



Minimal supersymmetric left–right model with automatic R -parity

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ABSTRACT

We revisit the minimal supersymmetric left–right model with $B - L = 2$ triplet Higgs fields and show that a self-consistent picture emerges with automatic R -parity conservation even in the absence of higher-dimensional operators. By computing the effective potential for the Higgs system including heavy Majorana neutrino Yukawa couplings we show that the global minimum of the model can lie in the charge and R -parity conserving domain. The model provides natural solutions to the SUSY phase problem and the strong CP problem and makes several interesting predictions. Quark mixing angles arise only after radiative corrections from the lepton sector are taken into account. A pair of doubly charged Higgs fields remain light below TeV with one field acquiring its mass entirely via renormalization group corrections. We find this mass to be not much above the Bino mass. In the supergravity framework for SUSY breaking, we also find similar upper limits on the stau masses. Natural solutions to the μ problem and the SUSY CP problem entails light $SU(2)_L$ triplet Higgs fields, leading to rich collider phenomenology.

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1. Introduction

Left–right symmetric extensions of the Standard Model (SM) based on the gauge group [1] $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ have many attractive features. These include an understanding of the origin of parity violation, and a compelling rationale for small neutrino masses via the seesaw mechanism. The enlarged gauge symmetry allows for parity to be defined as an exact symmetry, which is broken only spontaneously. Right-handed neutrino is required to exist in order to complete the $SU(2)_R$ multiplet, and so neutrino mass is natural. In the domain of flavor physics, the supersymmetric version of this theory resolves several problems of the popular Minimal Supersymmetric Standard Model (MSSM):

- (i) R -parity emerges as an exact symmetry of MSSM, preventing rapid proton decay and providing a naturally stable dark matter candidate [4]. This is possible if the $SU(2)_R \times U(1)_{B-L}$ gauge symmetry is broken down to $U(1)_Y$ by Higgs triplet fields carrying $B - L = \pm 2$. R -parity, which is part of the original $B - L$ symmetry, will remain unbroken even after symmetry breaking in this case.
- (ii) It solves the SUSY CP problem [2,3] because of parity invariance. Parity makes the Yukawa couplings and the corresponding SUSY breaking A terms Hermitian, and the gluino mass

and the μ term real. The electric dipole moments of fermions will then vanish at the scale of parity restoration.

- (iii) Finally, it has all the ingredients necessary to solve the strong CP problem without the need for an axion, again by virtue of parity symmetry [2,3]. This is achieved by ensuring that the quark mass matrix has a real determinant, which is possible since the Yukawa couplings are Hermitian.

Previous studies of this model focussed on two versions:

- (i) A TeV scale version where R -parity is spontaneously broken by the vacuum expectation value (VEV) of the right-handed sneutrino [5], or alternatively (ii) an R -parity conserving version [6,7] where non-renormalizable (NR) higher-dimensional operators were included and played an essential role. The reason for considering only these two versions is that in the absence of the above features, i.e., $\langle \tilde{\nu}^c \rangle \neq 0$, or the presence of NR operators, the global minimum of the theory that is both R -parity conserving and parity violating, breaks electric charge and is therefore unacceptable. In the first version with $\langle \tilde{\nu}^c \rangle \neq 0$, the W_R scale must necessarily be in the TeV range [5], whereas in the second one, it is necessarily above 10^{11} GeV. In the first version, SUSY dark matter candidate is lost. In the second version, the possibility of solving strong CP problem via parity symmetry is eliminated due to the essential presence of higher-dimensional operators which makes $\bar{\theta}$ large. It is also difficult to solve the SUSY phase problem, since these higher-dimensional operators typically generate parity violating effects in the fermion mass matrices. Extensions of the minimal model which use additional Higgs multiplets have been proposed. Ref. [8] introduces Higgs doublets in addition to triplets, but in

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such models R -parity conservation is exterior to parity symmetry. In Ref. [9] $B - L = 0$ Higgs triplets are introduced in addition to the $B - L = \pm 2$ triplets, which is clearly non-minimal.

In this Letter we revisit the minimal SUSYL model with $B - L = 2$ Higgs triplets. We assume that the higher-dimensional operators are absent or small, so that the solutions to the strong CP and the SUSY phase problems are still intact. The global minimum of the tree-level Higgs potential is either charge violating, or R -parity violating, as noted. However, we find that inclusion of the heavy Majorana neutrino Yukawa couplings in the effective potential automatically cures this problem. The vacuum that preserves both electric charge and R -parity can naturally be the global minimum of the full potential. We study the consequences of such a setup.¹

The main results of our investigation can be summarized as follows:

- (i) In this general class of models, there are two doubly charged Higgs and Higgsino fields with masses below a TeV. One combination of these doubly charged Higgs boson fields has a vanishing mass at the scale of $SU(2)_R \times U(1)_{B-L}$ breaking (denoted as ν_R). So its mass is calculable, arising through renormalization group effects between ν_R and the weak scale. We find its squared mass to be positive with the Higgs boson having a mass close to the Bino mass.
- (ii) There exist two pairs of Higgs doublets in the low energy, although one pair is unlikely to be observed directly at the LHC. This naturally leads to calculable flavor violation, which are within experimental limits.
- (iii) Renormalization group evolution plays a crucial role in the generation of quark mixing angles. In fact, an asymmetry in the μ terms of the Higgs doublets generated by the leptonic Yukawa couplings is what induces CKM mixings.
- (iv) In the version that solves the SUSY phase and the strong CP problems and which provides an understanding of the μ problem, there are also light $SU(2)_L$ triplet superfields with TeV to sub-TeV scale masses with interesting collider signature [10, 11]. These fields couple to left-handed leptons with the couplings proportional to the heavy Majorana neutrino masses.

2. The basic structure of the model

Quarks and leptons in the model have the following left-right symmetric assignment under the $SU(3)_C \times SU(2)_L \times SU(2)_R \times (1)_{B-L}$ gauge group:

$$\begin{aligned} Q\left(3, 2, 1, \frac{1}{3}\right) &= \begin{pmatrix} u \\ d \end{pmatrix}, & Q^c\left(3^*, 1, 2, -\frac{1}{3}\right) &= \begin{pmatrix} d^c \\ -u^c \end{pmatrix}, \\ L(1, 2, 1, -1) &= \begin{pmatrix} \nu_e \\ e \end{pmatrix}, & L^c(1, 1, 2, 1) &= \begin{pmatrix} e^c \\ -\nu_e^c \end{pmatrix}. \end{aligned} \quad (1)$$

The minimal Higgs sector consists of the following superfields:

$$\begin{aligned} \Delta(1, 3, 1, 2) &= \begin{pmatrix} \frac{\delta^{++}}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}, \\ \bar{\Delta}(1, 3, 1, -2) &= \begin{pmatrix} \frac{\bar{\delta}^-}{\sqrt{2}} & \bar{\delta}^0 \\ \bar{\delta}^{--} & -\frac{\bar{\delta}^-}{\sqrt{2}} \end{pmatrix}, \\ \Delta^c(1, 1, 3, -2) &= \begin{pmatrix} \frac{\delta^{c-}}{\sqrt{2}} & \delta^{c0} \\ \delta^{c--} & -\frac{\delta^{c-}}{\sqrt{2}} \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} \bar{\Delta}^c(1, 1, 3, 2) &= \begin{pmatrix} \frac{\bar{\delta}^{c+}}{\sqrt{2}} & \bar{\delta}^{c++} \\ \bar{\delta}^{c0} & -\frac{\bar{\delta}^{c+}}{\sqrt{2}} \end{pmatrix}, \\ \Phi_a(1, 2, 2, 0) &= \begin{pmatrix} \phi_1^+ & \phi_2^0 \\ \phi_1^0 & \phi_2^- \end{pmatrix}_a \quad (a = 1-2), \quad S(1, 1, 1, 0). \end{aligned} \quad (2)$$

This is the minimal Higgs system in the following sense. The $(\Delta^c + \bar{\Delta}^c)$ fields are needed for $SU(2)_R \times U(1)_{B-L}$ symmetry breaking without inducing R -parity violating couplings. The $(\Delta + \bar{\Delta})$ fields are their left-handed partners needed for parity invariance. Two bidoublet fields Φ_a are needed in order to generate quark and lepton masses and CKM mixings. The singlet field S is introduced so that $SU(2)_R \times U(1)_{B-L}$ symmetry breaking occurs in the supersymmetric limit.

The superpotential of the model is given by

$$\begin{aligned} W &= Y_u Q^T \tau_2 \Phi_1 \tau_2 Q^c + Y_d Q^T \tau_2 \Phi_2 \tau_2 Q^c \\ &\quad + Y_\nu L^T \tau_2 \Phi_1 \tau_2 L^c + Y_\ell L^T \tau_2 \Phi_2 \tau_2 L^c \\ &\quad + i(f^* L^T \tau_2 \Delta L + f L^{cT} \tau_2 \Delta^c L^c) \\ &\quad + S[\text{Tr}(\lambda^* \Delta \bar{\Delta} + \lambda \Delta^c \bar{\Delta}^c) + \lambda'_{ab} \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) - \mathcal{M}_R^2] + W', \end{aligned} \quad (3)$$

where

$$\begin{aligned} W' &= [M_\Delta \text{Tr}(\Delta \bar{\Delta}) + M_\Delta^* \text{Tr}(\Delta^c \bar{\Delta}^c)] \\ &\quad + \mu_{ab} \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) + \mathcal{M}_S S^2 + \lambda_S S^3. \end{aligned} \quad (4)$$

$Y_{u,d}$ and $Y_{\nu,\ell}$ in Eq. (3) are quark and lepton Yukawa coupling matrices, while f is the Majorana neutrino Yukawa coupling matrix. The W' term listed in Eq. (4) is optional, in fact when terms in W' are set to zero, the theory has an enhanced R -symmetry. Under this R -symmetry, $\{Q, Q^c, L, L^c\}$ fields have charge $+1$, S has charge $+2$, and all other fields have charge zero with W carrying charge $+2$. While the general setup of the minimal model includes W' , the special case of $W' = 0$ is interesting, as it leads to an understanding of the μ term. In the supersymmetric limit, the VEV of the singlet S is zero, but after SUSY breaking, $\langle S \rangle \sim m_{\text{SUSY}}$. Thus the μ term for the bidoublet Φ will arise from the coupling λ'_{ab} , with a magnitude of order m_{SUSY} [12]. It is also in the limit where $W' = 0$ that the SUSY CP problem and the strong CP problem can be explained naturally. The main difference between the cases $W' \neq 0$ and $W' = 0$ from the low energy perspective is that in the latter case the left-handed triplet superfields $(\Delta + \bar{\Delta})$ will remain light, also with masses of order m_{SUSY} .

The superpotential of Eq. (3) is invariant under the parity transformation under which $\Phi \rightarrow \Phi^\dagger$, $\Delta \rightarrow \Delta^{c*}$, $\bar{\Delta} \rightarrow \bar{\Delta}^{c*}$, $S \rightarrow S^*$, $Q \rightarrow Q^{c*}$, $L \rightarrow L^{c*}$, $\theta \rightarrow \bar{\theta}$, etc. Parity invariance implies that the Yukawa coupling matrices $Y_{u,d}, Y_{\nu,\ell}$ are Hermitian, i.e. $Y_u = Y_u^\dagger$, etc. Additionally, λ'_{ab} are real, as is \mathcal{M}_R^2 . This means that the effective μ terms of the bidoublet will be real, provided that $\langle S \rangle$ is real. If the Φ VEVs are also real, this setup will provide a solution to the SUSY CP problem and the strong CP problem [2,3]. Below we will study under what conditions this is achieved and what the implications of this theory are.

We will work in the ground state corresponding to the following charge preserving VEV pattern for the triplet fields:

$$\langle \Delta^c \rangle = \begin{pmatrix} 0 & \nu_R \\ 0 & 0 \end{pmatrix}, \quad \langle \bar{\Delta}^c \rangle = \begin{pmatrix} 0 & 0 \\ \bar{\nu}_R & 0 \end{pmatrix}. \quad (5)$$

The VEVs of the left-handed triplet fields $(\Delta + \bar{\Delta})$ are assumed to be zero since no interaction in the model induces such VEVs. There are two important implications of this setup:

¹ It is not strictly required that the vacuum we live in correspond to the global minimum of the potential. Metastable vacua are acceptable, provided that the tunnelling rate from that vacuum to the true vacuum is sufficiently slow in comparison to the age of the Universe.

- (i) Above the parity breaking scale \mathcal{M}_R , this model has an enhanced global $U(3, c)$ (complexified $U(3)$) symmetry which is broken by the above VEVs to $U(2, c)$. This leads to five massless superfields. Three of these superfields are absorbed by the gauge fields via the super-Higgs mechanism. There remains two light superfields, which are the doubly charged Higgs and Higgsino fields δ^{c--} and δ^{c++} . These fields will consistently acquire masses of order TeV or less, as we shall explicitly show in the next section. Even after soft SUSY breaking terms are turned on, there is a $U(3)$ symmetry in the potential, which leads to one massless doubly charged Higgs boson (and its conjugate). This field will acquire positive squared mass from the renormalization group evolution below v_R proportional to the Bino mass M_1 .
- (ii) The bi-doublet fields, when expressed in terms of the components $H_{u,a}, H_{d,a}$ ($a = 1, 2$), have a symmetric mass matrix in W due to parity symmetry which requires $\mu_{12} = \mu_{21}$. (When $W' = 0$, $\mu_{ij} = \lambda'_{ij} \langle S \rangle$.) Therefore, if we make one pair of doublets light at the scale v_R , it would lead to vanishing CKM mixing angle. This happens in spite of having two Yukawa coupling matrices. Consistency then requires that both pairs of doublets be light below v_R . In this case RGE extrapolation brings in an asymmetry between μ_{12} and μ_{21} . Thus, not only are the potential problems solved by RGE extrapolation, but the resulting scenario becomes very predictive.

3. Symmetry breaking and the mass of the doubly charged Higgs boson

To be specific, we will analyze the model with $W' = 0$ of Eq. (3). In the SUSY limit we have from the vanishing of D and F terms,

$$|v_R| = |\bar{v}_R|, \quad \lambda v_R \bar{v}_R = \mathcal{M}_R^2, \quad \langle S \rangle = 0. \quad (6)$$

It is easy to determine the VEV of S field that is generated after SUSY breaking. Only linear terms in SUSY breaking are relevant for this purpose. We have

$$V_{\text{soft}} = A_\lambda \lambda S \text{Tr}(\Delta^c \bar{\Delta}) - C_\lambda \mathcal{M}_R^2 S + \text{h.c.} \quad (7)$$

Minimization of the resulting potential yields

$$\langle S^* \rangle = \frac{1}{2|\lambda|} (C_\lambda - A_\lambda). \quad (8)$$

Note that this is of order m_{SUSY} . If the coupling $|\lambda|$ is somewhat small, then $\langle S \rangle$ can be above the SUSY breaking scale. This feature can be used to make one pair of Higgs doublet superfields somewhat heavier than the SUSY breaking scale. However, the masses of doubly charged fermionic fields, which are equal to $|\lambda| \langle S \rangle$ must remain below a TeV. Phenomenology of doubly charged Higgsino has been studied in Refs. [10,11,13,14].

Parity symmetry requires \mathcal{M}_R^2 and C_λ be real. If the trilinear soft breaking terms are proportional to the corresponding Yukawa coupling matrices, then we have A_λ real as well. Proportionality will require $A_\lambda \lambda = A_0 \lambda$, with the universal A_0 being real. Since the trilinear A terms in the quark sector must be Hermitian by parity, and since the Yukawa coupling matrices are Hermitian, A_0 must be real. This condition is realized in many models of SUSY breaking such as Poloni type supergravity breaking, gauge mediated SUSY breaking, anomaly mediated SUSY breaking, etc. We shall adopt this proportionality relation for all the A terms. We see that $\langle S \rangle$ is then real. The resulting μ terms will also be real. This helps solve the strong CP problem and the SUSY phase problem.

The full potential of the model relevant for symmetry breaking has F term, D term and soft SUSY breaking contributions. They are given by

$$\begin{aligned} V_F &= |\lambda \text{Tr}(\Delta^c \bar{\Delta}^c) + \lambda'_{ab} \text{Tr}(\Phi_a^T \tau_2 \Phi_b \tau_2) - \mathcal{M}_R^2|^2 \\ &\quad + |\lambda|^2 |\text{Tr}(\Delta^c \Delta^{c\dagger}) + \text{Tr}(\bar{\Delta}^c \bar{\Delta}^{c\dagger})|, \\ V_{\text{soft}} &= M_1^2 \text{Tr}(\Delta^{c\dagger} \Delta^c) + M_2^2 \text{Tr}(\bar{\Delta}^{c\dagger} \bar{\Delta}^c) + M_5^2 |S|^2 \\ &\quad + \{A_\lambda \lambda S \text{Tr}(\Delta^c \Delta^{c\dagger}) - C_\lambda \mathcal{M}_R^2 S + \text{h.c.}\}, \\ V_D &= \frac{g_R^2}{8} \sum_a |\text{Tr}(2\Delta^{c\dagger} \tau_a \Delta^c + 2\bar{\Delta}^{c\dagger} \tau_a \bar{\Delta}^c + \Phi_a \tau_a^T \Phi_a^\dagger)|^2 \\ &\quad + \frac{g'^2}{8} |\text{Tr}(2\Delta^{c\dagger} \Delta^c + 2\bar{\Delta}^{c\dagger} \bar{\Delta}^c)|^2. \end{aligned} \quad (9)$$

Minimizing the potential yields the following two complex conditions:

$$\begin{aligned} v_R^* \left[|\lambda|^2 |S|^2 + M_1^2 + g_R^2 \left(|v_R|^2 - |\bar{v}_R|^2 + \frac{X}{2} \right) + g'^2 (|v_R|^2 - |\bar{v}_R|^2) \right] \\ + \bar{v}_R \left[\lambda A_\lambda S + |\lambda|^2 \left(v_R \bar{v}_R - \frac{\mathcal{M}_R^2}{\lambda} \right)^* \right] &= 0, \\ \bar{v}_R^* \left[|\lambda|^2 |S|^2 + M_2^2 - g_R^2 \left(|v_R|^2 - |\bar{v}_R|^2 + \frac{X}{2} \right) - g'^2 (|v_R|^2 - |\bar{v}_R|^2) \right] \\ + v_R \left[\lambda A_\lambda S + |\lambda|^2 \left(v_R \bar{v}_R - \frac{\mathcal{M}_R^2}{\lambda} \right)^* \right] &= 0, \end{aligned} \quad (10)$$

where we defined $X = \sum_{a=1}^2 \langle |\phi_1^0|^2 - |\phi_2^0|^2 \rangle_a$. Applying these conditions, we obtain the following mass squared matrix for the doubly charged Higgs bosons ($\delta^{c--}, \delta^{c++}$).

$$\mathcal{M}_{\delta^{++}}^2 = \begin{pmatrix} -2g_R^2(|v_R|^2 - |\bar{v}_R|^2 + \frac{X}{2}) - \frac{\bar{v}_R}{v_R} Y & Y^* \\ Y & 2g_R^2(|v_R|^2 - |\bar{v}_R|^2 + \frac{X}{2}) - \frac{v_R}{\bar{v}_R} Y \end{pmatrix}, \quad (11)$$

where $Y = \lambda A_\lambda S + |\lambda|^2 (v_R \bar{v}_R - \frac{\mathcal{M}_R^2}{\lambda})^*$. It is clear that as the D term is set to zero, there is one massless mode in this sector. Actually, if v_R is much larger than the SUSY breaking terms, turning on the D term makes one of the masses negative. This is the pseudo-Goldstone boson of the model. There is no inconsistency, as this zero squared-mass will turn positive via RGE evolution.

Below the scale v_R , the mass matrix of the doubly charged Higgs boson fields has the form

$$\mathcal{M}_{\delta^{++}}^2 = \begin{pmatrix} M_{++}^2 + \mu_\delta^2 + \delta_1 & (B\mu)_\delta + \delta_{12} \\ (B\mu)_\delta^* + \delta_{12}^* & M_{--}^2 + \mu_\delta^2 + \delta_2 \end{pmatrix}, \quad (12)$$

where $\mu_\delta \delta^{++} \delta^{--}$ is the effective superpotential mass term, M_{++}^2 and M_{--}^2 are the soft mass parameters, and δ_i denote RGE correction factors corresponding to running from v_R down to the SUSY breaking scale. Eq. (12) should match Eq. (11) at v_R , which implies that $M_{++}^2 \simeq M_{--}^2$, $|(B\mu)_\delta| \simeq M_{++}^2 + \mu_\delta^2$ at v_R . In the large v_R limit, the light Higgs resulting from Eq. (11) is $(\delta^{*--} - \delta^{++})/\sqrt{2}$, so the squared mass of this state, including RGE corrections is $[\delta_1 + \delta_2 - 2\text{Re}(\delta_{12})]/2$. There is an upper limit on this mass, which can be derived as follows. Let us ignore the off-diagonal entry for the moment. The renormalization group equation for M_{++}^2 has the form [15]

$$\frac{dM_{++}^2}{dt} = -\frac{c}{16\pi^2} g_1^2 M_1^2 + \dots, \quad (13)$$

where $c = (96/5)$. Here we have displayed only the positive contributions to the mass-squared, which would be relevant for determining the upper limit. Along with

$$\frac{dg_1}{dt} = \frac{b_1}{16\pi^2} g_1^3, \quad \frac{dM_1}{dt} = \frac{2b_1}{16\pi^2} g_1^2 M_1^2, \quad (14)$$

we can solve for M_{++}^2 . In the present model $b_1 = (78/5)$ when the $(\Delta + \bar{\Delta})$ are light, and $b_1 = 12$ when these fields are heavy. We find

$$M_{++}^2(m_Z) < \frac{24}{5b_1} M_1^2(m_Z) \left[\frac{\alpha_1^2(v_R)}{\alpha_1^2(m_Z)} - 1 \right]. \quad (15)$$

The gauge couplings in this model will remain perturbative up to about 10^{12} GeV when $b_1 = (78/5)$ and up to about 10^{14} GeV when $b_1 = 12$. If we choose $\alpha_1(v_R) = 0.1$, we find the upper limit on $M_{++} < 3.7M_1$ (for the case where $b_1 = 12$). The running of the $(B\mu)_\delta$ will also contribute to the mass of this state, but this evolution depends on other SUSY breaking parameters. We expect the entire contribution to be of order few times M_1 .

4. Effective potential and the global minimum of the theory

Let us now turn attention to the electric charge and/or R -parity breaking global minimum of the model and see how this problem is cured by taking loop corrections induced by the heavy Majorana neutrino Yukawa couplings into account. We will show that the results of Ref. [5] gets significantly modified, allowing for the desired charge conserving minimum to be the global minimum for some domain of the parameters.

It is easy to see why the tree-level potential has a deeper minimum that violates electric charge and/or R -parity. In the desired minimum which preserves these quantum numbers, the VEVs of the triplet fields are as shown in Eq. (5). Consider the following alternative VEV configuration:

$$\langle \Delta^c \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & v_R \\ v_R & 0 \end{pmatrix}, \quad \langle \bar{\Delta}^c \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \bar{v}_R \\ \bar{v}_R & 0 \end{pmatrix}. \quad (16)$$

This pattern of course breaks electric charge. All terms in the scalar potential are exactly the same for this configuration of VEVs and that of Eq. (5), except in the $SU(2)_R$ D -terms. Since the VEVs of Eq. (16) are along τ_1 , the D -terms vanish for this configuration, while it is nonzero and positive for the desired configuration. This proves that the desired VEV pattern does not correspond to the global minimum of the potential.

We proceed to compute the Coleman–Weinberg potential of the model by keeping one family of neutrino Yukawa couplings to the Δ^c field, as shown by the f coupling in Eq. (3). To be able to compare different minima, we use the general background with the full Δ^c and $\bar{\Delta}^c$ fields. The field-dependent masses of the (e^c, ν^c) fermionic and scalar fields can be expressed in terms of the invariant combinations

$$D_{1,2}^2 = \frac{1}{2} \left[\text{Tr}(\Delta^{c\dagger} \Delta^c) \pm \sqrt{\text{Tr}(\Delta^{c\dagger} \Delta^c)^2 - \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\Delta^{c\dagger} \Delta^{c\dagger})} \right]. \quad (17)$$

We also define $\bar{D}_{1,2}^2$ in an analogous way, with the replacement of Δ^c by $\bar{\Delta}^c$ in Eq. (17). Including the soft SUSY breaking contributions, the F -term contributions, and the D -term contributions, the field-dependent masses of the sleptons $(\tilde{e}^c, \tilde{\nu}^c)$, and the corresponding fermions are found to be

$$\begin{aligned} m_{1,2}^2 &= |f|^2 D_1^2 + m_{Lc}^2 + \frac{g_R^2}{2} [(D_2^2 - \bar{D}_2^2) - (D_1^2 - \bar{D}_1^2)] \\ &\quad - \frac{g'^2}{2} [(D_1^2 - \bar{D}_1^2) + (D_2^2 - \bar{D}_2^2)] \pm |A_f f D_1 + \lambda^* S^* f \bar{D}_1|^2, \\ m_{3,4}^2 &= |f|^2 D_2^2 + m_{Lc}^2 + \frac{g_R^2}{2} [(D_1^2 - \bar{D}_1^2) - (D_2^2 - \bar{D}_2^2)] \\ &\quad - \frac{g'^2}{2} [(D_1^2 - \bar{D}_1^2) + (D_2^2 - \bar{D}_2^2)] \pm |A_f f D_2 + \lambda^* S^* f \bar{D}_2|^2, \\ m_{F_1}^2 &= |f D_1|^2, \quad m_{F_2}^2 = |f D_2|^2. \end{aligned} \quad (18)$$

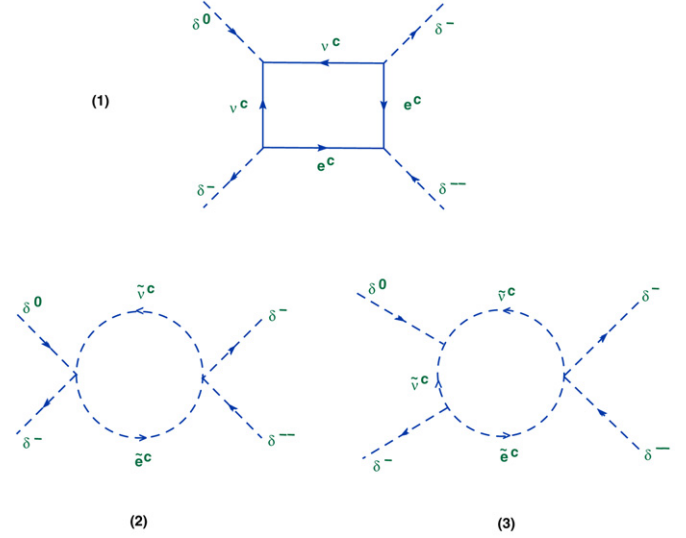


Fig. 1. Diagrams inducing effective quartic coupling of Eq. (21).

Here the m_{1-4} correspond to the masses of the four real scalar states, while $m_{F_{1,2}}$ are the masses of the two fermionic states.

With these mass eigenvalues, one can compute the effective potential in the Landau gauge in the \overline{DR} scheme from the expression

$$V_{\text{eff}}^{1\text{-loop}} = \frac{1}{64\pi^2} \sum_i (-1)^{2s} (2s+1) M_i^4 \left[\text{Log} \left(\frac{M_i^2}{\mu^2} \right) - \frac{3}{2} \right]. \quad (19)$$

We expand this potential in the limit where SUSY breaking parameters are small compared to the VEVs of the $(\Delta^c, \bar{\Delta}^c)$ fields. In the SUSY limit, vanishing of the D -terms require $D_1^2 = \bar{D}_1^2$, $D_2^2 = \bar{D}_2^2$. So we use the expansion

$$\bar{D}_1^2 - D_1^2 = a_1 m_{Lc}^2, \quad \bar{D}_2^2 - D_2^2 = a_2 m_{Lc}^2, \quad (20)$$

where m_{Lc}^2 denotes the soft SUSY breaking mass of the slepton doublet. Defining

$$x = \frac{\text{Tr}(\Delta^c \Delta^c) \text{Tr}(\Delta^{c\dagger} \Delta^{c\dagger})}{[\text{Tr}(\Delta^{c\dagger} \Delta^c)]^2} \quad (21)$$

we find the leading contribution to V_{eff} to be

$$\begin{aligned} V_{\text{eff}}^{1\text{-loop}} &= - \frac{|f|^2 m_{Lc}^2 \text{Tr}(\Delta^c \Delta^{c\dagger})}{64\pi^2} \\ &\quad \times \left[(4 + 2 \ln 2) + 2(a_1 - a_2) g_R^2 \sqrt{1-x} + 2(a_1 + a_2) g'^2 \right. \\ &\quad \left. - \{2 + (a_2 - a_1) g_R^2 + (a_2 + a_1) g'^2\} (1 - \sqrt{1-x}) \right] \\ &\quad \times \ln \left(\frac{|f|^2 \text{Tr}(\Delta^c \Delta^{c\dagger})}{2\mu^2} (1 - \sqrt{1-x}) \right) \\ &\quad + \{((a_2 - a_1) g_R^2 - (a_2 + a_1) g'^2) (1 + \sqrt{1-x}) - 2\sqrt{1-x}\} \\ &\quad \times \ln \left(\frac{|f|^2 \text{Tr}(\Delta^c \Delta^{c\dagger})}{2\mu^2} (1 + \sqrt{1-x}) \right) \\ &\quad \left. - 2 \ln \left(\frac{|f|^2 \text{Tr}(\Delta^c \Delta^{c\dagger})}{\mu^2} (1 + \sqrt{1-x}) \right) \right]. \end{aligned} \quad (22)$$

Clearly, these loop contributions vanish in the SUSY limit. The non-vanishing terms arise because the cancellation between the first two diagrams of Fig. 1 is no longer exact, once SUSY breaking is turned on. And diagram (c) has no fermionic counterpart. The most interesting aspect of the one-loop effective potential is the appearance of the structure $\text{Tr}(\Delta^c \Delta^c) \text{Tr}(\Delta^{c\dagger} \Delta^{c\dagger})$, which was absent in the tree-level potential. If we make a further expansion in small x , Eq. (22) will result in the following quartic coupling:

$$\begin{aligned}
V^{\text{quartic}} = & -\frac{|f|^2 m_{L^c}^2 \text{Tr}(\Delta^c \Delta^c) \text{Tr}(\Delta^{c\dagger} \Delta^{c\dagger})}{128\pi^2 |v_R|^2} \\
& \times \left[\{2 - (a_1 - a_2)g_R^2 - (a_1 + a_2)g'^2\} (1 + 2\ln 2) \right. \\
& + (a_1 - a_2)g_R^2 \ln \frac{|f v_R|^2}{\mu^2} \\
& \left. - \{2 - (a_1 - a_2)g_R^2 + (a_1 + a_2)g'^2\} \ln x \right] + \dots, \quad (23)
\end{aligned}$$

where the \dots indicates higher order terms in x and x -independent terms. In the desired vacuum we have $D_2 = \bar{D}_2 = 0$, so that the coefficient a_2 is zero. Minimization conditions (Eq. (10)) determine a_1 as

$$(g_R^2 + g'^2) a_1 m_{L^c}^2 \simeq \frac{1}{2} (M_1^2 - M_2^2 + g_R^2 X), \quad (24)$$

where $M_{1,2}^2$ are the soft mass squared of the $(\Delta^c, \bar{\Delta}^c)$ fields. In supergravity type SUSY breaking, one would expect $M_1^2 \leq M_2^2$, as Δ^c has the Majorana–Yukawa coupling which would lower its mass from the universal mass, while $\bar{\Delta}^c$ does not. Using this we find that for the charge conserving vacuum to be lower than the charge breaking vacuum, we would need $m_{L^c}^2$ to be negative. In such a situation, we can derive upper limits on the stau masses. (We assume that the third family fermions have the largest Majorana–Yukawa coupling f .) Note that the positive contributions to the masses of $\tilde{\tau}_R$ and $\tilde{\tau}_L$ arise from the gaugino masses M_1 and M_2 [15]:

$$\begin{aligned}
16\pi^2 \frac{dm_{\tilde{\tau}_R}^2}{dt} &= -\frac{24}{5} g_1^2 M_1^2 + \dots, \\
16\pi^2 \frac{dm_{\tilde{\tau}_L}^2}{dt} &= -\frac{6}{5} g_1^2 M_1^2 - 6g_2^2 M_2^2 + \dots, \quad (25)
\end{aligned}$$

where the \dots denote terms that would decrease the scalar mass in the evolution from v_R to m_Z . We have $b_2 = 6$ when $(\Delta + \bar{\Delta})$ fields are light, and $b_2 = 2$ when they are heavy. The upper limits on the stau masses are found to be

$$\begin{aligned}
M_{\tilde{\tau}_R}^2(m_Z) &< \frac{6}{5b_1} M_1^2(m_Z) \left[\frac{\alpha_1^2(v_R)}{\alpha_1^2(m_Z)} - 1 \right], \\
M_{\tilde{\tau}_L}^2(m_Z) &< \frac{3}{10b_1} M_1^2(m_Z) \left[\frac{\alpha_1^2(v_R)}{\alpha_1^2(m_Z)} - 1 \right] \\
&+ \frac{3}{2b_2} M_2^2(m_Z) \left[\frac{\alpha_2^2(v_R)}{\alpha_2^2(m_Z)} - 1 \right]. \quad (26)
\end{aligned}$$

Both these limits are in the acceptable range. For $\alpha_1(v_R) = 0.1$, we find the right-handed stau mass to be bounded by about $1.9M_1$ (for $b_1 = 12$), with the left-handed stau roughly two times heavier.

5. CKM angles out of radiative corrections

As noted earlier, our model predicts that the CKM angles vanish at the tree-level due to left–right symmetry. The reason for this is that the 2×2 (H_u, H_d) Higgsino mass matrix is symmetric. When one pair of light MSSM Higgs superfields is extracted from such a symmetric matrix, it follows that the up and down quark Yukawa coupling matrices to these light doublets will be the same. This is assuming that only one pair of doublets survives below v_R . Therefore once electroweak symmetry breaks, we have $M_u = \xi M_d$ and hence $V_{\text{CKM}} = 1$. Consistency with CKM mixings then requires that both pairs of Higgs doublets remain light below v_R . In that case, below v_R , the bidoublet mass terms μ_{ab} will receive asymmetric radiative RGE corrections, in the momentum range v_R to μ_ϕ , be-

cause parity is violated in this regime. (We denote the scale of the heavy doublet mass as μ_ϕ .) To leading order the quark Yukawa couplings do not induce an asymmetry in μ_{ab} . However, since the right-handed neutrinos decouple below v_R , the lepton sector induces an asymmetry. Only the charged lepton Yukawa couplings contribute to the evolution of μ_{ab} , making the RGE contribution to μ_{12} different from that of μ_{21} . As a result, when the H_u, H_d mass matrix is diagonalized at a scale μ_ϕ below v_R , so that only one pair of Higgs superfields remain light, the resulting light Higgs doublets couple to up and down quarks with different Yukawa coupling matrices.

The RGE for the asymmetry between μ_{12} and μ_{21} (to leading order) is

$$\frac{d}{dt}(\mu_{12} - \mu_{21}) = \frac{\mu_{12} + \mu_{21}}{32\pi^2} \text{Tr}(Y_\nu^\dagger Y_\nu - Y_\ell^\dagger Y_\ell), \quad (27)$$

which can be solved to determine the asymmetry in μ_{ij} . We obtain $(\mu_{12} - \mu_{21})/(\mu_{12} + \mu_{21}) \simeq 1/(16\pi^2) \text{Tr}(Y_\nu^\dagger Y_\nu - Y_\ell^\dagger Y_\ell) \ln(v_R/\mu_\phi)$, where μ_ϕ is the mass of the heavy bidoublet. The suppression factor that appears in the CKM angles is about 0.1 when one of the leptonic Yukawa coupling entries is of order one. This can lead to reasonable values for the CKM angles.

6. FCNC, the strong CP and the SUSY CP problems

The presence of a second pair of Higgs doublets coupling to fermions implies that there will be tree-level flavor changing neutral currents mediated by the Higgs. Experimental constraints will require that one pair of Higgs doublets be heavy, with mass of the order of few to 50 TeV [16,17]. This can be seen from the mass matrices of the quarks,

$$\begin{aligned}
M_u &= Y_u \kappa_u + Y_d \kappa'_u, \\
M_d &= Y_u \kappa'_d + Y_d \kappa_d, \quad (28)
\end{aligned}$$

where κ_i are the VEVs of the neutral components. These equations can be used to solve for the Yukawa coupling matrices. For example, $Y_d = (\kappa_u M_u - \kappa'_d M_d)/(\kappa_u \kappa_d - \kappa'_u \kappa'_d)$. In a basis where M_d is diagonal, $M_u = \hat{V}^T D_u \hat{V}^*$, where $\hat{V} = P \cdot V \cdot Q$, with V being the CKM matrix in the standard parametrization, and P, Q being phase matrices. D_u is the diagonal up-quark mass matrix. Flavor changing Higgs couplings can be then readily derived:

$$\mathcal{L}^{\text{FCNC}} = \frac{\kappa_u}{\kappa_u \kappa_d - \kappa'_u \kappa'_d} Q_i Q_j^* (D_u)_k V_{ki} V_{kj}^* H^0 + \text{h.c.} \quad (29)$$

Due to the hermiticity of this matrix, the unknown phase matrix Q disappears from processes such as $e\bar{e} \rightarrow K$. We find stringent limit on the mass of H^0 , $m_{H^0} \geq 30\text{--}50$ TeV, if there is no cancellation between the Higgs exchange and the SUSY squark–gluino exchange box diagram. If such cancellations are allowed, the limit on H^0 mass is considerably reduced [16]. As noted after Eq. (8), the model allows for one pair of Higgs doublets to be naturally heavier than the SUSY breaking scale, thus satisfying the FCNC constraint.

Since two pairs of Higgs doublets must survive below v_R , there are calculable FCNC via SUSY diagrams. The most significant ones are the gluino box diagram for $K^0\text{--}\bar{K}^0$ mixing. We find that these constraints are met in the model.

The basic idea behind parity as a solution for the strong CP problems is that left–right symmetry leads to Hermitian Yukawa couplings [18]. If the VEVs of bi-doublet Higgs fields are real, this would lead to a solution to the strong CP problem. The reality of the VEVs is not guaranteed by parity and always involves additional assumptions. Supersymmetry provides this extra symmetry in minimal left–right models without any singlet fields as shown in

[19]. When there are gauge singlet fields in the theory, this needs to be reinvestigated. As we noted in the symmetry breaking discussion, if $W' = 0$, which can be enforced by an R -symmetry, one can have a scenario where the singlet VEV is real. In such a setup not only is the strong CP problem solved, but the weak SUSY CP problem is also solved. The EDMs of the electron and the neutron will be vanishing due to parity at the scale v_R . Renormalization group evolution does induce small EDMs, but well within experimental limits.

One would expect higher-dimensional operators induced by gravity to be present in the theory at some level. It will be interesting to see if the solution to the strong CP problem provided by parity is stable in the presence of such operators. Operators such as $(Q^T \tau_2 \Phi_i \tau_2 Q^c)(\Delta^c \tilde{\Delta}^c / M_{\text{Pl}}^2)$ in the superpotential W will induce non-Hermitian Yukawa couplings. Such couplings are of order 10^{-14} for $v_R \leq 10^{11}$ GeV, and thus they do not destroy the solution to strong CP problem. Terms of the type $(\Phi_a^T \tau_2 \Phi_b \tau_2)(\Delta^c \tilde{\Delta}^c / M_{\text{Pl}})$ in W will induce complex phases in the μ terms. If $v_R \leq 10^7$ GeV, these phases will be less than 10^{-9} , and will leave the strong CP solution intact. With a parity even spurion field, which acquires a non-zero F -component leading to SUSY breaking, the constraints of parity invariance will be intact in the soft SUSY breaking parameters (such as gluino mass being real).

7. Conclusion

In conclusion, we have pointed out that the minimal renormalizable supersymmetric left–right model is completely consistent phenomenologically without any need for higher-dimensional operators or spontaneous R -parity violation. The scale of left–right symmetry can now be higher than TeV. The model can solve the strong CP problem without fear of large contributions to $\bar{\theta}$ from non-renormalizable terms (since they are now not needed). The model also provides a simple solution based on parity symmetry for the SUSY CP problem. The effective potential of the theory, which has important contributions from heavy Majorana–Yukawa couplings, allows for the charge conserving and R -parity conserving minimum to be the global minimum. The model predicts light (sub-TeV) doubly charged Higgs bosons and their superpartners.

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