Symmetry and decoupling in multi-Higgs boson models

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We study the scalar sector of the most general multi-Higgs doublet model satisfying explicitly an exact symmetry. We prove that such a model will exhibit decoupling if and only if the vacuum also satisfies the same symmetry. This general property is also shown independently and explicitly for three Higgs doublet models by considering in detail all symmetry-constrained models and their possible vacua. We also discuss some specific characteristics of different decoupling patterns.

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I. INTRODUCTION

There is great interest in models with extra scalars, for they can correct many of the shortcomings of the Standard Model (SM), such as the need for extra sources of *CP* violation to drive baryogenesis, the need for dark matter, or even, for example through a hierarchy of vacuum expectation values (vev), explain the smallness of neutrino masses. An independent source of interest lies in the fact that the ATLAS and CMS collaborations [1,2] have found a fundamental scalar (the 125 GeV Higgs boson h_{125}), prompting the obvious question: how many fundamental scalars are there in nature? Will it happen as in the fermion sector, where there are multiple families? As a result, many articles focus on N Higgs doublet models (NHDM)—for reviews, see, for example, [3–5] and references therein.

But multiscalar models are already constrained by data from LHC. In particular, the couplings of the h_{125} to gauge bosons and the heaviest charged fermions are known to coincide with couplings expected in the SM, with errors of order 20% or better [6–9]. This feature is easy to explain in models which have a so-called decoupling limit [10]. In that limit, the extra scalar fields have large masses and what is left at low energy is a state whose properties approach naturally those of the SM Higgs boson.[1] The most general NHDM does have a decoupling limit. However, it has too many parameters and may suffer from large flavor changing neutral scalar couplings, which are very constrained by flavor physics experiments. So, it has become commonplace to add specific symmetries to the NHDM.

Nevertheless, it has been appreciated for a while that many such symmetry-constrained models cannot accommodate a decoupling limit; see, for example, [10,15–17]. In this article, we use a very general method to show that for any NHDM with an exact symmetry, there will be a decoupling limit if and only if the vacuum also respects that symmetry. In contrast, Ref. [18] considered partial results applicable to NHDM with soft symmetry breaking.[2]

We present the notation in Sec. II and prove our theorem in Sec. III. An alternative to the method mentioned in Sec. III would be to identify *all* the symmetry-constrained NHDM models for a given N; and, within those, all the possible vacua. One would then study the existence (or lack thereof) of a decoupling limit for each case. This was the method mentioned in [18] in connection with the 2HDM. The 3HDM is the only other case where all symmetry-constrained models [20] and their respective vacua [21] have been identified. We present that alternative (and long) calculation in detail in Sec. IV. Of course, it confirms our general theorem, but it highlights how simple and elegant the general result is. We draw our conclusions in Sec. V.

II. NOTATION

A. The scalar potential

Consider a $SU(2)_L \times U(1)_Y$ gauge theory with N scalar doublets Φ_i with hypercharge $Y = 1/2$ ($Q = T_3 + Y$). The scalar potential can be written as [22]

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[1] There is also the alternative possibility, not considered here, that one has alignment without decoupling [11–14].

[2] It is interesting to note that that also bounded from below conditions deduced for the case with an exact symmetry can be invalidated by the introduction of soft symmetry breaking terms [19].

$$V_H = Y_{ij}(\Phi_i^\dagger \Phi_j) + Z_{ij,kl}(\Phi_i^\dagger \Phi_j)(\Phi_k^\dagger \Phi_l), \tag{1}$$

where, by Hermiticity,

$$Y_{ij} = Y_{ji}^*, \qquad Z_{ij,kl} = Z_{kl,ij} = Z_{ji,lk}^*. \tag{2}$$

We take the vacuum expectation values (vev) which preserve electromagnetism

$$\langle \Phi_i \rangle = \begin{pmatrix} 0 \\ v_i/\sqrt{2} \end{pmatrix}, \tag{3}$$

which may in general be complex. The stationary conditions are

$$(Y_{ij} + Z_{ij,kl} v_k^* v_l) v_j = 0. \tag{4}$$

Except where indicated otherwise, we use implicit summation of repeated indices. Since the mass matrix for the charged scalars is

$$(M_\pm^2)_{ij} = Y_{ij} + Z_{ij,kl} v_k^* v_l, \tag{5}$$

the stationarity conditions in (4) may be rewritten as

$$(M_\pm^2)_{ij} v_j = 0. \tag{6}$$

This expression will turn out to be quite useful.

B. Basis transformations and symmetries

It is very important to make clear the distinct concepts of basis transformations, on the one hand, and of symmetries, on the other. We start with the former. The theory was originally written in terms of fields Φ_i. These may be traded for new fields Φ_i':

$$\Phi_i \to \Phi_i' = U_{ij}\Phi_j. \tag{7}$$

This is a basis transformation which leaves the kinetic terms unchanged, making U a $N \times N$ unitary matrix. Under this transformation, the potential's parameters and the vevs become

$$Y_{ij} \to Y_{ij}' = U_{ik} Y_{kl} U_{jl}^*, \tag{8}$$

$$Z_{ij,kl} \to Z_{ij,kl}' = U_{im} U_{ko} Z_{mn,op} U_{jn}^* U_{lp}^*, \tag{9}$$

$$v_i \to v_i' = U_{ij} v_j. \tag{10}$$

Since such a transformation can have no effect on the physical predictions, only basis invariant combinations can be observed experimentally [22].

We now turn to the concept of symmetry. Take

$$\Phi_i \to \Phi_i^S = S_{ij}\Phi_j, \tag{11}$$

where S is also a $N \times N$ unitary matrix. If (11) is indeed a symmetry of the potential (1), then

$$Y_{ij} = Y_{ij}^S = S_{ik} Y_{kl} S_{jl}^*, \tag{12}$$

$$Z_{ij,kl} = Z_{ij,kl}^S = S_{im} S_{ko} Z_{mn,op} S_{jn}^* S_{lp}^*. \tag{13}$$

The symmetry may (or not) be spontaneously broken, depending on whether (or not)

$$v_i = v_i^S = S_{ij} v_j. \tag{14}$$

The crucial difference between a basis transformation and a symmetry is that in the former the potential parameters do not remain the same, while in the latter those coefficients must remain invariant.

Consider a theory in which V_H, when written in terms of the fields Φ_i, has the symmetry S. Now, perform the basis transformation in Eq. (7). When written in terms of the new fields Φ_i', V_H is no longer invariant under S; rather, it is now invariant under

$$S' = USU^\dagger. \tag{15}$$

C. The charged Higgs basis

The mass matrix for the charged scalars in Eq. (5) can be diagonalized via a unitary $N \times N$ matrix U^{ch}. But the basis freedom in Eq. (7) also involves a unitary $N \times N$ matrix. Thus, we may perform a basis change into a basis where the charged components of each doublet already correspond to mass eigenstates:

$$\Phi_1^{\mathrm{ch}} = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H^0 + iG^0) \end{pmatrix}, \qquad \Phi_k^{\mathrm{ch}} = \begin{pmatrix} H_k^+ \\ \frac{1}{\sqrt{2}}\varphi_k^{C0} \end{pmatrix}, \tag{16}$$

where H_k^+ $(k = 2\ldots N)$ are the physical charged Higgs mass eigenstate fields, with corresponding masses $m_{\pm,k}^2$. $H_1^\pm = G^\pm$ is the massless would-be Goldstone boson: $m_{\pm,1}^2 = 0$. This is known as the charged Higgs basis (CH basis) [23,24]. In this basis, only the first doublet has a vev,

$$v_1^{\mathrm{ch}} = v, \qquad v_{k\neq 1}^{\mathrm{ch}} = 0. \tag{17}$$

Thus, the CH basis is a particular case of what was dubbed a "Higgs basis" in Ref. [22]. The matrix that performs the transformation from the original basis into the CH basis clearly satisfies

$$U^{\rm ch}_{1k} = \frac{v_k^*}{v}. \tag{18}$$

We write the potential in the CH basis as

$$V_H = Y^{\rm ch}_{ij}(\Phi_i^{{\rm ch}\dagger}\Phi_j^{\rm ch}) + Z^{\rm ch}_{ij,kl}(\Phi_i^{{\rm ch}\dagger}\Phi_j^{\rm ch})(\Phi_k^{{\rm ch}\dagger}\Phi_l^{\rm ch}). \tag{19}$$

The quadratic and quartic coefficients in the CH basis are obtained by substituting U with $U^{\rm ch}$ in Eqs. (8) and (9), respectively. The matrix of the charged scalars of Eq. (5) becomes, in the CH basis,

$$(M^2_\pm)^{\rm ch}_{ij} = Y^{\rm ch}_{ij} + Z^{\rm ch}_{ij,kl}(v_k^{\rm ch})^* v_l^{\rm ch}, \tag{20}$$

$$= \delta_{ij} m^2_{\pm,i} \quad \text{(no sum)}. \tag{21}$$

Recall that we have chosen the transformation $U^{\rm ch}$ precisely such that the mass matrix is diagonal and the last equality holds.

As in any basis, the stationarity conditions may still be written as

$$(M^2_\pm)^{\rm ch}_{ij} v_j^{\rm ch} = 0, \tag{22}$$

c.f. Eq. (6). It is clear from Eqs. (17) and (21) that Eq. (22) indeed holds.

D. Decoupling

As shown in Ref. [18], the CH basis is particularly useful when investigating the decoupling limit. Looking back at Eq. (16), if one wishes to decouple the extra doublets, one merely needs to take the masses $m^2_{\pm,k}$ $(k \geq 2)$ to be much larger than v^2. Indeed, it was shown in [18] that taking the charged scalars very massive makes all extra neutral scalars very massive, and, simultaneously, suppresses any *CP* violation in scalar-pseudoscalar mixing. For completeness, we explain this result in the Appendix.

How can one make $(M^2_\pm)^{\rm ch}$ very large? Inspecting Eq. (20), one might think that there could be various ways to achieve that. However, this may only be achieved by making $Y^{\rm ch}$ large.[3] The point is that the quartic coefficients $Z^{\rm ch}$ are quite constrained by unitarity and perturbativity arguments. Common estimates take these to lie below 4π or 8π, with more precise statements possible—see, for example, Ref. [23]. So, the decoupling limit may be written schematically as

$$M^{2\,\rm ch}_\pm = Y^{\rm ch} + Z^{\rm ch} v^{{\rm ch}*} v^{\rm ch} \xrightarrow{\text{decoupling}} M^{2\,\rm ch}_\pm = Y^{\rm ch}. \tag{23}$$

Of course, the effective decoupling hinges on the possibility that (again schematically) $Y^{\rm ch}$ can be chosen much larger than v^2. Is this still possible in a symmetry-constrained potential? This is what we turn to next.

III. THEOREM AND PROOF

Let us imagine that the potential in Eq. (1) is constrained by requiring it invariant under a symmetry S, as in Eqs. (12) and (13). Then, according to Eq. (15), the potential (19) in the CH basis is invariant under

$$S^{\rm ch} = U^{\rm ch} S U^{{\rm ch}\dagger}. \tag{24}$$

In particular,

$$Y^{\rm ch} = S^{\rm ch} Y^{\rm ch} S^{{\rm ch}\dagger}. \tag{25}$$

Equation (24) is the crucial observation that has been missed before and, in particular, in Ref. [18]. We can learn quite a great deal by combining the simplicity of the CH basis with the form of the symmetry when written in the CH basis.

The possibility that the symmetry is not (is) spontaneously broken depends on whether (or not)

$$S^{\rm ch}_{ij} v_j^{\rm ch} = v_i^{\rm ch}. \tag{26}$$

This is just the CH basis version of Eq. (14). Since $S^{\rm ch}$ is $N \times N$ unitary and $v^{\rm ch}$ satisfies Eq. (17), one can show that Eq. (26) holds if and only if $S^{\rm ch}$ is of the form

$$S^{\rm ch}_{\text{vev preserving}} = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & \tilde{S}^{\rm ch} \end{array}\right), \tag{27}$$

where $\tilde{S}^{\rm ch}$ is any unitary $(N-1)\times(N-1)$ matrix. Notice that this must hold irrespectively of the specific form of the symmetry S chosen in the original basis. All such symmetries of the vacuum will map in the CH basis into symmetries $S^{\rm ch}$ of the form (27). Conversely, all symmetries of the type (27), will through $S = U^{{\rm ch}\dagger} S^{\rm ch} U^{\rm ch}$ map in the original basis onto symmetries of the vacuum, where Eq. (14) holds.

It is important to take a brief detour here. The last paragraph means that there is a very large set of symmetries of the vacuum in the original basis: a set that can be mapped onto $SU(N-1)$. Imagine that one has a group of symmetries $\{S_1, S_2, \ldots\}$ of the Lagrangian, and one wishes to inquire whether they survive spontaneous symmetry breaking. If this group is small, then it is conceivable that a vacuum may be found that breaks them all. In contrast, if the Lagrangian is invariant under a very large group, and given the fact that there are so many possible invariances of

[3] We are being slightly cavalier in this definition. Indeed, $m^2_{\pm,1} = 0$ cannot be "made large," and neither can $Y^{\rm ch}_{11}$, nor $Y^{\rm ch}_{ij\neq i}$. In fact, the latter must obey $Y^{\rm ch}_{11} + Z^{\rm ch}_{11,11} v^2 = 0$ and $Y^{\rm ch}_{ij\neq i} + Z^{\rm ch}_{ij,11} v^2 = 0$. But this subtlety does not affect our argument, so we will steer clear from overly well defined yet rather convoluted details. Whenever we mention "schematically" in the text, this is the detail we have in mind.

the vacuum, it becomes unlikely or even impossible for all symmetries of the Lagrangian to be spontaneously broken. Said otherwise, for a large enough group there will always be some remnant symmetries after spontaneous symmetry breaking. This is mentioned explicitly for the 3HDM in Sec. 5.2 of Ref. [21]. Our Eq. (27) shows why this must be true in general. Moreover, the existence or not of remnant symmetries is very important for a full theory including fermions. Indeed, it was shown in Refs. [25,26] that the only way of obtaining a physical CKM mixing matrix and, simultaneously, nondegenerate and nonzero quark masses is to require that the vevs of the Higgs fields break completely the full flavor group. This ends our detour into the importance of Eq. (27).

We now state our theorem.

Theorem: Given a Lagrangian with symmetry S, the theory has a decoupling limit if and only if the vacuum also has the same symmetry S.

Let us start our proof by assuming that S is a symmetry of the Lagrangian. Then, in the CH basis, Eq. (25) holds. We now assume that there is a decoupling limit, in the form of Eq. (23). Combining, we find

$$(M_\pm^2)^{\rm ch} = S^{\rm ch}(M_\pm^2)^{\rm ch} S^{\rm ch\dagger}. \tag{28}$$

But Eq. (28) holds if and only if $S^{\rm ch}$ is of the form (27). (We will even derive much stronger implications after Eq. (29) below.) This, in turn, holds if and only if $S^{\rm ch}$ is a symmetry of the vacuum $v^{\rm ch}$, as mentioned above. Thus, S is a symmetry of v_k in the original basis. In short, if S is a symmetry of the Lagrangian and we require a decoupling limit, then S must also be a symmetry of the vacuum.

Conversely, imagine that S is a symmetry of the Lagrangian and a symmetry of the vacuum. Then, we know from Eq. (5) that S is also a symmetry of the charged mass matrix. Thus, Eq. (28) holds in the CH basis, with $S^{\rm ch}$ given in Eq. (24). The question now is whether Eq. (28) allows or not for decoupling. We start by writing Eq. (28) as the commutator equation

$$[(M_\pm^2)^{\rm ch}, S^{\rm ch}] = 0. \tag{29}$$

Given Eq. (21), this translates into

$$(m_{\pm,i}^2 - m_{\pm,j}^2)\ S_{ij}^{\rm ch} = 0 \quad \text{(no sum)}. \tag{30}$$

If $i = 1$ and $j \neq 1$, then, because $G^\pm$ has mass $m_{\pm,1}^2 = 0$, while $m_{\pm,j}^2 \neq 0$ (assume this for the moment), one concludes that $S_{1j}^{\rm ch} = 0$. Similarly, $S_{i1}^{\rm ch} = 0$. Moreover, since $S^{\rm ch}$ is a unitary matrix, one is forced into $S_{11}^{\rm ch} = 1$, and $S^{\rm ch}$ must have the form in Eq. (27). This is the assertion we have used in the previous paragraph.

Equation (30) also means that if $S^{\rm ch}$ has any nonzero entry with $i \neq j$, then the corresponding charged scalars must be degenerate. In particular, one might consider theories with symmetries S corresponding to $S_{1,j\neq 1}^{\rm ch} \neq 0$. But those theories would have more than one massless scalar field, and, thus, be ruled out by experiment.[4] Excluding those cases, the charged scalars have masses that may be taken to infinity in a way consistent with the symmetry $S^{\rm ch}$ in the CH basis or (which is the same) S in the original basis. Thus, the theory does have a decoupling limit. In short, if S is a symmetry of the Lagrangian and of the vacuum, then the theory has a decoupling limit.

Given our theorem, and starting from a scalar potential invariant under S, Eq. (29) can be viewed as an effective definition of decoupling. Indeed, since commuting matrices do so in any basis, an alternative definition would be

$$[M_\pm^2, S] = 0. \tag{31}$$

Of course, it is much simpler to check whether or not $Sv = v$, as proposed in the theorem. Still Eq. (31) is interesting.

Equation (30) is even more powerful than it seems. It tells us exactly how the decoupling might be achieved. Let us concentrate on $i \neq 1$ and $j \neq 1$. To do so we, think of $\tilde{S}_{ab}^{\rm ch}$, where $a, b = 2, 3, \ldots N$, thus sidestepping the Goldstone boson issues already discussed. For a given $a \neq b$ there are two possibilities

(1) S is such that $\tilde{S}_{ab}^{\rm ch} \neq 0 \Rightarrow m_{\pm,a}^2 = m_{\pm,b}^2$ are degenerate.
(2) $m_{\pm,a}^2 \neq m_{\pm,b}^2$ are not degenerate $\Rightarrow S$ is such that $\tilde{S}_{ab}^{\rm ch} = 0$.

As an illustration, let us consider the 3HDM to be fully analyzed below. Requiring decoupling, there are two possibilities for S. It may lead into

$$S^{\rm ch} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}, \tag{32}$$

in which case the two charged scalars may have different masses $m_{\pm,2}^2 \neq m_{\pm,3}^2$ that can be taken to infinity independently. This includes in particular the possibility that one decouples the 3HDM into an effective 2HDM by taking only $m_{\pm,3}^2 \to \infty$. Alternatively, S may be such that

$$S^{\rm ch} = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & \tilde{S}^{\rm ch} \end{array}\right), \tag{33}$$

where now $\tilde{S}^{\rm ch}$ is a nondiagonal unitary 2×2 matrix. Then, $m_{\pm,2}^2 = m_{\pm,3}^2$ and the 3HDM can only decouple directly into the SM by taking $m_{\pm,2}^2 = m_{\pm,3}^2 \to \infty$.

[4]The presence of a second massless charged scalar field could possibly be solved by including an extra gauge group, of which this would be the corresponding would-be Goldstone boson. We will not consider that possibility here.

This study has an impact on non-Abelian symmetry groups. Indeed, take some symmetry S_k ($k = 1, 2, \ldots$), and the corresponding S'_k, obtained from S_k through Eq. (15) with the same basis transformation U for all. If $[S_1, S_2] = 0$ in one basis, then $[S'_1, S'_2] = 0$ in another. As a result, if the generators of a group are not simultaneously diagonal in some basis, neither will they be in any other basis. In particular

$$[S_1, S_2] \neq 0 \Rightarrow [S_1^{\mathrm{ch}}, S_2^{\mathrm{ch}}] \neq 0. \tag{34}$$

Thus, in such a case, one of the two S_k^{ch} will have off-diagonal entries and the decoupling charged scalars corresponding to those entries will be degenerate. We will illustrate both cases (32) and (33) with the full analysis of the 3HDM in the next section.

In contrast, Abelian groups always permit nondegenerate charged scalar masses. Indeed, if $[M_\pm^2, S] = 0$, S is in its diagonal basis, and S has nondegenerate eigenvalues, then $M_\pm^2$ is diagonal and we are already in the charged Higgs basis. If S has a degenerate subspace, then $M_\pm^2$ may be off diagonal in that subspace. But bringing $M_\pm^2$ into its CH basis diagonal form will not affect the diagonal form of S. Indeed, in that subspace S is proportional to the unit matrix and is unaffected by the mass diagonalization eventually required in that subspace. We are again in the CH basis. Thus proving our assertion.

IV. A FULL STUDY OF 3HDM

We have already proved for any NHDM that, given a Lagrangian with symmetry S, the theory has a decoupling limit if and only if the vacuum also has the same symmetry S. However, the 3HDM is the only NHDM besides the 2HDM where all the symmetries and corresponding vacua are known [20,21]. Thus, it is interesting to redo the proof of our theorem for $N = 3$ by (i) analyzing all possible symmetry-vacua pairs one by one, (ii) studying their mass matrices (charged and neutral), and (iii) probing whether (or not) they allow for decoupling in accordance with the theorem (as they must). This is also interesting because it will allow us to illustrate some of the remarks on the exact features of the alignment which we have made at the end of the previous section.

A. General method

Here, we describe the method used to test whether the masses of the particles predicted by several 3HDM have a decoupling limit or not.

The inputs to this method are a potential V_H and a respective vev.

(1) The stationarity equations impose conditions on the parameters of the potential, whose number depends on the degrees of freedom the vev has. These conditions will be referred to as t_a.

(2) Every doublet is expanded around the vev—c.f. Eq. (35)—and substituted back in the potential, such that the potential will have extra functional dependencies—c.f. Eq. (36):

$$\Phi_i = \begin{bmatrix} \varphi_i^+ \\ v_i + (H_i + i\chi_i)/\sqrt{2} \end{bmatrix}, \tag{35}$$

$$V_H = V_H(\varphi_i^+, \varphi_i^-, v_i, H_i, \chi_i). \tag{36}$$

(3) The mass (squared) matrices are calculated as being the Hessian of the potential in two different subspaces: the charged subspace and the neutral one:

$$(M_\pm^2)_{ij} = \left.\frac{\partial^2 V_H}{\partial\varphi_i^+ \partial\varphi_j^-}\right|_{\{t_a\},(\varphi_b^+\varphi_b^-, H_b, \chi_b)\to 0}, \tag{37}$$

$$(M_{\mathrm{neutral}}^2)_{ij} = \left.\frac{\partial^2 V_H}{\partial(H,\chi)_i \partial(H,\chi)_j}\right|_{\{t_a\},(\varphi_b^+\varphi_b^-, H_b, \chi_b)\to 0}, \tag{38}$$

where, recall, t_a are the conditions obtained from minimization of the potential in step 1.

(4) The eigenvalues of these matrices are the (squared) masses of the charged scalars in the first subspace (φ^+, φ^-) and of the neutral scalars in the second (H, χ).

(5) Whether the masses have a decoupling limit or not can only be decided by looking at the parametrical dependence of the eigenvalues. If any mass depends on a free parameter such as m_i (where $i = 1, 2, 3$), then there is a decoupling limit. Otherwise, the masses are said to be nondecoupling.

It is often the case where the matrix in the subspace (H, χ) is not diagonalizable analytically. Indeed, taking the obvious massless Goldstone boson out of the matrix, this still involves the solution of a polynomial of degree five. In such cases we evaluate the decoupling limit using the trace of the matrix. This works because the trace of the matrix is the sum of its eigenvalues, all of which are masses squared and, thus positive. Thus, the trace can be taken to infinity if and only if there is at least one massive state which can. In contrast, if no mass can decouple, then neither will the trace.

B. Some simple examples

Using the method described above, we can now show more concretely what is decoupling and nondecoupling. To this end we apply the method to a three Higgs doublet model with a $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2^*$ symmetry. The potential for the model may be written as

$$V_H = -\sum_{1\le i\le 3} m_i^2(\phi_i^\dagger\phi_i) + \sum_{1\le i\le j\le 3}\lambda_{ij}(\phi_i^\dagger\phi_i)(\phi_j^\dagger\phi_j) + \sum_{1\le i<j\le 3}\lambda'_{ij}(\phi_i^\dagger\phi_j)(\phi_j^\dagger\phi_i) + \lambda_1(\phi_2^\dagger\phi_3)^2 + \lambda_2(\phi_3^\dagger\phi_1)^2 + \lambda_3(\phi_1^\dagger\phi_2)^2 + \text{H.c.} \tag{39}$$

All possible vevs for this model (and to all other realizable symmetry-constrained 3HDM models) can be found in [21].

First, as an example of decoupling, we take a vev that does not break the $\mathbb{Z}_2\times\mathbb{Z}_2\times\mathbb{Z}_2^*$ symmetry: the vev $(v,0,0)$ (with v real). Following the method described in Sec. IVA, we obtain the mass matrices. For the charged fields ($\varphi_i^\pm$), the mass matrix is

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{\lambda_{12}v^2}{2}-m_2 & 0 \\ 0 & 0 & \frac{\lambda_{13}v^2}{2}-m_3 \end{pmatrix}. \tag{40}$$

The mass matrix for the neutral fields (i.e., H_i and χ_i) is

$$\begin{pmatrix} 2\lambda_{11}v^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{v^2}{2}(\lambda_{12}+\lambda'_{12})-m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{v^2}{2}(\lambda_{13}+\lambda'_{13})-m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{v^2}{2}(\lambda_{12}+\lambda'_{12})-m_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{v^2}{2}(\lambda_{13}+\lambda'_{13})-m_3 \end{pmatrix}. \tag{41}$$

The matrices obey the minimum condition $m_1=\lambda_{11}v^2$. We see that both matrices are immediately diagonal. And, from the eigenvalues, we notice that the fields Φ_2 and Φ_3 can decouple, because both have a free parameter (m_2 and m_3, respectively) that can be taken to be arbitrarily large. In this case, the vev does not break the symmetry and there is decoupling.

For the second example, we take the vev $(0,v_2,v_3)$ for the case $\lambda_1<0$. We note that this vev breaks the $\mathbb{Z}_2\times\mathbb{Z}_2\times\mathbb{Z}_2^*$ symmetry, leaving a residual symmetry of the type $\mathbb{Z}_2\rtimes\mathbb{Z}_2^*$. Following the method described in Sec. IVA, we obtain nondiagonal matrices. The mass eigenvalues for the charged fields are

$$\left\{0, -\frac{1}{2}(2\lambda_1+\lambda'_{23})(v_2{}^2+v_3{}^2), \frac{1}{2}(\lambda_{12}v_2{}^2+\lambda_{13}v_3{}^2-2m_1)\right\}, \tag{42}$$

with corresponding eigenvectors

$$\left\{\left(0,\frac{v_2}{v_3},1\right),\left(0,-\frac{v_3}{v_2},1\right),(1,0,0)\right\} \tag{43}$$

in the basis (φ_1^+, φ_2^+, φ_3^+). Note that the first two eigenvectors need a normalization constant, which is irrelevant for our purposes.

For the neutral fields the eigenvalues are

$$\begin{aligned}\Big\{&0, -2(\lambda_1 v_2{}^2+\lambda_1 v_3{}^2), \frac{1}{2}(v_2{}^2(\lambda_{12}+\lambda'_{12}-2\lambda_3)+v_3{}^2(\lambda_{13}+\lambda'_{13}-2\lambda_2)-2m_1),\\ &\frac{1}{2}(v_2{}^2(\lambda_{12}+\lambda'_{12}+2\lambda_3)+v_3{}^2(\lambda_{13}+\lambda'_{13}+2\lambda_2)-2m_1),\\ &-\sqrt{v_2{}^2v_3{}^2(2\lambda_1+\lambda_{23}+\lambda'_{23})^2+(\lambda_{22}v_2{}^2-\lambda_{33}v_3{}^2)^2}+\lambda_{22}v_2{}^2+\lambda_{33}v_3{}^2,\\ &\sqrt{v_2{}^2v_3{}^2(2\lambda_1+\lambda_{23}+\lambda'_{23})^2+(\lambda_{22}v_2{}^2-\lambda_{33}v_3{}^2)^2}+\lambda_{22}v_2{}^2+\lambda_{33}v_3{}^2\Big\}.\end{aligned} \tag{44}$$

The eigenvectors associated with these eigenvalues are, in the basis $(H_1,H_2,H_3,\chi_1,\chi_2,\chi_3)$,

$$\begin{aligned}\Big\{&\left\{0,0,0,0,\frac{v_2}{v_3},1\right\},\left\{0,0,0,0,-\frac{v_3}{v_2},1\right\},\{0,0,0,1,0,0\},\{1,0,0,0,0,0\},\\ &\left\{0,\frac{-\sqrt{v_2{}^2v_3{}^2(2\lambda_1+\lambda_{23}+\lambda'_{23})^2+(\lambda_{22}v_2{}^2-\lambda_{33}v_3{}^2)^2}+\lambda_{22}v_2{}^2-\lambda_{33}v_3{}^2}{v_2v_3(2\lambda_1+\lambda_{23}+\lambda'_{23})},1,0,0,0\right\},\\ &\left\{0,\frac{\sqrt{v_2{}^2v_3{}^2(2\lambda_1+\lambda_{23}+\lambda'_{23})^2+(\lambda_{22}v_2{}^2-\lambda_{33}v_3{}^2)^2}+\lambda_{22}v_2{}^2-\lambda_{33}v_3{}^2}{v_2v_3(2\lambda_1+\lambda_{23}+\lambda'_{23})},1,0,0,0\right\}\Big\}.\end{aligned} \tag{45}$$

We see from the eigenvalues that two of the fields do not decouple (these are a mixture of Φ_2 and Φ_3 in the eigenbasis), since there is no λ free term that we can make arbitrarily large. Here we see that the vev breaks the symmetry of the model and there is no decoupling limit.

We have only shown explicitly two simple examples. There are other possible vevs in the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2^*$ model. However, some of them will result in very complicated mass matrices, where determining analytically the eigenvalues is no longer possible. In such cases, it is necessary to evaluate the trace of the matrices to see if there is decoupling or not, as described at the end of the previous section.

C. Exhaustive list of symmetry-constrained 3HDM

Following the method and examples above, we have studied all the symmetry-vacua pairs identified in Ref. [21]. Our results are shown in Table I.

There are several things to note in Table I. The parameter $\omega = e^{i\frac{2\pi}{3}}$, meaning that $\omega^3 = 1$. The parameter λ in $\xi_i = \xi_i(v_1, v_2, v_3, \lambda)$ stands for all the coupling parameters in the potential. The dependence of ξ_i on these parameters can be determined following the procedure described in [21]. There are vevs that were not written down since they reduce trivially to the other cases studied. There are several other vevs that can be obtained from the ones listed by the group action; such a collection of vevs is dubbed "orbits" in Ref. [21]. Vevs on the same orbit lead to identical physical consequences.

TABLE I. List of all symmetry-constrained models via Higgs family symmetries in the 3HDM with corresponding vacua from [20,21]. The third column indicates whether or not the vacuum breaks the symmetry. For each pair, we have found the charged and neutral scalar mass matrices and (as explained in the text) have identified whether or not there is decoupling. This is noted in the fourth column.

Group G	vev	Breaks G?	Decoupling ?
$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2^*$	$(v, 0, 0)$	No	Y
	$(0, v_2 e^{i\frac{\pi}{4}}, v_3 e^{-i\frac{\pi}{4}})$	Y	No
	$(0, v_2, \pm v_3)$	Y	No
	$(v_1 e^{i\frac{k_1\pi}{2}}, v_2 e^{i\frac{k_2\pi}{2}}, v_3 e^{i\frac{k_3\pi}{2}}),\ k_i \in \mathbb{Z}$	Y	No
	$(v_1 e^{i\xi_1}, v_2 e^{i\xi_2}, v_3 e^{i\xi_3}),\ \xi_i = \xi_i(v_1, v_2, v_3, \lambda)$	Y	No
$\mathbb{Z}_3 \rtimes \mathbb{Z}_2^*$	$(v, 0, 0)$	No	Y
	$(v_1, v_2, 0)$	Y	No
	$(v_1, v_2 e^{i\frac{k_2\pi}{3}}, v_3 e^{i\frac{k_3\pi}{3}}),\ k_i \in \mathbb{Z}$	Y	No
	$(v_1 e^{i\xi_1}, v_2 e^{i\xi_2}, v_3 e^{i\xi_3}),\ \xi_i = \xi_i(v_1, v_2, v_3, \lambda)$	Y	No
$\mathbb{Z}_4 \rtimes \mathbb{Z}_2^*$	$(v, 0, 0)$	No	Y
	$(0, v_2 e^{i\frac{k_2\pi}{4}}, v_3 e^{i\frac{k_3\pi}{4}}),\ k_i \in \mathbb{Z}$	Y	No
	$(v_1, \pm v_2 e^{\mp i\frac{k\pi}{4}}, \mp v_3 e^{\mp i\frac{k\pi}{4}}),\ k \in \mathbb{Z}$	Y	No
D_4	$(v, 0, 0)$	No	Y
	(v_1, v_2, v_3)	Y	No
	$(v_1, \pm v_2 e^{i\xi}, \pm v_2 e^{-i\xi})$	Y	No
	$(v_1, v_2, i v_3)$	Y	No
S_3	$(v, 0, 0)$	No	Y
	(v_1, v_2, v_3)	Y	No
	$(v_1, v_2 e^{i\xi}, v_2 e^{i\xi})$	Y	No
S_4	$(v, 0, 0)$	Y	No
	(v, v, v)	Y	No
	$(\pm v, v\omega, v\omega^2)$	Y	No
	$(0, v, iv)$	Y	No
A_4	$(v, 0, 0)$	Y	No
	(v, v, v)	Y	No
	$(\pm v, v\omega, v\omega^2)$	Y	No
	$(0, v, v e^{i\alpha})$	Y	No
$\Delta(27)$ *family*	$(v\omega, v, v)$	Y	No
	$(v\omega^2, v, v)$	Y	No
	$(v, 0, 0)$	Y	No
	(v, v, v)	Y	No

Finally, from the table we can check that indeed for the 3HDM, whenever the vev breaks the group symmetry there is no decoupling. Conversely, if the vev is invariant under the group action, then there is a decoupling limit. This is a confirmation of our theorem via an explicit independent method, albeit it is only possible for the 3HDM.

D. The symmetry-constrained 3HDM models with decoupling

Inspection of Table I shows that there are only five symmetry-vacua pairs that do allow for a decoupling limit. These are

(1) $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2^*$ with vev $(v, 0, 0)$. The charged scalar masses has been presented in Eq. (40). This is an Abelian group, and there are two different charged scalar masses that, in agreement with the discussion after Eq. (32), may be taken to infinity independently.

(2) $\mathbb{Z}_3 \rtimes \mathbb{Z}_2^*$ with vev $(v, 0, 0)$. The charged scalar mass matrix is

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -m_2 + v^2\lambda_{12} & 0 \\ 0 & 0 & -m_3 + v^2\lambda_{13} \end{pmatrix}. \tag{46}$$

Again, in accordance with the discussion after Eq. (32), there are two different charged scalar masses that may be taken to infinity independently.

(3) $\mathbb{Z}_4 \rtimes \mathbb{Z}_2^*$ and vev $(v, 0, 0)$. The charged scalar mass matrix is

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -m_2 + v^2\lambda_{12} & 0 \\ 0 & 0 & -m_3 + v^2\lambda_{13} \end{pmatrix}, \tag{47}$$

following the decoupling pattern of the previous two cases.

(4) D_4 with vev $(v, 0, 0)$. Here the charged scalar mass matrix is

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & -m_2^2 + \lambda_3 v^2 + \lambda_3' v^2 & 0 \\ 0 & 0 & -m_2^2 + \lambda_3 v^2 + \lambda_3' v^2 \end{pmatrix}, \tag{48}$$

which is degenerate. Thus, both charged scalar masses must be taken to infinity simultaneously, and one can only reach the full 3HDM → SM decoupling limit. This is a confirmation of the discussion following Eq. (33), and it is related with the fact that the D_4 generators can be taken as

$$a_3 = \mathrm{diag}(1, i, -i), \qquad g_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{49}$$

which do not commute.

(5) S_3 with vev $(v, 0, 0)$. Here the charged scalar mass matrix coincides with Eq. (48). Thus, as for D_4, the charged scalar masses are degenerate and the generators of S_3,

$$a_3 = \mathrm{diag}(1, \omega, \omega^2), \qquad g_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \tag{50}$$

are also noncommuting.

One might be surprised by the fact that all the vevs in Table I that lead to decoupling are $(v, 0, 0)$, which, by definition, is equivalent to stating that the fields are already in a Higgs basis. In fact, noticing that the matrices in Eqs. (40) and (46)–(48) are already diagonal, we know that the fields are actually written from the start in the CH basis. (Recall that the CH basis is a particular case of a Higgs basis.) As far as we can tell, this has no profound physical justification. We know for certain that the vevs could have been written in any other form in the same orbit, had we changed the form of the symmetry generators. This is easily illustrated in the 2HDM. One can study the $\mathbb{Z}_2$ group generated by

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \text{with vev } (v, 0), \tag{51}$$

which does lead to decoupling, or one can study the group generated by[5]

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{with vev } (v, v)/\sqrt{2}, \tag{52}$$

which also leads to decoupling. In fact, the models and vacua in Eqs. (51) and (52) are exactly the same but written in different bases. Since the vev in Eq. (52) does not correspond to a Higgs basis, we see that the fact that all the situations in Table I that lead to decoupling are already in the CH basis is a red herring. But, at least in principle, it could happen that all vacua leading into decoupling should be in the same orbit as a vev with only one nonzero entry. We see no reason for that, but we cannot exclude it forthright, so this is an open problem.

[5]The model based on the group generated by the matrix in Eq. (52) was dubbed Π_2 in Refs. [27,28]. Of course, it is just $\mathbb{Z}_2$ in a different basis. But the distinction is interesting if one were going to impose a symmetry under both Eqs. (51) and (52) in the same basis.

V. CONCLUSIONS

We have studied the decoupling properties of the most general NHDM model. We have shown a very powerful theorem, stating that, given a Lagrangian with scalar family symmetry S, the theory has a decoupling limit if and only if the vacuum also has the same symmetry S. We have also produced an independent proof for the special case of the 3HDM. This was possible because, in the 3HDM, all the symmetry-constrained realizable models and their vacua are known [21]. This special 3HDM proof complements a proof along the same lines for the 2HDM, mentioned in [18]. Producing results along these lines for any NHDM with $N \geq 4$ would require the knowledge of all the symmetry-constrained models and corresponding vacua for those cases as well. This is unknown at the moment and is certainly exceedingly challenging. This highlights how elegant our proof for the general NHDM really is.

Along the way, we proved an interesting result concerning the behavior of the charged scalar mass matrix $M_\pm^2$ under an exact symmetry. Symmetry under an Abelian group can accommodate nondegenerate charged scalar masses, while a non-Abelian group will per force imply some degeneracy in charged scalar masses.

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APPENDIX: DECOUPLING CHARGED HIGGS MASSES FORCE ALIGNMENT

In this Appendix, we show that taking all charged scalar masses to infinity in NHDM forces alignment [18]. We write $\varphi_k^{C0} = \rho_k + i\chi_k$ in Eq. (16), and we substitute Eq. (16) in Eq. (19), expanding all terms. The neutral scalars' mass matrix is found as [23]

$$M_0^2 = \begin{pmatrix} M_{\rho\rho}^2 & M_{\rho\chi}^2 \\ (M_{\rho\chi}^2)^T & M_{\chi\chi}^2 \end{pmatrix}, \tag{A1}$$

$$(M_{\rho\rho}^2)_{ij} = \delta_{ij} m_{\pm,i}^2 + v^2 \mathrm{Re}\{Z_{i1,1j}^{\mathrm{ch}} + Z_{i1,j1}^{\mathrm{ch}}\}, \tag{A2}$$

$$(M_{\chi\chi}^2)_{ij} = \delta_{ij} m_{\pm,i}^2 + v^2 \mathrm{Re}\{Z_{i1,1j}^{\mathrm{ch}} - Z_{i1,j1}^{\mathrm{ch}}\}, \tag{A3}$$

$$(M_{\rho\chi}^2)_{ij} = -v^2 \mathrm{Im}\{Z_{i1,1j}^{\mathrm{ch}} - Z_{i1,j1}^{\mathrm{ch}}\}, \tag{A4}$$

where no sum over repeated indices is implied. Under the canonical definition of CP, $M_{\rho\rho}^2$ is the mass matrix of the CP-even scalars, $M_{\chi\chi}^2$ of the CP-odd scalars, and $M_{\rho\chi}^2$ gives the mixing between the CP-even and CP-odd scalars. Assuming for simplicity that all couplings and vevs are real, $M_{\rho\chi}^2 = 0$, there is no CP violation in scalar-pseudoscalar mixing, and M_0^2 becomes block diagonal. We now turn to $M_{\rho\rho}^2$ in this case.

Recall that $m_{\pm,1}^2 = 0$ corresponds to the massless $G^\pm$. Thus, $(M_{\rho\rho}^2)_{ij}$ is of order v^2 for all elements where $i \neq j$ or $i = j = 1$. Let us take the charged scalar masses very large, i.e., $m_{\pm,i\neq 1}^2 \sim M^2 \gg v^2$. Then, all elements along the diagonal of $M_{\rho\rho}^2$ (except for the first) become much larger than the rest. Under these circumstances, there is one eigenvalue of order v^2 and $N-1$ eigenvalues of order M^2. Moreover, the mixing angles between the first field (the one that carries all the vev and, thus, that couples as in the SM) and the rest, goes as v^2/M^2. That is, as we take larger and larger charged scalar masses, we find that the lightest neutral scalar particle becomes closer and closer to the SM Higgs [18]. This is what we name "decoupling," or, more to the point, "alignment from decoupling."

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