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A viable A_4 3HDM theory of quark mass matrices

Iris Brée[a], **Sérgio Carrôlo**[b], **Jorge C. Romão**[c], **João P. Silva**[d]

CFTP, Departamento de Física, Instituto Superior Técnico, Universidade de Lisboa, Avenida Rovisco Pais 1, 1049 Lisbon, Portugal

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Abstract It is known that a three Higgs doublet model (3HDM) symmetric under an exact A_4 symmetry is not compatible with nonzero quark masses and/or non-block-diagonal CKM matrix. We show that a 3HDM with softly broken A_4 terms in the scalar potential does allow for a fit of quark mass matrices. Moreover, the result is consistent with $m_h = 125$ GeV and the $h \to WW, ZZ$ signal. We also checked numerically that, for each point that passes all the constraints, the minimum is a global minimum of the potential.

1 Introduction

The observation in 2012 of a scalar particle with 125 GeV by the ATLAS and CMS collaborations [1,2] has incentivized experimental searches for beyond the Standard Model (SM) particles at the LHC. On par with these experimental endeavors, theoretical efforts in the search for extra scalar particles have been strengthened since this discovery. A promising framework is found in N-Higgs doublet models (NHDM).

Such models have many free parameters, which are often curtailed by imposing some discrete family symmetry. Here, we focus on the implementation of A_4 in a three Higgs doublet model (3HDM). The A_4 group is the group of even permutations on 4 elements. It is the smallest discrete group to contain a three-dimensional irreducible representation (irrep), which is ideal for describing the three families of quarks with a minimal number of independent Yukawa couplings. Thus, NHDM supplemented by the A_4 discrete symmetry has long been of interest in flavour physics research.

A number of early articles include: [3], mainly devoted to the leptonic sector and where the solution to the quark sector is briefly mentioned to include a fourth Higgs doublet and all quark fields in singlets (which is effectively the same as the Standard Model quark sector); [4], where A_4 is broken by dimension four Yukawa couplings, which, upon renormalization, will affect the scalar potential [5], which requires three Higgs doublets in the down-type quark sector and a further two in the up-type quark sector, consisting of a 5HDM; and [6], which is devoted to the leptonic sector, but has the interesting side query that it might be possible to recover a realistic CKM matrix through soft-breaking of A_4.

Quark mass matrices in the context of a 3HDM with Higgs doublets in the triplet representation of A_4 were studied in [7,8], with the vacuum expectation value (vev) structure $(e^{i\alpha}, e^{-i\alpha}, r)$, where α and r are real constants. This vacuum solution was also included in the original study of the A_4-3HDM vacua in Ref. [9]. Unfortunately, Degee et al. [10] proved in 2013 that such a vacuum can *never* be the global minimum of the A_4 symmetric 3HDM. In this beautiful paper, geometric techniques were used in order to identify all possible global minima (thus, all possible viable vacua) of the A_4 symmetric 3HDM. Immediately thereafter, those minima were used to show that all assignments of the quark fields into irreps of A_4, when combined with the possible vevs for the exact A_4 potential, yield vanishing quark masses and/or a CP conserving CKM matrix, both of which are forbidden by experiment. This is in fact a consequence of a much broader theorem, proved in [11,12]: given any flavour symmetry group, one can obtain a physical CKM mixing matrix and, simultaneously, non-degenerate and nonzero quark masses only if the vevs of the Higgs fields break completely the full flavour group. The idea is that a symmetry will reduce the number of redundant Yukawa couplings present in the SM, and it might even predict relations among observables which turn out to be consistent with experiment.

[a] e-mail: iris.bree.silva@tecnico.ulisboa.pt
[b] e-mail: sergio.carrolo@tecnico.ulisboa.pt
[c] e-mail: jorge.romao@tecnico.ulisboa.pt (corresponding author)
[d] e-mail: jpsilva@cftp.ist.utl.pt

When studying in detail the extensions of A_4 to the quark sector found by Ref. [13], we noticed that, in some of them, if it weren't for the particular form of the vevs allowed by the exact A_4 3HDM potential, the Yukawa matrices could allow for massive quarks, and for a realistic CKM matrix. Since the A_4 symmetric potential doesn't allow for minima other than those shown in [10], here we consider the case where the A_4 symmetry is softly broken by the addition of quadratic terms to the potential. Such terms do not spoil the theory's renormalizability, but break the A_4 symmetry.

Our article is organized as follows. We define the notation for the scalar potential in Sect. 2.1, discuss the Yukawa Lagrangian and the form of the possible mass matrices in Sect. 2.2, giving all the expressions needed for the fit in Sect. 2.3. In Sect. 3 we present our fit to the quarks mass matrices, while in Sect. 4 we discuss the viability of the vacuum found in the fit in terms of the scalar potential. Section 5 is devoted to the implementation of the theoretical constraints to be imposed, and in Sect. 6 we briefly discuss the constraints coming from the LHC. The results and conclusions are presented in Sects. 7 and 8, respectively. The Appendices contain some additional expressions that are needed for the fits.

2 Parameterization for the softly-broken A_4 3HDM

2.1 Potential and candidates for local minimum

The softly-broken potential of the 3HDM with an A_4 symmetry is given by

$$V_H = V_{4,\,A_4} + M^2_{ij}\left(\phi_i^\dagger \phi_j\right), \tag{1}$$

where $V_{4,\,A_4}$ is the quartic potential for the A_4 symmetric three Higgs doublet model (3HDM), which is, in the notation of [10],

$$\begin{aligned} V_{4,\,A_4} = {} & \frac{\Lambda_0}{3}\left(\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3\right)^2 \\ & + \Lambda_1\left[\left(\mathrm{Re}\left\{\phi_1^\dagger\phi_2\right\}\right)^2+\left(\mathrm{Re}\left\{\phi_2^\dagger\phi_3\right\}\right)^2\right. \\ & \left.+\left(\mathrm{Re}\left\{\phi_3^\dagger\phi_1\right\}\right)^2\right] + \Lambda_2\left[\left(\mathrm{Im}\left\{\phi_1^\dagger\phi_2\right\}\right)^2\right. \\ & \left.+\left(\mathrm{Im}\left\{\phi_2^\dagger\phi_3\right\}\right)^2+\left(\mathrm{Im}\left\{\phi_3^\dagger\phi_1\right\}\right)^2\right] \\ & + \frac{\Lambda_3}{3}\left[(\phi_1^\dagger\phi_1)^2+(\phi_2^\dagger\phi_2)^2+(\phi_3^\dagger\phi_3)^2\right. \\ & \left.-(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)-(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3)-(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1)\right] \\ & + \Lambda_4\left[\mathrm{Re}\left\{\phi_1^\dagger\phi_2\right\}\ \mathrm{Im}\left\{\phi_1^\dagger\phi_2\right\}+\mathrm{Re}\left\{\phi_2^\dagger\phi_3\right\}\right. \\ & \left.\times\ \mathrm{Im}\left\{\phi_2^\dagger\phi_3\right\}+\mathrm{Re}\left\{\phi_3^\dagger\phi_1\right\}\ \mathrm{Im}\left\{\phi_3^\dagger\phi_1\right\}\right]. \end{aligned} \tag{2}$$

The matrix M^2_{ij} is a general hermitian matrix, which can be parameterized by

$$(M^2_{ij}) = \begin{pmatrix} m_{11}^2 & m_{12}^2 e^{i\theta_{12}} & m_{13}^2 e^{i\theta_{13}} \\ m_{12}^2 e^{-i\theta_{12}} & m_{22}^2 & m_{23}^2 e^{i\theta_{23}} \\ m_{13}^2 e^{-i\theta_{13}} & m_{23}^2 e^{-i\theta_{23}} & m_{33}^2 \end{pmatrix}, \tag{3}$$

where m^2_{ij} are real parameters with the dimension of mass squared.[1]

Additionally, in the notation of [14], the exact A_4 potential can be written as

$$\begin{aligned} V_{A_4} = {} & \frac{r_1+2r_4}{3}\left[(\phi_1^\dagger\phi_1)+(\phi_2^\dagger\phi_2)+(\phi_3^\dagger\phi_3)\right]^2 \\ & + \frac{2(r_1-r_4)}{3}\left[(\phi_1^\dagger\phi_1)^2+(\phi_2^\dagger\phi_2)^2\right. \\ & +(\phi_3^\dagger\phi_3)^2-(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2)-(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) \\ & \left.-(\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1)\right] \\ & + 2r_7\left(|\phi_1^\dagger\phi_2|^2+|\phi_2^\dagger\phi_3|^2+|\phi_3^\dagger\phi_1|^2\right) \\ & + \left[c_3\left[(\phi_1^\dagger\phi_2)^2+(\phi_2^\dagger\phi_3)^2+(\phi_3^\dagger\phi_1)^2\right]+h.c.\right]. \end{aligned} \tag{4}$$

The relation between the two notations is

$$\begin{aligned} & r_1 = \frac{1}{3}(\Lambda_0+\Lambda_3), \quad r_4 = \frac{1}{6}(2\Lambda_0-\Lambda_3), \\ & r_7 = \frac{1}{4}(\Lambda_1+\Lambda_2), \\ & \mathrm{Re}(c_3) = \frac{1}{4}(\Lambda_1-\Lambda_2), \quad \mathrm{Im}(c_3) = -\frac{1}{4}\Lambda_4. \end{aligned} \tag{5}$$

We consider that the scalar fields can take complex vacuum expectation values (vevs), to be determined later. Thus, we write,

$$\phi_i = \begin{bmatrix} \varphi_i^+ \\ \frac{|v_i|e^{i\rho_i}}{\sqrt{2}} + \frac{1}{\sqrt{2}}(x_i + i x_{i+3}) \end{bmatrix}. \tag{6}$$

Because CP is spontaneously violated, the unrotated neutral fields have no definite CP, and for convenience we label them x_i, $i = 1, \ldots, 6$. We can also use the gauge freedom to absorb one of the phases in the vevs, that we choose to be ρ_1. Therefore we have the vector of vevs defined as

$$\vec{v} = (|v_1|, |v_2|e^{i\rho_2}, |v_3|e^{i\rho_3}). \tag{7}$$

This vev contributes with four free parameters to our model, because one of the parameters is constrained by the mass of the gauge bosons to match the observed SM values,

$$|v_1|^2 + |v_2|^2 + |v_3|^2 \equiv v^2 \simeq (246\,\mathrm{GeV})^2. \tag{8}$$

[1] In the quadratic terms, the combination $-\frac{M_0}{\sqrt{3}}\left(\phi_1^\dagger\phi_1+\phi_2^\dagger\phi_2+\phi_3^\dagger\phi_3\right)$ is also invariant under A_4. But, since we are keeping all soft-breaking terms, we find the notation in (3) more convenient.

Table 1 Extensions of A_4 to the Yukawa sector with non-vanishing determinant, and non-zero J for general, complex valued, vevs (v_1, v_2, v_3). In the table, I_d stands for the matrix M_d for case I and similarly for the other entries

Case	M_d	M_u
I	$\begin{pmatrix} ae^{i\alpha}v_1 & be^{i\beta}v_1 & ce^{i\gamma}v_1 \\ ae^{i\alpha}v_2 & \omega be^{i\beta}v_2 & \omega^2 ce^{i\gamma}v_2 \\ ae^{i\alpha}v_3 & \omega^2 be^{i\beta}v_3 & \omega ce^{i\gamma}v_3 \end{pmatrix}$	$\begin{pmatrix} A \to A', & A \in \{a,b,c\} \\ \Omega \to \Omega', & \Omega \in \{\alpha,\beta,\gamma\} \\ v_i \to v_i^*, & i \in \{1,2,3\} \end{pmatrix}$
II	I_d^T	I_u^T
III	$\begin{pmatrix} 0 & (ae^{i\alpha}-be^{i\beta})v_3 & (ae^{i\alpha}+be^{i\beta})v_2 \\ (ae^{i\alpha}+be^{i\beta})v_3 & 0 & (ae^{i\alpha}-be^{i\beta})v_1 \\ (ae^{i\alpha}-be^{i\beta})v_2 & (ae^{i\alpha}+be^{i\beta})v_1 & 0 \end{pmatrix}$	$\begin{pmatrix} A \to A', & A \in \{a,b\} \\ \Omega \to \Omega', & \Omega \in \{\alpha,\beta\} \\ v_i \to v_i^*, & i \in \{1,2,3\} \end{pmatrix}$
IV	I_d	III_u
V	III_d	I_u

The vev can also be parameterized as

$$\vec{v} = v\left(\cos(\beta_1)\cos(\beta_2), \cos(\beta_2)\sin(\beta_1)e^{ip_2}, \sin(\beta_2)e^{ip_3}\right). \tag{9}$$

Of the quantities arising out of the scalar potential, the vevs are the only relevant to the quark mass matrices. This leads many authors to just proclaim some vevs, without checking whether they can indeed be the global minima of a realistic Higgs potential. We will perform this crucial verification below, in Sect. 4.

2.2 Yukawa Lagrangian

As in Refs. [7,13], we consider that the Higgs doublets are in the **3** of A_4 as well as the three left-handed $SU(2)$ doublets Q_{Lj} of hypercharge 1/6. There are three right-handed $SU(2)$ singlets $n_{R,j}$ of hypercharge $-1/3$ and three right-handed $SU(2)$ singlets $p_{R,j}$ of hypercharge $2/3$. Our assignments for the singlets are as follows

$$n_{R1}, p_{R1} \to \mathbf{1}, \quad n_{R2}, p_{R2} \to \mathbf{1}', \quad n_{R3}, p_{R3} \to \mathbf{1}'' \text{ of } A_4. \tag{10}$$

Then, the A_4 transformations on the fields are generated by [7,13]

$$T: \begin{cases} \phi_1 \to \phi_2 \to \phi_3 \to \phi_1, \\ Q_{L1} \to Q_{L2} \to Q_{L3} \to Q_{L1}, \\ n_{R1} \to n_{R1}, n_{R2} \to \omega n_{R2}, n_{R3} \to \omega^2 n_{R3}, \\ p_{R1} \to n_{R1}, p_{R2} \to \omega p_{R2}, p_{R3} \to \omega^2 p_{R3}, \end{cases} \tag{11}$$

and

$$S: \begin{cases} \phi_1 \to \phi_1, \phi_2 \to -\phi_2, \phi_3 \to -\phi_3, \\ Q_{L1} \to Q_{L1}, Q_{L2} \to -Q_{L2}, Q_{L3} \to -Q_{L3}. \end{cases} \tag{12}$$

One can easily verify that the scalar potential in Eq. (4) is invariant under the previous transformations. Now we write the A_4 invariant Yukawa Lagrangian for quarks. We have

$$\begin{aligned} -\mathcal{L}_{\text{Yukawa}} &= \sqrt{2}\,\hat{a}\left(\overline{Q_{L1}}\phi_1 + \overline{Q_{L2}}\phi_2 + \overline{Q_{L3}}\phi_3\right) n_{R1} \\ &+ \sqrt{2}\,\hat{b}\left(\overline{Q_{L1}}\phi_1 + \omega\,\overline{Q_{L2}}\phi_2 + \omega^2\,\overline{Q_{L3}}\phi_3\right) n_{R2} \\ &+ \sqrt{2}\,\hat{c}\left(\overline{Q_{L1}}\phi_1 + \omega^2\,\overline{Q_{L2}}\phi_2 + \omega\,\overline{Q_{L3}}\phi_3\right) n_{R3} \\ &+ \sqrt{2}\,\hat{a}'\left(\overline{Q_{L1}}\tilde{\phi}_1 + \overline{Q_{L2}}\tilde{\phi}_2 + \overline{Q_{L3}}\tilde{\phi}_3\right) p_{R1} \\ &+ \sqrt{2}\,\hat{b}'\left(\overline{Q_{L1}}\tilde{\phi}_1 + \omega\,\overline{Q_{L2}}\tilde{\phi}_2 + \omega^2\,\overline{Q_{L3}}\tilde{\phi}_3\right) p_{R2} \\ &+ \sqrt{2}\,\hat{c}'\left(\overline{Q_{L1}}\tilde{\phi}_1 + \omega^2\,\overline{Q_{L2}}\tilde{\phi}_2 + \omega\,\overline{Q_{L3}}\tilde{\phi}_3\right) p_{R3} + \text{h.c.}, \end{aligned} \tag{13}$$

where, as usual,

$$\tilde{\phi}_j \equiv i\,\sigma_2 \phi_j^*, \tag{14}$$

and we define

$$\hat{a}=ae^{i\,\alpha},\ \hat{b}=be^{i\,\beta},\ \hat{c}=ce^{i\,\gamma},\ \hat{a}'=a'e^{i\,\alpha'},\ \hat{b}'=b'e^{i\,\beta'},\ \hat{c}'=c'e^{i\,\gamma'}, \tag{15}$$

where a, b, c, a', b', c' are real and positive. This choice of invariant Lagrangian corresponds to the case I identified in Ref. [13] (see the next section).

2.3 Yukawa matrices, masses and CKM

We aim to fit six quark masses and four CKM matrix elements to the currently accepted SM values for these observables. Therefore, we're interested in softly-broken A_4 symmetric models with up to ten parameters. Reference [13] has studied all of the possible extensions of A_4 to the fermion sector.

Using their results, we can check which of them can accommodate non-vanishing quark masses, CKM mixing angles and CP violation by considering a general vev $\vec{v}$. We take the Jarlskog invariant as a measure of CP violation [15]. Out of all possibilities, we are left with five of them, which we list in Table 1. There, A are real constants, Ω are constants in the $[0, 2\pi[$ interval, $\omega = e^{i\frac{2\pi}{3}}$ ($\omega^3 = 1$) and ^{T} is the transpose of the matrix.

In the table above, we have used the convention where the quarks' mass terms are written as

$$-\mathcal{L}_{\text{Yukawa}} \supset \overline{n_L} M_d n_R + \overline{p_L} M_u p_R + \text{h.c.}, \tag{16}$$

where h.c. stands for the hermitian conjugate.

In the Yukawa sector, there are ten observables, six masses, three mixing angles and one Jarlskog invariant, therefore, we would prefer to look for a case with ten parameters, or less. All possible neutral vevs of the 3HDM are consistent with the parameterization in Eq. (9), which consists of four free parameters that we can fit; two angles, and two phases. Looking at the cases in Table 1, we will see that it is possible to reduce the number of free parameters by performing both basis transformations to right-handed quarks and global $U(1)_Y$ rephasings, both of which have no effect on the physical predictions of the theory.

For case I, the down quark mass matrices read

$$M_d = \begin{pmatrix} ae^{i\alpha}v_1 & be^{i\beta}v_1 & ce^{i\gamma}v_1 \\ ae^{i\alpha}v_2 & \omega be^{i\beta}v_2 & \omega^2 ce^{i\gamma}v_2 \\ ae^{i\alpha}v_3 & \omega^2 be^{i\beta}v_3 & \omega ce^{i\gamma}v_3 \end{pmatrix} = D_v W D_a D_\alpha, \tag{17}$$

where (remember that the v_i are complex)

$$D_v = \text{diag}(v_1, v_2, v_3), \quad D_a = \text{diag}(a, b, c),$$
$$D_\alpha = \text{diag}(e^{i\alpha}, e^{i\beta}, e^{i\gamma}), \quad W = \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \tag{18}$$

We see that we can perform a unitary transformation to the right-handed quarks that removes all three phases α, β, γ. The same holds for M_u, by performing the substitution $A \to A'$, $\Omega \to \Omega'$ and $v_i \to v_i^*$. We note that the case I matrices were also used by Ref. [16] as the mass matrices for the charged leptons.

In this work, we study this case, that corresponds to the Lagrangian in Eq. (13). Then, given that $D_\alpha D_\alpha^\dagger = \mathbb{1}$ and $D_a D_a^\dagger = D_{a^2} = \text{diag}(a^2, b^2, c^2)$, we find

$$H_d \equiv M_d M_d^\dagger = D_v S_d D_v^\dagger,$$
$$H_u \equiv M_u M_u^\dagger = D_v^\dagger S_u D_v, \tag{19}$$

where $S_d = W D_{a^2} W^\dagger$ and $a^2 \to a'^2$ for the up quark case. This matrix can now be explicitly written out using appropriate parameters as

$$S_d = \begin{pmatrix} \Sigma_d & Z_d e^{i\phi_d} & Z_d e^{-i\phi_d} \\ Z_d e^{-i\phi_d} & \Sigma_d & Z_d e^{i\phi_d} \\ Z_d e^{i\phi_d} & Z_d e^{-i\phi_d} & \Sigma_d \end{pmatrix}, \tag{20}$$

where Σ_d and Z_d are real, and

$$\Sigma_d \equiv a^2 + b^2 + c^2,$$
$$Z_d\, e^{i\phi_d} \equiv a^2 + \omega^2 b^2 + \omega c^2, \tag{21}$$

with corresponding primes for the up case. For completeness, the specific forms for H_d and H_u found after using the parameterizations in Eqs. (9) and (21) are written in Appendix A. The eigenvalues of the matrices H_d and H_u will be fitted for the (square of the) quark masses, (m_d^2, m_s^2, m_b^2) and (m_u^2, m_c^2, m_t^2), respectively.

We now turn to the Cabibbo–Kobayashi–Maskawa (CKM) matrix. As found by Branco and Lavoura [17], the absolute values of the CKM matrix can be obtained through calculating the traces of appropriate powers of the matrices H_u and H_d. They observe that

$$\text{Tr}\left(H_u^a H_d^b\right) \equiv L_{ab} = \sum_{k,i} U_{ki} (D_u^a)_{kk} (D_d^b)_{ii}, \tag{22}$$

where $U_{ki} = |V_{ki}|^2$ and V is the CKM matrix. The CKM matrix is unitary and therefore U only has four independent entries. Consequently, in order to compute U, it is only necessary to resort to

$$L_{11} = U_{ki} (D_u)_{kk} (D_d)_{ii},$$
$$L_{12} = U_{ki} (D_u)_{kk} (D_d^2)_{ii},$$
$$L_{21} = U_{ki} (D_u^2)_{kk} (D_d)_{ii},$$
$$L_{22} = U_{ki} (D_u^2)_{kk} (D_d^2)_{ii}. \tag{23}$$

These equations are linear in U_{ik} and are, therefore, invertible for this variable. Thus, by picking U_{11}, U_{21}, U_{13}, and U_{23} (respectively, U_{ud}, U_{cd}, U_{ub}, and U_{cb}), we are able to obtain a unique solution for the magnitudes of the CKM elements as a function of L_{ab} and the quark masses. Namely,

$$U_{11} = \left(m_b{}^2 - m_s{}^2\right)\left(m_c{}^2 - m_t{}^2\right)\frac{a_{11}}{\det},$$
$$U_{21} = \left(m_b{}^2 - m_s{}^2\right)\left(m_u{}^2 - m_t{}^2\right)\frac{a_{21}}{\det},$$
$$U_{13} = \left(m_d{}^2 - m_s{}^2\right)\left(m_c{}^2 - m_t{}^2\right)\frac{a_{13}}{\det},$$
$$U_{23} = \left(m_d{}^2 - m_s{}^2\right)\left(m_u{}^2 - m_t{}^2\right)\frac{a_{23}}{\det}, \tag{24}$$

where

$$a_{11} = L_{11}\left(m_b{}^2 + m_s{}^2\right)\left(m_c{}^2 + m_t{}^2\right)$$
$$-L_{12}\left(m_c{}^2 + m_t{}^2\right) - L_{21}\left(m_b{}^2 + m_s{}^2\right) + L_{22}$$

$$+m_b^2\left(-m_c^2 m_t^2\left(m_d^2+m_s^2\right)-m_s^2 m_u^2\left(m_c^2+m_t^2\right)+m_s^2 m_u^4\right)$$
$$+m_c^2 m_d^2 m_t^2\left(m_d^2-m_s^2\right), \tag{25}$$

$$a_{21}=-L_{11}\left({m_b}^2+{m_s}^2\right)\left({m_t}^2+{m_u}^2\right)+L_{12}\left({m_u}^2+{m_t}^2\right)$$
$$+L_{21}\left({m_b}^2+{m_s}^2\right)-L_{22}$$
$$+m_b^2\left(m_c^2 m_s^2\left(m_t^2+m_u^2-m_c^2\right)+m_t^2 m_u^2\left(m_d^2+m_s^2\right)\right)$$
$$+m_d^2 m_t^2 m_u^2\left(m_s^2-m_d^2\right), \tag{26}$$

$$a_{13}=-L_{11}\left({m_d}^2+{m_s}^2\right)\left({m_t}^2+{m_c}^2\right)+L_{12}\left({m_c}^2+{m_t}^2\right)$$
$$+L_{21}\left({m_d}^2+{m_s}^2\right)-L_{22}$$
$$+m_b^2 m_c^2 m_t^2\left(m_d^2+m_s^2-m_b^2\right)+m_d^2 m_s^2\left(m_c^2\left(m_t^2+m_u^2\right)\right.$$
$$\left.+m_u^2\left(m_t^2-m_u^2\right)\right), \tag{27}$$

$$a_{23}=L_{11}\left({m_d}^2+{m_s}^2\right)\left({m_t}^2+{m_u}^2\right)-L_{12}\left({m_u}^2+{m_t}^2\right)$$
$$-L_{21}\left({m_d}^2+{m_s}^2\right)+L_{22}$$
$$+m_t^2 m_u^2\left(m_b^4-m_b^2\left(m_d^2+m_s^2\right)-m_d^2 m_s^2\right)$$
$$+m_c^4 m_d^2 m_s^2-m_c^2 m_d^2 m_s^2\left(m_t^2+m_u^2\right), \tag{28}$$

and

$$\det=\left({m_b}^2-{m_d}^2\right)\left({m_c}^2-{m_u}^2\right)\left({m_d}^2-{m_s}^2\right)$$
$$\times\left({m_u}^2-{m_t}^2\right)\left({m_b}^2-{m_s}^2\right)\left({m_c}^2-{m_t}^2\right). \tag{29}$$

In these equations, the L_{ij} are obtained by evaluating the left hand side of Eq. (22). Finally, we note that knowing these four CKM *magnitudes*, we can determine the Jarlskog invariant [15], up to its sign. Thus, given some phase convention, we are also able to determine the phases of all CKM matrix elements.

3 The fit to the quark mass matrices

3.1 Parameters and observables

We would like to fit 10 observables (6 quark masses and 4 CKM parameters) with the 10 free parameters that we have in this model,

$$\beta_1, \beta_2, \rho_2, \rho_3, \Sigma_d, \Sigma_u, Z_d, Z_u, \phi_d, \phi_u. \tag{30}$$

Notice that this is a huge improvement over the SM, where there are 18 complex Yukawa parameters. Similarly, in Ref. [4], there are 18 Yukawa couplings; in their notation $h_1^{u,d}$, $h_2^{u,d}$, $h_3^{u,d}$, and those with $h \to h'$ and $h \to h''$. These reduce to 12 complex parameters, even after the approximation in their equation (19). So, having only 10 real parameters is already excellent.

Moreover, our 10 parameters are *constrained*. Although we were not able to find an analytical relation which expresses such a constraint, we can show numerically that it does exist. We postpone this proof until the end of Sect. 3.3. The upshot is that it was not guaranteed a priori that our 10 parameters would be able to fit the 10 observables. Turning the argument around, the fact that the 10 experimental values do allow for a good fit in the A_4-3HDM can be viewed as a success for the model.

Table 2 Experimental values and fit results

Observable	Experimental value	Model prediction
m_u [MeV]	2.16 ± 0.50	2.15
m_c [MeV]	1270 ± 20	1271.6
m_t [GeV]	172.69 ± 0.30	172.68
m_d [MeV]	4.67 ± 0.50	4.66
m_s [MeV]	93.4 ± 8.6	92.08
m_b [MeV]	4180 ± 30	4180.39
$\|V_{11}\|$	0.97435 ± 0.00016	0.97434
$\|V_{21}\|$	0.22486 ± 0.00067	0.22479
$\|V_{13}\|$	0.00369 ± 0.00011	0.00369
$\|V_{23}\|$	0.04182 ± 0.00085	0.04178
J	$(3.08 \pm 0.15) \times 10^{-5}$	3.09×10^{-5}

3.2 The fitting procedure

We have implemented a χ^2 analysis of the model, through a minimization performed using the CERN Minuit library [18]. The observables employed in this analysis, labeled by $i = 1, \ldots, 11$ are specified in Table 2, where $\overline{X}_i$ represents the experimental mean value of the observable X_i and σ_i is the experimental error, which, when both left and right bounds are stated, is assumed to be the largest of the two.

The data on the quark masses as well as for the CKM matrix elements and the Jarlskog invariant experimental values were obtained from [19]. As mentioned, $|J|$ is fixed by $|V_{11}|$, $|V_{21}|$, $|V_{13}|$, and $|V_{23}|$. However, using it in the fit speeds the numerical convergence onto a good solution.

The χ^2 function depends on the 10 parameters of our model (31),

$$\beta_1, \beta_2, \rho_2, \rho_3, \Sigma_d, \Sigma_u, Z_d, Z_u, \phi_d, \phi_u \tag{31}$$

and is written as

$$\chi^2(\mathbf{p})=\sum_{i=1}^{11}\left(\frac{P_i(\mathbf{p})-\overline{X}_i}{\sigma_i}\right)^2, \tag{32}$$

where $P_i(\mathbf{p})$ is our model's prediction for each of the 11 $(10+J)$ observables. The fit is complicated by the fact that the masses (squared) are obtained from the eigenvalues of H_d, H_u but the elements of the CKM also depend on the masses, see Eq. (24). So, we start by calculating the eigenvalues of H_d and H_u, which depend only on the parameters in Eq. (31). Then, we evaluate the L_{ij} from the left hand side of Eq. (22), and finally the CKM elements are obtained from Eq. (24). In Appendix A we give the explicit expressions for the matrices H_d and H_u.

3.3 Results of the fit

We have found an excellent fit of our model to the data, given in the second column of Table 2. This fit results in $\chi^2 = 0.058$, for the parameters

$$\begin{aligned}
\beta_1 &= 1.4260868 \text{ radians},\\
\beta_2 &= 1.5424328 \text{ radians},\\
\rho_2 &= 4.2784971 \text{ radians},\\
\rho_3 &= 5.3682785 \text{ radians},\\
\Sigma_d &= 0.2889178 \times 10^{-3},\\
\Sigma_u &= 0.4927455,\\
Z_d &= 0.1816577 \times 10^{-3},\\
Z_u &= 0.4758317,\\
\phi_d &= -1.7324779 \text{ radians},\\
\phi_u &= 0.20644967 \times 10^{-1} \text{ radians}.
\end{aligned} \tag{33}$$

This fit also leads to the data in the third column of Table 2, as well as to the vevs

$$|v_i| = (1.00604, 6.90357, 245.901) \ (\text{GeV}). \tag{34}$$

We notice that the vevs obey $v_1 < v_2 \ll v_3$. This hierarchy of vevs is related to the hierarchy of the quark masses. This was also obtained in Ref. [7], although their model is not consistent, as their vev structure is not that of [10] for the symmetric A_4 potential they consider.

We can now perform a second (toy) fitting procedure, which illustrates the fact that the ten parameters in our model are *constrained*, as announced at the end of Sect. 3.1. In this fit, we take all experimental values in Table 2, *except* that we trade the correct experimental value of m_s for $m_s = (2 \pm 0.02)$ GeV. Now, the fit is very poor, having $\chi^2 = 600$. If these had been the correct experimental values for the 10 observables, then our model would not be able to fit them. Conversely, the fact that such a fit is possible is a success for the model.

4 Viability of the vacuum found in the fit

We start by defining the three doublets as in Eq. (6). Next we define the physical eigenstates for the charged Higgs as $(G^+, S_2^+, S_3^2)^T$, and for the neutral states we have $(G^0, S_2^0, S_3^0, S_4^0, S_5^0, S_6^0)^T$, identifying the would-be Goldstone bosons $G^+ \equiv S_1^+$ and $G^0 \equiv S_1^0$. With these conventions, and following the definitions in [20], we define the 3×3 matrix $\tilde{U}$ by

$$\varphi_i^+ \equiv \sum_{j=1}^{3} \tilde{U}_{ij} S_j^+, \tag{35}$$

and the 3×6 matrix $\tilde{V}$ by

$$x_i + i x_{i+3} = \sum_{j=1}^{6} \tilde{V}_{ij} S_j^0. \tag{36}$$

These matrices[2] are then related to the diagonalization matrices of the charged and neutral scalars, to which we now turn.

4.1 The minimization of the potential

In our procedure we already know the values of the vevs. So, we use the stationarity equations to solve for the soft parameters, and leave the quartic parameters of the potential Λ_i as free parameters. In this way we can solve for $m_{11}^2, m_{22}^2, m_{33}^2$ as well as for $\text{Im}(m_{12}^2), \text{Im}(m_{13}^2)$, leaving as free parameters the Λ_i and $\text{Re}(m_{12}^2)$, $\text{Re}(m_{13}^2)$, $\text{Re}(m_{23}^2)$, $\text{Im}(m_{23}^2)$. When evaluating the scalar mass matrices (see below) the conditions have to be applied to ensure that we are at the minimum. For completeness we write these conditions in Appendix B.

4.2 The charged mass matrix

The charged mass matrix is obtained from the second derivatives at the minimum,

$$\mathcal{M}_C^2 = \left.\frac{\partial^2 V_H}{\partial \varphi_i^+ \partial \varphi_j^-}\right|_{\text{Min}}. \tag{37}$$

The matrix $\mathcal{M}_C^2$ is an hermitian matrix, with real eigenvalues and satisfying, with our usual conventions,

$$R_{\text{ch}} \mathcal{M}_C^2 R_{\text{ch}}^\dagger = \text{diag}(0, m_{S_2^+}^2, m_{S_3^+}^2) \equiv \mathcal{M}_{D_{ch}}^2, \tag{38}$$

where R_{ch} is an unitary matrix that satisfies,

$$S_i^+ = \sum_{j=1}^{3} (R_{\text{ch}})_{ij}\, \varphi_j^+. \tag{39}$$

[2] From the point of view of a simultaneous fit of the Yukawa and scalar sectors, it is a pity that these matrices $\tilde{V}$ and $\tilde{U}$ have in the literature the same notation as the CKM matrix V and $U_{ki} = |V_{ki}|^2$.

This can be seen from

$$\mathcal{L}_{\text{mass}} = -\varphi_i^- \left(\mathcal{M}_C^2\right)_{ij} \varphi_j^+ = -\varphi_i^- \left(R_{\text{ch}}^\dagger R_{\text{ch}} \mathcal{M}_C^2 R_{\text{ch}}^\dagger R_{\text{ch}}\right)_{ij} \varphi_j^+$$
$$= -\varphi_i^- \left(R_{\text{ch}}^\dagger \mathcal{M}_{D_{ch}}^2 R_{\text{ch}}\right)_{ij} \varphi_j^+$$
$$= -S_i^- \left(\mathcal{M}_{D_{ch}}^2\right)_{ij} S_j^+, \tag{40}$$

where we have used Eq. (39).

We have checked both algebraically and numerically that we have a zero eigenvalue corresponding to G^+ and we require that all other masses squared are positive, a condition for a local minimum.

4.3 The neutral mass matrix

Since in our case CP is not conserved, we denote the unrotated neutral scalars by x_i, $i = 1, \ldots, 6$, as in Eq. (6). We therefore obtain the neutral mass matrix as,

$$\mathcal{M}_N^2 = \left.\frac{\partial^2 V_H}{\partial x_i \partial x_j}\right|_{\text{Min}}. \tag{41}$$

This is a symmetric real matrix diagonalized by an orthogonal 6×6 matrix,

$$R_{\text{neu}} \mathcal{M}_N^2 R_{\text{neu}}^T = \text{diag}(0, m_{S_2^0}^2, m_{S_3^0}^2, m_{S_4^0}^2, m_{S_5^0}^2, m_{S_6^0}^2)$$
$$\equiv \mathcal{M}_{D_{\text{neu}}}^2, \tag{42}$$

with

$$S_i^0 = \sum_{j=1}^{6} (R_{\text{neu}})_{ij}\, x_j. \tag{43}$$

As for the case of the charged scalars, we have checked both algebraically and numerically that we have a zero eigenvalue corresponding to G^0 and we require that all other masses squared are positive, a condition for a local minimum.

5 Theoretical constraints

After having shown that a solution exists for the vevs and parameters in the Yukawa sector that correctly fits the quarks masses and the CKM entries, we have to show that this is compatible with the scalar potential analysis. In particular we have to show that the vevs correspond to a local minimum of the potential and that both the theoretical constraints as well as those coming from LHC are satisfied. In this section we analyze the theoretical constraints.

5.1 Perturbative unitarity

This problem was already solved in [14], so we take the potential in the form of Eq. (4). From Ref. [14] we have the following expression for the eigenvalues λ_i[3]

$$\lambda_1 = 2\,(2\text{Re}(c_3) + r_1) \tag{44}$$
$$\lambda_2 = 2\left(\sqrt{3}\,|\text{Im}(c_3)| - \text{Re}(c_3) + r_1\right) \tag{45}$$
$$\lambda_3 = 2\left(-\sqrt{3}\,|\text{Im}(c_3)| - \text{Re}(c_3) + r_1\right) \tag{46}$$
$$\lambda_4 = 2(r_4 + r_7) \tag{47}$$
$$\lambda_5 = 2(r_4 - r_7) \tag{48}$$
$$\lambda_6 = 2(r_1 + 2r_7) \tag{49}$$
$$\lambda_7 = 2(r_1 - r_7) \tag{50}$$
$$\lambda_8 = 2(r_4 + |c_3|) \tag{51}$$
$$\lambda_9 = 2(r_4 - |c_3|) \tag{52}$$
$$\lambda_{10} = 6r_1 + 8r_4 + 4r_7 \tag{53}$$
$$\lambda_{11} = 6r_1 - 2(2r_4 + r_7) \tag{54}$$
$$\lambda_{12} = 6|c_3| + 2r_4 + 4r_7 \tag{55}$$
$$\lambda_{13} = -6|c_3| + 2r_4 + 4r_7. \tag{56}$$

Perturbative unitarity is satisfied if

$$|\lambda_i| < 8\pi, \quad \forall i. \tag{57}$$

5.2 The BFB conditions

For the A_4 symmetric potential, the conditions for boundedness from below along the neutral directions (BFB-n) have been conjectured in [21], and proved to hold in [22]. These are

$$\Lambda_0 + \Lambda_3 \geq 0, \tag{58}$$
$$\frac{4}{3}(\Lambda_0 + \Lambda_3) + \frac{1}{2}(\Lambda_1 + \Lambda_2) - \Lambda_3$$
$$- \frac{1}{2}\sqrt{(\Lambda_1 - \Lambda_2)^2 + \Lambda_4^2} \geq 0, \tag{59}$$
$$\Lambda_0 + \frac{1}{2}(\Lambda_1 + \Lambda_2) + \frac{1}{2}(\Lambda_1 - \Lambda_2)\cos(2k\pi/3)$$
$$+ \frac{1}{2}\Lambda_4 \sin(2k\pi/3) \geq 0 \quad (k = 1, 2, 3). \tag{60}$$

However, as shown in [21,23], a potential which is BFB-n is not necessarily BFB along the charge breaking directions (BFB-c). Necessary BFB-c conditions have yet to be found for the A_4 3HDM, but sufficient conditions have been proposed in [24] following the technique developed in [25]. They are,

$$A_d \geq 0, \quad A_o \geq -A_d/2, \tag{61}$$

[3] We use λ_i instead of Λ_i, in order to not confuse with the notation of Eq. (2).

where

$$
\begin{aligned}
A_d &= a = \frac{2}{3}(\Lambda_0 + \Lambda_3), \\
A_o &= b + \min(0, c) - d \\
&= \frac{1}{3}(2\Lambda_0 - \Lambda_3) + \frac{1}{2}(\Lambda_1 + \Lambda_2) \\
&\quad + \min(0, -\frac{1}{2}(\Lambda_1 + \Lambda_2)) - \frac{1}{2}\sqrt{(\Lambda_1 - \Lambda_2)^2 + \Lambda_4^2}.
\end{aligned}
\tag{62}
$$

It is important to remark that, since these are sufficient, but not necessary, conditions, some good points in parameter space may be excluded by this restriction.

5.3 The oblique parameters S, T, U

For this we use the notation and results from [20], which require the matrices $\tilde{U}$ and $\tilde{V}$. Comparing Eq. (39) with the definition in Eq. (35), we conclude that

$$
\tilde{U} = R_{\text{ch}}^{\dagger}, \tag{63}
$$

where the matrix R_{ch} is obtained from the numerical diagonalization of Eq. (38). Similarly, comparing Eq. (43) with the definition of $\tilde{V}$ in Eq. (36), we get,

$$
\tilde{V}_{ij} = \left(R_{\text{neu}}^T\right)_{ij} + i\left(R_{\text{neu}}^T\right)_{i+3,j}. \tag{64}
$$

Having $\tilde{U}$ and $\tilde{V}$, we can construct the needed matrices $\text{Im}\left(\tilde{V}^\dagger\tilde{V}\right)$, $\tilde{U}^\dagger\tilde{U}$, $\tilde{V}^\dagger\tilde{V}$ and $\tilde{U}^\dagger\tilde{V}$, and implement the procedure of [20].

5.4 Global minimum

After finding a set of $m_{i,j}$ and Λ_i which reproduce the vevs in Eq. (34) necessary for a good fit of the quark mass matrices, and after performing the previous theoretical checks on the scalar potential, we must still ensure that our minimum is indeed the global minimum. This step is almost never taken in studies of quark mass matrices, since there are no exact analytical formulae for it. Moreover, one must check that there are no lower minima both along the neutral directions and along the charge breaking directions. We follow the strategy discussed in Ref. [24]. Take a specific set of m_{ij}^2 and Λ_i. Then we parameterize the scalar doublets as [23,24],

$$
\begin{aligned}
\langle\phi_1\rangle &= \sqrt{r_1}\begin{pmatrix}0\\1\end{pmatrix}, \quad \langle\phi_2\rangle = \sqrt{r_2}\begin{pmatrix}\sin(\alpha_2)\\ \cos(\alpha_2)e^{i\beta_2}\end{pmatrix}, \\
\langle\phi_3\rangle &= \sqrt{r_3}e^{i\gamma}\begin{pmatrix}\sin(\alpha_3)\\ \cos(\alpha_3)e^{i\beta_3}\end{pmatrix},
\end{aligned}
\tag{65}
$$

where we have already used the gauge freedom. Now we let the vevs run free, for both charge conserving and charge violating directions. We give one seed point and perform a minimization of the potential using the CERN Minuit library [18]. We obtain not only the value of the potential at the minimum, but also the values of $r_i, \alpha_2, \beta_2, \alpha_3, \beta_3$ and γ. Then, we take one more (randomly generated) seed point and repeat the minimization. Finally, we take the minimum as the global one if it is found as the global minimum in each of 200 searches with randomly generated seed points. We have done this verification for every point that passed all the constraints. In all cases, we found that the local minimum was also a global minimum. In particular we always found that

$$
\sin(\alpha_2) = \sin(\alpha_3) = 0, \tag{66}
$$

showing that we do not have charged breaking directions[4] and, comparing with Eq. (6), we verified numerically that,

$$
\begin{aligned}
\frac{|v_i|}{\sqrt{2}} &= \sqrt{r_i}, \quad e^{i\,\rho_2} = \cos(\alpha_2)\, e^{i\,\beta_2}, \\
e^{i\,\rho_3} &= \cos(\alpha_3)\, e^{i\,(\beta_3+\gamma)}.
\end{aligned}
\tag{67}
$$

6 Simple LHC constraints

Up to now we have implemented the theoretical constraints on the model. The next step is to implement the LHC constraints. To do this completely one would have to implement all the decays of the neutral and charged Higgs as well as their branching ratios. One would also have to worry about the electric dipole moments (EDM) and the flavour-changing neutral couplings (FCNC), as the model does not have a structure of couplings of the Higgs to the fermions that automatically ensures vanishing FCNC [26–28]. This lies beyond the scope of the present work. Nonetheless, we can implement easily the constraints that come from $h \to WW/ZZ$ in the κ formalism, where the deviation from the coupling of the SM Higgs boson to a pair of W's (or Z's) is measured by κ_V.

In our model,

$$
\begin{aligned}
\kappa_V =& R_{21}^{\text{neu}} v_1 + R_{22}^{\text{neu}} v_2 \cos(\rho_2) + R_{23}^{\text{neu}} v_3 \cos(\rho_3) \\
&+ R_{25}^{\text{neu}} v_2 \sin(\rho_2) + R_{26}^{\text{neu}} v_3 \sin(\rho_3),
\end{aligned}
\tag{68}
$$

where R^{neu} is matrix defined in Eq. (42). We take the experimental constraint from ATLAS [29],

$$
\kappa_W = 1.0206\,^{+0.05172}_{-0.05087}, \quad \kappa_Z = 0.99\,^{+0.06136}_{-0.05214}. \tag{69}
$$

4 To cross check our numerical procedure we also considered points that violated the BFB conditions. And, indeed for these points, our algorithm showed that the potential was not BFB and could have charge breaking directions as well.

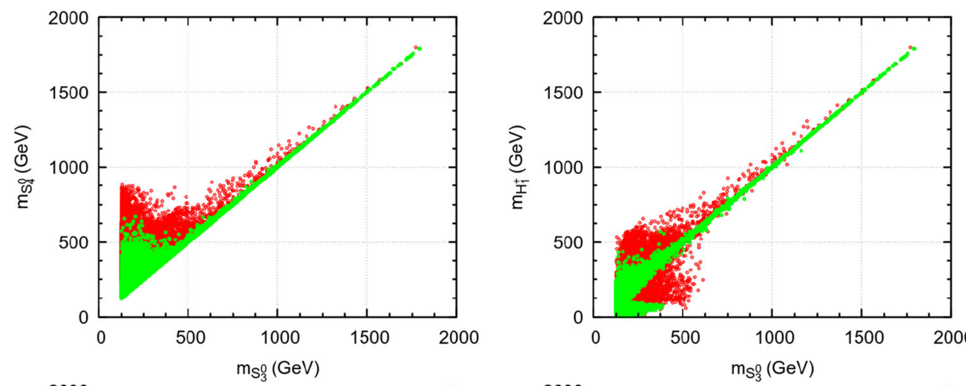

Fig. 1 Left panel: Relation between $m_{S_3^0}$ and $m_{S_4^0}$. Right panel: Relation between $m_{S_3^0}$ and $m_{H_1^+}$. Color conventions: No cuts (red); with cuts (green)

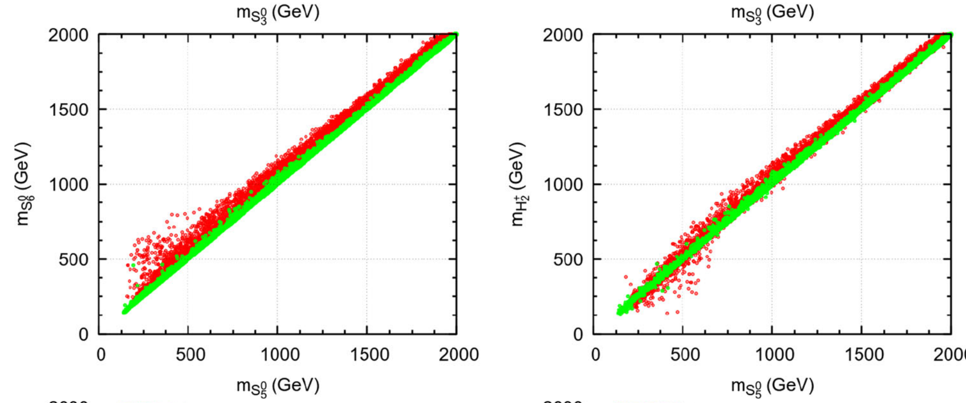

Fig. 2 Left panel: Relation between $m_{S_5^0}$ and $m_{S_6^0}$. Right panel: Relation between $m_{S_5^0}$ and $m_{H_2^+}$. Color conventions: No cuts (red); with cuts (green)

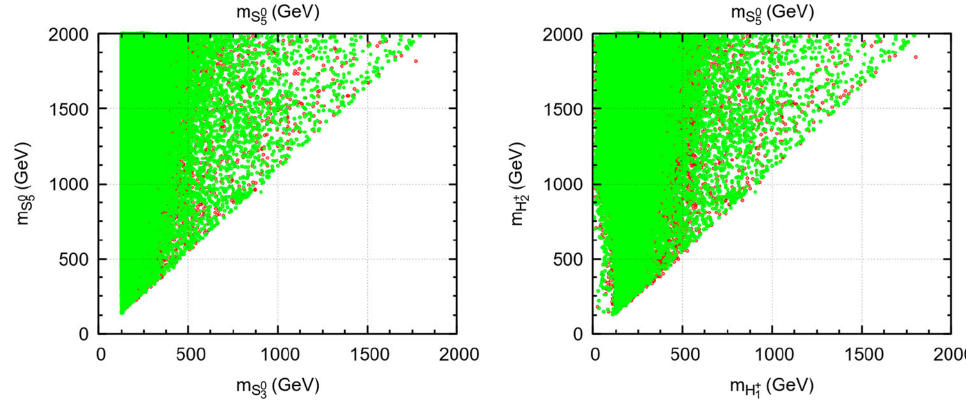

Fig. 3 Left panel: Relation between $m_{S_3^0}$ and $m_{S_5^0}$. Right panel: Relation between $m_{H_1^+}$ and $m_{H_2^+}$. Color conventions: No cuts (red); with cuts (green)

7 Results

In this section we present the results of the analysis of the scalar potential after imposing that we have a good solution for the fit of the quarks masses and CKM entries, as explained in Sect. 3.

7.1 Scanning strategy

We start by imposing the vevs obtained in the fit.

$$v_1 = 1.00604\,(\text{GeV}), \quad v_2 = 6.90357\, e^{i\,4.278497}\,(\text{GeV}),$$
$$v_3 = 245.901\, e^{i\,5.368278}\,(\text{GeV}). \tag{70}$$

Now we vary the free parameters of the potential in the following ranges,

$$\log_{10}|\Lambda_i| \in [-3, 1], \quad \log_{10}|\text{Im}(m_{23}^2)| \in [-1, 7]\,\text{GeV}^2,$$
$$\log_{10}|\text{Re}(m_{ij}^2)| \in [-1, 7]\,\text{GeV}^2, \tag{71}$$

where in the last equation we use

$$m_{ij}^2 \in \left\{m_{12}^2, m_{13}^2, m_{23}^2\right\}. \tag{72}$$

We randomly scan as in Eq. (71), and then:

1. Apply the theoretical constraints that only depend on the Λ_i, that is BFB and perturbative unitarity.
2. Then obtain the eigenvalues for the charged and neutral scalars. Verify that all the masses squared are positive, and that we have a zero eigenvalue corresponding to the Goldstone bosons, G^0 and G^+.
3. Verify the S, T and U oblique parameters.
4. Apply the LHC constraint on κ_V.
5. Check numerically that the vev is indeed a global minimum.

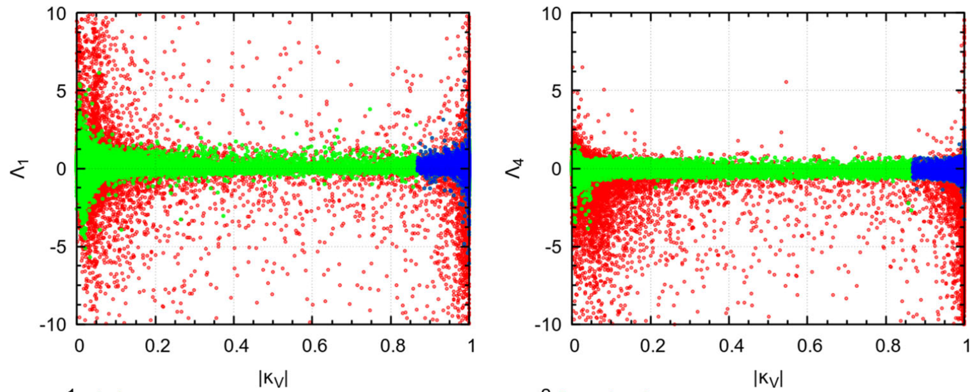

Fig. 4 Left panel: Relation between κ_V and Λ_1. Right panel: Relation between κ_V and Λ_4. Color conventions: No cuts (red), with theoretical cuts (green), and after the κ_V constraint (blue)

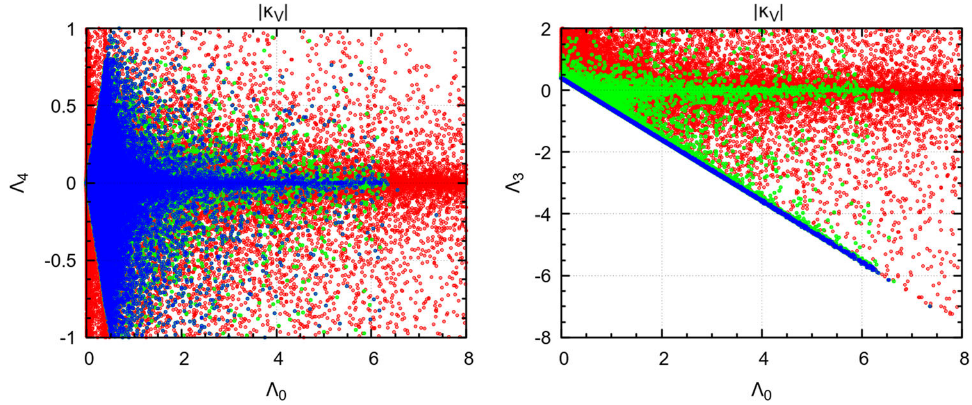

Fig. 5 Left panel: Relation between Λ_0 and Λ_4. Right panel: Relation between Λ_0 and Λ_3 Color conventions: No cuts (red), with theoretical cuts (green), and after the κ_V constraint (blue)

7.2 The scalar spectrum

We found that there is a strong correlation in the scalar masses. If we denote the masses of the neutral scalars by $(m_{G^0} = 0, m_{S_2^0}, m_{S_3^0}, m_{S_4^0}, m_{S_5^0}, m_{S_6^0})$, and $(m_{G^+} = 0, m_{H_1^+}, m_{H_2^+})$ for the charged scalars, we find numerically that

$$m_{S_3^0} \simeq m_{S_4^0} \simeq m_{H_1^+}, \quad m_{S_5^0} \simeq m_{S_6^0} \simeq m_{H_2^+}. \tag{73}$$

This is true even if we do not require $m_{S_2^0} = 125\,\text{GeV}$, and specially true after implementing the constraints of perturbative unitarity, BFB and STU. But, as we want to reproduce the LHC results, we also required that [19]

$$m_{S_2^0} = 125.25 \pm 0.17\,\text{GeV}. \tag{74}$$

In the following figures we show the correlation among the masses. Included in red are the points generated before the theoretical cuts were applied, and in green the points remaining after the constraints were implemented (Figs. 1, 2, 3).

7.3 The κ_V constraint

We can now implement the κ_V constraint on the model. In the following figures, in red are points without cuts, in green with cuts but no κ_V constraint, and finally in blue points remaining after this constraint is applied. We took the ATLAS result of Eq. (69) at 2σ. While the theoretical constraints cut around 88% of the points, the κ_V constraint only cuts 22% of the remaining points. In Fig. 4 we show the relation between κ_V and $\Lambda_{1,4}$ for the three sets of points as discussed above.

In fact it is not obvious from Fig. 4 that the κ_V constraint only cuts about 22% of the points that pass the other cuts. This is because there is a very large number of points with $|\kappa_V| \lesssim 1$, even without theoretical cuts, and this is even more so after imposing the theoretical cuts. In this figure, we have 200,000 points in the green region, but from these 156,516 are in the blue region. That is, after theoretical cuts, 78% of the points also satisfy the κ_V constraint. In Fig. 5 we show the relation between Λ_0 and $\Lambda_{3,4}$ for the same sets of points. We see that, while for $(\Lambda_0,\ \Lambda_4)$ there is not much difference before and after the κ_V constraint, the same is not true for $(\Lambda_0,\ \Lambda_3)$, where the constraints impose a linear relation between those two parameters. We note that, while Λ_0 is always positive, Λ_3 can be negative respecting the BFB condition in Eq. (58), $\Lambda_0 + \Lambda_3 \geq 0$, as it is clear in the right-handed panel of Fig. 5. Before we end this section, let us remark that we did not redraw the figures in Sect. 7.2 after imposing the κ_V constraint, as the blue points would just superimpose the green points, as we have checked.

8 Conclusions

It is known that the 3HDM symmetric under an exact A_4 symmetry is not compatible with non-zero quark masses and/or non-block-diagonal CKM matrix [13]. In this work, we studied a 3HDM with A_4 softly broken. This allows us to evade the above result, by enlarging the structure of the possible vacua.

We obtained an excellent fit of the quarks mass matrices, including the CP-violating Jarlskog invariant. This leads to a unique solution for the vevs. We showed that, with the solution for the vevs obtained from the fit, it is possible to have a local minimum of the potential. We enforce this by imposing that all squared masses are positive. As in our scheme the scalar masses are not input parameters, we have to restrict one of the neutral scalars to have the mass of the known Higgs boson.

We have implemented the BFB, perturbative unitarity and the oblique parameters S, T, U theoretical constraints. From LHC, we have considered the observed Higgs mass and the κ_V constraint.[5] After imposing the other constraints, we found that most of the points are close to the alignment required to respect the experimental κ_V constraint. We have discovered a strong correlation among the masses of the scalars, even before applying the theoretical constraints, especially for moderate to large scalar masses.

One important point is that we have numerically checked for all the points that pass our constraints, that for a given set of parameters of the potential, our minimum is the true global minimum.

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Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors' comment: The datasets generated during and/or analysed during the current study are available from the corresponding author upon request].

Funded by SCOAP^3. SCOAP^3 supports the goals of the International Year of Basic Sciences for Sustainable Development.

[5] The detailed study of other LHC constraints as well as those coming from FCNC and the EDM lies beyond the scope of the present work, and is left for a future publication.

Appendix A: The matrices H_d and H_u

$$H_d(1,1) = \Sigma_d v^2 \cos^2(\beta_1)\cos^2(\beta_2) \tag{75}$$

$$H_d(1,2) = v^2 Z_d \cos(\beta_1)\cos^2(\beta_2)\cos(\rho_2-\phi_d)\sin(\beta_1) - i\, v^2 Z_d \cos(\beta_1)\cos^2(\beta_2)\sin(\beta_1)\sin(\rho_2-\phi_d) \tag{76}$$

$$H_d(1,3) = v^2 Z_d \cos(\beta_1)\cos(\beta_2)\cos(\rho_3+\phi_d)\sin(\beta_2) - i\, v^2 Z_d \cos(\beta_1)\cos(\beta_2)\sin(\beta_2)\sin(\rho_3+\phi_d) \tag{77}$$

$$H_d(2,1) = (H_d(1,2))^* \tag{78}$$

$$H_d(2,2) = \Sigma_d v^2 \cos^2(\beta_2)\sin^2(\beta_1) \tag{79}$$

$$H_d(2,3)=v^2 Z_d \cos(\beta_2)\cos(\rho_2-\rho_3+\phi_d)\sin(\beta_1)\sin(\beta_2) +i\, v^2 Z_d \cos(\beta_2)\sin(\beta_1)\sin(\beta_2)\sin(\rho_2-\rho_3+\phi_d) \tag{80}$$

$$H_d(3,1) = (H_d(1,3))^* \tag{81}$$

$$H_d(3,2) = (H_d(2,3))^* \tag{82}$$

$$H_d(3,3) = \Sigma_d v^2 \sin^2(\beta_2) \tag{83}$$

$$H_u(1,1) = \Sigma_u v^2 \cos^2(\beta_1)\cos^2(\beta_2) \tag{84}$$

$$H_u(1,2) = v^2 Z_u \cos(\beta_1)\cos^2(\beta_2)\cos(\rho_2+\phi_u)\sin(\beta_1) + i\, v^2 Z_u \cos(\beta_1)\cos^2(\beta_2)\sin(\beta_1)\sin(\rho_2+\phi_u) \tag{85}$$

$$H_u(1,3) = v^2 Z_u \cos(\beta_1)\cos(\beta_2)\cos(-\rho_3+\phi_u)\sin(\beta_2) - i\, v^2 Z_u \cos(\beta_1)\cos(\beta_2)\sin(\beta_2)\sin(-\rho_3+\phi_u) \tag{86}$$

$$H_u(2,1) = (H_u(1,2))^* \tag{87}$$

$$H_u(2,2) = \Sigma_u v^2 \cos^2(\beta_2)\sin^2(\beta_1) \tag{88}$$

$$H_u(2,3)=v^2 Z_u \cos(\beta_2)\cos(-\rho_2+\rho_3+\phi_u)\sin(\beta_1)\sin(\beta_2) +i\, v^2 Z_u \cos(\beta_2)\sin(\beta_1)\sin(\beta_2) \times \sin(-\rho_2+\rho_3+\phi_u) \tag{89}$$

$$H_u(3,1) = (H_u(1,3))^* \tag{90}$$

$$H_u(3,2) = (H_u(2,3))^* \tag{91}$$

$$H_u(3,3) = \Sigma_u v^2 \sin^2(\beta_2). \tag{92}$$

Appendix B: The minimization conditions

$$\begin{aligned} m_{11}^2 = -\frac{\sec(\rho_2)\sec(\rho_3)}{24 v_1^2}\Big[&-12\mathrm{Im}(m_{23}^2)v_2 v_3 \sin(2(\rho_2-\rho_3)) \\ &+\cos(\rho_2-\rho_3)\Big(4\Lambda_0 v_1^2 v^2 \\ &+6\Lambda_1 v_1^2 v_2^2 + 6\Lambda_1 v_1^2 v_3^2 + 3\Lambda_1 v_2^2 v_3^2 - 3\Lambda_2 v_2^2 v_3^2 \\ &+2\Lambda_3 v_1^2\left(2v_1^2 - v_2^2 - v_3^2\right)\Big) \\ &+4\Lambda_0 v_1^2 v_2^2 \cos(\rho_2+\rho_3) + 4\Lambda_0 v_1^2 v_3^2 \cos(\rho_2+\rho_3) \\ &+4\Lambda_0 v_1^4 \cos(\rho_2+\rho_3) \end{aligned}$$

$$
\begin{aligned}
&+6\Lambda_1 v_1^2 v_2^2 \cos(\rho_2+\rho_3) + 6\Lambda_1 v_1^2 v_3^2 \cos(\rho_2+\rho_3)\\
&-3\Lambda_1 v_2^2 v_3^2 \cos(3(\rho_2-\rho_3))\\
&+3\Lambda_2 v_2^2 v_3^2 \cos(3(\rho_2-\rho_3)) - 2\Lambda_3 v_1^2 v_2^2 \cos(\rho_2+\rho_3)\\
&-2\Lambda_3 v_1^2 v_3^2 \cos(\rho_2+\rho_3)\\
&+4\Lambda_3 v_1^4 \cos(\rho_2+\rho_3) + 3\Lambda_4 v_1^2 v_2^2 \sin(\rho_2-\rho_3)\\
&+3\Lambda_4 v_1^2 v_2^2 \sin(\rho_2+\rho_3)\\
&+3\Lambda_4 v_1^2 v_3^2 \sin(\rho_2-\rho_3) - 3\Lambda_4 v_1^2 v_3^2 \sin(\rho_2+\rho_3)\\
&-3\Lambda_4 v_2^2 v_3^2 \sin(\rho_2-\rho_3)\\
&+3\Lambda_4 v_2^2 v_3^2 \sin(3(\rho_2-\rho_3)) - 12\mathrm{Re}(m_{23}^2) v_2 v_3 \cos(2(\rho_2-\rho_3))\\
&+24\mathrm{Re}(m_{13}^2) v_1 v_3 \cos(\rho_2) + 24\mathrm{Re}(m_{12}^2) v_1 v_2 \cos(\rho_3)\\
&+12\mathrm{Re}(m_{23}^2) v_2 v_3 \Big] \qquad (93)
\end{aligned}
$$

$$
\begin{aligned}
m_{22}^2 = &-\frac{1}{12 v_2}\Big[3\sec(\rho_2)\Big(-4\mathrm{Im}(m_{23}^2) v_3 \sin(\rho_3) + v_2 v_3^2 (\Lambda_1-\Lambda_2)\\
&\times \cos(\rho_2-2\rho_3) - \Lambda_4 v_2 v_3^2 \sin(\rho_2-2\rho_3)\\
&+4\mathrm{Re}(m_{23}^2) v_3 \cos(\rho_3) + 4\mathrm{Re}(m_{12}^2) v_1\Big) + v_2\Big(4\Lambda_0 v^2\\
&+6\Lambda_1 v_1^2 + 3\Lambda_1 v_3^2 + 3\Lambda_2 v_3^2\\
&-2\Lambda_3 v_1^2 + 4\Lambda_3 v_2^2 - 2\Lambda_3 v_3^2 + 3\Lambda_4 v_1^2 \tan(\rho_2)\Big)\Big] \qquad (94)
\end{aligned}
$$

$$
\begin{aligned}
m_{33}^2 = &-\frac{1}{12 v_3}\Big[3\sec(\rho_3)\Big(4\mathrm{Im}(m_{23}^2) v_2 \sin(\rho_2)\\
&+v_2^2 v_3 (\Lambda_1-\Lambda_2)\cos(2\rho_2-\rho_3) - \Lambda_4 v_2^2 v_3 \sin(2\rho_2-\rho_3)\\
&+4\mathrm{Re}(m_{23}^2) v_2 \cos(\rho_2) + 4\mathrm{Re}(m_{13}^2) v_1\Big)\\
&+ v_3\Big(4\Lambda_0\left(v_1^2+v_2^2+v_3^2\right) + 6\Lambda_1 v_1^2 + 3\Lambda_1 v_2^2\\
&+3\Lambda_2 v_2^2 - 2\Lambda_3 v_1^2 - 2\Lambda_3 v_2^2 + 4\Lambda_3 v_3^2 - 3\Lambda_4 v_1^2 \tan(\rho_3)\Big)\Big] \qquad (95)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Im}(m_{12}^2) = &\frac{1}{4 v_1}\Big[\sec(\rho_2)\Big(4\mathrm{Im}(m_{23}^2) v_3 \cos(\rho_2-\rho_3)\\
&-\Lambda_1 v_2 v_3^2 \sin(2(\rho_2-\rho_3)) - \Lambda_1 v_1^2 v_2 \sin(2\rho_2)\\
&+\Lambda_2 v_2 v_3^2 \sin(2(\rho_2-\rho_3)) + \Lambda_2 v_1^2 v_2 \sin(2\rho_2)\\
&-\Lambda_4 v_2 v_3^2 \cos(2(\rho_2-\rho_3))\\
&+\Lambda_4 v_1^2 v_2 \cos(2\rho_2) - 4\mathrm{Re}(m_{23}^2) v_3 \sin(\rho_2-\rho_3)\\
&-4\mathrm{Re}(m_{12}^2) v_1 \sin(\rho_2)\Big)\Big] \qquad (96)
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Im}(m_{13}^2) = &-\frac{1}{4 v_1}\Big[\sec(\rho_3)\Big(4\mathrm{Im}(m_{23}^2) v_2 \cos(\rho_2-\rho_3)\\
&-\Lambda_1 v_2^2 v_3 \sin(2(\rho_2-\rho_3)) + \Lambda_1 v_1^2 v_3 \sin(2\rho_3)\\
&+\Lambda_2 v_2^2 v_3 \sin(2(\rho_2-\rho_3)) - \Lambda_2 v_1^2 v_3 \sin(2\rho_3)\\
&-\Lambda_4 v_2^2 v_3 \cos(2(\rho_2-\rho_3))\\
&+\Lambda_4 v_1^2 v_3 \cos(2\rho_3) - 4\mathrm{Re}(m_{23}^2) v_2 \sin(\rho_2-\rho_3)\\
&+4\mathrm{Re}(m_{13}^2) v_1 \sin(\rho_3)\Big)\Big]. \qquad (97)
\end{aligned}
$$

References

1. ATLAS Collaboration, Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC. Phys. Lett. B **716**, 1 (2012). https://doi.org/10.1016/j.physletb.2012.08.020. arXiv:1207.7214
2. CMS Collaboration, Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC. Phys. Lett. B **716**, 30 (2012). https://doi.org/10.1016/j.physletb.2012.08.021. arXiv:1207.7235
3. E. Ma, G. Rajasekaran, Softly broken a(4) symmetry for nearly degenerate neutrino masses. Phys. Rev. D **64**, 113012 (2001). arXiv:hep-ph/0106291
4. E. Ma, Quark mass matrices in the a(4) model. Mod. Phys. Lett. A **17**, 627 (2002). arXiv:hep-ph/0203238
5. E. Ma, H. Sawanaka, M. Tanimoto, Quark masses and mixing with A4 family symmetry. Phys. Lett. B **641**, 301 (2006). https://doi.org/10.1016/j.physletb.2006.08.062. arXiv:hep-ph/0606103
6. E. Ma, Suitability of a(4) as a family symmetry in grand unification. Mod. Phys. Lett. A **21**, 2931 (2006). arXiv:hep-ph/0607190
7. L. Lavoura, H. Kuhbock, A(4) model for the quark mass matrices. Eur. Phys. J. C **55**, 303 (2008). https://doi.org/10.1140/epjc/s10052-008-0562-1. arXiv:0711.0670
8. S. Morisi, E. Peinado, An A4 model for lepton masses and mixings. Phys. Rev. D **80**, 113011 (2009). https://doi.org/10.1103/PhysRevD.80.113011. arXiv:0910.4389
9. R. de Adelhart Toorop, F. Bazzocchi, L. Merlo, A. Paris, Constraining flavour symmetries at the EW scale I: the A4 Higgs potential. JHEP **03**, 035 (2011). https://doi.org/10.1007/JHEP03(2011)035. arXiv:1012.1791 [Erratum: JHEP **01**, 098 (2013). https://doi.org/10.1007/JHEP01(2013)098]
10. A. Degee, I.P. Ivanov, V. Keus, Geometric minimization of highly symmetric potentials. JHEP **02**, 125 (2013). https://doi.org/10.1007/JHEP02(2013)125. arXiv:1211.4989
11. M. Leurer, Y. Nir, N. Seiberg, Mass matrix models. Nucl. Phys. B **398**, 319 (1993). https://doi.org/10.1016/0550-3213(93)90112-3. arXiv:hep-ph/9212278
12. R. González Felipe, I.P. Ivanov, C.C. Nishi, H. Serôdio, J.P. Silva, Constraining multi-Higgs flavour models. Eur. Phys. J. C **74**, 2953 (2014). https://doi.org/10.1140/epjc/s10052-014-2953-9. arXiv:1401.5807
13. R. González Felipe, H. Serôdio, J.P. Silva, Models with three Higgs doublets in the triplet representations of A_4 or S_4. Phys. Rev. D **87**, 055010 (2013). https://doi.org/10.1103/PhysRevD.87.055010. arXiv:1302.0861
14. M.P. Bento, J.C. Romão, J.P. Silva, Unitarity bounds for all symmetry-constrained 3HDMs. JHEP **08**, 273 (2022). https://doi.org/10.1007/JHEP08(2022)273. arXiv:2204.13130
15. C. Jarlskog, Commutator of the quark mass matrices in the standard electroweak model and a measure of maximal CP nonconservation. Phys. Rev. Lett. **55**, 1039 (1985). https://doi.org/10.1103/PhysRevLett.55.1039
16. N. Razzaghi, S.M.M. Rasouli, P. Parada, P. Moniz, Two-zero textures based on A_4 symmetry and unimodular mixing matrix. Symmetry **14**(11), 2410 (2022). arXiv:2203.05753
17. G.C. Branco, L. Lavoura, Rephasing invariant parametrization of the quark mixing matrix. Phys. Lett. B **208**, 123 (1988). https://doi.org/10.1016/0370-2693(88)91216-6
18. F. James, M. Roos, Minuit: a system for function minimization and analysis of the parameter errors and correlations. Comput. Phys. Commun. **10**, 343 (1975). https://doi.org/10.1016/0010-4655(75)90039-9
19. Particle Data Group Collaboration, Review of particle physics. PTEP **2022**, 083C01 (2022). https://doi.org/10.1093/ptep/ptac097
20. W. Grimus, L. Lavoura, O.M. Ogreid, P. Osland, A precision constraint on multi-Higgs-doublet models. J. Phys. G **35**, 075001 (2008). https://doi.org/10.1088/0954-3899/35/7/075001. arXiv:0711.4022

21. I.P. Ivanov, F. Vazão, Yet another lesson on the stability conditions in multi-Higgs potentials. JHEP **11**, 104 (2020). https://doi.org/10.1007/JHEP11(2020)104. arXiv:2006.00036
22. N. Buskin, I.P. Ivanov, Bounded-from-below conditions for A_4-symmetric 3HDM. J. Phys. A **54**, 325401 (2021). https://doi.org/10.1088/1751-8121/ac0e53. arXiv:2104.11428
23. F.S. Faro, I.P. Ivanov, Boundedness from below in the $U(1) \times U(1)$ three-Higgs-doublet model. Phys. Rev. D **100**, 035038 (2019). https://doi.org/10.1103/PhysRevD.100.035038. arXiv:1907.01963
24. S. Carrôlo, J.C. Romão, J.P. Silva, Conditions for global minimum in the A_4 symmetric 3HDM. Eur. Phys. J. C **82**, 749 (2022). https://doi.org/10.1140/epjc/s10052-022-10712-3. arXiv:2207.02928
25. R. Boto, J.C. Romão, J.P. Silva, Bounded from below conditions on a class of symmetry constrained 3HDM. Phys. Rev. D **106**, 115010 (2022). https://doi.org/10.1103/PhysRevD.106.115010. arXiv:2208.01068
26. S.L. Glashow, S. Weinberg, Natural conservation laws for neutral currents. Phys. Rev. D **15**, 1958 (1977). https://doi.org/10.1103/PhysRevD.15.1958
27. P.M. Ferreira, L. Lavoura, J.P. Silva, Renormalization-group constraints on Yukawa alignment in multi-Higgs-doublet models. Phys. Lett. B **688**, 341 (2010). https://doi.org/10.1016/j.physletb.2010.04.033. arXiv:1001.2561
28. K. Yagyu, Higgs boson couplings in multi-doublet models with natural flavour conservation. Phys. Lett. B **763**, 102 (2016). https://doi.org/10.1016/j.physletb.2016.10.028. arXiv:1609.04590
29. ATLAS Collaboration, A detailed map of Higgs boson interactions by the ATLAS experiment ten years after the discovery. Nature **607**, 52 (2022). https://doi.org/10.1038/s41586-022-04893-w. arXiv:2207.00092