boldsymbol{M}^2_{\tilde{d}LR}{}^{\dagger} \end{aligned} \tag{121}$$

We define the mass eigenstates

$$\tilde{q} = \boldsymbol{R}^{\tilde{q}}\, \tilde{q}' \tag{122}$$

which implies

$$\tilde{q}'_i = \boldsymbol{R}^{\tilde{q}}{}^{*}_{ji}\, \tilde{q}_j \tag{123}$$

The rotation matrices are obtained from

$$\boldsymbol{R}^{\tilde{q}\,\dagger}\left(\boldsymbol{M}_{\tilde{q}}^{\mathbf{diag}}\right)^2 \boldsymbol{R}^{\tilde{q}} = \boldsymbol{M}_{\tilde{q}}{}^2 \tag{124}$$

In many applications the matrices in Eq. 119 are real and therefore the rotation matrices $\boldsymbol{R}^{\tilde{q}}$ are orthogonal matrices.

6 Couplings in the MSSM

In this section we give a list of all the MSSM couplings. The following table is a guide.

Name	Type	Equation
Gauge Self-Interaction	VVV	Eq. (128)
	VVVV	Eq. (130), Eq. (131)
3-Point Gauge Coupling	Vff	Eq. (140), Eq. (150)
	$V\tilde{f}\tilde{f}$	
	$V\tilde{\chi}\tilde{\chi}$	Eq. (140), Eq. (150)
	VHH	
3-Point Higgs Coupling	Hff	Eq. (183)
	$H\tilde{f}\tilde{f}$	Eq. (192)
	$H\tilde{\chi}\tilde{\chi}$	Eq. (177)
	HVV	
	HHH	Eq. (203)
Other 3-Point	$\tilde{f}f\tilde{\chi}$	Eq. (168)
4-Point Coupling	$VV\tilde{f}\tilde{f}$	
	$HHVV$	
	$HHHH$	Eq. (209)
	$\tilde{f}\tilde{f}HH$	
	$\tilde{f}\tilde{f}\tilde{f}\tilde{f}$	
Ghost	$\overline{\omega}\omega$ V	
	$\overline{\omega}\omega$ H	

Table 8: Couplings in the MSSM. V, f $\tilde{f}$ and H are generic names. In particular H includes the Goldstone bosons.

6.1 Gauge Self-Interactions

The gauge sector of the MSSM is exactly the same as in the SM. We present it here both for completeness and to fix our notation.

6.1.1 VVV

From Eq. 39 we get

$$\mathcal{L}_{VVV} = -\frac{1}{2}\, g\, \left(\partial_\mu W^a_\nu - \partial_\nu W^a_\mu\right) \epsilon^{abc}\, W^{b\mu}\, W^{c\nu} \tag{125}$$

Now using

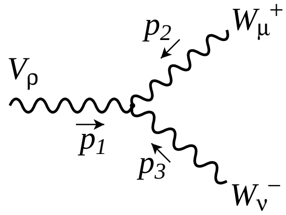
$$W^3_\mu \quad = \quad \sin\theta_W A_\mu + \cos\theta_W Z_\mu$$

$$W_\mu^\pm = \frac{W_\mu^1 \mp i\, W_\mu^2}{\sqrt{2}} \tag{126}$$

we obtain

$$\begin{aligned}
\mathcal{L} = \; & i\, e \Big[\left(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-\right) W^{+\mu} A^\nu - \left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\right) W^{-\mu} A^\nu \\
& \quad - \left(\partial_\mu A_\nu - \partial_\nu A_\mu\right) W^{+\mu} W^{-\nu} \Big] \\
& + i\, g \, \cos\theta_W \Big[\left(\partial_\mu W_\nu^- - \partial_\nu W_\mu^-\right) W^{+\mu} Z^\nu - \left(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+\right) W^{-\mu} Z^\nu \\
& \quad - \left(\partial_\mu Z_\nu - \partial_\nu Z_\mu\right) W^{+\mu} W^{-\nu} \Big]
\end{aligned} \tag{127}$$

which gives the following Feynman rule for the vertices,



$$i\, g_V \, \left[g^{\mu\nu}(p_2 - p_3)^\rho + g^{\nu\rho}(p_3 - p_1)^\mu + g^{\rho\mu}(p_1 - p_2)^\nu \right] \tag{128}$$

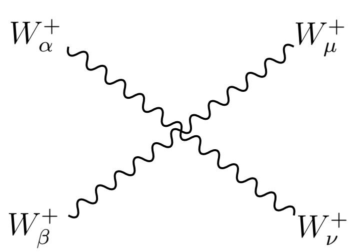
where $V = A(Z)$ and $g_A = e, g_Z = g \, \cos\theta_W$.

6.1.2 $VVVV$

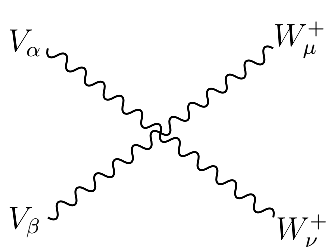
For the quartic self–interactions we have

$$\begin{aligned}
\mathcal{L}_{VVVV} = \; & \tfrac{1}{2}\, g^2 \left[\left(W_\mu^+ W^{+\mu}\right)\left(W_\nu^- W^{-\nu}\right) - \left(W_\mu^+ W^{-\mu}\right)^2 \right] \\
& + g^2 \, \cos^2\theta_W \left[W_\mu^+ W_\nu^- Z^\mu Z^\nu - W_\mu^+ W^{-\mu}\, Z_\nu Z^\nu \right] \\
& + e^2 \, \left[W_\mu^+ W_\nu^- A^\mu A^\nu - W_\mu^+ W^{-\mu}\, A_\nu A^\nu \right] \\
& + e\, g \, \cos\theta_W \, \left[W_\mu^+ W_\nu^- \left(Z^\mu A^\nu + Z^\nu A^\mu\right) - 2 W_\mu^+ W^{-\mu}\, A_\nu Z^\nu \right]
\end{aligned} \tag{129}$$

which gives the following Feynman rules for the quartic vertices,



$$i\, g^2 \, \left[2\, g^{\mu\alpha} g^{\nu\beta} - g^{\alpha\nu} g^{\mu\beta} - g^{\mu\nu} g^{\alpha\beta} \right] \tag{130}$$



$$i\, g_V \, g_V \, \left[g^{\mu\alpha} g^{\nu\beta} + g^{\mu\beta} g^{\alpha\nu} - 2\, g^{\mu\nu} g^{\alpha\beta} \right] \tag{131}$$

where $V = A(Z)$ and $g_A = e, g_Z = g\,\cos\theta_W$.

6.2 Charged Current Couplings

Using two component spinors and following the notation of ref. [5] the relevant part of the Lagrangian can be written as

$$
\begin{aligned}
\mathcal{L} \;=\;& g\,W^-_\mu \left[\left(\overline{\lambda^3}\,\overline{\sigma}^\mu\,\lambda^+ - \overline{\lambda^-}\,\overline{\sigma}^\mu\,\lambda^3 \right) - \frac{1}{\sqrt{2}} \left(\overline{\widetilde{H}^0_2}\,\overline{\sigma}^\mu\,\widetilde{H}^+_2 + \overline{\widetilde{H}^-_1}\,\overline{\sigma}^\mu\,\widetilde{H}^0_1 \right) \right. \\
&\left. - \frac{1}{\sqrt{2}} \left(\overline{\ell'_{Li}}\,\overline{\sigma}^\mu\,\nu'_{Li} + \overline{d'_{Li}}\,\overline{\sigma}^\mu\,u'_{Li} \right) \right] \\
+\;& g\,W^+_\mu \left[\left(\overline{\lambda^+}\,\overline{\sigma}^\mu\,\lambda^3 - \overline{\lambda^3}\,\overline{\sigma}^\mu\,\lambda^- \right) - \frac{1}{\sqrt{2}} \left(\overline{\widetilde{H}^+_2}\,\overline{\sigma}^\mu\,\widetilde{H}^0_2 + \overline{\widetilde{H}^0_1}\,\overline{\sigma}^\mu\,\widetilde{H}^-_1 \right) \right. \\
&\left. - \frac{1}{\sqrt{2}} \left(\overline{\nu'_{Li}}\,\overline{\sigma}^\mu\,\ell'_{Li} + \overline{u'_{Li}}\,\overline{\sigma}^\mu\,d'_{Li} \right) \right]
\end{aligned}
\qquad (132)
$$

To obtain the couplings in four component notation we first write Eq. 132 in terms of the mass eigenstates in two component notation, $F^\pm_i$ and F^0_i. In order to do that we recall that for the neutralinos

$$
\begin{aligned}
-i\lambda' &= \boldsymbol{N^*}_{i1} F^0_i & i\overline{\lambda'} &= \boldsymbol{N}_{i1} \overline{F^0_i} \\
-\,i\lambda^3 &= \boldsymbol{N^*}_{i2} F^0_i & i\overline{\lambda^3} &= \boldsymbol{N}_{i2} \overline{F^0_i} \\
\widetilde{H}^0_1 &= \boldsymbol{N^*}_{i3} F^0_i & \overline{\widetilde{H}^0_1} &= \boldsymbol{N}_{i3} \overline{F^0_i} \\
\widetilde{H}^0_2 &= \boldsymbol{N^*}_{i4} F^0_i & \overline{\widetilde{H}^0_2} &= \boldsymbol{N}_{i4} \overline{F^0_i}
\end{aligned}
\qquad (133)
$$

while for the charginos

$$
\begin{aligned}
-i\lambda^- &= \boldsymbol{U^*}_{j1} F^-_j & i\overline{\lambda^-} &= \boldsymbol{U}_{j1} \overline{F^-_j} \\
\widetilde{H}^-_1 &= \boldsymbol{U^*}_{j2} F^-_j & \overline{\widetilde{H}^-_1} &= \boldsymbol{U}_{j2} \overline{F^-_j} \\
-\,i\lambda^+ &= \boldsymbol{V^*}_{j1} F^+_j & i\overline{\lambda^+} &= \boldsymbol{V}_{j1} \overline{F^+_j} \\
\widetilde{H}^+_2 &= \boldsymbol{V^*}_{j2} F^+_j & \overline{\widetilde{H}^+_2} &= \boldsymbol{V}_{j2} \overline{F^+_j}
\end{aligned}
\qquad (134)
$$

We obtain then

$$
\begin{aligned}
\mathcal{L} = g\,W^-_\mu &\left[\; \overline{F^0_i}\,\overline{\sigma}^\mu\,F^+_j \left(\boldsymbol{N}_{i2}\boldsymbol{V^*}_{j1} - \frac{1}{\sqrt{2}}\boldsymbol{N}_{i4}\boldsymbol{V^*}_{j2} \right) + \overline{F^-_j}\,\overline{\sigma}^\mu\,F^0_i \left(-\boldsymbol{N^*}_{i2}\boldsymbol{U}_{j1} - \frac{1}{\sqrt{2}}\boldsymbol{N^*}_{i3}\boldsymbol{U}_{j2} \right) \right. \\
&\left. - \frac{1}{\sqrt{2}} \left(\overline{\ell'_{Li}}\,\overline{\sigma}^\mu\,\nu_{Li} + \overline{d'_{Li}}\,\overline{\sigma}^\mu\,u'_{Li} \right) \right] \\
+\, g\,W^+_\mu &\left[\; \overline{F^+_j}\,\overline{\sigma}^\mu\,F^0_i \left(\boldsymbol{N^*}_{i2}\boldsymbol{V}_{j1} - \frac{1}{\sqrt{2}}\boldsymbol{N^*}_{i4}\boldsymbol{V}_{j2} \right) + \overline{F^0_i}\,\overline{\sigma}^\mu\,F^-_j \left(-\boldsymbol{N}_{i2}\boldsymbol{U^*}_{j1} - \frac{1}{\sqrt{2}}\boldsymbol{N}_{i3}\boldsymbol{U^*}_{j2} \right) \right. \\
&\left. - \frac{1}{\sqrt{2}} \left(\overline{\ell'_{Li}}\,\overline{\sigma}^\mu\,\nu_{Li} + \overline{d'_{Li}}\,\overline{\sigma}^\mu\,u'_{Li} \right) \right]
\end{aligned}
\qquad (135)
$$

Finally using

$$
\begin{aligned}
\overline{F_i^0}\,\overline{\sigma}^\mu\, F_j^+ &= -\overline{\chi_j^-}\gamma^\mu P_R \chi_i^0 \\
\overline{F_j^-}\,\overline{\sigma}^\mu\, F_i^0 &= \overline{\chi_j^-}\gamma^\mu P_L \chi_i^0 \\
\overline{F_i^0}\,\overline{\sigma}^\mu\, F_j^- &= \overline{\chi_i^0}\gamma^\mu P_L \chi_j^- \\
\overline{F_j^+}\,\overline{\sigma}^\mu\, F_i^0 &= -\overline{\chi_i^0}\gamma^\mu P_R \chi_j^- \\
\overline{\ell'_{Li}}\,\overline{\sigma}^\mu\, \nu'_{Li} &= \overline{\ell'_{Li}}\gamma^\mu P_L \nu'_{Li} \\
\overline{\nu'_{Li}}\,\overline{\sigma}^\mu\, \ell'_{Li} &= \overline{\nu_{Li}}\gamma^\mu P_L \ell_{Li}
\end{aligned} \tag{136}
$$

we get

$$
\begin{aligned}
\mathcal{L} = g W^-_\mu \Big[\; \overline{\chi_j^-}\,\gamma^\mu \; \left(O^L_{ji} P_L + O^R_{ji} P_R\right) \chi_i^0 - \frac{1}{\sqrt{2}} \left(\overline{\ell'_{Li}}\gamma^\mu P_L \nu'_{Li} + \overline{d'_{Li}}\gamma^\mu P_L u'_{Li}\right) \Big] \\
+ g W^+_\mu \Big[\; \overline{\chi_i^0}\,\gamma^\mu \; \left((O^L_{ji})^* P_L + (O^R_{ji})^* P_R\right) \chi_j^- - \frac{1}{\sqrt{2}} \left(\overline{\nu'_{Li}}\gamma^\mu P_L \ell'_{Li} + \overline{u'_{Li}}\gamma^\mu P_L d'_{Li}\right) \Big]
\end{aligned} \tag{137}
$$

where

$$
\begin{aligned}
O^L_{ji} &= \left(-\boldsymbol{N^*}_{i2} \boldsymbol{U}_{j1} - \frac{1}{\sqrt{2}} \boldsymbol{N^*}_{i3} \boldsymbol{U}_{j2} \right) \\
O^R_{ji} &= \left(-\boldsymbol{N}_{i2} \boldsymbol{V^*}_{j1} + \frac{1}{\sqrt{2}} \boldsymbol{N}_{i4} \boldsymbol{V^*}_{j2} \right)
\end{aligned} \tag{138}
$$

Finally we rotate the leptons and quarks to the mass eigenstates using the relations[6]

$$
\begin{aligned}
u'_{Li} &= \boldsymbol{R^u_L}_{ij} u_{Lj} & \overline{u'_{Li}} &= \overline{u_{Lj}}\, (\boldsymbol{R^u_L})^*_{ij} \\
d'_{Li} &= \boldsymbol{R^d_L}_{ij} d_{Lj} & \overline{d'_{Li}} &= \overline{d_{Lj}}\, (\boldsymbol{R^d_L})^*_{ij} \\
\ell'_{Li} &= \boldsymbol{R^\ell_L}_{ij} \ell_{Lj} & \overline{\ell'_{Li}} &= \overline{\ell_{Lj}}\, (\boldsymbol{R^\ell_L})^*_{ij} \\
\nu'_{Li} &= \boldsymbol{R^\nu_L}_{ij} \nu_{Lj} & \overline{\nu'_{Li}} &= \overline{\nu_{Lj}}\, (\boldsymbol{R^\nu_L})^*_{ij}
\end{aligned} \tag{139}
$$

to get

$$
\begin{aligned}
\mathcal{L} = g W^-_\mu \Big[\overline{\chi_j^-}\,\gamma^\mu \; \left(O^L_{ji} P_L + O^R_{ji} P_R\right) \chi_i^0 - \frac{1}{\sqrt{2}} \left(\overline{\ell_{Li}}\gamma^\mu P_L \nu_{Li} + \boldsymbol{V}^{\text{CKM}*}_{ij}\, \overline{d_{Lj}}\gamma^\mu P_L u_{Li}\right) \Big] \\
+ g W^+_\mu \Big[\overline{\chi_i^0}\,\gamma^\mu \; \left((O^L_{ji})^* P_L + (O^R_{ji})^* P_R\right) \chi_j^- - \frac{1}{\sqrt{2}} \left(\overline{\nu_{Li}}\gamma^\mu P_L \ell_{Li} + \boldsymbol{V}^{\text{CKM}}_{ij}\, \overline{u_{Li}}\gamma^\mu P_L d_{Lj}\right) \Big]
\end{aligned} \tag{140}
$$

where the CKM matrix $\boldsymbol{V}^{\text{CKM}}$ was defined in Eq. 106.

[6]Notice that as the neutrinos are massless we can rotate them by the same matrix as the leptons. Then the charged current for neutrinos and leptons will remain diagonal.

6.3 Neutral Current Couplings

Using two component spinors and following the notation of ref. [5] the relevant part of the Lagrangian can be written as

$$\mathcal{L} = -\tfrac{1}{2}\left(gW^3_\mu + g'B_\mu\right)\left(\overline{\widetilde{H}^+_2}\,\overline{\sigma}^\mu\,\widetilde{H}^+_2 - \overline{\widetilde{H}^0_1}\,\overline{\sigma}^\mu\,\widetilde{H}^-_1\right) - gW^3_\mu\left(\overline{\lambda^+}\,\overline{\sigma}^\mu\,\lambda^+ - \overline{\lambda^-}\,\overline{\sigma}^\mu\,\lambda^-\right) \tag{141}$$

$$-\tfrac{1}{2}\left(-gW^3_\mu + g'B_\mu\right)\left(\overline{\widetilde{H}^0_2}\,\overline{\sigma}^\mu\,\widetilde{H}^0_2 - \overline{\widetilde{H}^0_1}\,\overline{\sigma}^\mu\,\widetilde{H}^0_1\right) \tag{142}$$

$$+\tfrac{1}{2}\left(-gW^3_\mu + g'B_\mu\right)\overline{\nu'_{Li}}\,\overline{\sigma}^\mu\,\nu'_{Li} + \tfrac{1}{2}\left(gW^3_\mu + g'B_\mu\right)\overline{\ell'_{Li}}\,\overline{\sigma}^\mu\,\ell'_{Li} - g'B_\mu\,\overline{\ell'^c_{Li}}\,\overline{\sigma}^\mu\,\ell'^c_{Li} \tag{143}$$

$$+\tfrac{1}{2}\left(-gW^3_\mu - \tfrac{1}{3}g'B_\mu\right)\overline{u'_{Li}}\,\overline{\sigma}^\mu\,u'_{Li} + \tfrac{1}{2}\left(gW^3_\mu - \tfrac{1}{3}g'B_\mu\right)\overline{d'_{Li}}\,\overline{\sigma}^\mu\,d'_{Li} \tag{144}$$

$$+\tfrac{2}{3}\,g'B_\mu\,\overline{u'^c_{Li}}\,\overline{\sigma}^\mu\,u'^c_{Li} - \tfrac{1}{3}\,g'B_\mu\,\overline{d'^c_{Li}}\,\overline{\sigma}^\mu\,d'^c_{Li} \tag{145}$$

To obtain the couplings in four component notation we first write Eq. 145 in terms of the mass eigenstates in two component notation, $F^\pm_i$ and F^0_i. We also use

$$gW^3_\mu = eA_\mu + \frac{g}{\cos\theta_W}\left(1-\sin^2\theta_W\right)Z_\mu \tag{146}$$

$$g'B_\mu = eA_\mu - \frac{g}{\cos\theta_W}\sin^2\theta_W\,Z_\mu \tag{147}$$

We get in two component notation

$$\begin{aligned}
\mathcal{L} \;=\; & -eA_\mu\Big[\boldsymbol{V}_{ik}\boldsymbol{V}^*{}_{jk}\overline{F^+_i}\,\overline{\sigma}^\mu\,F^+_j - \boldsymbol{U}_{ik}\boldsymbol{U}^*{}_{jk}\overline{F^-_i}\,\overline{\sigma}^\mu\,F^-_j - \overline{\ell'_{Li}}\,\overline{\sigma}^\mu\,\ell'_{Li} + \overline{\ell^c_{Li}}\,\overline{\sigma}^\mu\,\ell'^c_{Li} \\
& \quad +\tfrac{2}{3}\,\overline{u'_{Li}}\,\overline{\sigma}^\mu\,u'_{Li} - \tfrac{1}{3}\,\overline{d'_{Li}}\,\overline{\sigma}^\mu\,d'_{Li} - \tfrac{2}{3}\,\overline{u'^c_{Ri}}\,\overline{\sigma}^\mu\,u'^c_{Ri} + \tfrac{1}{3}\,\overline{d'^c_{Ri}}\,\overline{\sigma}^\mu\,d'^c_{Ri}\Big] \\
& + \frac{g}{\cos\theta_W}Z_\mu\Big[\tfrac{1}{2}\left(\boldsymbol{N}_{i4}\boldsymbol{N}^*{}_{j4} - \boldsymbol{N}_{i3}\boldsymbol{N}^*{}_{j3}\right)\overline{F^0_i}\overline{\sigma}^\mu\,F^0_j \\
& \quad + \left(\tfrac{1}{2}\boldsymbol{U}_{i2}\boldsymbol{U}^*{}_{j2} + \boldsymbol{U}_{i1}\boldsymbol{U}^*{}_{j1} - \sin^2\theta_W\boldsymbol{U}_{ik}\boldsymbol{U}^*{}_{jk}\right)\overline{F^-_i}\,\overline{\sigma}^\mu\,F^-_j \\
& \quad + \left(-\tfrac{1}{2}\boldsymbol{V}_{i2}\boldsymbol{V}^*{}_{j2} - \boldsymbol{V}_{i1}\boldsymbol{V}^*{}_{j1} + \sin^2\theta_W\boldsymbol{V}_{ik}\boldsymbol{V}^*{}_{jk}\right)\overline{F^+_i}\,\overline{\sigma}^\mu\,F^+_j \\
& \quad -\tfrac{1}{2}\overline{\nu'_{Li}}\,\overline{\sigma}^\mu\,\nu'_{Li} + (\tfrac{1}{2} - \sin^2\theta_W)\,\overline{\ell'_{Li}}\,\overline{\sigma}^\mu\,\ell'_{Li} + \sin^2\theta_W\,\overline{\ell^c_{Li}}\,\overline{\sigma}^\mu\,\ell'^c_{Li} \\
& \quad +(-\tfrac{1}{2} + \tfrac{2}{3}\sin^2\theta_W)\overline{u'_{Li}}\,\overline{\sigma}^\mu\,u'_{Li} + (\tfrac{1}{2} - \tfrac{1}{3}\sin^2\theta_W)\,\overline{d'_{Li}}\,\overline{\sigma}^\mu\,d'_{Li} \\
& \quad + \tfrac{2}{3}\sin^2\theta_W\,\overline{u'^c_{Li}}\,\overline{\sigma}^\mu\,u'^c_{Li} - \tfrac{1}{3}\sin^2\theta_W\,\overline{d'^c_{Li}}\,\overline{\sigma}^\mu\,d'^c_{Li}\Big]
\end{aligned} \tag{148}$$

Now using the unitarity of the diagonalization matrices we get, still in two component spinor notation,

$$\mathcal{L} = eA_\mu\Big[\left(\overline{F^-_i}\,\overline{\sigma}^\mu\,F^-_i - \overline{F^+_i}\,\overline{\sigma}^\mu\,F^+_i\right) + \left(\overline{\ell'_{Li}}\,\overline{\sigma}^\mu\,\ell'_{Li} - \overline{\ell'^c_{Li}}\,\overline{\sigma}^\mu\,\ell'^c_{Li}\right)$$

$$- \tfrac{2}{3} \left(\overline{u'_{Li}} \, \overline{\sigma}^\mu \, u'_{Li} - \overline{u'^c_{Li}} \, \overline{\sigma}^\mu \, u'^c_{Li} \right) + \tfrac{1}{3} \left(\overline{d'_{Li}} \, \overline{\sigma}^\mu \, d'_{Li} - \overline{d'^c_{Li}} \, \overline{\sigma}^\mu \, d'^c_{Li} \right) \Big]$$

$$+ \frac{g}{\cos\theta_W} Z_\mu \Big[\tfrac{1}{2} \left(\boldsymbol{N}_{i4} \boldsymbol{N^*}_{j4} - \boldsymbol{N}_{i3} \boldsymbol{N^*}_{j3} \right) \overline{F^0_i} \overline{\sigma}^\mu \, F^0_j$$

$$+ \left(\tfrac{1}{2} \boldsymbol{U}_{i2} \boldsymbol{U^*}_{j2} + \boldsymbol{U}_{i1} \boldsymbol{U^*}_{j1} - \sin^2\theta_W \, \delta_{ij} \right) \overline{F^-_i} \, \overline{\sigma}^\mu \, F^-_j$$

$$+ \left(-\tfrac{1}{2} \boldsymbol{V}_{i2} \boldsymbol{V^*}_{j2} - \boldsymbol{V}_{i1} \boldsymbol{V^*}_{j1} + \sin^2\theta_W \, \delta_{ij} \right) \overline{F^+_i} \, \overline{\sigma}^\mu \, F^+_j$$

$$- \tfrac{1}{2} \overline{\nu'_{Li}} \, \overline{\sigma}^\mu \, \nu'_{Li} + \left(\tfrac{1}{2} - \sin^2\theta_W \right) \overline{\ell'_{Li}} \, \overline{\sigma}^\mu \, \ell'_{Li} + \sin^2\theta_W \, \overline{\ell'^c_{Li}} \, \overline{\sigma}^\mu \, \ell'^c_{Li}$$

$$+ \left(-\tfrac{1}{2} + \tfrac{2}{3} \sin^2\theta_W \right) \overline{u'_{Li}} \, \overline{\sigma}^\mu \, u'_{Li} + \left(\tfrac{1}{2} - \tfrac{1}{3} \sin^2\theta_W \right) \overline{d'_{Li}} \, \overline{\sigma}^\mu \, d'_{Li}$$

$$+ \tfrac{2}{3} \sin^2\theta_W \, \overline{u'^c_{Li}} \, \overline{\sigma}^\mu \, u'^c_{Li} - \tfrac{1}{3} \sin^2\theta_W \, \overline{d'^c_{Li}} \, \overline{\sigma}^\mu \, d'^c_{Li} \Big] \tag{149}$$

Finally we get in four component notation[7]

$$\mathcal{L} = eA_\mu \left[\overline{\chi^-_i} \, \gamma^\mu \, \chi^-_i + \overline{\ell}_i \, \gamma^\mu \, \ell_i - \tfrac{2}{3} \, \overline{u}_i \, \gamma^\mu \, u_i + \tfrac{1}{3} \, \overline{d}_i \, \gamma^\mu \, d_i \right]$$

$$+ \frac{g}{\cos\theta_W} Z_\mu \Big[\tfrac{1}{2} \overline{\chi^0_i} \gamma^\mu \left(O^{L\prime\prime}_{ij} P_L + O^{R\prime\prime}_{ij} P_R \right) \chi^0_j + \overline{\chi^-_i} \, \gamma^\mu \left(O^{L\prime}_{ij} P_L + O^{R\prime}_{ij} P_R \right) \chi^-_j$$

$$+ \tfrac{1}{2} \overline{\nu}_i \, \gamma^\mu \left(-\tfrac{1}{2} P_L + \tfrac{1}{2} P_R \right) \nu_i$$

$$+ \sum_{f=\ell,u,d} \overline{f}_i \, \gamma^\mu \left[\left(-I^f_3 + Q^f \sin^2\theta_W \right) P_L + Q^f \sin^2\theta_W \, P_R \right] f_i \Big] \tag{150}$$

where

$$\begin{aligned} O^{L\prime\prime}_{ij} &= \tfrac{1}{2} \left(\boldsymbol{N}_{i4} \boldsymbol{N^*}_{j4} - \boldsymbol{N}_{i3} \boldsymbol{N^*}_{j3} \right) \\ O^{R\prime\prime}_{ij} &= - \left(O^{L\prime\prime}_{ji} \right)^* \\ O^{L\prime}_{ij} &= \tfrac{1}{2} \boldsymbol{U}_{i2} \boldsymbol{U^*}_{j2} + \boldsymbol{U}_{i1} \boldsymbol{U^*}_{j1} - \sin^2\theta_W \delta_{ij} \\ O^{R\prime}_{ij} &= \tfrac{1}{2} \boldsymbol{V}_{j2} \boldsymbol{V^*}_{i2} + \boldsymbol{V}_{j1} \boldsymbol{V^*}_{i1} - \sin^2\theta_W \delta_{ij} \end{aligned} \tag{151}$$

and I^f_3 and Q^f are, respectively, the weak isospin and the charge of fermion $f = \ell, u, d$.

6.4 3-point gauge boson couplings to scalars

In this section we give the 3-point couplings of gauge bosons with the scalars.

[7] We can see that the neutral current interaction of the leptons and quarks is diagonal in generation space after we perform the rotation into the mass eigenstates.

6.4.1 Gauge boson couplings to sfermions ($V\tilde{f}\tilde{f}$)

We write the Lagrangian as

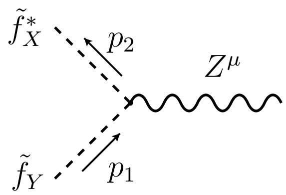
$$\mathcal{L} = i\, g^{XY}_{Z\tilde{f}\tilde{f}}\ \tilde{f}^*_X \overleftrightarrow{\partial}_\mu f_Y Z^\mu + i\, e\, Q_f\ \tilde{f}^*_X \overleftrightarrow{\partial}_\mu f_X A^\mu + \left[i\, g^{XY}_{W\tilde{f}\tilde{f}}\ \tilde{f}'^*_X \overleftrightarrow{\partial}_\mu f_Y W^{-\mu} + \text{ h.c} \right] \tag{152}$$

where in the last term $Q_{\tilde{f}} - Q_{\tilde{f}'} = 1$. We have

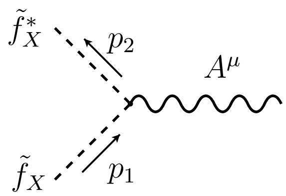
$$g^{XY}_{Z\tilde{f}\tilde{f}} = -\frac{g}{\cos\theta_W} \left[\boldsymbol{R}^{(\tilde{f})}_{Xi}\, \boldsymbol{R}^{(\tilde{f})\,*}_{Yi} \left(T^f_3 - \sin^2\theta_W Q_f \right) + \boldsymbol{R}^{(\tilde{f})}_{X,i+3}\, \boldsymbol{R}^{(\tilde{f})\,*}_{Y,i+3} \left(-\sin^2\theta_W Q_f \right) \right]$$

$$g^{XY}_{W\tilde{f}\tilde{f}} = -\frac{g}{\sqrt{2}} \boldsymbol{R}^{(\tilde{f})}_{Xi}\, \boldsymbol{R}^{(\tilde{f})\,*}_{Yi} \tag{153}$$

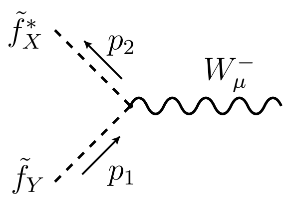
where $i = 1, 2, 3$. This in turn corresponds to the following Feynman rules.



$$i\ (p_1 + p_2)^\mu\ g^{XY}_{Z\tilde{f}\tilde{f}} \tag{154}$$



$$-\, i\ e\, Q_f\ (p_1 + p_2)^\mu \tag{155}$$



$$i\ (p_1 + p_2)_\mu\ g^{XY}_{W\tilde{f}\tilde{f}} \tag{156}$$

6.4.2 Gauge boson couplings to Higgs (VHH)

6.5 4-point gauge boson couplings to scalars

6.5.1 Gauge boson couplings to sfermions ($VV\tilde{f}\tilde{f}$)

6.5.2 Gauge boson couplings to Higgs ($VVHH$)

6.6 Scalar couplings to fermions

6.6.1 Charged scalars couplings to fermions

Using two component spinors and following the notation of ref. [5] the relevant part of the Lagrangian can be written as

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{Yukawa} \tag{157}$$

where the gauge part is determined by the gauge group and quantum numbers of the matter multiplets and is given by

$$\begin{aligned}
\mathcal{L}_{gauge} = H_1^+ & \left[\frac{g}{\sqrt{2}}\left(-i\lambda^3\right)\widetilde{H}_1^- + \frac{g'}{\sqrt{2}}\left(-i\lambda'\right)\widetilde{H}_1^- - g\left(-i\lambda^-\right)\widetilde{H}_1^0\right] \\
& + H_2^+\left[-\frac{g}{\sqrt{2}}\left(i\overline{\lambda^3}\right)\overline{\widetilde{H}_2^+} - \frac{g'}{\sqrt{2}}\left(i\overline{\lambda'}\right)\overline{\widetilde{H}_2^+} - g\left(i\overline{\lambda^+}\right)\overline{\widetilde{H}_2^0}\right] \\
& + \widetilde{\ell}_{Li}^*\left[\frac{g}{\sqrt{2}}\left(-i\lambda^3\right)\ell'_{Li} + \frac{g'}{\sqrt{2}}\left(-i\lambda'\right)\ell'_{Li} - g\left(-i\lambda^-\right)\nu'_{Li}\right] + \widetilde{\ell}_{Ri}\left[-g'\sqrt{2}\left(i\overline{\lambda'}\right)\overline{\ell^c_{Li}}\right] \\
& + \widetilde{u}_{Li}^*\left[-\frac{g}{\sqrt{2}}\left(-i\lambda^3\right)u'_{Li} - \frac{g'}{3\sqrt{2}}\left(-i\lambda'\right)u'_{Li} - g\left(-i\lambda^+\right)d'_{Li}\right] + \widetilde{u}_{Ri}\left[\frac{4g'}{3\sqrt{2}}\left(i\overline{\lambda'}\right)\overline{u^c_{Li}}\right] \\
& + \widetilde{d}_{Li}^*\left[+\frac{g}{\sqrt{2}}\left(-i\lambda^3\right)d'_{Li} - \frac{g'}{3\sqrt{2}}\left(-i\lambda'\right)d'_{Li} - g\left(-i\lambda^-\right)u'_{Li}\right] + \widetilde{d}_{Ri}\left[-\frac{2g'}{3\sqrt{2}}\left(i\overline{\lambda'}\right)\overline{d^c_{Li}}\right] \\
& + \text{h.c.}
\end{aligned} \tag{158}$$

The Yukawa part is derived from the superpotential W by use of Eq. 30. In order to use this equation it is helpful to write down explicitly the superpotential. We have

$$\begin{aligned}
W \;=\; & -\left(h_E\right)_{ij}\widehat{L}_i^1\widehat{R}_j\widehat{H}_1^2 - \left(h_D\right)_{ij}\widehat{Q}_i^1\widehat{D}_j\widehat{H}_1^2 - \left(h_U\right)_{ij}\widehat{Q}_i^2\widehat{U}_j\widehat{H}_2^1 \\
& + \left(h_E\right)_{ij}\widehat{L}_i^2\widehat{R}_j\widehat{H}_1^1 + \left(h_D\right)_{ij}\widehat{Q}_i^2\widehat{D}_j\widehat{H}_1^1 + \left(h_U\right)_{ij}\widehat{Q}_i^1\widehat{U}_j\widehat{H}_2^2
\end{aligned} \tag{159}$$

Then in two component spinor notation the Yukawa part of the couplings of charged scalar to fermions is

$$\begin{aligned}
\mathcal{L}_{Yukawa} \;=\; & -\left(h_E\right)_{ij}\widetilde{\ell}_{Li}\ell'^c_{Lj}\widetilde{H}_1^0 - \left(h_E\right)_{ij}\widetilde{\ell}_{Rj}\ell'_{Li}\widetilde{H}_1^0 + \left(h_E\right)_{ij}\widetilde{\ell}_{Rj}\nu'_{Li}\widetilde{H}_1^- + \left(h_E\right)_{ij}H_1^-\nu'_{Li}\ell'^c_{Lj} \\
& -\left(h_D\right)_{ij}\widetilde{d}_{Li}d'^c_{Lj}\widetilde{H}_1^0 - \left(h_D\right)_{ij}\widetilde{d}_{Rj}d'_{Li}\widetilde{H}_1^0 + \left(h_D\right)_{ij}\widetilde{d}_{Rj}u'_{Li}\widetilde{H}_1^- + \left(h_D\right)_{ij}H_1^-u'_{Li}d'^c_{Lj} \\
& -\left(h_U\right)_{ij}\widetilde{u}_{Li}u'^c_{Lj}\widetilde{H}_2^0 - \left(h_U\right)_{ij}\widetilde{u}_{Rj}u_{Li}\widetilde{H}_2^0 + \left(h_U\right)_{ij}\widetilde{u}_{Rj}d_{Li}\widetilde{H}_2^+ + \left(h_U\right)_{ij}H_2^+d'_{Li}u'^c_{Lj} \\
& +\left(h_D\right)_{ij}\widetilde{u}_{Li}d'^c_{Lj}\widetilde{H}_1^- + \left(h_U\right)_{ij}\widetilde{d}_{Li}u'^c_{Lj}\widetilde{H}_2^+ \\
& +\text{h.c.}
\end{aligned} \tag{160}$$

Now this can be written in four component spinor notation in terms of the mass eigenstates. We will do this separately for $\mathcal{L}_{gauge}$ and $\mathcal{L}_{Yukawa}$. We obtain[8]

$$\begin{aligned}
\mathcal{L}_{gauge} \;=\; & H_1^-\left[\left(\frac{g}{\sqrt{2}}\boldsymbol{U}_{A2}\left(\boldsymbol{N}_{B2} + \tan\theta_W\boldsymbol{N}_{B1}\right) - g\,\boldsymbol{U}_{A1}\boldsymbol{N}_{B3}\right)\overline{\chi^-}_A P_R\chi_B^0\right] \\
+ \; & H_2^-\left[\left(-\frac{g}{\sqrt{2}}\boldsymbol{V}^*_{A2}\left(\boldsymbol{N}^*_{B2} + \tan\theta_W\boldsymbol{N}^*_{B1}\right) - g\,\boldsymbol{V}^*_{A1}\boldsymbol{N}^*_{B4}\right)\overline{\chi^-}_A P_L\chi_B^0\right]
\end{aligned}$$

[8]We still keep the unrotated fields for the leptons and quarks. The final rotation will be done in Section 6.6.3.

$$
\begin{aligned}
&+ \quad \boldsymbol{R}^{\widetilde{\boldsymbol{\ell}}}{}_{X,i}\ \widetilde{\ell}^*_X \left[\frac{g}{\sqrt{2}}\ (\boldsymbol{N}^*_{A2}+\tan\theta_W \boldsymbol{N}^*_{A1})\ \overline{\chi^0}_A P_L \ell'_i - g\,\boldsymbol{U}^*_{B1}\ \overline{\chi^c_B} P_L \nu'_i\right] \\
&+ \quad \boldsymbol{R}^{\widetilde{\boldsymbol{\ell}}}{}_{X,i+3}\ \widetilde{\ell}^*_X \left[-g\sqrt{2}\tan\theta_W\ \boldsymbol{N}_{A1}\ \overline{\chi^0}_A P_R \ell'_i\right] \\
&+ \quad \boldsymbol{R}^{\widetilde{\boldsymbol{u}}}{}_{X,i}\ \widetilde{u}^*_X \left[-\frac{g}{\sqrt{2}}\ \left(\boldsymbol{N}^*_{A2}+\tfrac{1}{3}\tan\theta_W \boldsymbol{N}^*_{A1}\right)\ \overline{\chi^0}_A P_L u'_i - g\,\boldsymbol{V}^*_{B1}\ \overline{\chi^-_B} P_L d'_i\right] \\
&+ \quad \boldsymbol{R}^{\widetilde{\boldsymbol{u}}}{}_{X,i+3}\ \widetilde{u}^*_X \left[\frac{4}{3}\frac{g}{\sqrt{2}}\tan\theta_W\ \boldsymbol{N}_{A1}\ \overline{\chi^0}_A P_R u'_i\right] \\
&+ \quad \boldsymbol{R}^{\widetilde{\boldsymbol{d}}}{}_{X,i}\ \widetilde{d}^*_X \left[\frac{g}{\sqrt{2}}\ \left(\boldsymbol{N}^*_{A2}-\tfrac{1}{3}\tan\theta_W \boldsymbol{N}^*_{A1}\right)\ \overline{\chi^0}_A P_L d'_i - g\,\boldsymbol{U}^*_{B1}\ \overline{\chi^c}_B P_L u'_i\right] \\
&+ \quad \boldsymbol{R}^{\widetilde{\boldsymbol{d}}}{}_{X,i+3}\ \widetilde{d}^*_X \left[-\frac{2}{3}\frac{g}{\sqrt{2}}\tan\theta_W\ \boldsymbol{N}_{A1}\ \overline{\chi^0}_A P_R d'_i\right] \\
&+ \quad \text{h.c.}
\end{aligned}
\tag{161}
$$

and

$$
\begin{aligned}
\mathcal{L}_{Yukawa} \quad = \quad & -(h_E)_{ij}\,\boldsymbol{N}^*_{A3}\left(\boldsymbol{R}^{\widetilde{\boldsymbol{\ell}}\,*}_{X,i}\,\widetilde{\ell}_X\ \overline{\ell'}_j P_L \chi^0_A + \boldsymbol{R}^{\widetilde{\boldsymbol{\ell}}}{}_{X,j+3}\,\widetilde{\ell}^*_X\ \overline{\chi^0}_A P_L \ell'_i\right) \\
& + (h_E)_{ij}\left(\boldsymbol{U}^*_{A2}\,\boldsymbol{R}^{\widetilde{\boldsymbol{\ell}}}{}_{X,j+3}\,\widetilde{\ell}^*_X\ \overline{\chi^c_A} P_L \nu'_i + H^-_1\ \overline{\ell'}_j P_L \nu'_i\right) \\
& - (h_D)_{ij}\,\boldsymbol{N}^*_{A3}\left(\boldsymbol{R}^{\widetilde{\boldsymbol{d}}\,*}_{X,i}\,\widetilde{d}_X\ \overline{d'}_j P_L \chi^0_A + \boldsymbol{R}^{\widetilde{\boldsymbol{d}}}{}_{X,j+3}\,\widetilde{d}^*_X\ \overline{\chi^0}_A P_L d'_i\right) \\
& + (h_D)_{ij}\left(\boldsymbol{U}^*_{A2}\,\boldsymbol{R}^{\widetilde{\boldsymbol{d}}}{}_{X,j+3}\,\widetilde{d}^*_X\ \overline{\chi^c}_A P_L u'_i + H^-_1\ \overline{d'}_j P_L u'_i\right) \\
& - (h_U)_{ij}\,\boldsymbol{N}^*_{A4}\left(\boldsymbol{R}^{\widetilde{\boldsymbol{u}}\,*}_{X,i}\,\widetilde{u}_X\ \overline{u'}_j P_L \chi^0_A + \boldsymbol{R}^{\widetilde{\boldsymbol{u}}}{}_{X,j+3}\,\widetilde{u}^*_X\ \overline{\chi^0}_A P_L u'_i\right) \\
& + (h_U)_{ij}\left(\boldsymbol{V}^*_{A2}\,\boldsymbol{R}^{\widetilde{\boldsymbol{u}}}{}_{X,j+3}\,\widetilde{u}^*_X\ \overline{\chi^-}_A P_L d'_i + H^+_2\ \overline{u'}_j P_L d'_i\right) \\
& + (h_D)_{ij}\ \ \boldsymbol{U}^*_{A2}\,\boldsymbol{R}^{\widetilde{\boldsymbol{u}}\,*}_{X,i}\,\widetilde{u}_X\ \overline{d'}_j P_L \chi^-_A + (h_U)_{ij}\ \ \boldsymbol{V}^*_{A2}\,\boldsymbol{R}^{\widetilde{\boldsymbol{d}}\,*}_{X,i}\,\widetilde{d}_X\ \overline{u'}_j P_L \chi^c_A \\
& + \text{h.c.}
\end{aligned}
\tag{162}
$$

In Section 6.6.3 we will give the final formulas.

6.6.2 Neutral scalars couplings to fermions

Using two component spinors and following the notation of ref. [5] the relevant part of the Lagrangian can be written as

$$
\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{Yukawa} \tag{163}
$$

where the gauge part is

$$
\mathcal{L}_{gauge} = \left(H^0_1\right)^* \left[-\frac{g}{\sqrt{2}}\,(-i\lambda^3)\,\widetilde{H}^0_1 + \frac{g'}{\sqrt{2}}\,(-i\lambda')\,\widetilde{H}^0_1 - g\,(-i\lambda^+)\,\widetilde{H}^-_1\right]
$$

$$
\begin{aligned}
&+ \left(H_2^0\right)^* \left[\frac{g}{\sqrt{2}} \left(-i\lambda^3\right) \widetilde{H}_2^0 - \frac{g'}{\sqrt{2}} \left(-i\lambda'\right) \widetilde{H}_2^0 - g \left(-i\lambda^-\right) \widetilde{H}_2^+\right] \\
&+ \widetilde{\nu}_i^* \left[-\frac{g}{\sqrt{2}} \left(-i\lambda^3\right) \nu_i + \frac{g'}{\sqrt{2}} \left(-i\lambda'\right) \nu_i - g \left(-i\lambda^+\right) \ell_i^-\right] \\
&+ \text{h.c.}
\end{aligned}
\tag{164}
$$

and the Yukawa part is

$$
\begin{aligned}
\mathcal{L}_{Yukawa} =& \left(h_E\right)_{ij} \widetilde{\nu'}_{Li} \ell'^c_{Lj} \widetilde{H}_1^- - \left(h_E\right)_{ij} H_1^0 \ell'_{Li} \ell'^c_{Lj} - \left(h_D\right)_{ij} H_1^0 d'_{Li} d'^c_{Lj} - \left(h_U\right)_{ij} H_2^0 u'_{Li} u'^c_{Lj} \\
&+ \text{h.c.}
\end{aligned}
\tag{165}
$$

Now this can be written in four component spinor notation in terms of the mass eigenstates. We will do this separately for $\mathcal{L}_{gauge}$ and $\mathcal{L}_{Yukawa}$. We obtain

$$
\begin{aligned}
\mathcal{L}_{gauge} =& \left(H_1^0\right)^* \left[-\frac{g}{\sqrt{2}} \left(\boldsymbol{N}^*_{A2} - \tan\theta_W \boldsymbol{N}^*_{A1}\right) \boldsymbol{N}^*_{B3}\ \overline{\chi^0}_A P_L \chi^0_B - g \boldsymbol{V}^*_{A1} \boldsymbol{U}^*_{B2}\ \overline{\chi^-}_A P_L \chi^-_B\right] \\
&+ \left(H_2^0\right)^* \left[\frac{g}{\sqrt{2}} \left(\boldsymbol{N}^*_{A2} - \tan\theta_W \boldsymbol{N}^*_{A1}\right) \boldsymbol{N}^*_{B4}\ \overline{\chi^0}_A P_L \chi^0_B - g \boldsymbol{V}^*_{A2} \boldsymbol{U}^*_{B1}\ \overline{\chi^-}_A P_L \chi^-_B\right] \\
&+ \boldsymbol{R}^{\widetilde{\boldsymbol{\nu}}}{}_{X,i}\, \widetilde{\nu}^*_X \left[-\frac{g}{\sqrt{2}} \left(\boldsymbol{N}^*_{A2} - \tan\theta_W \boldsymbol{N}^*_{A1}\right)\ \overline{\chi^0}_A P_L \nu'_i - g \boldsymbol{V}^*_{A1}\ \overline{\chi^-}_A P_L \ell'_i\right] \\
&+ \text{h.c.}
\end{aligned}
\tag{166}
$$

and

$$
\begin{aligned}
\mathcal{L}_{Yukawa} =& -\left(h_E\right)_{ij} H_1^0\, \overline{\ell'}_j P_L \ell'_i - \left(h_D\right)_{ij} H_1^0\, \overline{d'}_j P_L d'_i - \left(h_U\right)_{ij} H_2^0\, \overline{u'}_j P_L u'_i \\
&+ \left(h_E\right)_{ij} \boldsymbol{U}^*_{A2}\, \boldsymbol{R}^{\widetilde{\boldsymbol{\nu}}*}{}_{X,i}\, \widetilde{\nu}_X\, \overline{\ell'}_j P_L \chi^-_A \\
&+ \text{h.c.}
\end{aligned}
\tag{167}
$$

In Section 6.6.3 we will give the final formulas.

6.6.3 Final formulas for the scalar couplings to fermions

In this Section we present the final formulas for the scalar couplings to fermions. To be more precise we are going to only to write down those for the couplings *fermion–sfermion–chargino* and *fermion–sfermion–neutralino*. These leave out the couplings of the neutral and charged Higgs bosons with the fermions. These can be read from Eqs. (161), (162), (166) and (167) not forgetting that we still have to perform the rotation into the Higgs bosons mass eigenstates.
We interaction lagrangian can be written as

$$
\mathcal{L} = \widetilde{\nu}_X\ \overline{\ell}_i \left[C^{L(\ell)}_{iAX} P_L + C^{R(\ell)}_{iAX} P_R\right] \chi^-_A + \widetilde{\ell}_X\ \overline{\nu_i} \left[C^{L(\nu)}_{iAX} P_L + C^{R(\nu)}_{iAX} P_R\right] \chi^c_A
$$

$$
\begin{aligned}
&+\quad \widetilde{u}_X\ \overline{d}_i \left[C^{L(d)}_{iAX} P_L + C^{R(d)}_{iAX} P_R\right] \chi^-_A + \widetilde{d}_X\ \overline{u}_i \left[C^{L(u)}_{iAX} P_L + C^{R(u)}_{iAX} P_R\right] \chi^c_A \\
&+\quad \widetilde{f}_X\ \overline{f}_i \left[N^{L(f)}_{iAX} P_L + N^{R(f)}_{iAX} P_R\right] \chi^0_A \\
&+\quad \text{h.c.}
\end{aligned}
\tag{168}
$$

where the indices i, A, X apply to generations, charginos (or neutralinos) and sfermions, respectively. All repeated indices are summed over and $f = \ell, \nu, u, d$. The coefficients C and N can be read from Eqs. (161), (162), (166) and (167). We list them below.

$$
\left\{
\begin{aligned}
C^{L(\ell)}_{iAX} &= (h_E)_{kj}\ \boldsymbol{U}^*_{A2}\, \boldsymbol{R^{\widetilde{\nu}}}^*_{X,k}\, \boldsymbol{R^{\ell}_{R}}_{ij} \\
C^{R(\ell)}_{iAX} &= -g\boldsymbol{V}_{A1}\, \boldsymbol{R^{\widetilde{\nu}}}^*_{X,k}\, \boldsymbol{R^{\ell}_{L}}_{ik}
\end{aligned}
\right.
\tag{169}
$$

$$
\left\{
\begin{aligned}
C^{L(\nu)}_{iAX} &= 0 \\
C^{R(\nu)}_{iAX} &= -g\boldsymbol{U}_{A1}\, \boldsymbol{R^{\widetilde{\ell}}}^*_{X,k}\, \boldsymbol{R^{\nu}_{L}}_{ik} + (h_E)^*_{kj}\ \boldsymbol{U}_{A2}\, \boldsymbol{R^{\widetilde{\ell}}}^*_{X,j+3}\, \boldsymbol{R^{\nu}_{L}}_{ik}
\end{aligned}
\right.
\tag{170}
$$

$$
\left\{
\begin{aligned}
C^{L(d)}_{iAX} &= (h_D)_{kj}\ \boldsymbol{U}^*_{A2}\, \boldsymbol{R^{\widetilde{u}}}^*_{X,k}\, \boldsymbol{R^{d}_{R}}_{ij} \\
C^{R(d)}_{iAX} &= -g\boldsymbol{V}_{A1}\, \boldsymbol{R^{\widetilde{u}}}^*_{X,k}\, \boldsymbol{R^{d}_{L}}_{ik} + (h^*_U)_{kj}\ \boldsymbol{V}_{A2}\, \boldsymbol{R^{\widetilde{u}}}^*_{X,j+3}\, \boldsymbol{R^{d}_{L}}_{ik}
\end{aligned}
\right.
\tag{171}
$$

$$
\left\{
\begin{aligned}
C^{L(u)}_{iAX} &= (h_U)_{kj}\ \boldsymbol{V}^*_{A2}\, \boldsymbol{R^{\widetilde{d}}}^*_{X,k}\, \boldsymbol{R^{u}_{R}}_{ij} \\
C^{R(u)}_{iAX} &= -g\boldsymbol{U}_{A1}\, \boldsymbol{R^{\widetilde{d}}}^*_{X,k}\, \boldsymbol{R^{u}_{L}}_{ik} + (h^*_D)_{kj}\ \boldsymbol{U}_{A2}\, \boldsymbol{R^{\widetilde{d}}}^*_{X,j+3}\, \boldsymbol{R^{u}_{L}}_{ik}
\end{aligned}
\right.
\tag{172}
$$

and for the $N's$

$$
\left\{
\begin{aligned}
N^{L(\ell)}_{iAX} &= -g\sqrt{2}\tan\theta_W\ \boldsymbol{N}^*_{A1}\, \boldsymbol{R^{\widetilde{\ell}}}^*_{X,k+3}\, \boldsymbol{R^{\ell}_{R}}_{ik}\ - (h_E)_{kj}\ \boldsymbol{N}^*_{A3}\, \boldsymbol{R^{\widetilde{\ell}}}^*_{X,k}\, \boldsymbol{R^{\ell}_{R}}_{ij} \\
N^{R(\ell)}_{iAX} &= \frac{g}{\sqrt{2}}\left(\boldsymbol{N}_{A2} + \tan\theta_W \boldsymbol{N}_{A1}\right) \boldsymbol{R^{\widetilde{\ell}}}^*_{X,k}\, \boldsymbol{R^{\ell}_{L}}_{ik}\ - (h^*_E)_{kj}\ \boldsymbol{N}_{A3}\, \boldsymbol{R^{\widetilde{\ell}}}^*_{X,j+3}\, \boldsymbol{R^{\ell}_{L}}_{ik}
\end{aligned}
\right.
\tag{173}
$$

$$
\left\{
\begin{aligned}
N^{L(\nu)}_{iAX} &= 0 \\
N^{R(\nu)}_{iAX} &= -\frac{g}{\sqrt{2}}\left(\boldsymbol{N}_{A2} - \tan\theta_W \boldsymbol{N}_{A1}\right) \boldsymbol{R^{\widetilde{\nu}}}^*_{X,k}\, \boldsymbol{R^{\nu}_{L}}_{ik}
\end{aligned}
\right.
\tag{174}
$$

$$
\left\{
\begin{aligned}
N^{L(u)}_{iAX} &= \frac{4}{3}\,\frac{g}{\sqrt{2}}\ \tan\theta_W\ \boldsymbol{N}^*_{A1}\, \boldsymbol{R^{\widetilde{u}}}^*_{X,k+3}\, \boldsymbol{R^{u}_{R}}_{ik} - (h_U)_{kj}\ \boldsymbol{N}^*_{A4}\, \boldsymbol{R^{\widetilde{u}}}^*_{X,k}\, \boldsymbol{R^{u}_{R}}_{ij} \\
N^{R(u)}_{iAX} &= -\frac{g}{\sqrt{2}}\left(\boldsymbol{N}_{A2} + \tfrac{1}{3}\tan\theta_W \boldsymbol{N}_{A1}\right) \boldsymbol{R^{\widetilde{u}}}^*_{X,k}\, \boldsymbol{R^{u}_{L}}_{ik}\ - (h^*_U)_{kj}\ \boldsymbol{N}_{A4}\, \boldsymbol{R^{\widetilde{u}}}^*_{X,j+3}\, \boldsymbol{R^{u}_{L}}_{ik}
\end{aligned}
\right.
\tag{175}
$$

$$
\left\{
\begin{aligned}
N^{L(d)}_{iAX} &= -\frac{2}{3}\,\frac{g}{\sqrt{2}}\ \tan\theta_W\ \boldsymbol{N}^*_{A1}\, \boldsymbol{R^{\widetilde{d}}}^*_{X,k+3}\, \boldsymbol{R^{d}_{R}}_{ik} - (h_D)_{kj}\ \boldsymbol{N}^*_{A3}\, \boldsymbol{R^{\widetilde{d}}}^*_{X,k}\, \boldsymbol{R^{d}_{R}}_{ij} \\
N^{R(d)}_{iAX} &= \frac{g}{\sqrt{2}}\left(\boldsymbol{N}_{A2} - \tfrac{1}{3}\tan\theta_W \boldsymbol{N}_{A1}\right) \boldsymbol{R^{\widetilde{d}}}^*_{X,k}\, \boldsymbol{R^{d}_{L}}_{ik}\ - (h^*_D)_{kj}\ \boldsymbol{N}_{A3}\, \boldsymbol{R^{\widetilde{d}}}^*_{X,j+3}\, \boldsymbol{R^{d}_{L}}_{ik}
\end{aligned}
\right.
\tag{176}
$$

6.6.4 Final formulas for the Higgs couplings to SUSY fermions

In this Section we present the final formulas for the Higgs couplings to supersymmetric fermions (charginos and neutralinos). These can be read from Eqs. (161), (162), (166) and (167).
We interaction lagrangian can be written as

$$\begin{aligned}\mathcal{L} =& \overline{\chi^-}_A \left[D^{L(S^+)}_{ABi} P_L + D^{R(S^+)}_{ABi} P_R\right] \chi^0_B\ S^-_i + \text{ h.c.}\\ &+ \frac{1}{2}\overline{\chi^0}_A \left[E^{L(S^0)}_{ABi} P_L + E^{R(S^0)}_{ABi} P_R\right] \chi^0_B\ S^0_i + \overline{\chi^-}_A \left[F^{L(S^0)}_{ABi} P_L + F^{R(S^0)}_{ABi} P_R\right] \chi^-_B\ S^0_i\\ &+ \frac{1}{2}\overline{\chi^0}_A \left[E^{L(P^0)}_{ABi} P_L + E^{R(P^0)}_{ABi} P_R\right] \chi^0_B\ P^0_i + \overline{\chi^-}_A \left[F^{L(P^0)}_{ABi} P_L + F^{R(P^0)}_{ABi} P_R\right] \chi^-_B\ P^0_i \end{aligned} \quad (177)$$

where the indices i, A, B apply to the Higgs, and charginos (or neutralinos), respectively. All repeated indices are summed over. The coefficients D, E and F can be read from Eqs. (161), (162), (166) and (167). We list them below [9].

$$\left\{\begin{aligned} D^{L(S^+)}_{ABi} &= \left[-\tfrac{g}{\sqrt{2}}\boldsymbol{V}^*_{A2}\left(\boldsymbol{N}^*_{B2} + \tan\theta_W \boldsymbol{N}^*_{B1}\right)\ - g\,\boldsymbol{V}^*_{A1}\boldsymbol{N}^*_{B4}\right]\boldsymbol{R}^{(S^+)\,*}_{i2}\\ D^{R(S^+)}_{ABi} &= \left[\tfrac{g}{\sqrt{2}}\boldsymbol{U}_{A2}\left(\boldsymbol{N}_{B2} + \tan\theta_W \boldsymbol{N}_{B1}\right)\ - g\,\boldsymbol{U}_{A1}\boldsymbol{N}_{B3}\right]\boldsymbol{R}^{(S^+)\,*}_{i1}\end{aligned}\right. \quad (178)$$

$$\left\{\begin{aligned} E^{L(S^0)}_{ABi} =&\ \tfrac{1}{2}\left[-g\boldsymbol{N}^*_{A2}\boldsymbol{N}^*_{B3} + g'\boldsymbol{N}^*_{A1}\boldsymbol{N}^*_{B3} - g\boldsymbol{N}^*_{B2}\boldsymbol{N}^*_{A3} + g'\boldsymbol{N}^*_{B1}\boldsymbol{N}^*_{A3}\right]\boldsymbol{R}^{(S^0)}_{i1}\\ &+\tfrac{1}{2}\left[+g\boldsymbol{N}^*_{A2}\boldsymbol{N}^*_{B4} - g'\boldsymbol{N}^*_{A1}\boldsymbol{N}^*_{B4} + g\boldsymbol{N}^*_{B2}\boldsymbol{N}^*_{A4} - g'\boldsymbol{N}^*_{B1}\boldsymbol{N}^*_{A4}\right]\boldsymbol{R}^{(S^0)}_{i2}\\ E^{R(S^0)}_{ABi} =&\ \left(E^{L(S^0)}_{ABi}\right)^*\end{aligned}\right. \quad (179)$$

$$\left\{\begin{aligned} E^{L(P^0)}_{ABi} =&\ -\tfrac{i}{2}\left[-g\boldsymbol{N}^*_{A2}\boldsymbol{N}^*_{B3} + g'\boldsymbol{N}^*_{A1}\boldsymbol{N}^*_{B3} - g\boldsymbol{N}^*_{B2}\boldsymbol{N}^*_{A3} + g'\boldsymbol{N}^*_{B1}\boldsymbol{N}^*_{A3}\right]\boldsymbol{R}^{(P^0)}_{i1}\\ &-\tfrac{i}{2}\left[+g\boldsymbol{N}^*_{A2}\boldsymbol{N}^*_{B4} - g'\boldsymbol{N}^*_{A1}\boldsymbol{N}^*_{B4} + g\boldsymbol{N}^*_{B2}\boldsymbol{N}^*_{A4} - g'\boldsymbol{N}^*_{B1}\boldsymbol{N}^*_{A4}\right]\boldsymbol{R}^{(P^0)}_{i2}\\ E^{R(P^0)}_{ABi} =&\ \left(E^{L(P^0)}_{ABi}\right)^*\end{aligned}\right. \quad (180)$$

$$\left\{\begin{aligned} F^{L(S^0)}_{ABi} &= -\tfrac{g}{\sqrt{2}}\left(\boldsymbol{V}^*_{A1}\boldsymbol{U}^*_{B2}\boldsymbol{R}^{(S^0)}_{i1} + \boldsymbol{V}^*_{A2}\boldsymbol{U}^*_{B1}\boldsymbol{R}^{(S^0)}_{i2}\right)\\ F^{R(S^0)}_{ABi} &= -\tfrac{g}{\sqrt{2}}\left(\boldsymbol{V}_{B1}\boldsymbol{U}_{A2}\boldsymbol{R}^{(S^0)}_{i1} + \boldsymbol{V}_{B2}\boldsymbol{U}_{A1}\boldsymbol{R}^{(S^0)}_{i2}\right)\end{aligned}\right. \quad (181)$$

[9]The rotation matrices $\boldsymbol{R}^{(S^0)}$ and $\boldsymbol{R}^{(P^0)}$ are real orthogonal matrices. However `SPheno`[9] considers $\boldsymbol{R}^{(S^+)}$ to be in general complex, so we follow this convention.

$$
\begin{cases}
F^{L(P^0)}_{ABi} = i\ \frac{g}{\sqrt{2}} \left(\boldsymbol{V}^*_{A1} \boldsymbol{U}^*_{B2} \boldsymbol{R}^{(P^0)}_{i1} + \boldsymbol{V}^*_{A2} \boldsymbol{U}^*_{B1} \boldsymbol{R}^{(P^0)}_{i2} \right) \\
F^{R(P^0)}_{ABi} = -i\ \frac{g}{\sqrt{2}} \left(\boldsymbol{V}_{B1} \boldsymbol{U}_{A2} \boldsymbol{R}^{(P^0)}_{i1} + \boldsymbol{V}_{B2} \boldsymbol{U}_{A1} \boldsymbol{R}^{(P^0)}_{i2} \right)
\end{cases}
\tag{182}
$$

6.6.5 Final formulas for the Higgs couplings to SM fermions

In this Section we present the final formulas for the Higgs couplings to Standard Model fermions. These can be read from Eqs. (162) and (167).
We write the interaction lagrangian as [10]

$$
\begin{aligned}
\mathcal{L} =& S^-_i\ \overline{\ell}_j \left(G^{L(\ell\nu)}_{ijk} P_L + G^{R(\ell\nu)}_{ijk} P_R \right) \nu_k + S^-_i\ \overline{d}_j \left(G^{L(qq')}_{ijk} P_L + G^{R(qq')}_{ijk} P_R \right) u_k \\
&+ \overline{\ell}_i \left(H^{L(\ell)}_{ijk} P_L + H^{R(\ell)}_{ijk} P_R \right) \ell_i\ S^0_i k \\
&+ \overline{u}_i \left(H^{L(u)}_{ijk} P_L + H^{R(u)}_{ijk} P_R \right) u_j\ S^0_k + \overline{d}_i \left(H^{L(d)}_{ijk} P_L + H^{R(d)}_{ijk} P_R \right) d_j\ S^0_k \\
&+ \overline{\ell}_i \left(I^{L(\ell)}_{ijk} P_L + I^{R(\ell)}_{ijk} P_R \right) \ell_j\ P^0_k \\
&+ \overline{u}_i \left(I^{L(u)}_{ijk} P_L + I^{R(u)}_{ijk} P_R \right) u_j\ P^0_k + \overline{d}_i \left(I^{L(d)}_{ijk} P_L + I^{R(d)}_{ijk} P_R \right) d_j\ P^0_k
\end{aligned}
\tag{183}
$$

The coefficients G, H and I can be read from Eqs. (162) and (167). We list them below.

$$
\begin{cases}
G^{L(qq')}_{ijk} = (h_D)_{k'j'} \boldsymbol{R}^{(S^+)\,*}_{i1}\ \boldsymbol{R_L}^{(u)\,*}_{kk'}\ \boldsymbol{R_R}^{(d)}_{jj'} \\
G^{R(qq')}_{ijk} = (h^*_U)_{j'k'} \boldsymbol{R}^{(S^+)\,*}_{i2}\ \boldsymbol{R_R}^{(u)\,*}_{kk'}\ \boldsymbol{R_L}^{(d)}_{jj'}
\end{cases}
\tag{184}
$$

$$
\begin{cases}
G^{L(\ell\nu')}_{ijk} = (h_E)_{k'j'} \boldsymbol{R}^{(S^+)\,*}_{i1}\ \boldsymbol{R_L}^{(\ell)\,*}_{kk'}\ \boldsymbol{R_R}^{(\ell)}_{jj'} \\
G^{R(\ell\nu)}_{ijk} = 0
\end{cases}
\tag{185}
$$

$$
\begin{cases}
H^{L(\ell)}_{ijk} = -\frac{1}{\sqrt{2}} (h_E)_{j'i'} \boldsymbol{R}^{(S^0)}_{k1}\ \boldsymbol{R_L}^{(\ell)\,*}_{jj'}\ \boldsymbol{R_R}^{(\ell)}_{ii'} \\
H^{R(\ell)}_{ijk} = \left(H^{L(\ell)}_{ijk} \right)^*
\end{cases}
\tag{186}
$$

$$
\begin{cases}
H^{L(d)}_{ijk} = -\frac{1}{\sqrt{2}} (h_D)_{j'i'} \boldsymbol{R}^{(S^0)}_{k1}\ \boldsymbol{R_L}^{(d)\,*}_{jj'}\ \boldsymbol{R_R}^{(d)}_{ii'} \\
H^{R(d)}_{ijk} = \left(H^{L(d)}_{ijk} \right)^*
\end{cases}
\tag{187}
$$

[10] The order of the fields was chosen to be in agreement with `SPheno`[9] conventions.

$$\begin{cases} H^{L(u)}_{ijk} = -\frac{1}{\sqrt{2}}(h_U)_{j'i'}\boldsymbol{R}^{(S^0)}_{k1}\ \boldsymbol{R_L}^{(u)\,*}_{jj'}\ \boldsymbol{R_R}^{(u)}_{ii'} \\ H^{R(u)}_{ijk} = \left(H^{L(u)}_{ijk}\right)^* \end{cases} \tag{188}$$

$$\begin{cases} I^{L(\ell)}_{ijk} = -i\ \frac{1}{\sqrt{2}}(h_E)_{j'i'}\boldsymbol{R}^{(S^0)}_{k1}\ \boldsymbol{R_L}^{(\ell)\,*}_{jj'}\ \boldsymbol{R_R}^{(\ell)}_{ii'} \\ I^{R(\ell)}_{ijk} = \left(I^{L(\ell)}_{ijk}\right)^* \end{cases} \tag{189}$$

$$\begin{cases} I^{L(d)}_{ijk} = -i\ \frac{1}{\sqrt{2}}(h_D)_{j'i'}\boldsymbol{R}^{(S^0)}_{k1}\ \boldsymbol{R_L}^{(d)\,*}_{jj'}\ \boldsymbol{R_R}^{(d)}_{ii'} \\ I^{R(d)}_{ijk} = \left(I^{L(d)}_{ijk}\right)^* \end{cases} \tag{190}$$

$$\begin{cases} I^{L(u)}_{ijk} = -i\ \frac{1}{\sqrt{2}}(h_U)_{j'i'}\boldsymbol{R}^{(S^0)}_{k1}\ \boldsymbol{R_L}^{(u)\,*}_{jj'}\ \boldsymbol{R_R}^{(u)}_{ii'} \\ I^{R(u)}_{ijk} = \left(I^{L(u)}_{ijk}\right)^* \end{cases} \tag{191}$$

6.7 Trilinear scalar couplings with Higgs bosons

In this section we will show the trilinear scalar couplings envolving the Higgs boson and the corresponding Goldstone bosons. The Higgs self interactions will be left for section 6.11.

6.7.1 Higgs − Sfermion − Sfermion

We write the lagrangian as

$$\begin{aligned} \mathcal{L} =& g^{(S^+\tilde{d}\tilde{u}^*)}_{ijk}\ S^+_i\tilde{d}_j\tilde{u}^*_k + g^{(S^+\tilde{\ell}\tilde{\nu}^*)}_{ijk}\ S^+_i\tilde{\ell}_j\tilde{\nu}^*_k + \text{ h.c.} \\ &+ g^{(S^0\tilde{d}\tilde{d}^*)}_{ijk}\ S^0_i\tilde{d}_j\tilde{d}^*_k + g^{(S^0\tilde{u}\tilde{u}^*)}_{ijk}\ S^0_i\tilde{u}_j\tilde{u}^*_k + g^{(S^0\tilde{\ell}\tilde{\ell}^*)}_{ijk}\ S^0_i\tilde{\ell}_j\tilde{\ell}^*_k + g^{(S^0\tilde{\nu}\tilde{\nu}^*)}_{ijk}\ S^0_i\tilde{\nu}_j\tilde{\nu}^*_k \\ &+ g^{(P^0\tilde{d}\tilde{d}^*)}_{ijk}\ P^0_i\tilde{d}_j\tilde{d}^*_k + g^{(P^0\tilde{u}\tilde{u}^*)}_{ijk}\ P^0_i\tilde{u}_j\tilde{u}^*_k + g^{(P^0\tilde{\ell}\tilde{\ell}^*)}_{ijk}\ P^0_i\tilde{\ell}_j\tilde{\ell}^*_k + g^{(P^0\tilde{\nu}\tilde{\nu}^*)}_{ijk}\ P^0_i\tilde{\nu}_j\tilde{\nu}^*_k \end{aligned} \tag{192}$$

We list below these couplings.

$$\begin{aligned} g^{(S^+\tilde{d}\tilde{u}^*)}_{ijk} =& \left[\frac{v_d}{\sqrt{2}}\left((h_D h_D^\dagger)_{j'k'} - \frac{g^2}{2}\delta_{j'k'}\right)\boldsymbol{R}^{(S^\pm)\,*}_{i1} + \frac{v_u}{\sqrt{2}}\left((h_U h_U^\dagger)_{j'k'} - \frac{g^2}{2}\delta_{j'k'}\right)\boldsymbol{R}^{(S^\pm)\,*}_{i2}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{jj'}\boldsymbol{R}^{(\tilde{u})}_{kk'} \\ &+ \left[\mu^*(h_U)_{j'k'}\boldsymbol{R}^{(S^\pm)\,*}_{i1} + (A_U)_{j'k'}\boldsymbol{R}^{(S^\pm)\,*}_{i2}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{jj'}\boldsymbol{R}^{(\tilde{u})}_{k,k'+3} \\ &+ \left[\mu(h_D^*)_{k'j'}\boldsymbol{R}^{(S^\pm)\,*}_{i2} + (A_D^*)_{k'j'}\boldsymbol{R}^{(S^\pm)\,*}_{i1}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde{u})}_{kk'} \end{aligned}$$

$$+\left[\frac{v_u}{\sqrt{2}}\boldsymbol{R}^{(S^\pm)\,*}_{i1}+\frac{v_d}{\sqrt{2}}\boldsymbol{R}^{(S^\pm)\,*}_{i2}\right](h_D^\dagger h_U)_{j'k'}\ \boldsymbol{R}^{(\tilde d)\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde u)}_{k,k'+3} \tag{193}$$

$$g^{(S^+\tilde\ell\tilde\nu^*)}_{ijk}=\left[\frac{v_d}{\sqrt{2}}\left((h_Eh_E^\dagger)_{j'k'}-\frac{g^2}{2}\delta_{j'k'}\right)\boldsymbol{R}^{(S^\pm)\,*}_{i1}-\frac{v_u}{\sqrt{2}}\frac{g^2}{2}\delta_{j'k'}\ \boldsymbol{R}^{(S^\pm)\,*}_{i2}\right]\boldsymbol{R}^{(\tilde\ell)\,*}_{jj'}\boldsymbol{R}^{(\tilde\nu)}_{kk'}$$

$$+\left[\mu(h_D^*)_{k'j'}\boldsymbol{R}^{(S^\pm)\,*}_{i2}+(A_E^*)_{k'j'}\boldsymbol{R}^{(S^\pm)\,*}_{i1}\right]\boldsymbol{R}^{(\tilde\ell)\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde\nu)}_{kk'} \tag{194}$$

$$g^{(S^0\tilde d\tilde d^*)}_{ijk}=\left[v_d\left(\frac{1}{12}\left(3g^2+g'^2\right)\delta_{j'k'}-(h_Dh_D^\dagger)_{j'k'}\right)\boldsymbol{R}^{(S^0)}_{i1}-\frac{v_u}{12}\left(3g^2+g'^2\right)\delta_{j'k'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{jj'}\boldsymbol{R}^{(\tilde d)}_{kk'}$$

$$+\left[-\frac{1}{\sqrt{2}}(A_D)_{j'k'}\boldsymbol{R}^{(S^0)}_{i1}+\frac{\mu^*}{\sqrt{2}}(h_D)_{j'k'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{jj'}\boldsymbol{R}^{(\tilde u)}_{k,k'+3}$$

$$+\left[-\frac{1}{\sqrt{2}}(A_D^*)_{k'j'}\boldsymbol{R}^{(S^0)}_{i1}+\frac{\mu}{\sqrt{2}}(h_D^*)_{k'j'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde u)}_{kk'}$$

$$+\left[v_d\left(\frac{g'^2}{6}-(h_D^\dagger h_D)_{j',k'}\right)\boldsymbol{R}^{(S^0)}_{i1}-v_u\frac{g'^2}{6}\delta_{j',k'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde u)}_{k,k'+3} \tag{195}$$

$$g^{(S^0\tilde u\tilde u^*)}_{ijk}=\left[-\frac{v_d}{12}\left(3g^2-g'^2\right)\delta_{j'k'}\boldsymbol{R}^{(S^0)}_{i1}+v_u\left(\frac{1}{12}\left(3g^2-g'^2\right)\delta_{j'k'}-(h_Uh_U^\dagger)_{j'k'}\right)\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde u)\,*}_{jj'}\boldsymbol{R}^{(\tilde u)}_{kk'}$$

$$+\left[\frac{\mu^*}{\sqrt{2}}(h_U)_{j'k'}\boldsymbol{R}^{(S^0)}_{i1}-\frac{1}{\sqrt{2}}(A_U)_{j'k'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{jj'}\boldsymbol{R}^{(\tilde u)}_{k,k'+3}$$

$$+\left[\frac{\mu}{\sqrt{2}}(h_U^*)_{k'j'}\boldsymbol{R}^{(S^0)}_{i1}-\frac{1}{\sqrt{2}}(A_U^*)_{k'j'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde u)}_{kk'}$$

$$+\left[-v_d\frac{g'^2}{3}\delta_{j',k'}\boldsymbol{R}^{(S^0)}_{i1}+v_u\left(\frac{g'^2}{3}-(h_U^\dagger h_U)_{j',k'}\right)\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde u)}_{k,k'+3} \tag{196}$$

$$g^{(S^0\tilde\ell\tilde\ell^*)}_{ijk}=\left[v_d\left(\frac{1}{4}\left(g^2-g'^2\right)\delta_{j'k'}-(h_Eh_E^\dagger)_{j'k'}\right)\boldsymbol{R}^{(S^0)}_{i1}-\frac{v_u}{4}\left(g^2-g'^2\right)\delta_{j'k'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{jj'}\boldsymbol{R}^{(\tilde d)}_{kk'}$$

$$+\left[-\frac{1}{\sqrt{2}}(A_E)_{j'k'}\boldsymbol{R}^{(S^0)}_{i1}+\frac{\mu^*}{\sqrt{2}}(h_E)_{j'k'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{jj'}\boldsymbol{R}^{(\tilde u)}_{k,k'+3}$$

$$+\left[-\frac{1}{\sqrt{2}}(A_E^*)_{k'j'}\boldsymbol{R}^{(S^0)}_{i1}+\frac{\mu}{\sqrt{2}}(h_E^*)_{k'j'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde d)\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde u)}_{kk'}$$

$$+\left[v_d\left(\frac{g'^2}{2}-(h_E^\dagger h_E)_{j',k'}\right)\boldsymbol{R}^{(S^0)}_{i1}-v_u\frac{g'^2}{2}\delta_{j',k'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde{u})}_{k,k'+3} \tag{197}$$

$$g^{(S^0\tilde{\nu}\tilde{\nu}^*)}_{ijk}=\left[-\frac{v_d}{4}\left(g^2+g'^2\right)\delta_{j'k'}\boldsymbol{R}^{(S^0)}_{i1}+\frac{v_u}{4}\left(g^2+g'^2\right)\delta_{j'k'}\boldsymbol{R}^{(S^0)}_{i2}\right]\boldsymbol{R}^{(\tilde{\nu})\,*}_{jj'}\boldsymbol{R}^{(\tilde{\nu})}_{kk'} \tag{198}$$

$$\begin{aligned}g^{(P^0\tilde{d}\tilde{d}^*)}_{ijk}=&-i\;\left[\frac{1}{\sqrt{2}}(A_D)_{j'k'}\boldsymbol{R}^{(P^0)}_{i1}+\frac{\mu^*}{\sqrt{2}}(h_D)_{j'k'}\boldsymbol{R}^{(P^0)}_{i2}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{jj'}\boldsymbol{R}^{(\tilde{u})}_{k,k'+3}\\&+i\;\left[\frac{1}{\sqrt{2}}(A_D^*)_{k'j'}\boldsymbol{R}^{(P^0)}_{i1}+\frac{\mu}{\sqrt{2}}(h_D^*)_{k'j'}\boldsymbol{R}^{(P^0)}_{i2}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde{u})}_{kk'}\end{aligned} \tag{199}$$

$$\begin{aligned}g^{(P^0\tilde{u}\tilde{u}^*)}_{ijk}=&-i\;\left[\frac{\mu^*}{\sqrt{2}}(h_U)_{j'k'}\boldsymbol{R}^{(P^0)}_{i1}+\frac{1}{\sqrt{2}}(A_U)_{j'k'}\boldsymbol{R}^{(P^0)}_{i2}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{jj'}\boldsymbol{R}^{(\tilde{u})}_{k,k'+3}\\&+i\;\left[\frac{\mu}{\sqrt{2}}(h_U^*)_{k'j'}\boldsymbol{R}^{(P^0)}_{i1}+\frac{1}{\sqrt{2}}(A_U^*)_{k'j'}\boldsymbol{R}^{(P^0)}_{i2}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde{u})}_{kk'}\end{aligned} \tag{200}$$

$$\begin{aligned}g^{(P^0\tilde{\ell}\tilde{\ell}^*)}_{ijk}=&-i\;\left[\frac{1}{\sqrt{2}}(A_E)_{j'k'}\boldsymbol{R}^{(P^0)}_{i1}+\frac{\mu^*}{\sqrt{2}}(h_E)_{j'k'}\boldsymbol{R}^{(P^0)}_{i2}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{jj'}\boldsymbol{R}^{(\tilde{u})}_{k,k'+3}\\&+i\;\left[\frac{1}{\sqrt{2}}(A_E^*)_{k'j'}\boldsymbol{R}^{(P^0)}_{i1}+\frac{\mu}{\sqrt{2}}(h_E^*)_{k'j'}\boldsymbol{R}^{(P^0)}_{i2}\right]\boldsymbol{R}^{(\tilde{d})\,*}_{j,j'+3}\boldsymbol{R}^{(\tilde{u})}_{kk'}\end{aligned} \tag{201}$$

$$g^{(P^0\tilde{\nu}\tilde{\nu}^*)}_{ijk}=0 \tag{202}$$

6.8 Quartic scalar couplings with Higgs bosons

In this section we will show the quartic scalar couplings envolving the Higgs boson and the corresponding Goldstone bosons. The Higgs self interactions will be left for section 6.11.

6.9 Quartic sfermion interactions

In this section we will show the quartic scalar couplings envolving the sfermions among themselves.

6.10 Higgs couplings with the gauge bosons

6.10.1 Higgs – Gauge Boson – Gauge Boson

6.10.2 Higgs – Higgs – Gauge Boson – Gauge Boson

6.11 Higgs boson self interactions

In this section we will show the self couplings envolving the Higgs boson among themselves. We give separately the 3-point and 4-point interactions.

6.11.1 Higgs – Higgs – Higgs

For the 3-point Higgs boson self interactions we write the Lagrangian as

$$\mathcal{L} = \frac{1}{6}\, g^{S^0S^0S^0}_{i,j,k} \; S^0_i S^0_j S^0_k + \frac{1}{2}\, g^{S^0P^0P^0}_{i,j,k} \; S^0_i P^0_j P^0_k + g^{S^0S^+S^-}_{i,j,k} \; S^0_i S^+_j S^-_k \tag{203}$$

where 1/6 and 1/2 are symmetry factors for the case of identical particles, chosen in such a way that, for instance, the coupling $g^{S^0S^0S^0}_{i,j,k}$[11] is really the Feynman rule for the vertice (after multiplying by i as usual), that is,

$$g^{S^0S^0S^0}_{i,j,k} \equiv \frac{\partial^3 \mathcal{L}}{\partial S^0_i \partial S^0_j \partial S^0_k} \tag{204}$$

and similarly for the other cases. To simplify the notation we write the values for the *unrotated* couplings defined by the relations

$$g^{S^0S^0S^0}_{i,j,k} = \hat{g}^{S^0S^0S^0}_{i',j',k'} \; \boldsymbol{R}^{(S^0)}_{i,i'} \boldsymbol{R}^{(S^0)}_{j,j'} \; \boldsymbol{R}^{(S^0)}_{k,k'} \tag{205}$$

and similar relations. We get

$$\hat{g}^{S^0S^0S^0}_{1,1,1} = -\frac{3g^2 v_d}{4} - \frac{3g'^2 v_d}{4} \qquad \hat{g}^{S^0S^0S^0}_{1,1,2} = \frac{g^2 v_u}{4} + \frac{g'^2 v_u}{4}$$

[11] We have that, obviously, $g^{S^0S^0S^0}_{i,j,k}$ is completely symmetric and $g^{S^0P^0P^0}_{i,j,k}$ is symmetric in the last two entries.

$$
\begin{aligned}
\hat{g}^{S^0S^0S^0}_{1,2,1} &= \frac{g^2 v_u}{4} + \frac{g'^2 v_u}{4} & \hat{g}^{S^0S^0S^0}_{1,2,2} &= \frac{g^2 v_d}{4} + \frac{g'^2 v_d}{4} \\
\hat{g}^{S^0S^0S^0}_{2,1,1} &= \frac{g^2 v_u}{4} + \frac{g'^2 v_u}{4} & \hat{g}^{S^0S^0S^0}_{2,1,2} &= \frac{g^2 v_d}{4} + \frac{g'^2 v_d}{4} \\
\hat{g}^{S^0S^0S^0}_{2,2,1} &= \frac{g^2 v_d}{4} + \frac{g'^2 v_d}{4} & \hat{g}^{S^0S^0S^0}_{2,2,2} &= -\frac{3g^2 v_u}{4} - \frac{3g'^2 v_u}{4}
\end{aligned} \tag{206}
$$

$$
\begin{aligned}
\hat{g}^{S^0P^0P^0}_{1,1,1} &= -\frac{g^2 v_d}{4} - \frac{g'^2 v_d}{4} & \hat{g}^{S^0P^0P^0}_{1,1,2} &= 0 \\
\hat{g}^{S^0P^0P^0}_{1,2,1} &= 0 & \hat{g}^{S^0P^0P^0}_{1,2,2} &= \frac{g^2 v_d}{4} + \frac{g'^2 v_d}{4} \\
\hat{g}^{S^0P^0P^0}_{2,1,1} &= \frac{g^2 v_u}{4} + \frac{g'^2 v_u}{4} & \hat{g}^{S^0P^0P^0}_{2,1,2} &= 0 \\
\hat{g}^{S^0P^0P^0}_{2,2,1} &= 0 & \hat{g}^{S^0P^0P^0}_{2,2,2} &= -\frac{g^2 v_u}{4} - \frac{g'^2 v_u}{4}
\end{aligned} \tag{207}
$$

$$
\begin{aligned}
\hat{g}^{S^0S^+S^-}_{1,1,1} &= -\frac{g^2 v_d}{4} - \frac{g'^2 v_d}{4} & \hat{g}^{S^0S^+S^-}_{1,1,2} &= -\frac{g^2 v_u}{4} \\
\hat{g}^{S^0S^+S^-}_{1,2,1} &= -\frac{g^2 v_u}{4} & \hat{g}^{S^0S^+S^-}_{1,2,2} &= -\frac{g^2 v_d}{4} + \frac{g'^2 v_d}{4} \\
\hat{g}^{S^0S^+S^-}_{2,1,1} &= -\frac{g^2 v_u}{4} + \frac{g'^2 v_u}{4} & \hat{g}^{S^0S^+S^-}_{2,1,2} &= -\frac{g^2 v_d}{4} \\
\hat{g}^{S^0S^+S^-}_{2,2,1} &= -\frac{g^2 v_d}{4} & \hat{g}^{S^0S^+S^-}_{2,2,2} &= -\frac{g^2 v_u}{4} - \frac{g'^2 v_u}{4}
\end{aligned} \tag{208}
$$

6.11.2 Higgs – Higgs – Higgs – Higgs

For the 4-point Higgs boson self interactions we write the Lagrangian as

$$
\begin{aligned}
\mathcal{L} =& \frac{1}{4!} g^{S^0S^0S^0S^0}_{i,j,k,l} \; S^0_i S^0_j S^0_k S^0_l + \frac{1}{(2!)^2} g^{S^0S^0P^0P^0}_{i,j,k,l} \; S^0_i S^0_j P^0_k P^0_l + \frac{1}{4!} g^{P^0P^0P^0P^0}_{i,j,k,l} \; P^0_i P^0_j P^0_k P^0_l \\
&+ \frac{1}{2!} g^{S^0S^0S^+S^-}_{i,j,k,l} S^0_i S^0_j S^+_k S^-_l + \frac{1}{2!} g^{P^0P^0S^+0S^-}_{i,j,k,l} P^0_i P^0_j S^+_k S^-_l + \frac{1}{(2!)^2} g^{S^+S^-S^+S^-}_{i,j,k,l} S^+_i S^-_j S^+_k S^-_l
\end{aligned} \tag{209}
$$

We get for the unrotated couplings

$$
\begin{aligned}
\hat{g}^{S^0S^0S^0S^0}_{1,1,1,1} &= -\frac{3g^2}{4} - \frac{3g'^2}{4}, & \hat{g}^{S^0S^0S^0S^0}_{1,1,1,2} &= \hat{g}^{S^0S^0S^0S^0}_{1,1,2,1} = 0, & \hat{g}^{S^0S^0S^0S^0}_{1,1,2,2} &= \frac{g^2}{4} + \frac{g'^2}{4} \\
\hat{g}^{S^0S^0S^0S^0}_{1,2,1,1} &= 0, & \hat{g}^{S^0S^0S^0S^0}_{1,2,1,2} &= \hat{g}^{S^0S^0S^0S^0}_{1,2,2,1} = \frac{g^2}{4} + \frac{g'^2}{4}, & \hat{g}^{S^0S^0S^0S^0}_{1,2,2,2} &= 0 \\
\hat{g}^{S^0S^0S^0S^0}_{2,1,1,1} &= 0, & \hat{g}^{S^0S^0S^0S^0}_{2,1,1,2} &= \hat{g}^{S^0S^0S^0S^0}_{2,1,2,1} = \frac{g^2}{4} + \frac{g'^2}{4}, & \hat{g}^{S^0S^0S^0S^0}_{2,1,2,2} &= 0
\end{aligned} \tag{210}
$$

$$\hat{g}^{S^0S^0S^0S^0}_{2,2,1,1} = \frac{g^2}{4} + \frac{g'^2}{4}, \qquad \hat{g}^{S^0S^0S^0S^0}_{2,2,1,2} = \hat{g}^{S^0S^0S^0S^0}_{2,2,2,1} = 0, \qquad \hat{g}^{S^0S^0S^0S^0}_{2,2,2,2} = -\frac{3g^2}{4} - \frac{3g'^2}{4}$$

$$\begin{aligned}
\hat{g}^{S^0S^0P^0P^0}_{1,1,1,1} &= -\frac{g^2}{4} - \frac{g'^2}{4}, & \hat{g}^{S^0S^0P^0P^0}_{1,1,1,2} &= \hat{g}^{S^0S^0P^0P^0}_{1,1,2,1} = 0, & \hat{g}^{S^0S^0P^0P^0}_{1,1,2,2} &= \frac{g^2}{4} + \frac{g'^2}{4},\\
\hat{g}^{S^0S^0P^0P^0}_{1,2,1,1} &= 0, & \hat{g}^{S^0S^0P^0P^0}_{1,2,1,2} &= \hat{g}^{S^0S^0P^0P^0}_{1,2,2,1} = 0, & \hat{g}^{S^0S^0P^0P^0}_{1,2,2,2} &= 0,\\
\hat{g}^{S^0S^0P^0P^0}_{2,1,1,1} &= 0, & \hat{g}^{S^0S^0P^0P^0}_{2,1,1,2} &= \hat{g}^{S^0S^0P^0P^0}_{2,1,2,1} = 0, & \hat{g}^{S^0S^0P^0P^0}_{2,1,2,2} &= 0,\\
\hat{g}^{S^0S^0P^0P^0}_{2,2,1,1} &= \frac{g^2}{4} + \frac{g'^2}{4}, & \hat{g}^{S^0S^0P^0P^0}_{2,2,1,2} &= \hat{g}^{S^0S^0P^0P^0}_{2,2,2,1} = 0, & \hat{g}^{S^0S^0P^0P^0}_{2,2,2,2} &= -\frac{g^2}{4} - \frac{g'^2}{4}
\end{aligned} \tag{211}$$

$$\begin{aligned}
\hat{g}^{P^0P^0P^0P^0}_{1,1,1,1} &= -\frac{3g^2}{4} - \frac{3g'^2}{4}, & \hat{g}^{P^0P^0P^0P^0}_{1,1,1,2} &= \hat{g}^{P^0P^0P^0P^0}_{1,1,2,1} = 0, & \hat{g}^{P^0P^0P^0P^0}_{1,1,2,2} &= \frac{g^2}{4} + \frac{g'^2}{4},\\
\hat{g}^{P^0P^0P^0P^0}_{1,2,1,1} &= 0, & \hat{g}^{P^0P^0P^0P^0}_{1,2,1,2} &= \hat{g}^{P^0P^0P^0P^0}_{1,2,2,1} = \frac{g^2}{4} + \frac{g'^2}{4}, & \hat{g}^{P^0P^0P^0P^0}_{1,2,2,2} &= 0,\\
\hat{g}^{P^0P^0P^0P^0}_{2,1,1,1} &= 0, & \hat{g}^{P^0P^0P^0P^0}_{2,1,1,2} &= \hat{g}^{P^0P^0P^0P^0}_{2,1,2,1} = \frac{g^2}{4} + \frac{g'^2}{4}, & \hat{g}^{P^0P^0P^0P^0}_{2,1,2,2} &= 0,\\
\hat{g}^{P^0P^0P^0P^0}_{2,2,1,1} &= \frac{g^2}{4} + \frac{g'^2}{4}, & \hat{g}^{P^0P^0P^0P^0}_{2,2,1,2} &= \hat{g}^{P^0P^0P^0P^0}_{2,2,2,1} = 0, & \hat{g}^{P^0P^0P^0P^0}_{2,2,2,2} &= -\frac{3g^2}{4} - \frac{3g'^2}{4}
\end{aligned} \tag{212}$$

$$\begin{aligned}
\hat{g}^{S^0S^0S^+S^-}_{1,1,1,1} &= -\frac{g^2}{4} - \frac{g'^2}{4}, & \hat{g}^{S^0S^0S^+S^-}_{1,1,1,2} &= \hat{g}^{S^0S^0S^+S^-}_{1,1,2,1} = 0, & \hat{g}^{S^0S^0S^+S^-}_{1,1,2,2} &= -\frac{g^2}{4} + \frac{g'^2}{4},\\
\hat{g}^{S^0S^0S^+S^-}_{1,2,1,1} &= 0, & \hat{g}^{S^0S^0S^+S^-}_{1,2,1,2} &= \hat{g}^{S^0S^0S^+S^-}_{1,2,2,1} = -\frac{g^2}{4}, & \hat{g}^{S^0S^0S^+S^-}_{1,2,2,2} &= 0,\\
\hat{g}^{S^0S^0S^+S^-}_{2,1,1,1} &= 0, & \hat{g}^{S^0S^0S^+S^-}_{2,1,1,2} &= \hat{g}^{S^0S^0S^+S^-}_{2,1,2,1} = -\frac{g^2}{4}, & \hat{g}^{S^0S^0S^+S^-}_{2,1,2,2} &= 0,\\
\hat{g}^{S^0S^0S^+S^-}_{2,2,1,1} &= -\frac{g^2}{4} + \frac{g'^2}{4}, & \hat{g}^{S^0S^0S^+S^-}_{2,2,1,2} &= \hat{g}^{S^0S^0S^+S^-}_{2,2,2,1} = 0, & \hat{g}^{S^0S^0S^+S^-}_{2,2,2,2} &= -\frac{g^2}{4} - \frac{g'^2}{4}
\end{aligned} \tag{213}$$

$$\begin{aligned}
\hat{g}^{P^0P^0S^+S^-}_{1,1,1,1} &= -\frac{g^2}{4} - \frac{g'^2}{4}, & \hat{g}^{P^0P^0S^+S^-}_{1,1,1,2} &= \hat{g}^{P^0P^0S^+S^-}_{1,1,2,1} = 0, & \hat{g}^{P^0P^0S^+S^-}_{1,1,2,2} &= -\frac{g^2}{4} + \frac{g'^2}{4},\\
\hat{g}^{P^0P^0S^+S^-}_{1,2,1,1} &= 0, & \hat{g}^{P^0P^0S^+S^-}_{1,2,1,2} &= \hat{g}^{P^0P^0S^+S^-}_{1,2,2,1} = \frac{g^2}{4}, & \hat{g}^{P^0P^0S^+S^-}_{1,2,2,2} &= 0,\\
\hat{g}^{P^0P^0S^+S^-}_{2,1,1,1} &= 0, & \hat{g}^{P^0P^0S^+S^-}_{2,1,1,2} &= \hat{g}^{P^0P^0S^+S^-}_{2,1,2,1} = \frac{g^2}{4}, & \hat{g}^{P^0P^0S^+S^-}_{2,1,2,2} &= 0,\\
\hat{g}^{P^0P^0S^+S^-}_{2,2,1,1} &= -\frac{g^2}{4} + \frac{g'^2}{4}, & \hat{g}^{P^0P^0S^+S^-}_{2,2,1,2} &= \hat{g}^{P^0P^0S^+S^-}_{2,2,2,1} = 0, & \hat{g}^{P^0P^0S^+S^-}_{2,2,2,2} &= -\frac{g^2}{4} - \frac{g'^2}{4}
\end{aligned} \tag{214}$$

$$\hat{g}^{S^+S^-S^+S^-}_{1,1,1,1} = \hat{g}^{S^+S^-S^+S^-}_{2,2,2,2} = -\frac{g^2}{2} - \frac{g'^2}{2}$$

$$
\begin{aligned}
&\hat{g}^{S^+S^-S^+S^-}_{1,1,2,2} = \hat{g}^{S^+S^-S^+S^-}_{1,2,2,1} = \hat{g}^{S^+S^-S^+S^-}_{2,1,1,2} = \hat{g}^{S^+S^-S^+S^-}_{2,2,1,1} = \frac{g^2}{4} + \frac{g'^2}{4} \\
&\hat{g}^{S^+S^-S^+S^-}_{1,1,1,2} = \hat{g}^{S^+S^-S^+S^-}_{1,1,2,1} = \hat{g}^{S^+S^-S^+S^-}_{1,2,1,1} = \hat{g}^{S^+S^-S^+S^-}_{1,2,1,2} = \hat{g}^{S^+S^-S^+S^-}_{1,2,2,2} = 0 \\
&\hat{g}^{S^+S^-S^+S^-}_{2,1,1,1} = \hat{g}^{S^+S^-S^+S^-}_{2,1,2,1} = \hat{g}^{S^+S^-S^+S^-}_{2,1,2,2} = \hat{g}^{S^+S^-S^+S^-}_{2,2,1,2} = \hat{g}^{S^+S^-S^+S^-}_{2,2,2,1} = 0
\end{aligned}
\tag{215}
$$

6.12 Ghost interactions

6.12.1 Ghost – Ghost – Gauge Boson

6.12.2 Ghost – Ghost – Higgs

A Changelog

- 24/6/2011
  - Corrected misprints in Eq. 161, Eq. 162, Eq. 168, Eq. 169 and Eq. 170. We thank R. Fonseca for pointing out these.
- 23/11/2010
  - Added the 3-point and 4-point Higgs self-interaction in sections 6.11.1 and 6.11.2.
- 18/11/2010
  - Added the Higgs–Sfermion–Sfermion couplings of section 6.7.1
- 16/11/2010
  - Changed the convention in Eq. (161) to write the couplings for H_i^- instead of H_i^+. This leads to Eq. (177) and Eq. (178) in agreement with `SPheno`.
- 15/11/2010
  - Changed the convention of the left-handed rotation matrices for leptons, Eq. (90).
  - Corrected a misprint in Eq. (161).
  - Changed notation in Eq. (168) for $C_{iAX}^{L(\nu)}$ and $C_{iAX}^{R(\nu)}$ to be uniform with the other terms.
  - Changed some of the couplings in Eqs. (169)-(176) to be consistent with the previous changes.
  - All the couplings in Eqs (169)-(176) were checked against SPheno[9] with complete agreement.
- 16/2/2005
  - $\sin^2$ and $\cos^2$ were exchanged in Eq. 78.
- 28/11/2003
  - Corrected misprints in Eq. 96, Eq. 170, Eq. 171 and Eq. 172.
- 27/10/2002
  - Changed Version Number to 1.6.xxx corresponding to the year 2002.
  - Changed minor points in notation in Eq. 62 and related expressions.

- 31/8/2001
  - Corrected $Q = T_3 + Y$ in Eq. 43.
- 17/6/2001
  - Introduced the VVV and $VVVV$ couplings.
- 27/5/2001
  - Introduced the version numbering convention.

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