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Matter–antimatter oscillations and CP violation as manifested through quantum mysteries

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Abstract

Meson–antimeson oscillations are a pure quantum phenomenon involving many of quantum mechanics' 'spooky' features like state mixing and EPR correlations; the subtle implementation of (approximate) symmetries also plays a crucial role. The analysis of oscillations represents a high sensitivity probe of nature's fundamental forces and at the same time provides experimental validation of subtle features of quantum mechanics. The theoretical framework and experimental signatures are described in detail. Meson–antimeson oscillations have also formed essential ingredients in the discovery of **CP** violation, a delicate, yet profound feature of our universe. These phenomena have been crucial for the evolution of the Standard Model of high energy physics and have more recently provided impressive validation for its CKM dynamics. Nevertheless these successes do not invalidate the arguments for the Standard Model being incomplete already at 'nearby' energy scales. Oscillation phenomena and **CP** violation open up new portals for the emergence of the anticipated new physics. Other incarnations of matter–antimatter oscillations, as for neutrons, are briefly commented on.

(Some figures in this article are in colour only in the electronic version)

Dedicated to Lalit Sehgal on the occasion of his 67th birthday—an admired colleague, excellent cook and wonderful friend for more than 30 years.

This article was invited by Professor P Zerwas.

Contents

Prologue

The narrative to be given below will contain many concrete numbers and explicit mathematical expressions, as it has to be and for which I do not apologize. Instead I will offer 'active repentance' by providing also intuitive arguments as much as possible. However there are limitations to that. For the phenomena cannot be understood outside a quantum mechanical description. Yet quantum mechanics is as counter-intuitive as it was in the early twentieth century when it was born; we just got used to it—in this case familiarity has bred acceptance (often without questioning) rather than contempt—and follow the guidance by a mature mathematical framework. Some of the most counter-intuitive features of quantum mechanics are actually at the core of matter–antimatter oscillations like 'spooky action-at-a-distance' that was criticized by Einstein, Podolsky and Rosen (EPR) in their seminal 1935 paper [1]. Some of the most precise measurements in high energy physics exploit EPR correlations in a crucial way and actually might not be possible without them. Talking of an EPR 'paradox' misses the point. As explained later on, a better formulation is: 'EPR correlations are surely spooky, yet they are real in an empirical sense.'

The reader will forgive, I hope, if I indulge myself in one personal remark before beginning to describe the physics of oscillations: their tale and that of **CP** violation provides a multilayered illustration of the paradigm of high energy physics (HEP) or of basic science in general. Stated in a nutshell it contains four elements: a fundamental question is at stake; long periods of *apparent* stagnation are followed by intervals of unexpected twists and turns, even breakthroughs; the conclusion of one chapter often comes with the first message from the next chapter; and finally—and even as a theorist I view it as most fascinating—it is driven by an intense interplay between theory, experiment and technology with each taking turns in the lead. I also want to emphasize that the whole development—starting from the observation that the production rates of 'strange' hadrons exceed their decay rates by several orders of magnitude to verifying the difference between K^0 and $\bar{\mathrm{K}}^0$ through meson–antimeson oscillations and to leading to the discovery of **CP** violation, a story that has repeated itself for B mesons—was a rational and almost logical one at least in hindsight, although contemporaries did not and probably could not see it that way.

This paper will be organized as follows. In section 1 I introduce the general phenomenology of matter–antimatter oscillations emphasizing the special role of **CP** violation; I list different examples, mention the central role they have played in the emergence of the Standard Model (SM) of HEP and sketch the flavor dynamics of the SM. In section 2 I give a more explicit theoretical description of oscillations as an exercise in basic quantum mechanics and describe the theoretical and experimental landscape of flavor dynamics as it existed just before the turn of the millennium; I state the salient features of its description within the SM. In section 3 I address two of the 'quantum jumps' that occurred in our knowledge of nature's basic dynamics: *direct* **CP** violation and the validation of the SM's paradigm of large **CP** violation in beauty decays, for which the existence of EPR correlations turned out to be a routine precision tool rather than a paradox; after briefly recapitulating the theoretical and empirical arguments for the SM being incomplete I conclude with describing strategic elements of searching for dynamics beyond the SM in section 4 followed by a short epilogue. While I will attempt to stay away from HEP jargon as much as possible, I sketch frequently used terms in a glossary in appendix A; I discuss the technical points of phase conventions and the signs of ΔM and $\Delta\Gamma$ in appendix B: in appendix C I describe how the characteristic time behavior of oscillation phenomena is inferred experimentally and address the issue of whether states of equal energy or of equal momenta interfere; in appendix D I present the argument by Kobayashi and Maskawa for counting the number of observable angles and phases as a function of the number of families; appendix E contains an intriguing example for the subtle role played by $K^0 - \bar{K}^0$ oscillations in restoring **CPT** invariance; the solutions to the problems posed in the main text are given in appendix F. Sections where the title is enclosed by the symbol ♠ are more technical in nature, meant for the dedicated reader and can be left out for a first reading.

The presentation is aimed at a wider audience of interested physicists and reasonably self-contained with references often to overviews of a similar style rather than the original papers. I should warn the reader that I will use the unit system for Planck's quantum and the speed of light conveniently employed in HEP:

$$\hbar = 1 = c \tag{1}$$

This leads to mass, energy and momentum having the same dimension, which in turn is inverse to the dimension of time and length. I will also follow the practice common in HEP to refer to experimental collaborations by the name of their experiment.

A general comment on experimental references in particular: there is a stream of high quality data on matter–antimatter oscillations and **CP** violation coming from different experiments, which is expected to increase even further with the beginning of the LHC program. One can best stay up-to-date with all measurements by consulting the homepage of the Heavy Flavour Averaging Group (HFAG): http://www.slac.stanford.edu/xorg/hfag/. Yet referring only to the latter would not be fair to the experimental groups that obtained the data involved. As a practical compromise I will list some, though not all of the seminal individual papers.

1. Introduction to oscillations and the Standard Model

Elementary quantum mechanics is all that is needed to describe the *phenomenology* of oscillations. The inverse is true as well: oscillations represent the quintessential laboratory for quantum mechanics exhibiting the peculiar features of the latter. Precision studies of oscillations thus provide high sensitivity tests of quantum mechanics' foundations; harnessing in turn quantum mechanics over macroscopic distances leads to measurements of impressive quantitative accuracy that open a window on nature's fundamental forces as we will see. Yet to interpret the true meaning of these perspectives requires quantum field theory.

In this section 1 I will introduce the concepts most relevant for matter–antimatter oscillations and **CP** violation and summarize the data on oscillations of K, B and D mesons. Most explicit expressions will be postponed to section 2; here I will appeal to very general arguments and to an analogy from classical mechanics.

1.1. Matter–antimatter oscillations in the evolution of the SM

1.1.1. Qualitative introduction. The discovery of antimatter was the unforeseen consequence of a seemingly unrelated theoretical development, namely, Dirac's construction of a relativistic wave equation for the electron. It was realized that the electron had to have a partner of exactly opposite electric charge and equal mass. The latter showed this partner could not be the proton, which would have been an attractive solution to a puzzle that is still with us, namely, why the electron and proton charges are equal in magnitude and opposite in sign. The subsequent discovery of the positron by Anderson in 1932 provided a striking example of the 'unreasonable efficiency of mathematics', as formulated by Hertz: 'We make inner images or symbols of the external objects, and we make them in such a way that the consequence of the images dictated by thinking are always the images of the consequences dictated by nature of the mapped objects'. One could replace 'dictated by thinking' with 'dictated by mathematics'.

It was realized subsequently that combining the requirements of quantum mechanics and special relativity, as implemented through quantum field theory, necessarily leads to the existence of antimatter for bosons and fermions alike. Particles like neutral pions and photons are their own antiparticle, whereas charged pions, electrons and nucleons come in mass degenerate **CP** conjugate pairs; P stands for parity and C for charge conjugation, i.e. the exchange of particle and antiparticle.

K mesons (or kaons) and Λ baryons were discovered more than fifty years ago: they were called 'strange', since their *production* rate exceeded their *decay* rates by many orders of magnitude. This puzzle was solved by introducing the concept of 'associated production': a quantum number 'strangeness' S was introduced with all previously known hadrons—pions, protons, neutrons—carrying $S = 0$, whereas values $-1[+1]$ were assigned to Λ, $\bar{\mathrm{K}}^0$, K^- [$\bar{\Lambda}$, K^0, K^+]; for the strong and electromagnetic forces the selection rule $\Delta S = 0$ was imposed—i.e. conservation of strangeness—meaning they can produce only pairs of strange hadrons; their decays, which obviously require $\Delta S = 1$, can be driven only by weak forces.

Such a scenario raised an intriguing challenge: how can one establish empirically that the postulated two neutral mesons K^0 and $\bar{\mathrm{K}}^0$ are indeed distinct? The answer to this challenge has been the phenomenon of meson–antimeson oscillations first given by Gell-Mann and Pais [2], which has been one of the most impressive success stories of basic science. It is a straightforward, yet powerful exercise in basic nonrelativistic quantum mechanics that per se can be discussed irrespective of the fundamental physical degrees of freedom (like quarks in the SM). I will give here a simplified and more qualitative discussion; a detailed description will follow in section 2.1.

With strangeness describing a conserved quantum number the two states of definite strangeness K^0 and $\bar{\mathrm{K}}^0$ are also mass eigenstates; i.e. they possess a definite mass and lifetime with the latter being infinite at this point. Since they are particle and antiparticle **CPT** invariance—an almost inescapable consequence of any local quantum field theory—requires their masses and lifetimes to be the same. (For that reason *any* linear combination of $|\mathrm{K}^0\rangle$ and $|\bar{\mathrm{K}}^0\rangle$ will be a mass eigenstate as well.)

The intervention of $\Delta S \neq 0$ weak forces changes the situation fundamentally in a two-fold way:

(A) Since $\mathrm{K}^0 \leftrightarrow \bar{\mathrm{K}}^0$ transitions can now occur, K^0 and $\bar{\mathrm{K}}^0$ will cease to be mass eigenstates. Those, denoted by $\mathrm{K_S}$ and $\mathrm{K_L}$, will be linear combinations of K^0 and $\bar{\mathrm{K}}^0$ and thus will *not* possess definite strangeness. There is no reason, why they should possess equal mass; i.e. violation of the conservation law—here $\Delta S \neq 0$—lifts the degeneracy of the system: $M(\mathrm{K_L}) \neq M(\mathrm{K_S})$. These features are illustrated in figure 1.

(B) Kaons can decay into two classes of final states without strangeness:
   (i) (semi)leptonic ones–$\mathrm{K}^-/\bar{\mathrm{K}}^0 \to l^-\bar{\nu}(\pi^0/\pi^+)$, $\mathrm{K}^+/\mathrm{K}^0 \to l^+\nu(\pi^0/\pi^-)$;
   (ii) nonleptonic ones–$\mathrm{K}^\pm \to \pi^\pm\pi^0, 3\pi$, $\mathrm{K}^0/\bar{\mathrm{K}}^0 \to 2\pi, 3\pi$.

The first class is controlled by the SM selection rule $\Delta Q = \Delta S$ for the hadron charge Q. Those final states are flavor specific, i.e. reveal by their nature whether they came from the decay of a strange particle or antiparticle and thus allow to track the flavor of the meson *at the time of decay*.

The nonleptonic decays for the *neutral* kaons on the other hand are flavor-*non*specific, i.e are common to K^0 and $\bar{\mathrm{K}}^0$ transitions. Since there are several decay channels, there is no reason why the decays width should be the same for $\mathrm{K_S}$ and $\mathrm{K_L}$: $\Gamma(\mathrm{K_S}) \neq \Gamma(\mathrm{K_L})$.

Other symmetries allow us to make more specific statements. *If* the **CP** transformation operator describes a symmetry of the Hamilton operator **H**, they have to commute: $[\mathbf{CP}, \mathbf{H}] = 0$. Hence the mass eigenstates have to be **CP** eigenstates as well. Using the *definition* (see appendix B for a general discussion)

$$\mathbf{CP}|\mathrm{K}^0\rangle \equiv |\bar{\mathrm{K}}^0\rangle \tag{2}$$

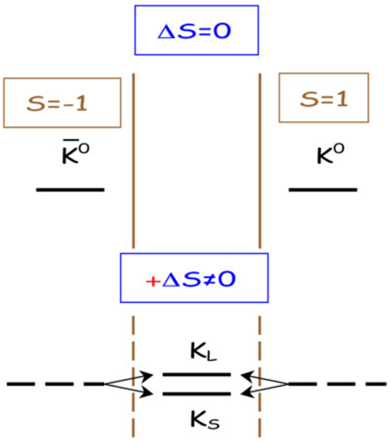

Figure 1. The mass spectrum for neutral kaons without and with weak forces.

we have for the **CP** even and odd eigenstates

$$|K_\pm\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle \pm |\bar{K}^0\rangle) \qquad \text{with } \Delta M_K \equiv M_{K_-} - M_{K_+} \neq 0. \tag{3}$$

CP symmetry also constrains the decay modes

$$|K_+\rangle \to 2\pi \;\; \not\leftarrow |K_-\rangle \to 3\pi \; . \tag{4}$$

For with kaons and pions being pseudoscalar mesons the two pions from a K decay have to form an S wave and therefore $\mathbf{CP}|2\pi\rangle = (-1)^2(-1)^l|2\pi\rangle = +|2\pi\rangle$. On the other hand $\pi^+\pi^-\pi^0$ can be **CP** odd and $3\pi^0$ has to be [6]. (With $M_K < 4m_\pi$ $K \to 4\pi$ cannot occur.) Such a difference enforces $\Gamma(K_+) \neq \Gamma(K_-)$. A kinematical 'accident' intervenes at this point: since the kaon mass is barely above the three pion threshold and thus $K_- \to 3\pi$ greatly suppressed by phase space, its lifetime is much longer than for K_+. Their lifetime ratio is actually as large as 570; accordingly one refers to them as K_L and K_S with the subscripts L and S referring to 'long'- and 'short'-lived. Thus one predicts the following nontrivial scenario: if one starts with a pure beam of, say, K^0, which is a linear combination of K_+ and K_-, one finds different components in the decay rate evolution depending on the nature of the final state:

- In $K^{\text{neut}} \to$ pions two components will emerge, namely, $K_+ \to 2\pi$ and $K_- \to 3\pi$ controlled by the (vastly different) lifetimes $\tau(K_S)$ and $\tau(K_L)$, respectively.
- Tracking the flavor specific (semi)leptonic modes instead, one encounters a considerably more complex situation as shown in figure 2: the decay rate for the 'right-sign' leptons $K^0 \to l^+\nu\pi^+$ at first drops off faster than follows from $e^{-\Gamma_S t}$, an exponential dependence on the time of decay, then bounces back up, etc, i.e. 'oscillates'—hence the name. The deviation from the exponential is described by a functional dependence $\cos \Delta M_K t$, meaning it takes a period $T = 2\pi/\Delta M_K$ for the K^0 to transmogrify itself into a $\bar{K}^0$ and then back into a K^0 again (unless it decays in the meantime). The rate for the 'wrong-sign' transitions $K^0 \to l^-\nu\pi^+$, which has to start out at zero for $t = 0$ rises quickly, yet turns around dropping down, before bouncing back up again etc. It provides the complement for $K^0 \to l^+\nu\pi^-$, i.e. the rate for the sum of both modes should exhibit a simple exponential behavior.

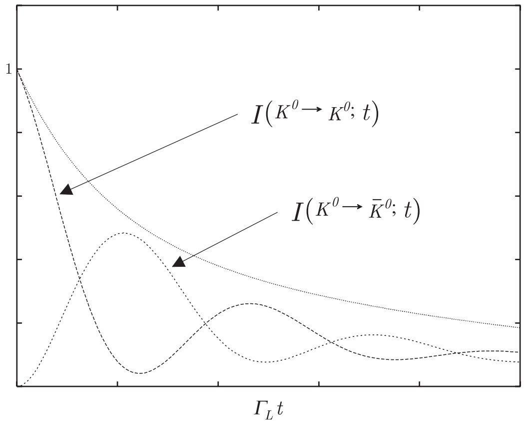

Figure 2. The probabilities of finding a K^0 and a $\bar{K}^0$ in an *initial* K^0 beam as a function of time.

These predictions given by Gell-Mann and Pais first assuming **C** invariance and relaxing it later to **CP** symmetry were verified experimentally with impressive numerical sensitivity [3]:

$$\Delta M_K = (3.483 \pm 0.006) \times 10^{-12}\,\text{MeV} = (0.5290 \pm 0.0016) \times 10^{10}\,\text{s}^{-1}. \tag{5}$$

(English speakers can rely on a simple mnemonic to remember which state is heavier: L stands for *larger* mass and *longer* lifetime, whereas S denotes *smaller* and *shorter*.) This number is a striking demonstration for the sensitivity reached when quantum mechanical interference can be tracked over macroscopic distances, i.e. flight paths of metres or even hundreds of metres. Using the kaon mass as yardstick one can re-express equation (5)

$$\frac{\Delta M_K}{M_K} = 7.7 \times 10^{-15}, \tag{6}$$

which is obviously a most striking number. A hard-nosed reader can point out that equation (6) vastly overstates the point since the kaon mass generated largely by the *strong* interactions has *no* intrinsic connection with ΔM_K generated by the *weak* interactions and that calibrating ΔM_K by M_K is arbitrary.

Such sentiment might actually go too far, as can be illustrated by the following consideration within the context of an antigravity ansatz: there one assumes gravity to couple to matter and antimatter with opposite signs. The gravitational potential Φ_{grav} would then produce a *relative* phase of $2M_K\Phi_{\text{grav}}t$ between K^0 and $\bar{K}^0$. In the Earth's potential this would generate a gravitational oscillation time of 10^{-15} s [4], which is much shorter than the *observed* oscillation period $\sim 10^{-9}$ s; i.e. kaons and antikaons couple to gravity with equal strength to a very good approximation. The attentive reader will notice some essential flaws in this simple argument: having gravity couple differently to matter and antimatter violates the equivalence principle and makes the observable oscillation phase depend on the value of the potential rather than a potential difference. Suffice it to say that in *extended* supergravity theories [5] one has a spin one partner of the spin two graviton creating an antigravity component. One could argue that antigravity is universal with respect to quark flavors (described below in some detail). This would mean that K^0 as a bound state of an s antiquark and a d quark experiences the same gravitational coupling as a $\bar{K}^0$ consisting of an s quark and d antiquark even with antigravity present.

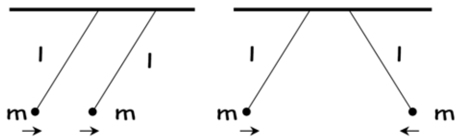

Figure 3. Two special configurations of the uncoupled double pendulum.

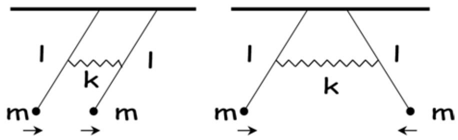

Figure 4. The two fundamental modes of the coupled double pendulum.

The more relevant yardstick for the oscillation rates is provided by the weak decay width [3]

$$x_K = \frac{\Delta M_K}{\bar{\Gamma}_K} \simeq 0.945 \pm 0.003 \tag{7}$$

$$y_K = \frac{\Delta \Gamma_K}{2\bar{\Gamma}_K} \simeq 0.996 \qquad \text{with } \bar{\Gamma}_K = \tfrac{1}{2}(\Gamma_{K_S} + \Gamma_{K_L}). \tag{8}$$

1.1.2. A mathematical analogue from classical mechanics, part I. Although matter–antimatter oscillations represent an intrinsically quantum mechanical phenomenon, we can find a simple mathematical analogue from classical mechanics. Let us consider two identical pendula—i.e. with equal length l and mass m—side by side. They have equal oscillation frequency $\omega = \sqrt{g/l}$ and damping time $1/\gamma$. Since no energy can be transferred between the two pendula, they undergo independent oscillations. Yet there are two special configurations: they swing back and forth (a) 'in phase', i.e. in parallel or (b) 'out of phase', i.e. both swinging inward or both outward at any one time with frequency ω; see figure 3.

Next one couples the two pendula weakly to each other by connecting them through a spring with spring constant k. Now energy can be transferred from one pendulum to the other. The two special configurations mentioned above are the fundamental modes; see figure 4: (a) When the two identical pendula swing *in phase*, the presence of the spring connecting them is irrelevant, and the frequency is still $\omega_{in} = \sqrt{g/l}$. (b) For the *out of phase* oscillations the frequency gets affected and actually enhanced by the coupling spring: $\omega_{out} = \sqrt{g/l + 2k/m}$. The dynamical coupling between the two systems has thus lifted the degeneracy (in frequency). Furthermore internal friction and air resistance, which dampen the oscillations, affect the two fundamental modes differently due to $\omega_{in} \neq \omega_{out}$ and lead to different damping times: $\gamma_{in} \neq \gamma_{out}$. In section 2.2 I will give explicit solutions.

Let us finally consider what happens with the coupled double pendulum, when the initial configuration consists of, say, the right pendulum being perpendicular and the left one non-perpendicular, as shown in figure 5. The oscillations of the left pendulum will at first be little affected by the weak coupling to the other pendulum. Yet some of the left pendulum's energy will be transferred through the spring causing the right (left) pendulum to oscillate with increasing (descreasing) amplitude, at first hardly perceptible, yet then becoming pronounced. There will be a point in time when all the energy has been transferred to the right pendulum with the left one coming to a rest. Afterwards the process will reverse itself with the energy being transferred from the right to the left pendulum and so

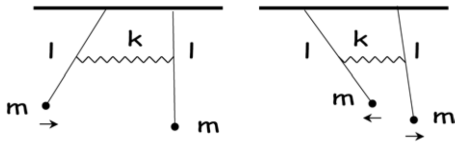

Figure 5. An asymmetric initial configuration.

forth, till the initial energy has been totally dissipated due to friction and both pendula come to a rest.

The analogy to the $K^0 - \bar{K}^0$ system is rather obvious. The motions of the left and right pendula correspond to the K^0 and $\bar{K}^0$ mesons, respectively. The parallel and antiparallel motions represent the K_L and K_S, respectively. The behavior sketched in figure 5 provides a close analogy to $K^0-\bar{K}^0$ oscillations occurring in an initially pure K^0 beam as shown in figure 2. The similarities actually go further. One can define a parity transformation exchanging the two pendula, which obviously represents a symmetry of the coupled system. Since the latter is non-degenerate, the fundamental modes have to be an even or odd eigenfunction under this parity, analogous to the $K^0 \leftrightarrow \bar{K}^0$ case. The coupling between the two pendula lifts the degeneracy in the frequency of the two normal modes, again in analogy to $(M_{K_L}, \Gamma_{K_L}) \neq (M_{K_S}, \Gamma_{K_S})$.

1.1.3. $B_d - \bar{B}_d$ *oscillations—An essential test case.* Like a patient teacher does with a slow student nature has provided us with more examples of meson–antimeson oscillations to make sure we learn our lesson. It had been recognized from the start that neutral B or 'beauty' mesons (which are about ten times heavier than kaons) should exhibit oscillations [6]. There are actually two neutral B mesons, namely, written in quark language $B_d = [\bar{b}d]$ and $B_s = [\bar{b}s]$, i.e. the latter also carries one unit of strangeness.

The general phenomenology posed no mystery, since it follows a close qualitative—though not quantitative—analogy with kaon oscillations. One obvious difference arises in the lifetime ratios of the two mass eigenstates: the kinematical 'accident' that leads to the huge disparity in the K_L and K_S lifetimes as described above does not repeat itself for the much heavier B mesons with their multitude of decay channels, and one predicts $\Delta\Gamma_B/\Gamma_B \ll 1$.

When ARGUS discovered $B_d - \bar{B}_d$ oscillations in 1986 [7]—a seminal discovery soon confirmed by CLEO [8]—it caused quite a stir for two main reasons, the details of which will be explained in section 2:

(i) They found $x_d \equiv \Delta M_{B_d}/\Gamma_{B_d} \simeq 0.7$, which was much larger than the quantitative theoretical predictions given before. Yet in all fairness one should understand the main reason behind this considerable underestimate: ΔM_{B_d} is very sensitive to the value of the top quark mass m_t. In the early 1980s there had been an experimental claim by UA1 to have discovered top quarks in $p\bar{p}$ collisions with $m_t = 40 \pm 10$ GeV. To their later chagrin theorists by and large had accepted this claim. Yet after the ARGUS discovery theorists quickly concluded that top quarks had to be much heavier than previously thought, namely, $m_t > 100\,\text{GeV}$ [6]; this was the first indirect evidence for top quarks being 'super-heavy' before they were discovered in $p\bar{p}$ collisions at Fermilab. Present data tell us [3]

$$\Delta M_{B_d} = (0.507 \pm 0.005)\text{ps}^{-1} = (3.337 \pm 0.033) \times 10^{-10}\,\text{MeV}, \tag{9}$$

$$x_d \equiv \frac{\Delta M_{B_d}}{\Gamma_{B_d}} = 0.776 \pm 0.008. \tag{10}$$

The measured value of ΔM_{B_d} is completely compatible with the SM.

(ii) Since, as explained later, **CP** violation can enter the underlying dynamics only through complex phases, one needs two different amplitudes contributing *coherently* to the same overall transition for a **CP** asymmetry to surface. Since $x = \Delta M_B/\Gamma_B$ denotes the ratio between the oscillation and decay rates, $x = 1$ represents an optimal situation for satisfying such a requirement for final states f that can be fed by B_d as well as $\bar{B}_d$ decays:

$$B_d \Rightarrow \bar{B}_d \to f \leftarrow B_d \tag{11}$$

with $\Rightarrow$ denoting the oscillation. While it had been predicted several years earlier [9, 10] that some B_d decay channels should exhibit large **CP** asymmetries based on oscillations anticipated to take place, it was ARGUS' discovery of an almost 'optimal' oscillation rate that convinced many physicists that one could search for such asymmetries with high experimental sensitivity.

1.1.4. The 'hot' news: $B_s - \bar{B}_s$ *oscillations.* Nature actually provided us with an 'encore'. As explained later, within the SM one predicts $\Delta M(B_s) \gg \Delta M(B_d)$, i.e. that B_s mesons oscillate much faster than B_d mesons. Those rapid oscillations have been resolved now:

$$\Delta M_{B_s} = \begin{cases} (19 \pm 2)\ \text{ps}^{-1} = (1.25 \pm 0.13) \times 10^{-2}\,\text{eV} & \text{D0 [11]}, \\ (17.77 \pm 0.12)\,\text{ps}^{-1} = (1.17 \pm 0.01) \times 10^{-2}\,\text{eV} & \text{CDF [12]}, \end{cases} \tag{12}$$

$$x_s = \frac{\Delta M_{B_s}}{\Gamma_{B_s}} \simeq 25 \tag{13}$$

to be compared with the theoretical prediction as explained in section 3.2.8:

$$\Delta M_{B_s} = (18.3^{+6.5}_{-1.5})\,\text{ps}^{-1} = (1.20^{+0.43}_{-0.10}) \times 10^{-2}\,\text{eV} \tag{14}$$

This finding represents another triumph of the SM even more impressive than a mere comparison of the observed and predicted values of ΔM_{B_s}, as explained later.

There is marginal evidence for $\Delta\Gamma_{B_s} \neq 0$ from an overall fit to the data [52]

$$\Delta\Gamma_{B_s} = (0.084^{+0.055}_{-0.050})\,\text{ps}^{-1}\ \text{HFAG '07}. \tag{15}$$

1.1.5. 'Stop the press': evidence for $D^0 - \bar{D}^0$ *oscillations.* In the spring of 2007 both BABAR and BELLE have reported most intriguing signals for oscillations of neutral mesons [13, 14]. A 'preliminary' average by the Heavy Flavour Averaging Group over all relevant data [52] yields

$$x_D \equiv \frac{\Delta M_D}{\bar{\Gamma}_D} = (0.85 \pm 0.32) \times 10^{-2}, \qquad y_D \equiv \frac{\Delta\Gamma_D}{2\bar{\Gamma}_D} = (0.71 \pm 0.21) \times 10^{-2},$$
$$\Delta M_D = (2.07 \pm 0.78) \times 10^{10}\,\text{s}^{-1}, \qquad \Delta\Gamma_D = (3.46 \pm 1.02) \times 10^{10}\,\text{s}^{-1} \tag{16}$$

with $5\,\sigma$ significance for $[x_D, y_D] \neq [0, 0]$—and the caveat that averaging over the existing data sets has to be taken with quite a grain of salt at present due to the complicated likelihood functions [15].

1.1.6. Some observable characteristics. Within the SM *instantaneous* semileptonic decays of kaons (antikaons) can produce only positively (negatively) charged leptons:

$$l^-\bar{\nu}\pi^+ \not\leftarrow \mathrm{K}^0 \to l^+\nu\pi^-, \qquad l^+\nu\pi^- \not\leftarrow \bar{\mathrm{K}}^0 \to l^-\bar{\nu}\pi^+, \qquad l = \mathrm{e}, \mu. \tag{17}$$

This fact is expressed compactly through the selection rule $\Delta S = \Delta Q$ meaning the decay of a meson with strangeness $S = -1[+1]$ will increase (decrease) the electric charge of the hadron in the final state. (A more transparent formulation would be $\Delta S = -\Delta Q_l$ relating the change in S to that in the charge of the lepton pair.) Beauty mesons B and charm mesons D—heavier than kaons by a factor of about ten and three, respectively, and much shorter lived—exhibit similar features

$$\begin{aligned} &l^-\bar{\nu}\mathrm{D}^+ \not\leftarrow \mathrm{B}^0 \to l^+\nu\mathrm{D}^- \not\leftarrow \bar{\mathrm{B}}^0 \to l^-\bar{\nu}\mathrm{D}^+, \\ &l^-\bar{\nu}\mathrm{K}^+ \not\leftarrow \mathrm{D}^0 \to l^+\nu\mathrm{K}^- \not\leftarrow \bar{\mathrm{D}}^0 \to l^-\bar{\nu}\mathrm{K}^+\ ; \end{aligned} \tag{18}$$

i.e. they obey the selection rules $\Delta B = \Delta Q$ and $\Delta C = \Delta Q$.

Oscillations lead to a violation of these selection rules 'on average' due to the two-step process expressed generically for a pseudoscalar meson $P^0 = \mathrm{K}^0, \mathrm{D}^0, \mathrm{B}^0$ with quark flavor F:

$$P^0 \Longrightarrow \bar{P}^0 \to l^-\nu + X^+, \qquad \bar{P}^0 \Longrightarrow P^0 \to l^+\nu X^-, \qquad X = \pi, \mathrm{K}, \mathrm{D}, \tag{19}$$

where '$\Longrightarrow$' and '$\to$' denote the $\Delta F = 2$ oscillation and $\Delta F = 1$ direct transitions, respectively. Integrating over all times of decay yields, as shown in section 2.1, for the ratio of wrong- to right-sign leptons and for the probability of wrong-sign leptons

$$\begin{aligned} &r_P = \frac{\Gamma(\bar{P}^0 \to l^+\nu X^-)}{\Gamma(\bar{P}^0 \to l^-\nu X^+)} = \frac{x_P^2}{2 + x_P^2 + y_P^2}, \qquad x_P \equiv \frac{\Delta M_P}{\bar{\Gamma}_P}, \qquad y_P \equiv \frac{\Delta\Gamma_P}{2\bar{\Gamma}_P}, \\ &\chi_P = \frac{\Gamma(\bar{P}^0 \to l^+\nu X^-)}{\Gamma(\bar{P}^0 \to l^\pm\nu X^\mp)} = \frac{r_P}{1 + r_P}. \end{aligned} \tag{20}$$

Maximal oscillations can be defined as $x_P \gg 1$ and thus $r_P \to 1$ and $\chi_P \to 1/2$. For $\mathrm{B_s}$ oscillations with $x_s \simeq 25$ we have $r_s \simeq 0.997$ and $\chi_s \simeq 0.499$; i.e. they are maximal for all practical purposes.

Huge samples of beauty mesons can be obtained in $\mathrm{p\bar{p}}$ or pp collisions at high energies, which yield *in*coherent pairs of B mesons. Three cases have to be distinguished:

(i) $$\mathrm{p\bar{p}} \to \mathrm{B^+\bar{B}_d} + X/\mathrm{B^-B_d} + X \tag{21}$$
leading to a single beam of neutral B mesons, for which equation (20) applies.

(ii) $$\mathrm{p\bar{p}} \to \mathrm{B_d\bar{B}_d} + X, \tag{22}$$
when both B mesons can oscillate—actually into each other leading to *like*-sign di-leptons $\mathrm{p\bar{p}} \to \mathrm{B_d\bar{B}_d} + X \Longrightarrow \mathrm{B_dB_d}/\mathrm{\bar{B}_d\bar{B}_d} + X \to l^\pm l^\pm + X'$ with
$$\frac{\mathrm{Rate}(\mathrm{p\bar{p}} \to \mathrm{B_d\bar{B}_d} + X \to l^\pm l^\pm + X')}{\mathrm{Rate}(\mathrm{p\bar{p}} \to \mathrm{B_d\bar{B}_d} + X \to ll + X')} = 2\chi_d(1 - \chi_d)\ ; \tag{23}$$
it means that *like*-sign di-leptons require one B meson to have oscillated into its antiparticle at its time of decay, while the other one has not.

(iii) In
$$\mathrm{e^+e^-} \to \mathrm{B_d\bar{B}_d} \tag{24}$$
one encounters the *coherent* production of two neutral beauty mesons. As discussed in section 3.2.3 EPR correlations combine with the requirement of Bose–Einstein statistics to make the pair act as a *single* oscillating system leading to [10]
$$\frac{\mathrm{Rate}(\mathrm{e^+e^-} \to \mathrm{B_d\bar{B}_d} \to l^\pm l^\pm + X)}{\mathrm{Rate}(\mathrm{e^+e^-} \to \mathrm{B_d\bar{B}_d} \to ll + X')} = \chi_d. \tag{25}$$

*1.2. **CP** violation in* K_L *decays*

Oscillations per se had been predicted by Gell-Mann and Pais [2]—yet we have obtained from nature more than we had bargained for: not only was **CP** violation not predicted, there was strong sentiment in the community (as advocated by Landau) that it could not exist, despite maximal **P** violation having been established in 1957. I know of only one 'heretic', namely, Okun, who in his 1963 textbook [18] explicitly listed the search for $K_L \to \pi^+\pi^-$ as a priority, i.e. one year before its discovery. I will explain the reasons behind the orthodoxy in section 1.3.

A long series of experiments [21] exploring the production and decay of neutral kaons culminated in 1964 with the discovery of **CP** violation through the observation of

$$K_L \to \pi^+\pi^- \tag{26}$$

by the Fitch–Cronin experiment [16]. Why does the mere existence of $K_L \to \pi^+\pi^-$ prove the existence of **CP** violation? The argument goes as follows: **CP** invariance implies that mass eigenstates come either as *mass degenerate* pairs of states—K^0 and $\bar{K}^0$—or as even and odd **CP** eigenstates K_+ and K_-, respectively. Those two scenarios are truly different only for $M(K_+) \neq M(K_-)$. K_L is defined as K_- by its major nonleptonic modes $K_L \to \pi^+\pi^-\pi^0, 3\pi^0$. Yet $K_L \to 2\pi$ produces a **CP** even final state; K_L is no longer a pure **CP** eigenstate implying **CP** violation; see the discussion around equation (4).

The best demonstration of the shock the community felt is given by two radical *alternatives* to **CP** violation that were suggested:

- One can postulate the existence of a new very light neutral particle U with $\mathbf{CP}|U\rangle = -|U\rangle$ such that

$$K_S \to 2\pi \qquad \text{versus} \qquad K_L \to 2\pi + U \tag{27}$$

  occur. Due to the odd **CP** parity of U the symmetry is thus restored, yet observing U directly would pose quite an experimental challenge. Here one is actually mimicking Pauli's brilliant introduction of the neutrino to reconcile the continuous lepton electron spectrum observed in β decay with conservation of energy and momentum: in both cases one postulates the existence of a hitherto unobserved neutral and very light particle to save an invariance. Alas—it did not work this time. For more careful studies revealed that the decay rate evolution of $K^0 \to 2\pi$ was not described by a simple sum of the $K_S \to 2\pi$ and $K_L \to 2\pi$ rates, but also exhibited an *interference* region between the two transitions. The latter could not happen, if equation (27) described the underlying processes, since interference requires in principle indistinguishable final states. These features are shown in figure 6: the solid line represents the expected distribution of decays as a function of time if equation (27) applied; it is manifestly inconsistent with the experimental points, which exhibit an unambiguous interference pattern. In appendix E I present an example for how the subtle interplay between these regions restores the equality between $\tau^+ \to K_{neut}\pi^+\bar{\nu}$ and $\tau^- \to K_{neut}\pi^-\nu$, as required by **CPT** invariance.
  This story represents a modern example of the ancient Roman saying:

  "Quod licet Jovi, non licet bovi."
  "What is permitted Jove, is not permitted a bull."

  That is we mere mortals cannot get away with speculations like 'Jove = Jupiter' Pauli.
- An essential element of this argument rests on the *linear superposition principle* of quantum mechanics. It was suggested [22] to introduce a judiciously chosen *non*-linear term into the Schrödinger equation to induce $K_L \to 2\pi$ with **CP** conserving dynamics. This intriguing option could however be falsified, since it predicted very different oscillation rates in **CP** conserving and violating transitions. More importantly, comparing

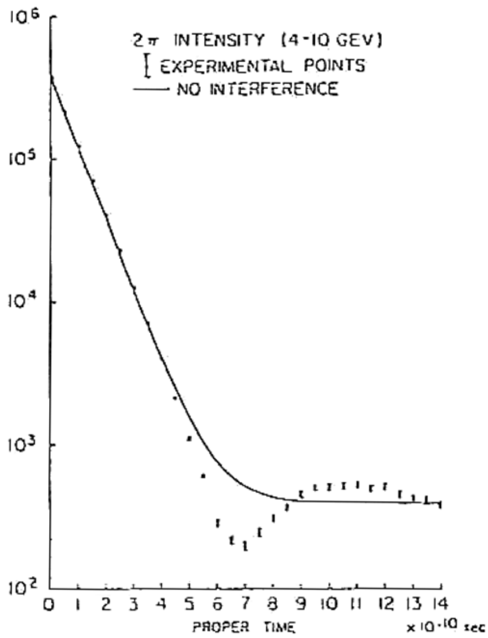

Figure 6. $K^0(t) \to \pi^+\pi^-$ as a function of proper time of decay t [17]; a clear interference pattern is visible for t between 5 and 13 $\times$ 10^{-10} s.

$K^0(t) \to \pi^+\pi^-$, as shown in figure 6, with the conjugate $\bar{K}^0(t) \to \pi^+\pi^-$ established **CP** violation unequivocally.

In quantum mechanics one does not always need to compare **CP** conjugate transitions to establish **CP** violation. This can be expressed through the following

Theorem. *If one finds that the evolution of the decays of an arbitrary linear combination of neutral mesons into a* ***CP*** eigenstate *as a function of (proper) time of decay cannot be described by a* single exponential*, then* ***CP*** *invariance must be broken. Or formulated more concisely for the case at hand:*

$$\frac{\mathrm{d}}{\mathrm{d}t} \mathrm{e}^{\Gamma t} \Gamma(K^{\mathrm{neut}}(t) \to \pi^+\pi^-) \neq 0 \qquad \text{for all real } \Gamma \Longrightarrow \mathbf{CP} \text{ violation!} \tag{28}$$

The proof is elementary: with ***CP*** *being conserved, mass eigenstates have to be either even or odd* ***CP*** *eigenstates as well and can decay only into final states of the same* ***CP*** *parity. Their decay rate evolution thus has to be given by a single exponential in time; q.e.d.*

Figure 6 provides a nice illustration of the reach of this theorem. Assume the rate for $K^0(t) \to \pi^+\pi^-$ could be described exactly as the *sum of two exponential functions of time*, say $\mathrm{e}^{-\Gamma_{K_L} t} G_L + \mathrm{e}^{-\Gamma_{K_S} t} G_S$, i.e. *no* interference term were observed. **CP** invariance would still be broken, since the two mass eigenstates of the $K^0 - \bar{K}^0$ complex could both decay into an even **CP** eigenstate—meaning they cannot be **CP** eigenstates—or the **CP**-odd mass eigenstate can decay into a **CP** even final state (or both). It would merely show that ΔM_K were zero, a conceivable (though 'unnatural') value. The observation of the interference effect closes one loop hole of an experimental nature (apart from confirming $\Delta M_K \neq 0$): As already stated, it demonstrates that the final state indeed consists only of two pions without a 'missing' particle.

The fact that the mass eigenstate K_L is not a **CP** eigenstate manifests itself in nonleptonic as well as semileptonic decays.

- The first observation of **CP** violation stated through the branching ratio [3]

$$\mathcal{B}(K_L \to \pi^+\pi^-) \equiv \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma_{K_L}} = (1.976 \pm 0.008) \times 10^{-3} \tag{29}$$

can be best analyzed through two ratios of amplitudes with practically equal phase space:

$$\eta_{+-} \equiv \frac{T(K_L \to \pi^+\pi^-)}{T(K_S \to \pi^+\pi^-)}, \qquad \eta_{00} \equiv \frac{T(K_L \to \pi^0\pi^0)}{T(K_S \to \pi^0\pi^0)} \tag{30}$$

η_{+-} and/or $\eta_{00} \neq 0$ represent **CP** violation, since it shows the mass eigenstate K_L defined by its dominant channel $K_L \to 3\pi$ and $\Gamma(K_L)$ can decay into final states of both **CP** parities. Yet there is more to it as emphasized by the following notation:

$$\eta_{+-} \equiv \epsilon + \epsilon', \qquad \eta_{00} \equiv \epsilon - 2\epsilon'. \tag{31}$$

The quantities $\eta_{+-,00}$, ϵ and ϵ' are complex with the phases of $\eta_{+-,00}$ and ϵ determined by **CPT** invariance to be close to 45^o and ϵ'/ϵ being basically real [90] (see also section 3.1.1). From equation (29) one infers

$$|\eta_{+-}| = (2.236 \pm 0.007) \times 10^{-3}. \tag{32}$$

  – If

$$\eta_{+-} - \eta_{00} = 3\epsilon' \neq 0, \tag{33}$$

  then one has a **CP** asymmetry that depends on the final state, which means **CP** violation has to reside (also) in $\Delta S = 1$ dynamics. This is referred to as *direct* **CP** violation.
  – The quantity ϵ on the other hand describes a **CP** asymmetry that characterizes the nature of K_L and is thus common to both modes. It reflects **CP** violation in $\Delta S = 2$ dynamics and is referred to as *indirect* **CP** violation.

The two terms direct and indirect **CP** violation are universally used and generalized to any flavor F to classify **CP** violation in $\Delta F = 1$ and $\Delta F = 2$ dynamics, respectively. While indirect **CP** violation involving matter–antimatter oscillations in an essential way—$\epsilon \neq 0$—was discovered in 1964, it took another 35 years of dedicated and ingenious experimentation to establish direct **CP** violation through $\eta_{+-} \neq \eta_{00}$.

- The second manifestation came in semileptonic K_L decays:

$$\delta_l \equiv \frac{\Gamma(K_L \to l^+\nu\pi^-) - \Gamma(K_L \to l^-\bar{\nu}\pi^+)}{\Gamma(K_L \to l^+\nu\pi^-) + \Gamma(K_L \to l^-\bar{\nu}\pi^+)} = (3.32 \pm 0.06) \times 10^{-3}. \tag{34}$$

With neither the SM nor most of its extensions being considered generating observable **CP** asymmetries in semileptonic decays, one encounters purely indirect **CP** violation there; thus its strength can be inferred from $K_L \to 2\pi$:

$$\delta_l|_{K_L \to 2\pi} \simeq 2\mathrm{Re}\,\epsilon = (3.16 \pm 0.01) \times 10^{-3}, \tag{35}$$

which is quite consistent with equation (34).
An even more obvious signature for **CP** violation is the observation of an asymmetry in neutral kaons decaying into 'wrong-sign' leptons:

$$\Gamma(\bar{K}^0 \to l^+\nu\pi^-) \neq \Gamma(K^0 \to l^-\bar{\nu}\pi^+). \tag{36}$$

Yet such a comparison requires the determination of the initial state as a K^0 or $\bar{K}^0$; I will discuss it in section 1.2.1.

The fact that $\mathrm{K}^0 - \bar{\mathrm{K}}^0$ oscillations are involved in an essential way can most directly be established by comparing $\mathrm{K}^0(t) \to \pi^+\pi^-$, see figure 6—i.e. the decay rate evolution as a function of the (proper) time of decay t of a beam that initially contained only K^0 mesons—with its **CP** conjugate $\bar{\mathrm{K}}^0 \to \pi^+\pi^-$. It reveals t *dependent* asymmetries; to be more specific: while the pure $\mathrm{K_S}$ domain exhibits no observable asymmetry, both the pure $\mathrm{K_L}$ as well as the $\mathrm{K_S} - \mathrm{K_L}$ interference domains do, yet in a way that the *time integrated* rates $\Gamma(\mathrm{K}^0 \to \pi^+\pi^-)$ and $\Gamma(\bar{\mathrm{K}}^0 \to \pi^+\pi^-)$ are equal, as required by **CPT** invariance, see section 4.3.1.

1.2.1. Time reversal invariance and the Kabir test. **CPT** invariance tells us that for every violation of **CP** symmetry there has to be a commensurate one for **T** invariance. Verifying this statement experimentally is far from straightforward though. For in a decay process $A \to B+C$ practical considerations prevent one from creating the time reversed sequence $B + C \to A$.

Matter–antimatter oscillations (and likewise neutrino oscillations) provide unique opportunities to probe **T** violations. For one can compare directly the rates for $\mathrm{K}^0 \Rightarrow \bar{\mathrm{K}}^0$ and $\bar{\mathrm{K}}^0 \Rightarrow \mathrm{K}^0$, which is referred to as 'Kabir test' [23]. For that purpose one has to determine the flavor of the final state—K^0 or $\bar{\mathrm{K}}^0$ – as well as tag the flavor of the initial one. Semileptonic channels can achieve the former through the SM $\Delta S = \Delta Q$ selection rule. For the latter one can rely on associated production in, say, proton-antiproton annihilation: $\mathrm{p\bar{p}} \to \mathrm{K}^+\bar{\mathrm{K}}^0\pi^-$ versus $\mathrm{p\bar{p}} \to \mathrm{K}^-\mathrm{K}^0\pi^+$, i.e., one compares the sequences $\mathrm{p\bar{p}} \to \mathrm{K}^+\bar{\mathrm{K}}^0\pi^- \to \mathrm{K}^+(l^+\nu\pi^-)\pi^-$ and $\mathrm{p\bar{p}} \to \mathrm{K}^-\mathrm{K}^0\pi^+ \to \mathrm{K}^-(l^-\bar{\nu}\pi^-)\pi^-$. Using this technique the CPLEAR collaboration found [24][1]:

$$A_T = \frac{\mathrm{rate}(\bar{\mathrm{K}}^0 \to \mathrm{K}^0) - \mathrm{rate}(\mathrm{K}^0 \to \bar{\mathrm{K}}^0)}{\mathrm{rate}(\bar{\mathrm{K}}^0 \to \mathrm{K}^0) + \mathrm{rate}(\mathrm{K}^0 \to \bar{\mathrm{K}}^0)} = (6.6 \pm 1.6) \times 10^{-3} \tag{37}$$

in full agreement with what is inferred from $\mathrm{BR}(\mathrm{K_L} \to \pi^+\pi^-)$:

$$A_T|_{\mathrm{K_L}\to 2\pi} \simeq 4\mathrm{Re}\,\epsilon = (6.32 \pm 0.02) \times 10^{-3}. \tag{38}$$

At first sight it might be surprising to find **T** violation in a setting that is completely controlled by non-relativistic quantum mechanics. For we have dealt with even a *free* Schrödinger equation, which is manifestly **T** invariant.

This apparent paradox is resolved by remembering that the dynamical evolution of any system is controlled both by the equations of motion and the boundary conditions. The latter provides the portal for **T** violation to enter on the level of quantum mechanics. We will see below that on the underlying level described by relativistic quantum field theory it is the dynamics expressed through the Lagrangian that harbours **CP** and **T** violation. Since **CP** violation allows $\mathrm{K_S}$ and $\mathrm{K_L}$ to decay into the same final state, namely, two pions, they are no longer orthogonal to each other:

$$\langle \mathrm{K_S}|\mathrm{K_L}\rangle \simeq 2\mathrm{Re}\epsilon \neq 0. \tag{39}$$

The fact that the relative weight of $\mathrm{K_S}$ and $\mathrm{K_L}$ is different in the K^0 and $\bar{\mathrm{K}}^0$ wave functions therefore leads to an observable effect, namely, $A_T \neq 0$; i.e. in the language of quantum mechanics the observed **T** violation follows from a (**CP**) asymmetry in the possible *initial conditions*. Yet in the underlying SM it reflects **T** and **CP** violating *dynamics*. However it is conceivable that in the ultimate theory it might revert to being the consequence of some asymmetry in the initial conditions. The relevant formalism is described in section 2.

T and **CP** violation can be emulated in *dissipative classical* dynamics like a two-dimensional oscillator [25], a Foucault pendulum or electric circuits [26]. In a less elegant

[1] The value quoted here is from [3].

way one can embed the breaking of **CP** and **T** invariance also in the boundary conditions for our toy model of the two weakly coupled pendula by making them not quite identical.

*1.3. On the special role of **CP** violation*

The realization in 1956 that parity is violated (maximally) in the weak interactions certainly caused a great shock in the community. One might then think that the discovery of **CP** violation in 1964 caused only a minor stir under Yogi Berra's dictum: 'Deja vu all over again.' Instead it generated another huge shock—as illustrated above by the actions taken at first to avoid accepting **CP** violation as an established fact. There were profound scientific reasons for that:

- For **P** violation being maximal (in the weak sector)—*no right*-handed neutrinos couple to the weak interactions—and likewise for **C** violation; yet with their combined **CP** transformation describing an *exact* symmetry one had an attractive 'fall back' position of pairing left-handed neutrinos with right-handed antineutrinos, as described by Oscar Wilde: '...people are attracted to men with a future and women with a past...'. Yet the discovery of **CP** violation shattered this balanced picture. Furthermore even Luther's redemption of last resort 'peccate fortiter' ('sin boldly') could not be invoked, since **CP** violation announced its arrival with a mere whimper: characterized by $\mathrm{Im}M_{12}/M_\mathrm{K} = 2.2 \times 10^{-17}$ it appears as the feeblest *observed* violation of any symmetry. **CP** symmetry as a 'near-miss' is rather puzzling in view of its fundamental consequences listed next.
- Parity violation tells us that nature makes a difference between 'left' and 'right'—but not which is which! For the statement that neutrinos emerging from pion decays are left- rather than right-handed implies the use of positive instead of negative pions. 'Left' and 'right' are thus defined in terms of 'positive' and 'negative', respectively. This is like saying that your left thumb is on your right hand—certainly correct, yet circular and thus not overly useful. The fact that maximal violations of **P** and **C** symmetries seemed to be exactly matched maintaining **CP** invariance gave considerable solace to theorists' belief in nature's predilection for symmetry.

  On the other hand **CP** violation was found manifesting itself also through

$$\frac{\mathcal{B}(\mathrm{K_L} \to l^+ \nu \pi^-)}{\mathcal{B}(\mathrm{K_L} \to l^- \bar{\nu} \pi^+)} \simeq 1.006 \neq 1, \tag{40}$$

  which is a re-formulation of equation (34). It allows us to define 'positive' and 'negative' in terms of *observation* rather than *convention*, and subsequently likewise for 'left' and 'right'. In that sense **CP** violation constitutes a more radical breakdown.
- Due to **CPT** invariance **CP** breaking implies a violation of time reversal invariance **T**. *Operationally* one defines time reversal as the reversal of *motion*: $\vec{p} \stackrel{\mathbf{T}}{\rightarrow} -\vec{p}$, $\vec{j} \stackrel{\mathbf{T}}{\rightarrow} -\vec{j}$ for momenta $\vec{p}$ and angular momenta $\vec{j}$. That nature makes an intrinsic distinction between past and future on the *microscopic* level that cannot be explained by statistical considerations represents an amazing observation.
- The fact that time reversal represents a very peculiar operation can also be expressed in a less emotional way, namely, through *Kramers' Degeneracy* [32]. The **T** operator changes the LHS of the canonical commutation relation

$$[\mathbf{X}, \mathbf{P}] = i\mathbf{1}. \tag{41}$$

  T invariance thus requires the RHS to change sign as well—which can be satisfied only for **T** being *anti*linear: $\mathbf{T}(\alpha|a\rangle + \beta|b\rangle) = \alpha^*\mathbf{T}|a\rangle + \beta^*\mathbf{T}|b\rangle$. This implies that $\mathbf{T}^2$ has eigenvalues ± 1. Consider the sector of the Hilbert space with $\mathbf{T}^2 = -1$ and assume the dynamics to conserve **T**; i.e. the Hamilton operator **H** and **T** commute. It is easily shown that if $|E\rangle$ is an eigenvector of $\mathbf{H}$, so is $\mathbf{T}|E\rangle$—with the *same* eigenvalue. Yet $|E\rangle$ and $\mathbf{T}|E\rangle$ are—that is the main substance of this theorem—orthogonal to each other. Each energy eigenstate in the Hilbert sector with $\mathbf{T}^2 = -1$ is therefore at least doubly degenerate. This degeneracy is realized in nature through *fermionic spin* degrees. It is quite remarkable that the time reversal operator $\mathbf{T}$ already anticipates this option—and the qualitative difference between fermions and bosons—through $\mathbf{T}^2 = \pm 1$—*without* any explicit reference to spin.

- *Baryogenesis*: while we observe about one nucleon for roughly every 10^9 or so photons (the latter mostly from the cosmic background radiation carrying 'the echo from the big bang'), no evidence has been found for any *primary* antimatter in our universe. The amount of antimatter observed in cosmic rays is fully consistent with it being produced in collisions with primary matter cosmic rays.

$$\text{number}(\overline{\text{nucleons}}) \ll \text{number}(\text{nucleons}) \ll \text{number}(\text{photons}). \tag{42}$$

To understand this matter–antimatter asymmetry—or baryon number of the universe—not as an arbitrary initial condition of our universe, but as a *dynamically generated* quantity requires the three so-called Sakharov conditions [33]:

  - baryon number violation—otherwise the baryon number observed today has to equal the one existing at the time of the big bang;
  - **CP** violation—otherwise any production of baryons has to coincide with that for antibaryons and no *net* baryon number gets generated;
  - temporary lack of thermal equilibrium—for thermal equilibrium effectively rules out any violation of time reversal invariance 'on balance' and thus, due to **CPT** symmetry, likewise for **CP**.

The connection between **CP** and **T** symmetries has a practical consequence as well: with **T** being antilinear, transformations under **T** will not represent a symmetry, if the effective couplings in the dynamics are complex—nor will **CP** transformations, since **CPT** invariance connects the two. Since only relative phases are measurable, **CP** violation becomes observable only if two different amplitudes with different phases contribute coherently to a given transition. Meson–antimeson oscillations can provide such a second amplitude in a natural way with the added advantage that the observable asymmetry exhibits a very peculiar dependence on the time of decay. Oscillations were instrumental in discovering **CP** violation both in kaon and B decays.

1.4. ♠ Selected special items ♠

1.4.1. Trading time for space. The SM has the selection rule $\Delta B = \Delta Q$ allowing $\bar{\mathrm{B}}_\mathrm{d} \to l^- \bar{\nu} X^+$ to occur, but not $\bar{\mathrm{B}}_\mathrm{d} \to l^+ \nu \tilde{X}^-$. A prominent example for X^+ is D^+. Finding $\bar{\mathrm{B}}_\mathrm{d} \to l^+ \nu \tilde{X}^-$ would then seem to establish the intervention of new physics. Alas—this selection rule can be circumvented in the SM by an oscillation taking place between production and decay. One can distinguish between the two scenarios

$$\bar{\mathrm{B}}_\mathrm{d} \overset{\Delta B=2}{\Longrightarrow} \mathrm{B}_\mathrm{d} \overset{\Delta B=1}{\longrightarrow} l^+ \nu X^- \qquad \text{versus} \qquad \bar{\mathrm{B}}_\mathrm{d} \overset{\Delta B=1}{\longrightarrow} l^+ \nu \tilde{X}^- \tag{43}$$

by analyzing the composition of the states X^- and $\tilde{X}^-$ or more specifically by tracing the transition as a function of the time of decay. For oscillations betray their presence by their tell-tale time dependence.

One can differentiate between the two scenarios even when the time dependence can*not* be traced. Consider the *time integrated* ratio of like-sign di-leptons to all di-leptons emerging from the semileptonic decays of a coherently or incoherently produced B_d–$\bar{\mathrm{B}}_\mathrm{d}$ pair, see equations (25) and (23). For the measured value $x_\mathrm{d} = 0.776 \pm 0.008$ we have

$$\chi_\mathrm{d} = 0.188 \pm 0.003 \qquad \text{versus} \qquad 2\chi_\mathrm{d}(1 - \chi_\mathrm{d}) \simeq 0.305. \tag{44}$$

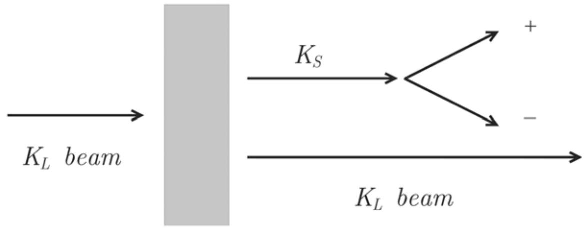

Figure 7. Regeneration of K_S from a K_L beam traversing a slab of matter.

That means the production rate of such like-sign di-leptons differ greatly, namely, by a factor of almost two due to the EPR correlations in the coherently produced pair. In the absence of oscillations on the other hand—$x_d = 0 = y_d$—the relative rate of like-sign di-leptons is the same for coherent and incoherent production, since in all likelihood one has $X \neq \tilde{X}$; e.g. $X^{\pm} = D^{\pm}$ versus $\tilde{X}^{\mp} = K^{\mp}$. Thus we see that if one spends the time to study the production and subsequent decay of B mesons in these two different environments, one can infer from the data whether the violation of the selection rule is driven by a *one-step* transition or a *two-step process involving oscillations* with the latter characterized by a time scale $(\Delta M(B_d))^{-1} = \tau(B_d)/x_d$ *without* having resolved the short flight paths of the B mesons. In that sense one has traded time (spent) for space (distances resolved), as stated in the heading. The fact that one measures quite different ratios of like-sign di-leptons in the production scenarios of equations (23,25), as given by equation (44), shows there are two rather than one time scales involved in those transitions, namely, $1/\Delta M(B_d)$ and $1/\Gamma(B_d)$ with their ratio determined. This is another demonstration of the power of quantum mechanical interference and EPR effects as routine experimental precision tools.

1.4.2. Spontaneous and matter-enhanced regeneration. It is one of quantum mechanics' most radical departures from classical physics that a component of a particle's state that a measurement has shown to be absent can be resurrected by a later measurement, if a non-commuting component has been measured in between. For example if one has found that the spin of an electron 'points' in the *positive* z direction—i.e. $S_z = 1/2$–, then measured its y component before turning to *re*measure the z component, one has even odds to find $S_z = -1/2$. Oscillations add a new twist to such phenomena. As figure 2 illustrates, a state that was totally absent in the ensemble under study at $t = 0$—a $\bar{K}^0$ in this case—emerges at later times, and its intensity can at times even exceed that of the original component—K^0. Oscillations are said to *regenerate* the originally absent component 'in vacuum' or *spontaneously*, i.e. without the intervention of an intermediate measurement.

The term 'regeneration' is usually reserved for an even more complex scenario: starting with a pure K^0 beam and waiting long enough, namely, $1/\Gamma(K_S) \ll t \sim \mathcal{O}(1/\Gamma(K_L))$, one obtains a practically pure K_L beam—if the beam travels through vacuum. Yet after passing through a slab of matter the beam will again contain a K_S component—the K_S has been *regenerated*, see figure 7. For kaons and antikaons interact differently with matter (as they do with antimatter). These interactions act like a measurement by changing the phase relation between the K^0 and $\bar{K}^0$ components in the K_L wave function. This regeneration can proceed in a coherent way and allows one to determine also the sign of ΔM_K [6].

1.4.3. On other incarnations of matter–antimatter oscillations. There are other incarnations of matter–antimatter oscillations possible, although within the SM they are not predicted to occur, namely, neutrino–antineutrino, neutron–antineutron and mesino–antimesino oscillations listed in rising 'exoticity'. They have not been observed yet.

(a) *Neutrino oscillations*: These come in two variants, namely, oscillations transforming one neutrino into another one—they have been observed in the flux of solar and atmospheric neutrinos—and those turning a neutrino into an antineutrino. The mathematical formalism for both variants, although not the underlying dynamics, is the same as for meson–antimeson oscillations, with only the qualitative difference that there are no **CPT** constraints for the first variant as there are for the second variant and meson–antimeson oscillations[2].

Among quarks and leptons only neutrinos have the potential to oscillate into their antiparticles, since they carry no electric charge. The transition $\nu \to \bar{\nu}$ requires that lepton number can be changed by two units. While it does not happen in the SM, we know of no fundamental objection against it. More specifically neutrinos can possess a Majorana mass. Through the 'see-saw' mechanism Majorana masses can provide a natural explanation why neutrinos have unusually tiny masses; they represent lepton-number violation by two units and thus lead to $\nu - \bar{\nu}$ transitions. Such Majorana neutrinos are actually their own antiparticles. One might then think that there can be no difference between, say, $\nu_e \to \nu_\mu$ and $\bar{\nu}_e \to \bar{\nu}_\mu$ oscillations and that $\nu \to \bar{\nu}$ oscillations cannot occur. This would, however, be a fallacious conclusion. For neutrinos and antineutrinos can still be distinguished by their association with a charged lepton at their birth and their death [27]:

$$[\mathrm{e}^+]\nu_\mathrm{e} \longrightarrow \nu_\mu[\mu^+] \qquad \text{versus} \qquad [\mathrm{e}^-]\bar{\nu}_\mathrm{e} \longrightarrow \bar{\nu}_\mu[\mu^-], \tag{45}$$

$$[\mathrm{e}^+]\nu_\mathrm{e} \longrightarrow \nu_\mathrm{e}[\mathrm{e}^+] \qquad \text{versus} \qquad [\mathrm{e}^+]\nu_\mathrm{e} \longrightarrow \bar{\nu}_\mathrm{e}[\mathrm{e}^-]. \tag{46}$$

It thus makes eminent sense to search for **CP** violation in neutrino oscillations. On the other hand we do not know of a feasible way to identify $\nu - \bar{\nu}$ oscillations since they require a spin flip thus highly suppressing them [27, 28].

(b) *Neutron oscillations*: Neutron–antineutron oscillations can likewise be described in close analogy to the $\mathrm{K}^0 - \bar{\mathrm{K}}^0$ case, yet require rather exotic extensions of the SM. For while the transition $\mathrm{K}^0 \Rightarrow \bar{\mathrm{K}}^0$ changes *strangeness* by two units, $n \Rightarrow \bar{n}$ changes *baryon number* by two units. No baryon number violating process has been observed so far, and proton stability imposes very tight bounds on it.

Since baryon number changes by one unit—like $\mathrm{p} \to \mathrm{e}^+\pi^0$ or $\mathrm{n} \to \mathrm{e}^+\pi^-$—are highly constrained due to the observed stability of nuclei, it would appear not to make any sense to search for neutron–antineutron oscillations. However one can (if so inclined) construct models of new physics, where baryon number violation can proceed only (or mainly) by two rather than one unit and still achieve baryogenesis in the Universe [29]; proton stability then provides no (or little) constraints. One might argue this is not completely ad hoc by invoking an analogy with the lepton sector where neutrino masses are likely to contain a Majorana component, which represents a coupling changing lepton number by two units and gives rise to neutrino-antineutrino oscillations as outlined just above.

There is a novel aspect in searches for neutron–antineutron oscillations. For there are two ways to probe them, namely, in vacuum and in nuclear matter:

(i) One tracks a neutron beam in vacuum and searches for the tell-tale sign of antineutrons, namely, their violent annihilation with a neutron or proton. No effect has been found yet

[2] I believe that if one had treated $\mathrm{K}^0 - \bar{\mathrm{K}}^0$ oscillations not quasi automatically with the **CPT** restrictions imposed from the start, one might have discovered the possibility of *matter-enhanced* neutrino oscillations much sooner in analogy to the coherent regeneration of kaons.

leading to a lower bound on the oscillation time of

$$\tau_{\text{vacuum}}(n \to \bar{n}) \geqslant 0.9 \times 10^8\ \text{s}. \tag{47}$$

(ii) Most neutrons are found in nuclei of course. If a neutron transmogrified itself into an antineutron in that environment, it would annihilate with one of the other nucleons wreaking havoc on nuclear stability which is characterized by

$$\tau_{\text{nuclei}} \geqslant (5.4 \pm 1.1) \times 10^{31}\ \text{years}. \tag{48}$$

Noting that the bound in equation (48) exceeds that in equation (47) by a 'mere' 31 orders of magnitude, one would view in vacuum searches as truly quixotic. However a more careful consideration shows that the two bounds in equations (47) and (48) are actually quite equivalent in their sensitivity. This most surprising conclusion is based on some subtle quantum mechanical features.

A first orientation can be obtained by the following hand-waving argument invoking the 'collapse of the wavefunction' or the 'quantum Zeno effect'. A neutron bound inside a neutron will move around with a certain mean free path. Thus there is an average time t_{between} *between* collisions with other nucleons. The neutron can oscillate only during this (brief) interval; for the next collision with another nucleon represents a measurement of the baryon number of the neutron in question: its annihilation would reveal it had transmogrified itself into an antineutron; no annihilation would mean it had again become validated as a neutron thus setting its oscilllation clock back to zero. Thus

$$\tau_{\text{nuclei}} \sim \tau_{\text{vacuum}} \cdot \frac{\tau_{\text{vacuum}}}{\tau_{\text{between}}} \sim 10^{38}\ \text{s} \simeq 3 \times 10^{31}\ \text{years} \tag{49}$$

using a typical nuclear reaction time of 10^{-23} s for τ_{between}.

One has of course to offer a less hand-waving reasoning, and one can. Inside the nuclear medium neutrons and antineutrons are obviously no longer fully degenerate: for they experience different potentials due to their differences in the third component of their isospin, magnetic moments etc. State-of-the-art calculations can be expressed as follows: from the observed nuclear stability, equation (48), one infers [30]

$$\tau_{\text{vacuum}}(n \to \bar{n}) \simeq 2\sqrt{\tau_{\text{nuclei}}/\Gamma_{\bar{n}}} \geqslant (2.1 \pm 0.2) \times 10^8\ \text{s}, \tag{50}$$

where $\Gamma_{\bar{n}} \sim 100\,\text{MeV}$ is a typical nuclear annihilation width for antineutrons. This bound is only slightly better than the one from the direct analysis of free neutron beams and suggests the latter still has a future.

(c) *Mesino oscillations*: Some implementations of supersymmetry (SUSY) allow for the gravitino—the spin-3/2 partner of the spin-2 graviton—to be the lightest new SUSY particle, and squarks—the scalar partners of the quarks—the next to lightest ones. In such admittedly unconventional scenarios a new class of hadrons can emerge, namely, 'mesinos'—the bound states of a squark and an antiquark (or their **CP** conjugates)—that are sufficiently long-lived to undergo effectively weak decays. As far as the strong interactions are concerned, mesinos resemble mesons, yet carry spin one half and thus represent fermions; hence the name. The squark and thus also the hadron could carry a heavy flavor like top or beauty. There is no fundamental reason preventing a neutral mesino oscillating into its antimesino [31]. In hadronic collisions the strong interactions produce a squark and its **CP** conjugate antisquark with quark flavor F and $-F$, respectively. The, say, squark can hadronize into a neutral mesino by picking up a light flavor antiquark. Mesino-antimesino oscillations can then occur allowing two antimesinos (and likewise two mesinos) to emerge. The fact that one has a $F = 2$ or $F = -2$ final state can be demonstrated by like-sign di-leptons, a pair of beauty or of antibeauty hadrons, etc.

Table 1. Compilation of oscillations.

P^0	χ	$x = \Delta M(P^0)/\Gamma(P^0)$	$2y = \Delta\Gamma(P^0)/\Gamma(P^0)$	$\Delta M(P^0)$
K^0 ($d\bar{s}$)	0.187	0.946 ± 0.003	0.9963 ± 0.0036	$(0.5290 \pm 0.0016) \times 10^{10}\,s^{-1}$
D^0 ($c\bar{u}$)	$\simeq 3.6 \times 10^{-5}$	0.0085 ± 0.0032	0.0071 ± 0.0021	$(2.07 \pm 0.78) \times 10^{10}\,s^{-1}$
B^0_d ($d\bar{b}$)	0.188 ± 0.003	0.776 ± 0.008		$(0.507 \pm 0.005) \times 10^{12}\,s^{-1}$
B^0_s ($s\bar{b}$)	$\simeq 0.5$	25.4 ± 0.7	0.31 ± 0.13	$(17.77 \pm 0.12) \times 10^{12}\,s^{-1}$

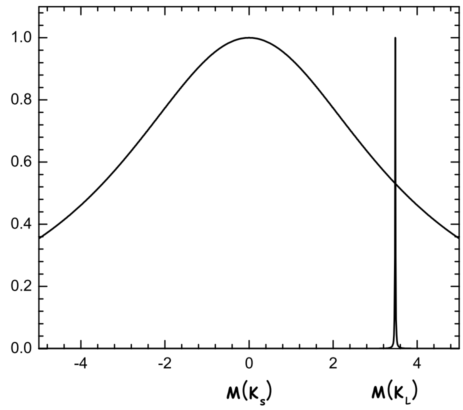

Figure 8. The K_S and K_L mass distributions in units of 10^{-6} eV.

Such SUSY scenarios are certainly unorthodox, yet conceivable. On the other hand mesino-antimesino oscillations, if they occur, could tell us a lot about salient features of flavor dynamics in SUSY.

1.5. The data-status in 2006

The present experimental situation is summarized in table 1. A few comments are in order:

- Nature was kind enough to provide us with more than one meson system exhibiting oscillations, namely, neutral kaons and the two types of beauty mesons, and still holds out the promise for a fourth one, namely, neutral charm mesons. The oscillation parameters implement very different patterns as illustrated by figures 8 and 9:

$$\Delta M_K \simeq \Delta\Gamma_K \simeq \Gamma_{K_S}, \tag{51}$$

$$\Delta M_{B_d} \sim \Gamma_{B_d} \gg \Delta\Gamma_{B_d}, \tag{52}$$

$$\Delta M_{B_s} \gg \Gamma_{B_s} > \Delta\Gamma_{B_s}, \tag{53}$$

$$\Delta M_D \sim \Delta\Gamma_D \ll \Gamma_D. \tag{54}$$

- $\Delta M_K \simeq \Delta\Gamma_K \simeq \Gamma_{K_S}$ is due to two facts, namely, the aforementioned dynamical accident that there is very little phase space available for $K_L \to 3\pi$ and thus $\Gamma_{K_S} \gg \Gamma_{K_L}$ and that **CP** invariance holds to a good approximation. This will not be repeated in other systems. The typical situation is $\Delta M \gg \Delta\Gamma$, as long as short distance dynamics generates the $P^0 - \bar{P}^0$ transition operators, as it happens for $P^0 = B^0$. For charm we predict $\Delta M_D \sim \Delta\Gamma_D \ll \Gamma_D$.

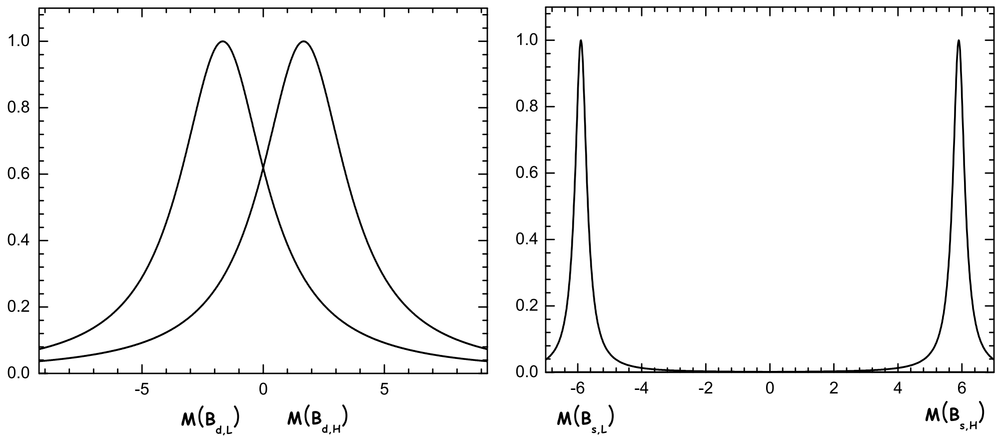

Figure 9. The $B_{d,L}$ [$B_{s,L}$] and $B_{d,H}$ [$B_{s,H}$] mass distributions on the left (right) in units of 10^{-4} [10^{-3}] eV.

1.6. A first summary on oscillations

To summarize the discussion so far and anticipate further related points:

- Matter–antimatter oscillations for kaons are a glorious demonstration of quantum mechanics' subtleties in its implementation of full and approximate symmetries. Starting out from a conservation law—in this case that of strangeness for the strong and electromagnetic forces—one obtains *degenerate stationary* states. Violating this conservation law—in this case by weak forces—leads to new mass eigenstates, which are a generalization of stationary states since they can decay away. Yet those are no longer degenerate, i.e. they have different masses and lifetimes. Furthermore they do not have well-defined values of the quantum number that corresponds to the original invariance—strangeness in this case.
- As explained later the existence and theoretical control we have over EPR correlations between rapidly oscillating neutral B mesons was essential for the observation of **CP** violation in $B_d \rightarrow J/\psi K_S$.
- It has become very common to refer to 'oscillations' as 'mixing' in an interchangable way. Both are concepts deeply embedded in quantum mechanics. Yet I will distinguish between them in this review.
  'Mixing' means that classically distinct states are not necessarily so in quantum mechanics and therefore can *interfere*. For example in atomic physics wave functions are said to be *mixtures* of 'right' and 'wrong' parity components whose interference generates parity odd observables; it is the weak neutral current that induces such a wrong parity component. Above we have described how K^0 and $\bar{K}^0$ 'mix' to form the K_L and K_S wave functions. In section 2.3 we will describe how the generalized mass matrices of quarks will in general be non-diagonal leading to the mass eigenstates of quarks (and leptons) containing components of different flavours. Such mixing creates a plethora of observable effects that can be expressed compactly through a non-trivial CKM matrix.
  The most intriguing effects arise when the violation of a certain quantum number—like strangeness—leads to the stationary or mass eigenstates not being eigenstates under that quantum number. This induces 'oscillations' like for mesons discussed above. 'Oscillations' thus require 'mixing', but go beyond it in the sense that they generate

transitions with a very peculiar time evolution, namely, an oscillatory one rather than the usual exponentially damped one.

- Oscillations have been and will continue to be harnessed to teach us essential lessons about nature's basic structure on the qualitative as well as the quantitative level: the concept of quark families emerged from here; they showed that while strangeness-changing neutral currents do exist, they are greatly suppressed as inferred from the smallness of ΔM_K; daring minds postulated that therefore yet another internal quantum number had to exist, namely, charm. These issues will be addressed below.

1.7. The SM and its flavor dynamics

The SM was not born like the goddess Athena who jumped fully grown and in full armour out of the head of her father Zeus. Many of the central elements and concepts of the SM actually precede it by many years.

The term 'Standard Model' is attached to a large number of diverse theoretical frameworks in all branches of physics that even change their identity over time. Here I refer to the SM of HEP aiming at a theoretical description of nature's fundamental forces. It is based on the (local) gauge group

$$SU(3)_C \times SU(2)_L \times U(1)^* \tag{55}$$

meaning the following: forces are mediated by the exchange of gauge bosons forming regular representations of the group with $SU(3)_C$ for the strong forces, $SU(2)_L \times U(1)$ for the electroweak interactions with the latter being *realized spontaneously*, i.e. having degenerate groundstates leaving only $U(1)_{QED}$ fully intact. The matter fields, namely, (anti)quarks and leptons, form (anti)triplets and singlets under $SU(3)_C$ and doublets and singlets under $SU(2)_L$. There is more as indicated by the asterisks in equation (55): to engineer the spontaneous realization of $SU(2)_L \times U(1)$—usually more ambiguously referred to as spontaneous breaking—an $SU(2)_L$ doublet of scalar Higgs fields is introduced: its vacuum expectation value generates masses for the gauge bosons and, with nifty efficiency, for the quarks and charged leptons as well. This is a very important feature for the later discussion. All fundamental states of the SM have been found experimentally except for the one footprint left by the Higgs mechanism, namely, the emergence of one neutral scalar field in the physical spectrum, usually referred to as 'the' Higgs field.

The SM contains more layers than it would appear to need: the minimal structure requires u and d quarks and the electron and its neutrino composing one quark-lepton *family* with two quark and two lepton flavours. Yet we have found two more families, i.e. copies of two quarks and two leptons identical in all their properties except for their mass parameters. Thus we have six quark and six lepton flavours arranged in three families

$$\text{1st family: (u, d)\&(e, } \nu_e); \qquad \text{2nd family: (c, s)\&}(\mu, \nu_\mu);$$
$$\text{3rd family: (t, b)\&}(\tau, \nu_\tau) \tag{56}$$

with the three up-type quarks u, c, t called *up*, *charm*, *top* and down-type quarks d, s, b *down*, *strange*, *beauty* [3]. Since we have measured that the Z^0, the neutral weak boson, decays only into 3 neutrino–antineutrino pairs—i.e. the mass of a fourth neutrino had to exceed half the Z^0 mass—it is plausible (though not conclusive) that only three families exist. Two other findings

[3] Two sets of names are usually used for the quarks of the third family, namely, (top, bottom) or (truth,beauty). Due to M K Gaillard's dictum that 'truth' is pretentious and 'bottom' vulgar I employ 'top' and 'beauty'; it might not be consistent, but 'Consistency is the last refuge of the unimaginative' (O Wilde).

also point to that number: nucleosynthesis does not allow for more than three families, and, as explained below, **CP** violation requires at least that many.

We do not understand this family replication. It is not even clear whether the number of families represents a fundamental quantity or is due to the more or less accidental interplay of complex forces as one encounters when analyzing the structure of nuclei.

With the strong, electromagnetic and neutral weak forces conserving quark-lepton flavor, only charged weak currents create non-diagonal flavor transitions and induce even flavor-changing neutral currents, as explained later.

1.8. Other meson–antimeson oscillations

Oscillations observed for neutral kaons have to occur for other neutral mesons as well—merely their rate and **CP** properties are at question. Because of the conservation of electric charge and the SM quarks being charged, only neutral hadrons can undergo oscillations. In the absence of top *hadrons*—top quarks decay before they can hadronize [34]—there are only four candidates for oscillations, namely, neutral kaons, beauty mesons with and without strangeness and charm mesons. As already stated above, see table 1, they have been *established* experimentally for all except for D^0 mesons; strong experimental evidence has been found now even for their oscillations [13–15].

2. The theoretical description of flavor dynamics and the experimental landscape in 1999

2.1. Oscillations—a basic exercise in quantum mechanics

The phenomenology of oscillations can be given as a basic exercise in nonrelativistic quantum mechanics without recourse to the fundamental degrees of freedom involved; i.e., it can adequately be described in terms of the observable hadrons rather than the more fundamental quarks. The subsequent discussion is straightforward, yet admittedly on the rather technical side. It will lay important groundwork for a proper understanding of **CP** asymmetries in B decays as well.

Consider a neutral meson P^0 with flavor quantum number F; it can denote a K^0, D^0 or B^0. The most general time evolution for the $P^0 - \bar{P}^0$ complex, including its decays, is given by an infinite-component vector in Hilbert space, which reads for $P^0 = K^0$

$$|\tilde{\Psi}(t)\rangle = a(t)|K^0\rangle + b(t)|\bar{K}^0\rangle + c(t)|2\pi\rangle + d(t)|3\pi\rangle + e(t)|\pi l \bar{\nu}_l\rangle + \dots, \tag{57}$$

which is the solution of the Schrödinger equation

$$i\hbar \frac{d}{dt}|\tilde{\Psi}(t)\rangle = \tilde{\mathbf{H}}|\tilde{\Psi}(t)\rangle; \tag{58}$$

$\tilde{\mathbf{H}}$ denotes an *infinite*-dimensional Hamilton operator in the Hilbert space. We do not know how to solve this infinite set of coupled differential equations affected by strong dynamics. Fortunately we do not have to. For our purposes it suffices to treat a more special case [35,36][4]:

- The initial state is a linear combination of P^0 and $\bar{P}^0$ alone: $|\Psi(0)\rangle = a(0)|P^0\rangle + b(0)|\bar{P}^0\rangle$.
- Likewise we are interested only in $a(t)$ and $b(t)$.
- We restrict ourselves to times much longer than typical strong interaction times as appropriate for the Weisskopf–Wigner approximation [38].

[4] Another formulation proposed by R G Sachs [37] relies on the analysis of the neutral kaon propagator in which the proper self-energy diagram $\Pi^*(k^2)$ must be approximated by $\Pi^*(m^2_{P^0} + i0^+)$ near poles of the propagator.

The Schrödinger equation then simplifies dramatically to two coupled differential equations that can easily be treated [39]:

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix} = \begin{pmatrix} M_{11} - \frac{\mathrm{i}}{2}\Gamma_{11} & M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12} \\ M_{12}^* - \frac{\mathrm{i}}{2}\Gamma_{12}^* & M_{22} - \frac{\mathrm{i}}{2}\Gamma_{22} \end{pmatrix} \begin{pmatrix} P^0 \\ \bar{P}^0 \end{pmatrix}, \tag{59}$$

CPT invariance imposes [40]

$$M_{11} = M_{22}, \qquad \Gamma_{11} = \Gamma_{22}. \tag{60}$$

It should be noted that the case of neutrino oscillations between two neutrino flavors, say $\nu_e \leftrightarrow \nu_\mu$, is described by exactly the same mathematical formalism except for these **CPT** constraints. In principle one had to allow also for neutrino decays.

The mass eigenstates obtained through diagonalising this matrix are given by [6]

$$\begin{aligned} |P_A\rangle &= \frac{1}{\sqrt{|p|^2 + |q|^2}}(p|P^0\rangle + q|\bar{P}^0\rangle), \\ |P_B\rangle &= \frac{1}{\sqrt{|p|^2 + |q|^2}}(p|P^0\rangle - q|\bar{P}^0\rangle) \end{aligned} \tag{61}$$

with eigenvalues

$$\begin{aligned} M_A - \frac{\mathrm{i}}{2}\Gamma_A &= M_{11} - \frac{\mathrm{i}}{2}\Gamma_{11} + \frac{q}{p}\left(M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12}\right) \\ M_\mathrm{B} - \frac{\mathrm{i}}{2}\Gamma_\mathrm{B} &= M_{11} - \frac{i}{2}\Gamma_{11} - \frac{q}{p}\left(M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12}\right) \end{aligned} \tag{62}$$

as long as

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - \frac{\mathrm{i}}{2}\Gamma_{12}^*}{M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12}} \tag{63}$$

holds. I am using letter subscripts A and B for labeling the mass eigenstates rather than numbers 1 and 2 as is usually done. For they should not be confused with the matrix indices 1, 2 in $M_{ij} - \frac{\mathrm{i}}{2}\Gamma_{ij}$. In expressing the mass eigenstates P_A and P_B explicitly in terms of the flavor eigenstates—equations (61)—one needs q/p. To do that correctly, one has to pay attention to how phases can emerge in quantum mechanics. While dealing properly with this issue is important—and tracking it provides useful cross checks in lengthy computations—addressing it in detail is rather tedious. Therefore I relegate its discussion to appendix B. I will adopt the *conventions*

$$\mathbf{CP}\,|P^0\rangle \equiv |\bar{P}^0\rangle, \tag{64}$$

$$\frac{q}{p} \equiv +\sqrt{\frac{M_{12}^* - \frac{\mathrm{i}}{2}\Gamma_{12}^*}{M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12}}}. \tag{65}$$

Equation (62) yield for the differences in mass and width

$$\Delta M \equiv M_\mathrm{B} - M_A = -2\mathrm{Re}\left[\frac{q}{p}\left(M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12}\right)\right], \tag{66}$$

$$\Delta\Gamma \equiv \Gamma_A - \Gamma_\mathrm{B} = -2\mathrm{Im}\left[\frac{q}{p}\left(M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12}\right)\right]. \tag{67}$$

Note that the subscripts A, B have been swapped in going from ΔM to $\Delta\Gamma$. This is done to have both quantities *positive* for kaons. Up to this point the two states $|P_{A,B}\rangle$ are merely *labeled* by their subscripts. One can define the labels A and B such that

$$\Delta M \equiv M_{\mathrm{B}} - M_A > 0 \tag{68}$$

holds. Once this *convention* has been adopted, it becomes a sensible question whether

$$\Gamma_{\mathrm{B}} > \Gamma_A \qquad \text{or} \qquad \Gamma_{\mathrm{B}} < \Gamma_A \tag{69}$$

holds, i.e. whether the heavier state is shorter or longer lived; I return to this point in appendix B.

In the limit of **CP** invariance there is more we can say: since the mass eigenstates are **CP** eigenstates as well, we can raise another meaningful question: is the heavier state **CP** even or odd? With **CP** invariance requiring absence of relative weak phases we have $\arg(\Gamma_{12}/M_{12}) = 0$ and $|q/p| = 1$, i.e. q/p becomes a pure phase, for which we choose $(q/p) = 1$. Thus

$$|P_{A[B]}\rangle = \frac{1}{\sqrt{2}}(|P^0\rangle + [-]|\bar{P}^0\rangle) = |P_{+[-]}\rangle \tag{70}$$

with P_A and P_B being **CP** even and odd, respectively: $\mathbf{CP}|P_\pm\rangle = \pm|P_\pm\rangle$.

$$M_{\mathrm{odd}} - M_{\mathrm{even}} = M_{\mathrm{B}} - M_A = -2\mathrm{Re}\left[\frac{q}{p}\left(M_{12} - \frac{\mathrm{i}}{2}\Gamma_{12}\right)\right] = -2M_{12}. \tag{71}$$

Allowing for **CP** violation the general mass eigenstates P_A and P_B can be written in terms of the **CP** eigenstates $P_\pm$:

$$|P_{A[B]}\rangle = \frac{1}{\sqrt{1+|\bar{\epsilon}|^2}}\left(|P_{+[-]}\rangle + \bar{\epsilon}|P_{-[+]}\rangle\right); \tag{72}$$

$\bar{\epsilon} = 0$ means that the mass and **CP** eigenstates coincide, i.e. **CP** is conserved in $\Delta F = 2$ dynamics driving $P - \bar{P}$ oscillations. With the phase between the orthogonal states $|P_+\rangle$ and $|P_-\rangle$ arbitrary, the phase of $\bar{\epsilon}$ can be changed at will and is not an observable; $\bar{\epsilon}$ can be expressed in terms of $\frac{q}{p}$, yet in a way that depends on the convention for the phase of antiparticles, see appendix B. With our conventions one has

$$\bar{\epsilon} = \frac{1 - \frac{q}{p}}{1 + \frac{q}{p}}. \tag{73}$$

The lack of orthogonality between P_A and P_B is a measure of **CP** violation in $\Delta F = 2$ dynamics:

$$\langle P_B|P_A\rangle = \frac{1 - \left|\frac{q}{p}\right|^2}{1 + \left|\frac{q}{p}\right|^2} = \frac{2\mathrm{Re}\bar{\epsilon}}{1+|\bar{\epsilon}|^2}. \tag{74}$$

Let me recapitulate the relevant points:

- The labels of the two mass eigenstates P_A and P_B can be chosen such that

$$M_{P_B} > M_{P_A} \tag{75}$$

holds.

- Then it becomes an *empirical* question whether P_A or P_B is longer lived:

$$\Gamma_{P_A} > \Gamma_{P_B} \qquad \text{or} \qquad \Gamma_{P_A} < \Gamma_{P_B}? \tag{76}$$

- In the limit of **CP** invariance one can also raise the question whether it is the **CP** even or the odd state that is heavier.

2.1.1. Time evolution of single beams of mesons. Knowing the mass eigenstates $P_{A[B]}$ one can write down the time evolution for the flavor eigenstates:

$$|P^0(t)\rangle = g_+(t)|P^0\rangle + \frac{q}{p}g_-(t)|\bar{P}^0\rangle, \tag{77}$$

$$|\bar{P}^0(t)\rangle = g_+(t)|\bar{P}^0\rangle + \frac{p}{q}g_-(t)|P^0\rangle, \tag{78}$$

with

$$g_\pm(t) = \tfrac{1}{2}\mathrm{e}^{-\mathrm{i}M_1 t}\mathrm{e}^{-\frac{1}{2}\Gamma_1 t}[1 \pm \mathrm{e}^{-\mathrm{i}\Delta M t}\mathrm{e}^{\frac{1}{2}\Delta\Gamma t}]. \tag{79}$$

Denoting by $A(f)$ and $\bar{A}(f)$ the amplitude for the decay of P^0 and $\bar{P}^0$, respectively, into a final state f, and by $\bar{\rho}_f$ and ρ_f their ratios, i.e.

$$A(f) = \langle f|H_{\Delta F=1}|P^0\rangle, \qquad \bar{A}(f) = \langle f|H_{\Delta F=1}|\bar{P}^0\rangle,$$
$$\bar{\rho}_f = \frac{\bar{A}(f)}{A(f)} = \frac{1}{\rho_f}, \tag{80}$$

we write down

$$\Gamma(P^0(t) \to f) \propto \mathrm{e}^{-\Gamma_1 t}|A(f)|^2\left[\mathrm{K}_+(t) + \mathrm{K}_-(t)\left|\frac{q}{p}\right|^2|\bar{\rho}_f|^2 + 2\mathrm{Re}\left[L^*(t)\left(\frac{q}{p}\right)\bar{\rho}_f\right]\right], \tag{81}$$

$$\Gamma(\bar{P}^0(t) \to f) \propto \mathrm{e}^{-\Gamma_1 t}|\bar{A}(f)|^2\left[\mathrm{K}_+(t) + \mathrm{K}_-(t)\left|\frac{p}{q}\right|^2|\rho_f|^2 + 2\mathrm{Re}\left[L^*(t)\left(\frac{p}{q}\right)\rho_f\right]\right], \tag{82}$$

where

$$|g_\pm(t)|^2 = \tfrac{1}{4}\mathrm{e}^{-\Gamma_1 t}\mathrm{K}_\pm(t), \tag{83}$$

$$g_-(t)g_+^*(t) = \tfrac{1}{4}\mathrm{e}^{-\Gamma_1 t}L^*(t), \tag{84}$$

$$\mathrm{K}_\pm(t) = 1 + \mathrm{e}^{\Delta\Gamma t} \pm 2\mathrm{e}^{\frac{1}{2}\Delta\Gamma t}\cos\Delta M t, \tag{85}$$

$$L^*(t) = 1 - \mathrm{e}^{\Delta\Gamma t} + 2\mathrm{i}\mathrm{e}^{\frac{1}{2}\Delta\Gamma t}\sin\ \Delta M t. \tag{86}$$

This is the master equation describing the time evolution of a single beam of any oscillating meson. While it enhances one's peace of mind to have the most general expression, it is usually not very illuminating. Therefore I will sketch two special, yet typical cases.

(A) $\Delta\Gamma = 0$ with f being a flavor specific final state as in equations (17) and (18), which is condensed into the following notation:

$$P^0 \to l^+ + X \not\leftarrow \bar{P}^0, \qquad P^0 \not\to l^- + X \leftarrow \bar{P}^0, \tag{87}$$

i.e.

$$|A(l^+X)| = |\bar{A}(l^-X)| \equiv A_{\mathrm{SL}}, \qquad A(l^-X) = \bar{A}(l^+X) = 0, \tag{88}$$

with **CPT** invariance enforcing $|A(l^+X)| = |\bar{A}(l^-X)|$.

The master equation then yields

$$\Gamma(P^0(t) \to l^+X) \propto \mathrm{e}^{-\Gamma_1 t}\mathrm{K}_+(t)|A_{\mathrm{SL}}|^2 \propto 2\mathrm{e}^{-\Gamma_1 t}(1 + \cos\Delta M t), \tag{89}$$

$$\Gamma(P^0(t) \to l^-X) \propto \mathrm{e}^{-\Gamma_1 t}\mathrm{K}_-(t)\left|\frac{q}{p}\right|^2|A_{\mathrm{SL}}|^2 \propto 2\mathrm{e}^{-\Gamma_1 t}(1 - \cos\Delta M t)\left|\frac{q}{p}\right|^2, \tag{90}$$

$$\Gamma(\bar{P}^0(t) \to l^-X) \propto \mathrm{e}^{-\Gamma_1 t}\mathrm{K}_+(t)|A_{\mathrm{SL}}|^2 \propto 2\mathrm{e}^{-\Gamma_1 t}(1 + \cos\Delta M t), \tag{91}$$

$$\Gamma(\bar{P}^0(t) \to l^+X) \propto \mathrm{e}^{-\Gamma_1 t}\mathrm{K}_-(t)\left|\frac{p}{q}\right|^2|A_{\mathrm{SL}}|^2 \propto 2\mathrm{e}^{-\Gamma_1 t}(1 - \cos\Delta M t)\left|\frac{p}{q}\right|^2. \tag{92}$$

Integrating over time of decay t then yields the equations (20). Note that

- $P^0(t) \to l^- X$ and $\bar{P}^0(t) \to l^+ X$ can occur through oscillations—$\Delta M \neq 0$—with time dependent rates
- that can exhibit a **CP** asymmetry

$$a_{\mathrm{SL}} \equiv \frac{\Gamma(P^0(t) \to l^- X) - \Gamma(\bar{P}^0(t) \to l^+ X)}{\Gamma(P^0(t) \to l^- X) + \Gamma(\bar{P}^0(t) \to l^+ X)} = \frac{1 - |p/q|^4}{1 + |p/q|^4} \tag{93}$$

- which however is independent of time.

(B) $\Delta\Gamma = 0$ with f being a flavour *non*-specific final state, i.e. one fed by both P^0 and $\bar{P}^0$ decays, although not necessarily with the same rate:

$$P^0 \to f \leftarrow \bar{P}^0. \tag{94}$$

Examples are

$$\mathrm{K}^0 \to \pi\pi \leftarrow \bar{K}^0, \tag{95}$$

$$\mathrm{B}^0 \to \mathrm{J}/\psi \mathrm{K_S}, \quad \pi\pi \leftarrow \bar{\mathrm{B}}^0. \tag{96}$$

Equations (81) and (82) simplify considerably, if the flavor non-specific states are also **CP** eigenstates as is the case for the examples—$\mathbf{CP}|\pi\pi\rangle = +|\pi\pi\rangle$, $\mathbf{CP}|\mathrm{J}/\psi \mathrm{K_S}\rangle = -|\mathrm{J}/\psi \mathrm{K_S}\rangle$—especially if

$$|A(f)| = |\bar{A}(f)|; \qquad \text{i.e. } |\bar{\rho}_f| = 1 \tag{97}$$

holds. One also has $|q/p| = 1$ to a very good approximation.

$$\Gamma(P^0(t) \to f) \propto 4\mathrm{e}^{-\Gamma_1 t}|A(f)|^2 \times \left(1 - \mathrm{Im}\left(\frac{q}{p}\bar{\rho}_f\right)\sin\Delta M t\right), \tag{98}$$

$$\Gamma(\bar{P}^0(t) \to f) \propto 4\mathrm{e}^{-\Gamma_1 t}|A(f)|^2 \times \left(1 + \mathrm{Im}\left(\frac{q}{p}\bar{\rho}_f\right)\sin\Delta M t\right). \tag{99}$$

The $\cos\Delta M t$ term that in general enters through the functions $\mathrm{K}_\pm(t)$, see equation (85), drops out in this case. This is the scenario for $\mathrm{B_d}(t) \to \mathrm{J}/\psi \mathrm{K_S}$, where a large **CP** asymmetry had been been predicted.

These expressions allow a simple illustration of the theorem given above equation (28):

$$\Gamma(\mathrm{B}^{\mathrm{neut}}(t) \to \mathrm{J}/\psi \mathrm{K_S}) \propto \mathrm{e}^{-\Gamma_1 t} \times \left(1 - (n - \bar{n})\mathrm{Im}\left(\frac{q}{p}\bar{\rho}_{\mathrm{J}/\psi \mathrm{K_S}}\right)\sin\Delta M t\right) \tag{100}$$

with $n[\bar{n}] = N[\bar{N}]/(N + \bar{N})$, where N and $\bar{N}$ denote the initial number of B^0 and $\bar{\mathrm{B}}^0$ mesons in the beam of neutral B mesons. **CP** violation can be established by observing that the decay rate into a **CP** final state is *not* described by a single exponential function in time t even for an *untagged* beam of neutral B mesons. A necessary condition is the existence of a production asymmetry: $n \neq \bar{n}$.

The time of decay t that figures so prominently in these equations cannot be measured directly; it is inferred from the length of the flight paths of the decaying mesons. A proper discussion of how to do this requires nothing more than elementary quantum mechanics; yet since one can find misleading or even erroneous statements in the literature about it, I give a more detailed description in appendix C.

2.2. A mathematical analogue from classical mechanics, part II

As sketched in section 1.1.2 classical mechanics has a well-known analogue of meson–antimeson oscillations despite the fact that the latter is intrinsically quantum mechanical, namely the system of two identical pendula weakly coupled by a spring, see figure 4. The Newtonian equations of motion are given by

$$\frac{\mathrm{d}^2}{(\mathrm{d}t)^2}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = -\begin{pmatrix} \frac{g}{l}+\frac{k}{m} & -\frac{k}{m} \\ -\frac{k}{m} & \frac{g}{l}+\frac{k}{m} \end{pmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}, \tag{101}$$

where we have ignored the damping of the pendula due to internal friction and air resistance. The rest position of the pendula on the left and on the right are labeled by $x_1 = 0 = x_2$. The symmetric matrix in equation (101) is easily diagonalized; the eigenvalues are

$$\omega_{\mathrm{in}} = \sqrt{\frac{g}{l}}, \qquad \omega_{\mathrm{out}} = \sqrt{\frac{g}{l}+\frac{2k}{m}} \tag{102}$$

and the normal modes

$$\vec{x}_{\mathrm{in}}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\mathrm{e}^{\mathrm{i}\omega_{\mathrm{in}}t}, \qquad \vec{x}_{\mathrm{out}}(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}\mathrm{e}^{\mathrm{i}\omega_{\mathrm{out}}t}. \tag{103}$$

Let us consider the special solution

$$\vec{x}(t) = x_1^{(0)}[\vec{x}_{\mathrm{in}}(t) + \vec{x}_{\mathrm{out}}(t)]; \tag{104}$$

its components read

$$x_1(t) = x_1^{(0)}\cos\tfrac{1}{2}\delta\omega t\cos\bar{\omega}t, \qquad x_2(t) = x_1^{(0)}\sin\tfrac{1}{2}\delta\omega t\sin\bar{\omega}t \tag{105}$$

with

$$\bar{\omega} = \tfrac{1}{2}(\omega_{\mathrm{in}}+\omega_{\mathrm{out}}) \simeq \sqrt{\frac{g}{l}}\left(1+\frac{kl}{mg}\right), \qquad \delta\omega = \omega_{\mathrm{out}} - \omega_{\mathrm{in}} \simeq \frac{k}{m}\sqrt{\frac{l}{g}}, \tag{106}$$

where we have already assumed weak coupling between the two pendula—$k/m \ll g/l$—leading to two very different oscillation periods: $T_{\bar{\omega}} = 2\pi/\bar{\omega} \ll T_{\delta\omega} = 4\pi/\delta\omega$. This solution has $x_1(0) = x_1^{(0)}$, $x_2(0) = 0$ as initial condition; i.e. the first pendulum starts out oscillating with a displacement $x_1^{(0)}$, while the second one is initially at rest. Yet over time also the second pendulum will oscillate with period $2\pi/\bar{\omega}$. After the (longer) time $\sim T_{\delta\omega}/4$ we have

$$x_1(t \sim \pi/\delta\omega) \sim 0, \qquad x_2(t \sim \pi/\delta\omega) \sim x_1^{(0)}\sin\bar{\omega}t; \tag{107}$$

i.e. the whole oscillatory motion (and its kinetic energy) has been transferred from the pendulum on the left to the one on the right. Then the process reverses itself and starts to shift the motion back to the left pendulum and hence back and forth. This shifting of the oscillatory energy happens irrespective of the size of the spring constant k (as long as $k \neq 0$), yet its rate depends on it.

The analogy should be obvious: the oscillations of the left pendulum correspond to a K^0 and those of the right one to a $\bar{\mathrm{K}}^0$:

$$|\mathrm{K}^0\rangle \hat{=} \vec{x}(t) = C(\vec{x}_{\mathrm{in}}(t) + \vec{x}_{\mathrm{out}}(t)) = C\begin{pmatrix} \mathrm{e}^{\mathrm{i}\omega_{\mathrm{in}}t} + \mathrm{e}^{\mathrm{i}\omega_{\mathrm{out}}t} \\ \mathrm{e}^{\mathrm{i}\omega_{\mathrm{in}}t} - \mathrm{e}^{\mathrm{i}\omega_{\mathrm{out}}t} \end{pmatrix}, \tag{108}$$

$$|\bar{\mathrm{K}}^0\rangle \hat{=} \vec{x}(t) = \bar{C}(\vec{x}_{\mathrm{in}}(t) - \vec{x}_{\mathrm{out}}(t)) = \bar{C}\begin{pmatrix} \mathrm{e}^{\mathrm{i}\omega_{\mathrm{in}}t} - \mathrm{e}^{\mathrm{i}\omega_{\mathrm{out}}t} \\ \mathrm{e}^{\mathrm{i}\omega_{\mathrm{in}}t} + \mathrm{e}^{\mathrm{i}\omega_{\mathrm{out}}t} \end{pmatrix}. \tag{109}$$

The coupling through the spring represents the $\Delta S = 2$ weak interactions leading to the normal modes in analogy to $K_{\pm}$; left-right switching of the pendula corresponds to **CP** transformation with the normal modes being even and odd under this transformation. $C \neq \bar{C}$ is the analogue of indirect **CP** violation. Finally including the damping of the pendula due to their internal friction and air resistance completes the analogy by playing the role of $\Delta S = 1$ decay dynamics.

2.3. CKM dynamics—an 'accidental miracle'

The existence of three quark-lepton families that differ only in their mass-related parameters—and within the SM thus only in their Yukawa couplings—is one of the profound puzzles about the SM. The latter's Yukawa sector is indeed its most unsatisfactory feature. Yet this three family structure is an observed fact, and it gives rise to a very rich phenomenology in weak dynamics based on a huge body of data—including **CP** violation—that so far is fully consistent with the SM's predictions.

*2.3.1. Quark masses, the GIM & CKM mechanisms and **CP** violation.* The six quark flavours of the SM are arranged in three up-type and three down-type quarks fields that can be written as vectors $U^F = (\mathrm{u}, \mathrm{c}, \mathrm{t})^F$ and $D^F = (\mathrm{d}, \mathrm{s}, \mathrm{b})^F$, respectively, in terms of the *flavor* eigenstates denoted by the superscript F. One can form two 3×3 mass matrices

$$\mathcal{L}_M \propto \bar{U}_{\mathrm{L}}^F \mathcal{M}_U U_{\mathrm{R}}^F + \bar{D}_{\mathrm{L}}^F \mathcal{M}_{\mathrm{D}} D_{\mathrm{R}}^F. \tag{110}$$

There is no a priori reason why the matrices $\mathcal{M}_{U/D}$ should be diagonal. Applying bi-unitary rotations $\mathcal{J}_{U/D,\mathrm{L}}$ will allow to diagonalize them

$$\mathcal{M}_{U/D}^{\mathrm{diag}} = \mathcal{J}_{U/D,\mathrm{L}} \mathcal{M}_{U,D} \mathcal{J}_{U/D,\mathrm{R}}^{\dagger} \tag{111}$$

and obtain the *mass* eigenstates of the quark fields:

$$U_{\mathrm{L/R}}^m = \mathcal{J}_{U,\mathrm{L/R}} U_{\mathrm{L/R}}^F, \qquad D_{\mathrm{L/R}}^m = \mathcal{J}_{D,\mathrm{L/R}} D_{\mathrm{L/R}}^F, \tag{112}$$

i.e. the flavor eigenstates 'mix' to form the mass eigenstates. The eigenvalues of $\mathcal{M}_{U/D}$ represent the masses of the quark fields. The measured values exhibit a very peculiar hierarchical pattern for up- and down-type quarks, charged and neutral leptons that hardly appears to be accidental.

There is much more to it. Consider the neutral current coupling

$$\mathcal{L}_{\mathrm{NC}}^{U[D]} \propto \bar{g}_Z \bar{U}^F [\bar{D}^F] \gamma_\mu U^F [D^F] Z^\mu. \tag{113}$$

It keeps its form when expressed in terms of the mass eigenstates

$$\mathcal{L}_{\mathrm{NC}}^{U[D]} \propto \bar{g}_Z \bar{U}^m [\bar{D}^m] \gamma_\mu U^m [D^m] Z^\mu; \tag{114}$$

i.e., there are *no elementary* flavor-changing neutral currents. This important property is referred to as the 'generalized' GIM mechanism [42].

The *charged* currents do change their form when going from flavor to mass eigenstates;

$$\mathcal{L}_{\mathrm{CC}} \propto \bar{g}_W \bar{U}_{\mathrm{L}}^F \gamma_\mu D^F W^\mu = \bar{g}_W \bar{U}_{\mathrm{L}}^m \gamma_\mu V_{\mathrm{CKM}} D^m W^\mu \tag{115}$$

with

$$V_{\mathrm{CKM}} = \mathcal{J}_{U,\mathrm{L}} \mathcal{J}_{D,\mathrm{L}}^{\dagger}. \tag{116}$$

While the matrix V_{CKM} has to be unitary (within the SM), there is no known reason why it should be the identity matrix or even diagonal. It means the charged current couplings of the mass eigenstates will be modified in an observable way. In which way and by how much this happens requires further analysis since the phases of fermion fields are not necessarily observables. This analysis was given by Kobayashi and Maskawa [43]; accordingly the matrix is named after them (or their initials). C stands for Cabibbo, who was the first to analyze quark mixing when only u, d and s quarks were discussed; he realized that the mixing of three quarks could be described by a single angle later named the Cabibbo angle θ_C.

For three families the universality of the CKM matrix

$$\boldsymbol{V}_{\rm CKM} = \begin{pmatrix} V(\mathrm{ud}) & V(\mathrm{us}) & V(\mathrm{ub}) \\ V(\mathrm{cd}) & V(\mathrm{cs}) & V(\mathrm{cb}) \\ V(\mathrm{td}) & V(\mathrm{ts}) & V(\mathrm{tb}) \end{pmatrix} \tag{117}$$

yields three *universality* relations

$$\sum_{j=\mathrm{d,s,b}} |V(ij)|^2 = 1, \qquad i = \mathrm{u, c, t} \tag{118}$$

as well as six *orthogonality* conditions

$$\sum_{j=\mathrm{u,c,t}} V^*(ji)V(jk) = 0, \qquad i \neq k = \mathrm{d, s, b}. \tag{119}$$

Equations (119) represent triangle relations in the complex plane. Changing the phase conventions for the quark fields will change the orientations of these triangles in the complex plane, but *not* their internal angles. Those represent the *relative* phases of the elements of $V_{\rm CKM}$, which in turn can give rise to observable **CP** asymmetries.

This graphic interpretation also makes it transparent why the charged currents cannot generate **CP** violation with two families. In that case the orthogonality relations of the corresponding 2×2 matrix are trivial stating that two products of a priori complex matrix elements had to add to zero, i.e. cannot exhibit a *non*trivial phase.

For the three families of the SM there are six triangles. They can and do vary greatly in their shapes, as will be described in appendix D. One can write

$$\boldsymbol{V}_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\mathrm{e}^{-\mathrm{i}\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\mathrm{e}^{\mathrm{i}\delta_{13}} & c_{13}c_{23} \end{pmatrix}, \tag{120}$$

where

$$c_{ij} \equiv \cos\theta_{ij}, \qquad s_{ij} \equiv \sin\theta_{ij} \tag{121}$$

with $i, j = 1, 2, 3$ label the families.

This is a completely general, yet not unique parametrisation: a different set of Euler angles could be chosen; the phases can be shifted around among the matrix elements by using a different phase convention.

The CKM implementation of **CP** violation depends on the form of the quark mass matrices $\mathcal{M}_{U,D}$, not so much on how those are generated. Nevertheless something can be inferred about the latter: within the SM all fermion masses are driven by a *single* vacuum expectation value of a neutral Higgs field (VEV); to obtain an irreducible relative phase between different quark couplings thus requires such a phase in quark Yukawa couplings; this means that in the SM **CP** violation arises in dimension-*four* couplings, i.e. is ‘hard’ in the language of quantum field theory.

*2.3.2. ‘Maximal’ **CP** violation?* Charged current couplings with their $V - A$ structure break parity and charge conjugation *maximally*. Since due to **CPT** invariance **CP** violation is expressed through couplings with complex phases, one might say that maximal **CP** violation is characterized by complex phases of 90^o. However this would be fallacious: for by changing the phase *convention* for the quark fields one can change the phase of a given CKM matrix element and even rotate it away; it will of course re-appear in other matrix elements. For example $|s\rangle \to e^{i\delta_s}|s\rangle$ leads to $V_{qs} \to e^{i\delta_s}V_{qs}$ with $q = u, c, t$. In that sense the CKM phase is like the 'Scarlet Pimpernel': 'Sometimes here, sometimes there, sometimes everywhere.'

One can actually illustrate with a general argument why there can be no straightforward definition for *maximal* **CP** violation. Consider neutrinos: maximal **P** violation means there are ν_L and $\bar{\nu}_R$, yet no ν_R or $\bar{\nu}_L$. (To be more precise: ν_L and $\bar{\nu}_R$ couple to weak gauge bosons, ν_R or $\bar{\nu}_L$ do not.) Likewise for maximal **C** violation: there are ν_L and $\bar{\nu}_R$, but not $\bar{\nu}_L$ or ν_R. One might then suggest that maximal **CP** violation means that ν_L exists, but $\bar{\nu}_R$ does not. Alas—**CPT** invariance already enforces the existence of both.

Similarly—and maybe more obviously—it is not clear what maximal **T** violation would mean although some formulations have entered daily language like the 'no future generation' and the 'woman without a past'.

2.4. Evaluating the oscillation parameters

$P^0 - \bar{P}^0$ oscillations are driven by an effective flavor-changing neutral current. The latter does not exist in the SM on the tree level, yet can be generated by iterating the $\Delta F = 1$ charged currents as a loop effect, namely the so-called quark box diagram, i.e. as a pure quantum correction. The situation is most easily discussed for $\mathrm{B}^0 - \bar{\mathrm{B}}^0$ oscillations, which are dominated by a single contribution, written on the quark level as

$$\mathrm{B}_q = [\bar{\mathrm{b}}q] \to \text{`}\mathrm{t}\bar{\mathrm{t}}/W^+W^-\text{'} \to [\bar{q}\mathrm{b}] = \bar{\mathrm{B}}_q, \qquad q = \mathrm{d, s}, \tag{122}$$

i.e. the transition is mediated by a virtual top quark-antiquark and a W^+W^- pair. The transition operator is obtained by 'integrating out ' these virtual states in the diagram:

$$\mathcal{H}_{\mathrm{eff}}^{\mathrm{box}}(\Delta B = 2) \simeq \left(\frac{G_F}{4\pi}\right)^2 M_W^2 \cdot \xi_t^2 E(x_t)\eta_{tt}\left(\bar{q}\gamma_\mu(1-\gamma_5)\mathrm{b}\right)^2 + h.c.; \tag{123}$$

η_{tt} denote radiative QCD corrections, $\xi_t = V(\mathrm{tb})V^*(\mathrm{t}q)$ the CKM factor and $E(x_t)$ the dependence on the top quark mass [44, 45]:

$$E(x_t) = \frac{x_t}{4}\left(1 + \frac{9}{1-x_t} - \frac{6}{(1-x_t)^2}\right) - \frac{3}{2}\left(\frac{x_t}{1-x_t}\right)^3 \log x_t, \qquad x_t = \frac{m_t^2}{M_W^2} \tag{124}$$

The off-diagonal elements of the generalized mass matrix $\mathbf{M} - \frac{i}{2}\Gamma$ of equation (59) are then obtained from

$$M_{12} - \frac{i}{2}\Gamma_{12} = \langle B_q|\mathcal{H}_{\mathrm{eff}}^{\mathrm{box}}(\Delta B = 2)|\bar{B}_q\rangle. \tag{125}$$

With it one can express ΔM_D (equation(66)), $\Delta\Gamma_B$ (equation(67)) and q/p (equation(65)).

There remains one nontrivial theoretical challenge, namely, to evaluate the hadronic matrix element of the $(\bar{q}\gamma_\mu(1-\gamma_5)\mathrm{b})^2$ operator, which depends on nonperturbative dynamics. This is done these days by invoking lattice QCD.

Problem 1. When one calculates ΔM_B as a function of the top mass employing the quark box diagram, one finds (see equation (124))

$$\Delta M_B \propto \left(\frac{m_t}{M_W}\right)^2 \qquad \text{for } m_t \gg M_W. \tag{126}$$

The factor on the right hand side of equation (126) for $m_t \ll M_W$ reflects the familiar GIM suppression; yet for $m_t \gg M_W$ it constitutes a (huge) enhancement! It means that a low energy observable, namely, ΔM_B, is controlled more and more by a state or field at asymptotically high scales. Does it violate decoupling—and if so, why is it allowed to do so—or not?

2.5. *The SM paradigm of large **CP** asymmetries in B decays*

The first indication that the B lifetime is significantly longer and thus $|V(\mathrm{cb})|$ smaller than anticipated came in 1982 [6]. It was then confirmed that B mesons live about 1 ps. This pointed to $|V(\mathrm{cb})| \sim \mathcal{O}(\lambda^2)$ with $\lambda = \sin\theta_C$. Together with the (expected) observation $|V(\mathrm{ub})| \ll |V(\mathrm{cb})|$ and coupled with the assumption of three-family unitarity this allows us to expand the CKM matrix in powers of λ, which yields the following most intriguing result through order λ^5, as first recognized by Wolfenstein [46]:

$$\boldsymbol{V}_{\mathrm{CKM}} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3\left(\rho - \mathrm{i}\eta + \frac{\mathrm{i}}{2}\eta\lambda^2\right) \\ -\lambda & 1-\frac{1}{2}\lambda^2 - \mathrm{i}\eta A^2\lambda^4 & A\lambda^2(1+\mathrm{i}\eta\lambda^2) \\ A\lambda^3(1-\rho-\mathrm{i}\eta) & -A\lambda^2 & 1 \end{pmatrix}. \tag{127}$$

The three Euler angles and one complex phase of the representation given in equation (120) are replaced by the four real quantities λ, A, ρ and η; λ is the expansion parameter with $\lambda \ll 1$, whereas A, ρ and η are *a priori* of order unity, as will be discussed in some detail later on. That is the ‘long’ lifetime of beauty hadrons of around 1 ps together with beauty’s affinity to transform itself into charm and the assumption of only three quark families tell us that the CKM matrix exhibits a very peculiar hierarchical pattern in powers of λ:

$$V_{\mathrm{CKM}} = \begin{pmatrix} 1 & \mathcal{O}(\lambda) & \mathcal{O}(\lambda^3) \\ \mathcal{O}(\lambda) & 1 & \mathcal{O}(\lambda^2) \\ \mathcal{O}(\lambda^3) & \mathcal{O}(\lambda^2) & 1 \end{pmatrix}, \qquad \lambda = \sin\theta_C. \tag{128}$$

This matrix has to be unitary. Yet in addition it is almost the identity matrix, almost symmetric in the moduli of its elements and those shrink with the distance from the diagonal. It has to contain a message from nature—albeit in a highly encoded form.

My view of the situation is best described by a poem by the German poet Joseph von Eichendorff from the late romantic period[5]:

Schläft ein Lied in allen Dingen,	There sleeps a song in all things
die da träumen fort und fort,	that dream on and on,
und die Welt hebt an zu singen,	and the world will start to sing,
findst Du nur das Zauberwort.	if you find the magic word.

The sides of the triangle shown in figure 10 are given by $\lambda \cdot V(\mathrm{cb})$, $V(\mathrm{ub})$ and $V^*(\mathrm{td})$. Therefore their lengths are of the same order λ^3 and their angles thus naturally large, i.e. ~several × 10 degrees. The sides control the rates for CKM favored and disfavored $B_{\mathrm{u,d}}$ decays and $\mathrm{B_d} - \bar{\mathrm{B}}_\mathrm{d}$ oscillations, and the angles their **CP** asymmetries, as described in section 3. This triangle is usually referred to as *‘the’ CKM unitarity triangle*.

[5] I have been told that early romantic writers would have used the term ‘symmetry’ instead of ‘song’.

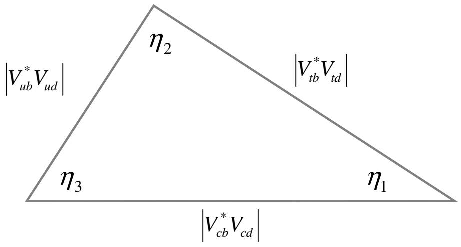

Figure 10. The CKM Unitarity Triangle with (ϕ_1, ϕ_2, ϕ_3) (a.k.a. (β, α, γ)).

For **CP** violation and its signature complex phases to become observable, we need two different, yet coherent amplitudes to contribute to the same process. The best and most spectacular implementation of this requirement is provided by $\mathrm{B}^0 - \bar{\mathrm{B}}^0$ oscillations. As mentioned before, such oscillations for $\mathrm{B_d}$ mesons were discovered by the ARGUS collaboration [47] in 1986 with

$$x_\mathrm{d} = \frac{\Delta M_{\mathrm{B_d}}}{\Gamma_{\mathrm{B_d}}} \simeq 0.776 \pm 0.008; \tag{129}$$

in today's numbers; i.e. the oscillation rate $\Delta M_{\mathrm{B_d}}$ and decay rate $\Gamma_{\mathrm{B_d}}$ are very close to each other, which is optimal. The discovery of $\mathrm{B_d} - \bar{\mathrm{B}}_\mathrm{d}$ oscillations was the last central element in establishing the 'CKM Paradigm of Large **CP** Violation in B Decays' that had been *anticipated* in 1980 [10].

- A host of nonleptonic B channels has to exhibit sizable **CP** asymmetries. In particular the 'golden modes' $\mathrm{B_d}/\bar{\mathrm{B}}_\mathrm{d} \to \mathrm{J}/\psi \mathrm{K_S}$, where $\mathrm{J}/\psi \to l^+l^-$ provides a striking signature and well defined decay vertex, are described by the simple expressions as in equation (99):

$$\mathrm{Rate}(\mathrm{B_d}(t)[\bar{\mathrm{B}}_\mathrm{d}] \to \mathrm{J}/\psi \mathrm{K_S}) \propto \mathrm{e}^{-t/\tau_\mathrm{B}}(1 - [+]S_{\mathrm{J}/\psi \mathrm{K_S}} \sin \Delta M_{\mathrm{B_d}} t), \tag{130}$$

  where the asymmetry parameter $S_{\mathrm{J}/\psi \mathrm{K_S}}$ is expressed by one of the angles in the CKM unitarity triangle (figure 10):

$$S_{\mathrm{J}/\psi \mathrm{K_S}} \equiv \frac{q}{p} \bar{\rho}_{\mathrm{J}/\psi \mathrm{K_S}} = \sin 2\phi_1. \tag{131}$$

  The size of the observable asymmetry is thus a non-trivial function of the time of decay. There are then two independent observables: (i) The *period* of the asymmetry is given by $1/\Delta M_{\mathrm{B_d}}$. The fact that it can be and has been extracted from **CP** *in*sensitive observables provides powerful validation for this measurement. (ii) The *amplitude* of the asymmetry is reliably related to a fundamental CKM parameter. This feature is actually the exception rather than the rule in B^0 decays and provides one justification for the name 'golden mode'.
- The size of the **CP** asymmetries should typically be measured in units of 10% rather than 0.1%.
- To borrow a phrase from the US political scene in the 1980s, *there is no plausible deniability for the CKM description, if such asymmetries are not found.*
- For $m_\mathrm{t} \geqslant 150\,\mathrm{GeV}$ the SM prediction for ϵ_K is dominated by the top quark contribution like $\Delta M_{\mathrm{B_d}}$. It thus drops largely out from their ratio, and $\sin 2\phi_1$ can be predicted within the SM irrespective of the (super-heavy) top quark mass. In the early 1990s, i.e. *before* the direct discovery of top quarks, it was predicted [49]

$$\frac{\epsilon}{\Delta M_{\mathrm{B_d}}} \propto \sin 2\phi_1 \sim 0.6 \tag{132}$$

with values for the relevant hadronic matrix elements inserted as now estimated by lattice QCD. It shows the intrinsic size of the **CP** asymmetry is rather insensitive to the value of m_t; its observability, however, is not, since ΔM_{B_d} and thus the coefficient $\sin\Delta M_{B_d}t$ depend strongly on m_t. The more noteworthy point here is the following: the estimates of the hadronic matrix elements from lattice QCD available at *that* time were such that $\sin 2\phi_1 \sim 0.3$ was actually expected. Accordingly, the planned B factories were designed such that their experiments should reveal an asymmetry of that size in a nominal year of operating at the anticipated luminosity. As it turned out, the real asymmetry was much larger as described below. In that sense the B factories had been 'over'-designed; yet that has led to a much richer experimental program than originally counted on.

- The **CP** asymmetry in the Cabibbo-favored channels $B_s \to J/\psi\phi$, $J/\psi\eta$ is Cabibbo-suppressed, i.e. below 4%, for reasons very specific to CKM theory, as already pointed out in 1980 [10].

There was one more obstacle to overcome before this paradigm could be probed. The reaction that provides the cleanest experimental environment

$$e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d \tag{133}$$

leads to a pair of oscillating mesons rather than a single beam of B_d or $\bar{B}_d$ mesons as described in section 2.1.1. As explained in section 3.2.3 the $B_d\bar{B}_d$ pair forms a coherent quantum system. Since the final state $J/\psi K_S$ as a matter of principle does not signal whether it came from a B_d or $\bar{B}_d$ decay, one has to infer that information from the decay of the other B meson via an EPR correlation. The asymmetry is then controlled by the time factor $\sin\Delta M_{B_d}(t_1 - t_2)$ with $t_{1,2}$ denoting the time of decay of the two B mesons; thus it has to average to zero, if one integrates over all t_1, t_2. On the other hand the previously existing e^+e^- accelerators operating at these energies had symmetric energies for its two beams. The lab frame thus coincides basically with the C.M. frame. Since the two B mesons are produced barely above their threshold they move rather slowly in the C.M. frame and their flight path between production and decay is too short to be resolved. Thus their time of decay cannot be inferred from it. It had already been suggested in 1980 [10] to harness the magic powers of quantum mechanics to overcome this hurdle. In the reaction

$$e^+e^- \to B_d^*\bar{B}_d/B_d\bar{B}_d^* \to B_d\bar{B}_d\gamma \tag{134}$$

the $B_d\bar{B}_d$ pair is produced in a **C** even configuation and the aforementioned time dependence of the **CP** asymmetry is changed to $\sin\Delta M_{B_d}(t_1 + t_2)$, which does *not* average to zero upon integrating over all times of decay. However this idea turned out to be impractical, since no sufficiently strong source for this reaction was found.

It was a seemingly crazy idea that provided the solution to this impasse. Since one cannot change the energetics of $e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d$, one could change the kinematics by building an *asymmetric* e^+e^- collider, where the e^+ and e^- beams had considerably different energies, as first suggested by Oddone [48]. This would boost the C.M. frame relative to the lab frame and thus extend the flight paths of the B mesons. This had never been done before, yet it succeeded beyond most expectations. This shows that truly formidable obstacles can be overcome through human ingenuity coupled with persistence, if the prize is attractive enough.

There is another more quantitative lesson to be learnt as well. To obtain 100 events of the type $e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d \to [l^\pm X^\mp]_B[(l^+l^-)_{J/\psi}(\pi^+\pi^-)_{K_S}]_B$ during one year of running to observe a 30% asymmetry with about 3 sigma significance one needs a luminosity $L \sim 3 \times 10^{33}\,\mathrm{s}^{-1}\,\mathrm{cm}^{-2}$. We know now the asymmetry is significantly larger than 30%. Yet it has turned out to be most beneficial that the SLAC B factory design aimed for

$L \sim 3 \times 10^{33}\,\mathrm{s}^{-1}\,\mathrm{cm}^{-2}$—a goal they have exceeded by more than a factor of 3!—and the KEK B factory even for $L \sim 10 \times 10^{33}\,\mathrm{s}^{-1}\,\mathrm{cm}^{-2}$—and achieving actually almost twice that. For these two B factories have already brought in a much richer crop of results than even optimists had expected. The lesson here is that precision tools cannot be overdesigned!

3. The new discoveries

Within a few years around the turn of the millennium several discoveries of far-reaching consequences were made (or conclusively confirmed) in high energy physics and astrophysics. They added a new quality to our knowledge about 'Nature's Grand Design', yet so far have not enhanced our understanding of it.

*3.1. Completion of an heroic era: direct **CP** violation*

As explained below equation (30) the nonleptonic decays of neutral K mesons can exhibit two types of **CP** violation: (i) the indirect variety in $\Delta S = 2$ dynamics that shapes the formation of the $\mathrm{K_L}$ state and (ii) the direct kind in $\Delta S = 1$ dynamics which drive the decays to different final states. The latter has been established in 1999, i.e. 35 years after the first one [50]. Observing the four transitions $K_{\mathrm{L,S}} \to \pi^+\pi^-, \pi^0\pi^0$ one can determine a double ratio, where systematic uncertainties will largely cancel:

$$\frac{\Gamma(\mathrm{K_L} \to \pi^0\pi^0)}{\Gamma(\mathrm{K_S} \to \pi^0\pi^0)}\frac{\Gamma(\mathrm{K_S} \to \pi^+\pi^-)}{\Gamma(\mathrm{K_L} \to \pi^+\pi^-)} = \left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 \simeq 1 - 6\,\mathrm{Re}\frac{\epsilon'}{\epsilon}. \tag{135}$$

The 2006 world average yields [3]

$$\mathrm{Re}\frac{\epsilon'}{\epsilon} = (1.66 \pm 0.26) \times 10^{-3}. \tag{136}$$

The overall strength of direct **CP** violation as expressed through ϵ' (and the experimental achievement necessary to find it) can be better illustrated by quoting the asymmetry in the $\Delta S = 1$ transitions rather than relating it to the already tiny quantity ϵ:

$$\frac{\Gamma(\mathrm{K}^0 \to \pi^+\pi^-) - \Gamma(\bar{\mathrm{K}}^0 \to \pi^+\pi^-)}{\Gamma(\mathrm{K}^0 \to \pi^+\pi^-) + \Gamma(\bar{\mathrm{K}}^0 \to \pi^+\pi^-)} = (5.04 \pm 0.82) \times 10^{-6}. \tag{137}$$

Numerically it is a miniscule effect, which is not inconsistent with the rather imprecise prediction from CKM dynamics. The seminal importance of this effect can be characterized as follows:

- it was the first **CP** asymmetry observed beyond the one in $\mathrm{K}^0 - \bar{\mathrm{K}}^0$ oscillations and
- the **CP** violation required for baryogenesis is of the direct variety.

Stated in a nutshell: the significance of this discovery is irrespective of whether theory can or cannot predict it.

*3.1.1. ♠ Compact parametrization of $\Delta S \neq 0$ **CP** violation ♠.* **CP** violation in $\Delta S \neq 0$ dynamics has been characterized by the complex quantities ϵ_{K} and ϵ'. Yet their phases contain no information on **CP** violation. The latter can be characterized more compactly for $\mathrm{K} \to 2\pi$, $l\nu\pi$ transitions as follows [55].

- *Indirect* **CP** violation is measured in the semileptonic decay asymmetry as expressed through $|q/p| \neq 1$, see equation (93). The latter condition requires, see equation (65), that M_{12} and Γ_{12} possess a relative complex phase:

$$\left|\frac{q}{p}\right| \simeq 1 + \frac{1}{2}\arg\frac{M_{12}}{\Gamma_{12}}. \tag{138}$$

Therefore **CP** violation in $\Delta S = 2$ dynamics can be stated through a phase

$$\Phi(\Delta S = 2) \equiv \arg \frac{M_{12}}{\Gamma_{12}}. \tag{139}$$

- For a *direct* **CP** asymmetry to emerge in $K_L \to 2\pi$ the two amplitudes A_0 and A_2 for producing the pion pair in an state of isospin 0 and 2, respectively, have to exhibit a weak relative phase

$$\Phi(\Delta S = 1) \equiv \arg \frac{A_2}{A_0}. \tag{140}$$

- The **CP** odd amplitude ratio η_{+-}, see equation (30), and ϵ'/ϵ_K can then be written as

$$\eta_{+-} \simeq \frac{1}{2} \frac{x_K}{1 - \mathrm{i}x_K} [\Phi(\Delta S = 2) + 2\omega \Phi(\Delta S = 1)], \tag{141}$$

$$\mathrm{Re} \frac{\epsilon'}{\epsilon} \simeq 2\omega \frac{\Phi(\Delta S = 1)}{\Phi(\Delta S = 2)}, \tag{142}$$

with $x_K = \frac{\Delta M_K}{\Gamma_K}$ and $\omega \equiv \left|\frac{A_2}{A_0}\right| \simeq 0.05$; the latter reflects the observed enhancement of the $I = 0$ over the $I = 2$ amplitude.
- The data tell us

$$\Phi(\Delta S = 2) = (6.64 \pm 0.12) \times 10^{-3}, \tag{143}$$

$$\Phi(\Delta S = 1) = (1.10 \pm 0.17) \times 10^{-4}. \tag{144}$$

This compact notation also reflects better the true strength of the underlying forces by factoring out the suppression factor ω that is not intrinsically related to **CP** violation.

3.2. *Validation of the SM paradigm of large **CP** asymmetries in B decays*

3.2.1. Act I: $B_d \to J/\psi K_S$. After some early indications from OPAL and CDF the existence of a large **CP** asymmetry in $B_d \to J/\psi K_S$ has been established in 2001 by two experiments—BABAR [56] and BELLE [57]—working at the Stanford Linear Accelerator Center in the USA and at KEK in Japan, respectively. Those two groups have continued to accumulate more data and refine their analysis leading to the 2006 world average [52]

$$\sin 2\phi_1 = 0.674 \pm 0.026 \hat{=} 3.9\% \text{ uncertainty} \tag{145}$$

to be compared with CKM predictions from 1998 and 2005:

$$\sin 2\phi_1 = \begin{cases} 0.72 \pm 0.07 & [51], \\ 0.725 \pm 0.065 & [68]. \end{cases} \tag{146}$$

Later I will comment on how good an agreement these numbers reflect.

*3.2.2. **CP** violation in K and B decays—exactly the same, only different.* Looking at table 1 there are several similarities between $K^0 - \bar{K}^0$ and $B_d - \bar{B}_d$ oscillations even on the quantitative level. Their values for $x = \Delta M/\Gamma$ and thus for χ are very similar. It is even more intriguing that also their pattern of **CP** asymmetries in $K^0(t)/\bar{K}^0(t) \to \pi^+\pi^-$ and $B_d(t)/\bar{B}_d(t) \to J/\psi K_S$ is very similar. Consider the two lower plots in figure 11, which show the asymmetry directly as a function of Δt: it looks intriguingly similar qualitatively and even quantitatively. The lower left plot shows that the difference between $K^0 \to \pi^+\pi^-$ and $\bar{K}^0 \to \pi^+\pi^-$ is actually measured in units of 10% for $\Delta t \sim (8 - 16)\tau_{K_S}$, which is the $K_S - K_L$ interference region.

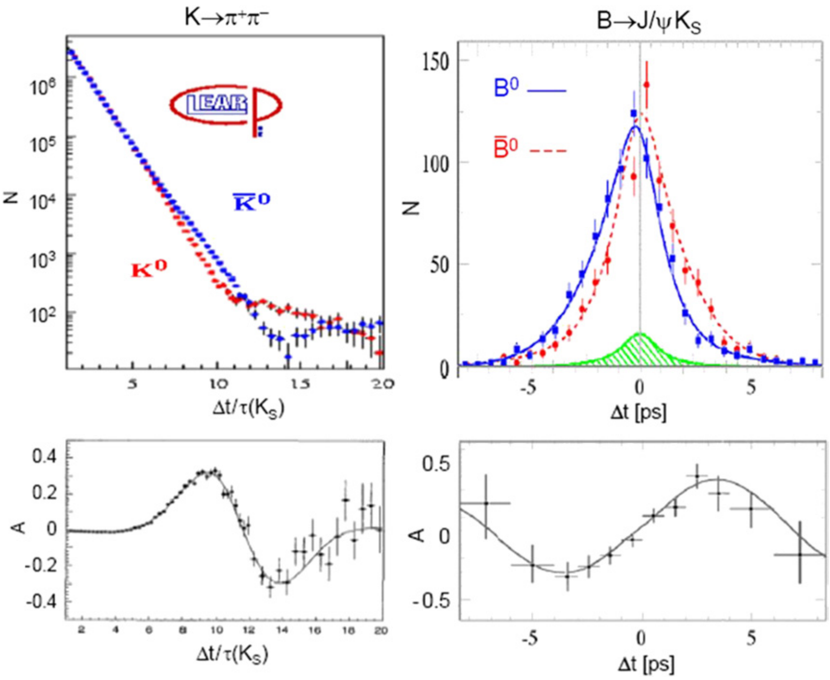

Figure 11. The observed decay time distributions for K^0 versus $\bar{K}^0$ from CPLEAR on the left and for B_d versus $\bar{B}_d$ from BABAR on the right.

Clearly one can find domains in $K \to \pi^+\pi^-$ that exhibit a truly large **CP** asymmetry. Nevertheless it is an empirical fact that **CP** violation in B decays is much larger than in K decays. For the mass eigenstates of neutral kaons are very well approximated by **CP** eigenstates, as can be read off from the upper left plot: it shows that the vast majority of $K \to \pi^+\pi^-$ events follow a single exponential decay law that coincides for K^0 and $\bar{K}^0$ transitions. This is in marked contrast to the $B_d \to J/\psi K_S$ and $\bar{B}_d \to J/\psi K_S$ transitions, which in no domain are well approximated by a single exponential law and do not coincide at all, except for $\Delta t = 0$, as it has to be, as explained next.

3.2.3. On the practical importance of EPR correlations. The BABAR and BELLE collaborations did not have a beam of single B_d or $\bar{B}_d$ mesons at their disposal, for which equation (130) applies. For both experiments operate in e^+e^- annihilation on the $\Upsilon(4S)$ resonance, where one produces an exclusive $B_d - \bar{B}_d$ pair in a $\mathbf{J^{PC}} = 1^{--}$ configuration corresponding to the intermediate one-photon state:

$$e^+e^- \to \gamma^* \to \Upsilon(4S) \to B_d\bar{B}_d. \tag{147}$$

Two complications seem to arise here: (i) Both neutral mesons undergo oscillations. (ii) As explained in appendix C in detail, one infers the time of decay t from the length of the flight path between production and decay. Yet in this reaction the point where the B meson pair is produced is ill-determined due to the finite size of the electron and positron beam spots: the latter amounts to about 1 mm in the longitudinal direction, while a B meson typically travels only about a quarter of that distance before it decays. It would then seem that the length of the flight path of the B mesons is poorly known and that averaging over this ignorance would greatly dilute or even eliminate the signal.

It turns out that both complications taken together actually provide the solution for measuring t: it is based on a glorious application of quantum mechanics, where the existence of an EPR correlation [1] comes to the rescue. While the two B mesons in the reaction of equation (147) oscillate back and forth between a $\mathrm{B_d}$ and $\bar{\mathrm{B}}_\mathrm{d}$, they change their flavor identity in a *completely correlated* way. For the $\mathrm{B\bar{B}}$ pair forms a **C** *odd* state; Bose statistics then tells us that there cannot be two identical flavor hadrons in the final state:

$$e^+e^- \to \Upsilon(4S) \to \mathrm{B_d}\bar{\mathrm{B}}_\mathrm{d} \not\to \mathrm{B_d}\mathrm{B_d}, \bar{\mathrm{B}}_\mathrm{d}\bar{\mathrm{B}}_\mathrm{d}. \tag{148}$$

Once one of the B mesons decays through a flavor specific mode, say $\mathrm{B_d} \to l^+\nu X$ $[\bar{\mathrm{B}}_\mathrm{d} \to l^-\bar{\nu}X]$, then we know unequivocally that the other B meson was a $\bar{\mathrm{B}}_\mathrm{d}$ $[\mathrm{B_d}]$ at *that* time. The time evolution of $\bar{\mathrm{B}}_\mathrm{d}(t)[\mathrm{B_d}(t)] \to \mathrm{J}/\psi\mathrm{K_S}$ as described by equation (130) starts at *that* time as well, i.e. the relevant time parameter is the *interval between* the two times of decay, not those times themselves. That time interval is related to—and thus can be inferred from—the distance between the two decay vertices, which is well defined and can be measured.

To make these general considerations explicit, consider a **C** odd $\mathrm{B_d}\bar{\mathrm{B}}_\mathrm{d}$ pair being created at time $t = 0$ with momenta $\vec{k}$ and $-\vec{k}$ in the center of mass system. The probability amplitude for this initial state evolving into a state with $\mathrm{B_d}$ or $\bar{\mathrm{B}}_\mathrm{d}$ at time t_k carrying momentum $\vec{k}$ and $\mathrm{B_d}$ or $\bar{\mathrm{B}}_\mathrm{d}$ at time t_{-k} with momentum $-\vec{k}$ is given by (for $\Delta\Gamma_\mathrm{B} = 0$) [6]

$$\begin{aligned} |[\mathrm{B_d}\bar{\mathrm{B}}_\mathrm{d}]_{\mathrm{C=odd}}(t_k, k), (t_{-k}, -k)\rangle &= \frac{1}{\sqrt{2}} e^{-\frac{1}{2}\Gamma_\mathrm{B}(t_k+t_{-k})} \\ &\cdot \left(\mathrm{i}\sin\frac{\Delta M_{\mathrm{B_d}}\Delta t}{2} \left[\frac{p}{q}|\mathrm{B_d}(k)\mathrm{B_d}(-k)\rangle - \frac{q}{p}|\bar{\mathrm{B}}_\mathrm{d}(k)\bar{\mathrm{B}}_\mathrm{d}(-k)\rangle \right] \right. \\ &\left. + \cos\frac{\Delta M_{\mathrm{B_d}}\Delta t}{2} \left[|\mathrm{B_d}(k)\bar{\mathrm{B}}_\mathrm{d}(-k)\rangle - |\bar{\mathrm{B}}_\mathrm{d}(k)\mathrm{B_d}(-k)\rangle \right] \right) \end{aligned} \tag{149}$$

with $\Delta t = t_k - t_{-k}$. We can read off from this expression that for $\Delta t = 0$ there is *no* $\mathrm{B_d}\mathrm{B_d}$ or $\bar{\mathrm{B}}_\mathrm{d}\bar{\mathrm{B}}_\mathrm{d}$ component, as already stated in equation (148).

The 'entanglement' implied by the EPR correlation is even more amazing in this situation than in the traditional EPR scenario. In Bohm's version of the original EPR argument one considers

$$\pi^0 \to e^+e^-. \tag{150}$$

With the pion carrying zero angular momentum, the spins of the electron and positron have to be antiparallel. Quantum mechanics allow to predict neither the direction of the spin of the electron nor that of the positron. Yet once one of the spins has been measured, then we 'know' immediately and with certainty that the other spin has to point in the opposite direction, no matter how far apart the electron and positron are at the time of measurement. This 'spooky action-at-a-distance' most offended Einstein and co-workers about quantum mechanics and suggested to them that quantum mechanics had to be *in*complete, that 'hidden variables' had to exist. The fact that nobody ever succeeded in showing how this would cause a problem for causality—no information could be relayed by this method—did not mollify them. An analogous mathematical treatment applies to $\Upsilon(4S) \to \mathrm{B_d}\bar{\mathrm{B}}_\mathrm{d}$ when analyzing B flavor instead of spin. Yet there is a major complexity added: the $\mathrm{B_d}$ and $\bar{\mathrm{B}}_\mathrm{d}$ swap their flavor identity back and forth on the time scale of about 1 ps due to their oscillations; yet even so they maintain their entanglement till one decays. This corresponds to the electron and positron emerging from reaction (equation (150)) traversing a magnetic field of about 10 T causing the lepton spins to precess on the time scale of a ps! Quantum entanglement of the oscillating pair of B mesons in $\Upsilon(4S) \to \mathrm{B_d}\bar{\mathrm{B}}_\mathrm{d}$ thus represents an even more amazing realization of EPR correlations.

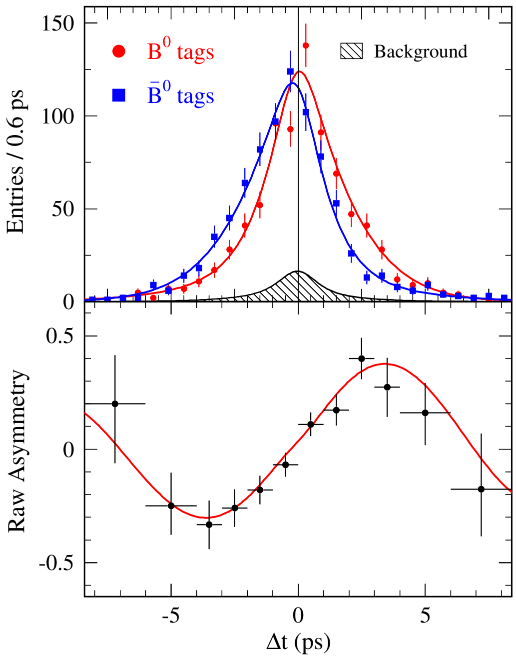

Figure 12. The observed decay time distributions for B^0 (red circles) and $\bar{B}^0$ (blue squares) decays.

*3.2.4. **T** violation in* $B_d \to J/\psi K_S$. The great *practical* value of the EPR correlation is instrumental for another consideration as well, namely how to see directly from the data that **CP** violation is matched by **T** violation. Figure 12 shows two distributions, one for the interval Δt between the times of decays $B_d \to l^+X$ and $\bar{B}_d \to J/\psi K_S$ and the other one for the **CP** conjugate process $\bar{B}_d \to l^-X$ and $B_d \to J/\psi K_S$. They are clearly different proving that **CP** is violated. Yet they show more: the shape of the two distributions is actually the same (within experimental uncertainties) the only difference being that the average of Δt is *positive* for $(l^-X)_{\bar{B}}(J/\psi K_S)$ and *negative* for $(l^+X)_B(J/\psi K_S)$ events. That is, there is a preference for $B_d \to J/\psi K_S$ [$\bar{B}_d \to J/\psi K_S$] to occur after [before] and thus more [less] slowly (rather than just more rarely) than $\bar{B} \to l^-X$ [$B \to l^+X$]. An EPR correlation synchronizes the B_d and $\bar{B}_d$ decay 'clocks'; invoking **CPT** invariance merely for semileptonic B decays—yet not for nonleptonic transitions—provides a common calibration point for both decay clocks. We thus see that **CP** and **T** violation are 'just' different sides of the same coin. EPR correlations are essential for this argument.

The reader can be forgiven for feeling that this argument is of academic interest only, since **CPT** invariance of all processes is based on very general arguments. Yet the main point to be noted is that EPR correlations, which represent some of quantum mechanics' most puzzling features, serve as an essential precision tool, which is routinely used in these measurements. I feel it is thus inappropriate to refer to EPR correlations as a paradox. While they might run counter to one's intuition, they are both an unequivocal consequence of quantum mechanics and an empirical fact relied upon routinely.

3.2.5. Future applications of EPR correlations. **CP** violation was first discovered in K decays through the *existence* of a transition, namely $K_L \to \pi^+\pi^-$, before it was seen through an asymmetry, namely, $K_L \to l^+\nu\pi^-$ versus $K_L \to l^-\bar{\nu}\pi^+$ or $K^0 \to \pi^+\pi^-$ vs. $\bar{K}^0 \to \pi^+\pi^-$.

In B^0 decays it could also be seen through the *existence* of a transition based on an EPR correlation. Consider

$$e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d \to f_a f_b, \tag{151}$$

where f_a and f_b are both even or both odd **CP** eigenstates. Such a transition can occur only through **CP** violation. For while the initial state—a 1^{--} resonance—is **CP** even, the final state is **CP** odd, since f_a and f_b have to form a P-wave [6]:

$$\mathbf{CP}[\Upsilon(4S)] = +1 \neq \mathbf{CP}[f_a f_b] = (-1)^{l=1}\mathbf{CP}[f_a]\mathbf{CP}[f_b] = -1. \tag{152}$$

It is *not* necessary for the two states f_a and f_b to be the same.

Starting from equation (149), applying the formalism of section 2.1 and integrating over all times of decay one arrives after some lengthy algebra at

$$\mathcal{B}(B_d\bar{B}_d|_{C=-} \to f_a f_b) \simeq \mathcal{B}(B \to f_a))\mathcal{B}(B \to f_b)$$
$$\cdot\left[\left(1+\frac{1}{1+x_d^2}\right)|\bar{\rho}_{f_a}-\bar{\rho}_{f_b}|^2 + \frac{x_d^2}{1+x_d^2}\left|1-\frac{q}{p}\bar{\rho}_{f_a}\frac{q}{p}\bar{\rho}_{f_b}\right|^2\right]. \tag{153}$$

Several important results can be read off equation (153).

- In the absence of **CP** violation—$\frac{q}{p}\bar{\rho}(f_a) = \pm 1 = \frac{q}{p}\bar{\rho}_{f_b}$—the reaction cannot occur, as already stated.
- In the absence of oscillations—$x_d = 0$—it can proceed only, if $\bar{\rho}_{f_a} \neq \bar{\rho}_{f_b}$, i.e. if $B \to f_a$ and $B \to f_b$ show a different amount of direct **CP** violation.
- Otherwise the transition requires $x_d \neq 0$.
- Equation (153) can actually be applied also when f_a and f_b are not **CP** eigenstates. While the mere existence of the transition does not imply **CP** violation, its measurement will yield important dynamical information, as on strong phase shifts [6].

3.2.6. Act II: $B_d \to \pi^+\pi^-$. Another large asymmetry involving oscillations has been found by BELLE in $B_d \to \pi^+\pi^-$. Its decay rate evolution is more complex than that for $B_d \to J/\psi K_S$ [6]:

$$\mathrm{Rate}(B_d(t) \to \pi^+\pi^-) \propto e^{-t/\tau_B}[1 + C_{\pi^+\pi^-}\cos\Delta M_{B_d}t - S_{\pi^+\pi^-}\sin\Delta M_{B_d}t]$$

versus

$$\mathrm{Rate}(\bar{B}_d(t) \to \pi^+\pi^-) \propto e^{-t/\tau_B}[1 - C_{\pi^+\pi^-}\cos\Delta M_{B_d}t + S_{\pi^+\pi^-}\sin\Delta M_{B_d}t],$$

where

$$S_{\pi^+\pi^-} = \frac{2\mathrm{Im}\frac{q}{p}\bar{\rho}_{\pi^+\pi^-}}{1+\left|\frac{q}{p}\bar{\rho}_{\pi^+\pi^-}\right|^2}, \qquad C_{\pi^+\pi^-} = \frac{1-\left|\frac{q}{p}\bar{\rho}_{\pi^+\pi^-}\right|^2}{1+\left|\frac{q}{p}\bar{\rho}_{\pi^+\pi^-}\right|^2} \tag{154}$$

with

$$S^2_{\pi^+\pi^-} + C^2_{\pi^+\pi^-} \leqslant 1. \tag{155}$$

The coefficients of the sin and cos$\Delta M_{B_d}t$ terms represent **CP** violation, $C_{\pi^+\pi^-}$ of the direct variety and $S_{\pi^+\pi^-}$ of both the indirect and direct variants. As for $B_d \to J/\psi K_S$ these decays are being studied at the KEK and SLAC B factories in $e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d$, where the B meson pair oscillates in a completely correlated way leading to a time dependent **CP** asymmetry:

$$\frac{R_+(\Delta t) - R_-(\Delta t)}{R_+(\Delta t) + R_-(\Delta t)} = S_{\pi^+\pi^-}\sin(\Delta M_d\Delta t) - C_{\pi^+\pi^-}\cos(\Delta M_d\Delta t); \tag{156}$$

$R_{+[-]}(\Delta t)$ denotes the rate for $\mathrm{B}^{tag}(t)\bar{\mathrm{B}}_\mathrm{d}(t+\Delta)[\bar{\mathrm{B}}^{tag}(t)\mathrm{B}_\mathrm{d}(t+\Delta)]$. As before, it is the *relative* time interval Δt between the two B decays that matters, not their lifetime. The new feature is that one has also a cosine dependence on Δt.

BABAR and BELLE find [52, 58, 59]

$$S_{\pi^+\pi^-} = \begin{cases} -0.60 \pm 0.11 \pm 0.03 & \text{BABAR '07,} \\ -0.61 \pm 0.10 \pm 0.04 & \text{BELLE '06,} \\ -0.61 \pm 0.08 & \text{HFAG '07,} \end{cases} \tag{157}$$

$$C_{\pi^+\pi^-} = \begin{cases} -0.21 \pm 0.09 \pm 0.02 & \text{BABAR '07,} \\ -0.55 \pm 0.08 \pm 0.05 & \text{BELLE '06,} \\ -0.38 \pm 0.07 & \text{HFAG '07.} \end{cases} \tag{158}$$

While BABAR and BELLE agree nicely on $S_{\pi^+\pi^-}$ making the HFAG average straightforward, their findings on $C_{\pi^+\pi^-}$ indicate different messages thus making the HFAG average less than compelling .

$S_{\pi^+\pi^-} \neq 0$ has been established and thus **CP** violation also in this channel. While BELLE finds $C_{\pi^+\pi^-} \neq 0$ as well, BABAR's number is still consistent with $C_{\pi^+\pi^-} = 0$. $C_{\pi^+\pi^-} \neq 0$ obviously represents *direct* **CP** violation. Yet it is often overlooked that also $S_{\pi^+\pi^-}$ can reveal such **CP** violation. For if one studies $\mathrm{B_d}$ decays into two **CP** eigenstates f_a and f_b and finds

$$S_{f_a} \neq \eta_a \eta_b S_{f_b} \tag{159}$$

with η_i denoting the **CP** parities of f_i, then one has established *direct* **CP** violation. For the case under study that means even if $C_{\pi\pi} = 0$, yet $S_{\pi^+\pi^-} \neq -S_{\mathrm{J}/\psi \mathrm{K_S}}$, one has observed unequivocally *direct* **CP** violation. One should note that such direct **CP** violation might not necessarily induce $C \neq 0$. For the latter requires that two different amplitudes contribute coherently to $\mathrm{B_d} \to f_b$ with non-zero relative weak as well as strong phases. $S_{f_a} \neq \eta_a \eta_b S_{f_b}$ on the other hand only requires that the two overall amplitudes for $\mathrm{B_d} \to f_a$ and $\mathrm{B_d} \to f_b$ possess a relative phase. This can be illustrated with a familiar example from CKM dynamics: If there were no penguin operators for $\mathrm{B_d} \to \pi^+\pi^-$ (or they could be ignored quantitatively), one would have $C_{\pi^+\pi^-} = 0$, yet at the same time $S_{\mathrm{J}/\psi \mathrm{K_S}} = \sin(2\phi_1)$ together with $S_{\pi^+\pi^-} = \sin(2\phi_2) \neq -\sin(2\phi_1)$, i.e. *without* direct **CP** violation one would have to find $C_{\pi^+\pi^-} = 0$ and $S_{\pi^+\pi^-} = -\sin 2\phi_1$ [69]. This would be an example where oscillations are needed to make *direct* **CP** violation observable. Yet since the measured value of $S_{\pi^+\pi^-}$ is within one sigma of $-\sin 2\phi_1$ this distinction is mainly of academic interest at the moment.

Beyond the general constraint $C^2 + S^2 \leqslant 1$ the connection between the observables $S_{\pi^+\pi^-}$ and $C_{\pi^+\pi^-}$ and the basic parameters of the underlying dynamics are not yet known precisely. Yet their measured values are consistent with their predicted ones within their considerable uncertainties. In particular one can extract ϕ_2, the second angle of the CKM unitarity triangle, from the **CP** asymmetry in this and other multipion channels.

3.2.7. Act III: $\mathrm{B_d} \to \mathrm{K}^+\pi^-$ *and* $\mathrm{B}^\pm \to \mathrm{D_{neut}K}^\pm$. Another manifestation of direct **CP** violation—and one seen by both BABAR and BELLE—is the asymmetry between $\mathrm{B_d} \to \mathrm{K}^+\pi^-$ and $\bar{\mathrm{B}}_\mathrm{d} \to \mathrm{K}^-\pi^+$, which is independent of oscillations. It was pointed out in a seminal paper in 1979 [70] that rare transitions like $\mathrm{B} \to \mathrm{K}^+ + \pi$s have the ingredients for truly sizable direct **CP** asymmetries. To translate this statement into accurate numbers represents a formidable task we have not mastered yet. In [71] an early and detailed effort was made to treat $\bar{B}_d \to K^-\pi^+$ quantitatively with the following results: $\mathcal{B}(\bar{B}_d \to K^-\pi^+) \sim 10^{-5}$, $A_{\mathbf{CP}} \sim -0.10$. Those numbers are in gratifying agreement with the data

$$\mathcal{B}(\bar{B}_d \to K^-\pi^+) = (1.85 \pm 0.11) \times 10^{-5},$$

$$A_{\mathbf{CP}} = \begin{cases} -0.108 \pm 0.024 \pm 0.008 & \text{BABAR} \quad [58], \\ -0.093 \pm 0.018 \pm 0.008 & \text{BELLE} \quad [60], \\ -0.086 \pm 0.023 \pm 0.009 & \text{CDF} \quad [61], \\ -0.095 \pm 0.013 & \text{HFAG} \quad [52]. \end{cases} \tag{160}$$

Skeptics can point out that the authors in [71] did not give a specific estimate of the theoretical uncertainties in their prediction. More recent authors have been more ambitious in predicting **CP** asymmetries in $B \to K\pi$ modes including error estimates based on systematic treatments of hadronization effects referred to as pQCD [62] and QCD factorization [63]—with somewhat mixed success. Others have relied on $SU(3)_{Fl}$ symmetry and some specific assumptions how it is broken to derive sum rules relating **CP** asymmetries in different $B \to K\pi$ and $B \to \pi\pi$ modes [64, 65]; those will provide important constraints in the future once more precise data are available.

The experimental findings of equations (160) are compatible with the SM predictions within rather sizable theoretical uncertainties. This asymmetry tells us that the angle ϕ_3 has to differ from zero, yet provide little further constraint on it.

Quantitatively much more reliable values of ϕ_3 can be inferred from measuring $B^\pm \to D^0/\bar{D}^0K^\pm$. There are actually six transitions that can be measured: $B^+ \to D^0K^+$, $\bar{D}^0K^+$, $D_{neut}K^+$ and $B^- \to D^0K^-$, $\bar{D}^0K^-$, $D_{neut}K^-$; D^0 and $\bar{D}^0$ mean they have been identified as such by their decay into a flavor *specific* final state, whereas D_{neut} denotes events where the neutral D meson has decayed into a flavor *non*specific way, i.e. into a nonleptonic final state f_{common} that is common to D^0 and $\bar{D}^0$ decays. Since $B^\pm \to D^0K^\pm$ and $B^\pm \to \bar{D}^0K^\pm$ therefore contribute *coherently* to $B^\pm \to f_{common}K^\pm$, a (direct) **CP** asymmetry can arise there driven by the relative phase between the two amplitudes, which is sensitive to ϕ_3 [9, 72]. On the other hand one has $\Gamma(B^+ \to D^0K^+) = \Gamma(B^- \to \bar{D}^0K^-)$ and $\Gamma(B^+ \to \bar{D}^0K^+) = \Gamma(B^- \to D^0K^-)$, since those channels are driven by a single amplitude. Yet the rates for these modes are measured to determine the size of hadronic matrix elements that are also involved in the aforementioned asymmetry, which in turn allows one to isolate ϕ_3 there [73, 74]. The observed **CP** asymmetry is not yet significant experimentally. Yet the overall description permits extracting a value for ϕ_3, albeit with sizable uncertainties [52]:

$$\phi_3 = \begin{cases} 53^\circ {+15^\circ}_{-18^\circ}(\text{stat}) \pm 3^\circ(\text{syst}) \pm 9^\circ(\text{model}) & \text{BELLE '06}, \\ 92^\circ \pm 41^\circ(\text{stat}) \pm 11^\circ(\text{syst}) \pm 12^\circ(\text{model}) & \text{BABAR '06}. \end{cases} \tag{161}$$

Since one is dealing with a charged B meson, no $B^0 - \bar{B}^0$ oscillations are involved here. There are two reasons why I treat it here.

- It provides the best measurement of the third angle of the CKM unitarity triangle, and thus is needed to obtain a complete picture.
- It nicely demonstrates—for neutral D mesons—the difference between 'mixing' and 'oscillation' mentioned before. The crucial point is that in the limit of **CP** invariance—a very good approximation for charm mesons—D^0 and $\bar{D}^0$ *mix* to form the mass eigenstates as **CP** eigenstates $|D_\pm\rangle = \frac{1}{\sqrt{2}}(|D^0\rangle \pm |\bar{D}^0\rangle)$. $D^0 - \bar{D}^0$ oscillations are *not* required—they would actually present a complication, which however can safely be ignored due to the low upper bounds on the oscillation rate.

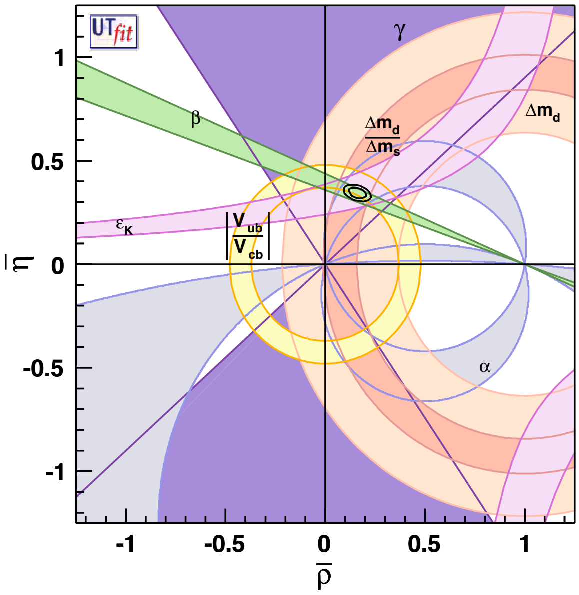

Figure 13. The 2006 constraints on the CKM triangle with $\beta = \phi_1, \alpha = \phi_2, \gamma = \phi_3$ as obtained from the UT_{fit} group; the CKM_{fitter} results are very similar [68].

3.2.8. The 'expected' triumph of a peculiar theory. We can construct the CKM triangle from what we know about its angles, sides and other constraints like ϵ_K. Mathematically one needs only three pieces to determine it. Yet we have information on several more, as shown in figure 13, i.e. the triangle is highly over-constrained by data[6]. The fact that despite these over-constraints we find a consistent solution for the triangle represents a great qualitative as well as quantitative success for CKM theory. While combining all constraints in one figure looks impressive, it does not always truly illuminate the situation. Let me emphasize a few points in this context.

- Based on a tiny effect in K_L decays, namely, a few $\times 10^{-3}$ **CP** asymmetry, CKM theory leads to the prediction that some channels of the ten times heavier B mesons have to exhibit asymmetries two orders of magnitude larger, i.e. very close to the maximal value mathematically possible. This is quite non-trivial.
- CKM theory explains **CP** symmetry being a 'near miss' in K_L decays by having the first and second quark families almost decoupled from the third one.
- The success of CKM theory in describing flavor dynamics is often *under*stated by saying the constraints on the CKM triangle are given by 'broad' bands due mainly to theoretical uncertainties. While such a statement is factually correct, it misses the deeper message. The observables $\Gamma(B \to l\nu X_{c,u})$, $\Gamma(K \to l\nu\pi)$, ΔM_K, ΔM_B, ϵ_K and $\sin 2\phi_1$ etc. represent very different dynamical regimes that proceed on time scales spanning several orders of magnitude. The fact that CKM theory can accommodate such diverse observables always within a factor two or mostly better and relate them in such a manner that its parameters can be plotted as meaningful constraints on a triangle is highly significant

[6] There are two groups–their websites are listed under [68]—constructing and continuously updating the CKM triangle based on input from a comprehensive set of data. While they rely on the same data base, the two groups employ different statistical tools. The quality of the available data is such that the results from the two groups more and more converge.

and, in my view, must reflect some underlying, yet unknown dynamical layer. This is achieved with just a handful of parameters, namely four CKM quantities and a few quark masses. Furthermore the CKM parameters exhibit an unusual hierarchical pattern—$|V(\mathrm{ud})| \sim |V(\mathrm{cs})| \sim |V(\mathrm{tb})| \sim 1$, $|V(\mathrm{us})| \simeq |V(\mathrm{cd})| \simeq \lambda$, $|V(\mathrm{cb})| \sim |V(\mathrm{ts})| \sim \mathcal{O}(\lambda^2)$, $|V(\mathrm{ub})| \sim |V(\mathrm{td})| \sim \mathcal{O}(\lambda^3)$—as do the quark masses culminating in $m_t \simeq 175\,\mathrm{GeV}$. Picking such values for these parameters would have been seen as frivolous at best—had they not been forced upon us by (independent) data. Thus I view it already as a big success for CKM theory that the experimental constraints on its parameters can be represented through triangle plots in a meaningful way, even if the constraints were represented by broad bands due largely to theoretical uncertainties.

- The width of these bands has been reduced significantly since 2000 with no inconsistency surfacing. The 2006 *measured* value of $\sin 2\phi_1$ is fully consistent with the CKM predictions of 1991 and 1998 and still compatible with that of 2005:

$$\sin 2\phi_1 = \begin{cases} 0.678 \pm 0.025 & \text{HFAG '07} \quad [52], \\ \sim 0.6 & \text{prediction '91} \quad [49], \\ 0.72 \pm 0.07 & \text{prediction '98} \quad [51], \\ 0.725 \pm 0.065 & \text{prediction '05} \quad [68]. \end{cases} \tag{162}$$

- The very recent resolution of $\mathrm{B_s} - \bar{\mathrm{B}}_\mathrm{s}$ oscillations, which allowed to measure $\Delta M_{\mathrm{B_s}}$, brought a novel and impressive success of CKM theory. With the base line of the triangle normalized to unity, the **CP** *in*sensitive observables $|V(\mathrm{ub})/V(\mathrm{cb})|$ and $\Delta M_{\mathrm{B_d}}/\Delta M_{\mathrm{B_s}}$, i.e. observables *not* requiring **CP** violation for acquiring a non-zero value, imply
  - a non-flat CKM triangle and thus **CP** violation, see the left of figure 14,
  - that is fully consistent with the observed **CP** sensitive observables ϵ_K and $\sin 2\phi_1$, see the right of figure 14.
- These impressive successes did not come as a total surprise, since CKM theory had provided a successful description of K, D and B transitions including oscillations already before 2001, albeit for fewer observables and with larger uncertainties. This is the reason why I speak about an *expected* triumph.
- By extension we cannot *count* on future measurements of **CP** asymmetries to typically show *numerically large* deviations from the CKM predictions. There are some exceptions—in particular concerning $\mathrm{B_s}(t) \to \mathrm{J}/\psi\phi$ to be discussed later—where one predicts unusually small effects for reasons very specific to CKM theory, while new physics could have an unusually large impact there.
- We can conclude now that CKM theory describes at least the lion's share of the **CP** violation observed in particle decays. The future emphasis therefore has shifted from searching for *alternatives* to CKM dynamics to *corrections* to it.
- All these successes should not make us forget that CKM theory is a very peculiar one with the mysterious hierarchical pattern in its basic parameters.

The subtle phenomenon of meson–antimeson oscillations—even coupled through EPR correlations—has played a crucial role in these studies, has been sharpened into a routine high accuracy tool for measurements and will continue to be a great and essential asset.

*3.2.9. Hadronization—the unsung hero rather than the villain in the tale of oscillations and **CP** violation.* Hadronization and nonperturbative dynamics in general are usually viewed as unwelcome complication, if not outright nuisances. A case in point was already

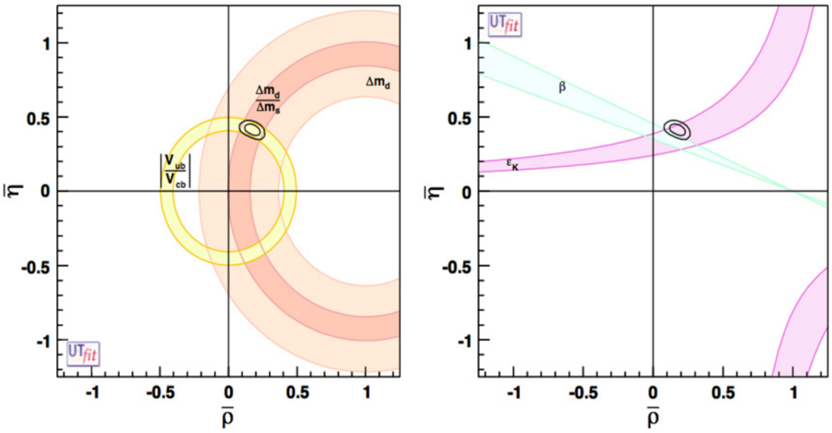

Figure 14. CKM unitarity triangle from $|V(\mathrm{ub})/V(\mathrm{cb})|$ and $\Delta M_{\mathrm{B_d}}/\Delta M_{\mathrm{B_s}}$ on the left and compared with constraints from ϵ_K and $\sin 2\phi_1/\beta$ on the right (courtesy of M Pierini).

mentioned: while I view the CKM predictions for ΔM_K, ΔM_B and ϵ_K to be in remarkable agreement with the data, significant contributions from new physics could be hiding there behind the theoretical uncertainties due to lack of computational control over hadronization. Yet *without* hadronization bound states of quarks and antiquarks will not form; without the existence of kaons $K^0 - \bar{K}^0$ oscillations obviously cannot occur. It is hadronization that provides the 'cooling' of the (anti)quark degrees of freedom, which allows subtle quantum mechanical effects to add up coherently over macroscopic distances. Otherwise one would not have access to a super-tiny energy difference $\mathrm{Im}\mathcal{M}_{12} \sim 10^{-8}$ eV, which is very sensitive to different layers of dynamics, and indirect **CP** violation could not manifest itself. The same would hold for B mesons and $B^0 - \bar{B}^0$ oscillations.

To express it in a more down to earth way:

- Hadronization leads to the formation of kaons and pions with masses greatly exceeding (current) quark masses. It is the *hadronic* phase space that suppresses the **CP** *conserving* rate for $K_L \to 3\pi$ by a factor ~ 500, since the K_L barely resides above the three pion threshold.
- It rewards 'patience'; i.e. one can 'wait' for a pure K_L beam to emerge after starting out with a beam consisting of K^0 and $\bar{K}^0$.
- It enables **CP** violation to emerge in the *existence* of a reaction, namely $K_L \to 2\pi$ rather than an asymmetry; this greatly facilitates its observation.
- For the **CP** asymmetry to emerge in $B_d(t) \to J/\psi K_S$ it is essential that both B_d and $\bar{B}_d$ can decay into the same final state $J/\psi K_S$. On the pure quark level it would appear not to happen, since one has $\bar{B}_d = [b\bar{d}] \to [c\bar{c}][s\bar{d}]$ versus $B_d = [\bar{b}d] \to [\bar{c}c][\bar{s}d]$ with seemingly $[s\bar{d}] \neq [\bar{s}d]$. Again it is hadronization into K^0 and $\bar{K}^0$ coupled with their mixing into the mass eigenstates K_S and K_L that saves the day. It also leads to

$$S_{J/\psi K_S} = -S_{J/\psi K_L}; \tag{163}$$

  i.e. there is no asymmetry if one averages over K_S and K_L.

For these reasons we should praise hadronization as the hero in the tale of **CP** violation rather than the villain it is all too often portrayed as.

3.2.10. Nature's gift. The existence of the 'CKM paradigm of large **CP** violation' and our ability to probe it experimentally is due to several favorable factors, whose confluence must be seen as a generous gift from nature, who had

(i) arranged for a huge top quark mass,
(ii) a 'long' B lifetime,
(iii) the $\Upsilon(4S)$ resonance being above $B\bar{B}$, yet below $B\bar{B}^*$ thresholds and
(iv) had presented us previously with charm hadrons.

The last two gifts have not been explained yet. The discovery of charm hadrons prompted the development of microvertex detectors with an effective resolution that was needed for B decays: B mesons being about three times heavier than D mesons have a three times smaller time dilatation factor, yet that is compensated by their about three times longer lifetimes.

The third item in the list is the most subtle one. As described before the $B_d - \bar{B}_d$ pair is produced in a **C** odd configuration on the $\Upsilon(4S)$ resonance. While it forces the **CP** asymmetry in $B_d \to J/\psi K_S$ to vanish if integrated over all times of decay, it makes it depend on $\sin \Delta M_{B_d} \Delta t$ with Δt denoting the *difference* between the decay times of the two B mesons. If on the other hand $\Upsilon(4S) \to B_d\bar{B}_d^*/B_d^*\bar{B}_d$ were kinematically allowed, it would lead to $\Upsilon(4S) \to B_d\bar{B}_d + \gamma$, since $B_d^* \to B_d\gamma$ is by far the leading B_d^* channel, and thus produce the $B_d\bar{B}_d$ pair in a **C** *even* configuration. Integrating over all times of decay for those events would *not* average the **CP** asymmetry to zero, yet the uncertainty in the production point would greatly hinder tracking of the time evolution of the signal.

4. Searching for dynamics beyond the SM

I have repeatedly emphasized that the SM of high energy physics has been more successful phenomenologically than we had reason to expect. With its dynamics based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)$ it has not achieved full unification, yet seems primed for that with the quantization of electric charge that it accommodates and needs, but does not explain. Its family replication would seem to call for an explanation as does the hierarchical pattern in its fermion masses and the mass-related quantities, namely the CKM parameters. Its whole mass generation process through Higgs fields is widely viewed as an exercise in theoretical engineering, which is not meant as a flattering label. The community's long search for physics beyond the SM has led to many frustrations. One way to characterize it is to quote Samuel Beckett:

'Ever tried? Ever failed?
No matter.
Try again. Fail again. Fail better.'

Only an Irishman can express profound skepticism concerning the world in such a poetic way. Beckett actually spent most of his life in Paris, since Parisians like to listen to someone expressing such a world view, even while they do not share it.

We know now that we will not fail forever, that physics beyond the SM has to exist. Apart from the theoretical deficits of the SM mentioned above, there are also experimental signatures for new physics, mostly of a heavenly nature, namely, coming from cosmology and astrophysics.

- The observed neutrino oscillations imply the need for non-degenerate neutrino masses, which points beyond the SM.

- The bulk of the gravitationally 'felt' dark matter has to be non-baryonic, for which the SM has no satisfactory candidate.
- Standard CKM dynamics cannot drive baryogenesis in our Universe.
- Dark energy is the most mysterious phenomenon, which we cannot even categorize yet.

I find the theoretical arguments compelling that suggest new physics has to exist around the few TeV scale to solve the gauge hierarchy problem and am confident the LHC scheduled to begin its running in 2008 will find it.

4.1. Indirect versus direct searches

In quantum theory new physics can be characterized by new, i.e. additional fields, their properties like mass, spin and couplings to other fields. There are basically two ways to experimentally verify their existence.

- '*Direct' searches*. Particles corresponding to the new fields are produced in high energy collisions. Since the fact that they have not been observed before probably means that they are heavy, one has to utilize accelerators with the highest possible energy in the center of mass system. This is called the 'high energy frontier', and the LHC represents the next attack on it.
- '*Indirect' searches*. Through their exchanges these additional fields can also generate *forces* between known particles, which will change the SM forces in quantitative or even qualitative ways. One prime example is the observation of $K_L \to \pi^+\pi^-$ in 1964. Both the kaons and the pions were already well-known particles, yet this process required **CP** violating forces unknown in the theoretical framework of that time that was based on up, down and strange quarks: it required the existence of three more quarks with the quantum numbers charm, beauty and top (although this realization was not made till 1972 [43]). These quarks have all been found, the top quark only in 1995 with a mass almost 350 times that of the K_L. Since such indirect searches involve the production only of known particles, one does not need to go to the highest energy accelerators; furthermore one can become sensitive to mass scales beyond the reach of the most powerful available accelerator. For this advantage one has to pay a price, however: obviously one is searching for reactions that are either forbidden in the SM or at least highly suppressed. Therefore high statistics and excellent control over systematics are essential. There are actually two branches of such searches: if a process is absolutely forbidden in the SM, then any signal will establish the intervention of new physics; in such a case one has a *high sensitivity* probe. The above example of $K_L \to \pi^+\pi^-$ fell into that category at the time of its discovery. Yet very often SM forces can generate the observable as well, albeit on the highly suppressed level; then one has to search for a quantitative deviation from the predicted value, which could be rather small; these are *high accuracy* searches. Its prime example is $g - 2$, the muon anomalous magnetic moment: the SM makes a tiny, yet extremely well calculated contribution to it; data show a value very close to the predicted one, yet with a tantalizing deviation.

  I will argue that the indirect searches described below need to combine both aspects, namely high sensitivity as well as high accuracy on the experimental as well as the theoretical side. *Oscillation phenomena offer an optimal stage for such an undertaking.*

The next central challenge facing the high energy physics community is to identify the dynamics driving the electroweak phase transition. This is the motivation—an excellent one in my judgement—for building and operating the LHC. This new physics—be it SUSY, Technicolour, Extra Dimensions—is unlikely to be intrinsically connected with flavor dynamics and their mysteries. Yet our goal has to go beyond uncovering the existence of new physics—we have to aim at establishing its salient features. This is a highly demanding task, since we have come up with a multitude of possible scenarios; one should note that broken SUSY represents more an organizing principle than a specific theory. We should also keep in mind that nature might have quite a few more tricks up her sleeves. We should therefore strive to obtain all the experimental information we can from nature. It is my considered judgement that without studying the possible impact of this new physics on heavy flavor decays we will fall significantly short of our goal to find out which version of new physics underlies the electroweak phase transition. Even finding that impact to be negligible would be an important lesson—albeit one most frustrating for the experimentalists involved. Dedicated and accurate studies of heavy flavour decays are thus related to the core mission of the LHC rather than a luxury.

*4.2. Instrumentalizing matter–antimatter oscillations and **CP** studies*

As illustrated above meson–antimeson oscillations and **CP** asymmetries provide highly sensitive probes for new physics, and even more so when these two phenomena co-operate as in $K_L \to \pi^+\pi^-$ and $B_d \to J/\psi K_S$; it is future manifestations of those effects I will focus on. For these oscillations provide two necessarily coherent amplitudes and build up quantum mechanical effects over macroscopic distances. Furthermore since the SM provides at least one amplitude, observable rates can be merely linear in a new physics amplitude and thus become much more sensitive. Without short-changing the intrinsic fascination of and profound interest in oscillations and **CP** violation one instrumentalizes those precision tools for probing for new physics.

*4.2.1. Act IV: **CP** asymmetries in $B_d(t) \to \phi K_S$, $\eta' K_S$, etc—snatching victory from the jaws of defeat or defeat from the jaws of victory?* Analyzing **CP** violation in $B_d(t) \to \phi K_S$ or $B_d(t) \to \phi\eta'$ is a most promising way to search for new physics. For the underlying quark-level transition $b \to s\bar{s}s$ constitutes a loop effect in the SM, i.e. a pure quantum correction. Thus it represents a highly forbidden SM transition and therefore a priori possesses an enhanced sensitivity to new physics. Furthermore within the SM these two modes are closely related to the well measured transition $B_d(t) \to J/\psi K_S$ [80]. While their branching ratios are very different, they should, to a good approximation, exhibit the same **CP** pattern:

$$\mathrm{Rate}(B_d[\bar{B}_d](t) \to f_-) \propto e^{-t/\tau_B}(1 - [+]S_{f_-} \sin \Delta M_{B_d} t) \tag{164}$$

with

$$S_{f_-} \simeq S_{J/\psi K_S}, \qquad \text{where } f_- = \phi K_S,\ \eta' K_S \tag{165}$$

for these odd **CP** eigenstates and $C_{f_-} \simeq 0$. If S_{f_-} or C_{f_-} or both differ significantly from these values we have uncovered an unequivocal manifestation of new physics that leads to direct **CP** violation, i.e. enters through $\Delta B = 1$ dynamics. Yet this would then strongly suggest that new physics should also affect the $\Delta B = 2$ amplitude—a conjecture that can be tested experimentally as well as theoretically, namely by constructing the CKM unitarity triangle even more accurately and with even more overconstraints.

The experimental situation is tantalizing, yet inconclusive—hence the heading. Great excitement was created when BELLE reported a large discrepancy between the predicted and the observed **CP** asymmetry in $B_d \to \phi K_S$ in the summer of 2003. However, based on more data taken, this discrepancy has shrunk considerably with the 2006 values reading as follows [52]:

$$\sin 2\phi_1(\mathrm{B_d} \to \mathrm{J}/\psi \mathrm{K_S}) = 0.678 \pm 0.025 \tag{166}$$

compared with

$$\sin 2\phi_1(\mathrm{B_d} \to \phi \mathrm{K_S}) = \begin{cases} 0.50 \pm 0.21 \pm 0.06 & \text{BELLE '06,} \\ 0.12 \pm 0.31 \pm 0.10 & \text{BABAR '06,} \\ 0.39 \pm 0.18 & \text{HFAG '06.} \end{cases} \tag{167}$$

The 2006 data are still low relative to the prediction, yet not conclusively so. At the same time the existence of a **CP** asymmetry has not been established yet either.

The situation for the similar channel $\mathrm{B_d}(t) \to \phi\eta'$ carries a different 'flavor:

$$\sin 2\phi_1(\mathrm{B_d} \to \eta' \mathrm{K_S}) = \begin{cases} 0.64 \pm 0.10 \pm 0.04 & \text{BELLE '06} & [66], \\ 0.58 \pm 0.10 \pm 0.03 & \text{BABAR '06} & [67], \\ 0.61 \pm 0.07 & \text{HFAG '06} & [52], \end{cases} \tag{168}$$

i.e.

- a **CP** asymmetry has been clearly established in this rare mode
- that so far is consistent with the prediction.

These transitions have to be studied in the most vigorous way, because they provide a natural portal for new physics. The lack of a conclusive deviation from the predictions should not discourage us at all. For the multi-faceted successes of CKM theory suggest that the impact of new physics in B transitions will typically be no more than moderate. Accordingly we are only now entering a sensitivity level, where one can realistically expect new physics contributions to surface.

There are other rare channels that receive large or even dominant contributions from the similar quark-level operator $\mathrm{b} \to \mathrm{s}\bar{q}q$ like $\mathrm{B_d}(t) \to f^0\mathrm{K_S}$, $\pi^0\mathrm{K_S}$ and $\omega\mathrm{K_S}$ that despite widely varying branching ratios should exhibit a pattern of **CP** violation close to that in $\mathrm{B_d}(t) \to \mathrm{J}/\psi\mathrm{K_S}$ according to the SM. The results for these other modes are similar to that for $\mathrm{B_d}(t) \to \phi\mathrm{K_S}$: the data, which are severely limited by statistics still, show neither a significant deviation from expectations nor a clear signal for any **CP** asymmetry. However sizable deviations might hide in the experimental uncertainties.

Naively—actually very naively—averaging over all these channels would answer both questions: it would lead to a claim that 'on average' there is a **CP** asymmetry that (a) differs from zero and (b) falls significantly below expectations. At the present level of statistical accuracy I see no serious justification for such a procedure—it is hardly more than a game, yet admittedly an intriguing one.

There is an important issue for further study. The two modes $\mathrm{B_d} \to \phi\mathrm{K_S}$ and $\mathrm{B_d} \to f^0\mathrm{K_S}$ have to be extracted from the measured rate and distribution for $\mathrm{B_d} \to \mathrm{K^+K^-K_S}$. To do this by merely imposing a cut on the $\mathrm{K^+K^-}$ pair mass is not quite up to the task when one wants to undertake the required precision studies. One reason why is the following: the final states $\phi\mathrm{K_S}$ and $f^0\mathrm{K_S}$ carry opposite **CP** parity. As long as $\mathrm{B_d} \to \phi\mathrm{K_S}$ and $\mathrm{B_d} \to f^0\mathrm{K_S}$ are driven by the same transition operator, they have to exhibit an opposite asymmetry due to their being **CP** odd and even, respectively:

$$\sin 2\phi_1(\mathrm{B_d} \to \phi\mathrm{K_S}) = -\sin 2\phi_1(\mathrm{B_d} \to f^0\mathrm{K_S}). \tag{169}$$

Even a small admixture of $\mathrm{B_d} \to f^0\mathrm{K_S}$ in supposedly $\mathrm{B_d} \to \phi\mathrm{K_S}$ events, say 10% in *amplitude*, would have a sizable impact on the **CP** asymmetry, which would be *linear* in the 'wrong' amplitude—20% in this example. Thus one has to separate those two contributions carefully. The most satisfactory solution is to perform a full *time dependent* Dalitz plot analysis of $\mathrm{B_d}(t) \to \mathrm{K^+K^-K_S}$ and likewise for $\mathrm{B_d} \to 3\mathrm{K_S}$ and other channels like that. This is of course easier said than done, since it requires very large statistics. Yet it will presumably be essential: for the internal consistency relations that a Dalitz plot study has to satisfy will be of great value in controlling even small uncertainties.

4.2.2. $\mathrm{B_s}$ *decays—an independent chapter in nature's book.* As described before, the resolution of the fast $\mathrm{B_s} - \bar{\mathrm{B}}_\mathrm{s}$ oscillations is a remarkable experimental achievement and provided another major triumph for CKM theory. Yet there could still be significant contributions from new physics lurking below the surface of $\mathrm{B_s}$ transitions. They present some advantages concerning searches for new physics: (i) they are 'calibrated' by our findings from $\mathrm{B_d}$ decays; one should keep in mind that the SM connections between those two systems might not hold in new physics scenarios. (ii) CKM theory makes some very unusual predictions for **CP** violation in $\mathrm{B_s}$ decays, as discussed now.

The transition $\mathrm{B_s}(t) \to \mathrm{J}/\psi\phi, \mathrm{J}/\psi\eta$ is a close 'cousin' of $\mathrm{B_d}(t) \to \mathrm{J}/\psi\mathrm{K_S}$ qualitatively

$$\mathrm{Rate}(\mathrm{B_s}[\bar{\mathrm{B}}_\mathrm{s}](t) \to \mathrm{J}/\psi\eta) \propto \mathrm{e}^{-t/\tau_\mathrm{B}}(1 - [+]S_{\mathrm{J}/\psi\eta} \sin \Delta M_{\mathrm{B_s}} t) \tag{170}$$

—i.e. no $C_{\mathrm{J}/\psi\eta}$ term—but not quantitatively. For its oscillations proceed much faster and $S_{\mathrm{J}/\psi\eta}$ is much smaller than $S_{\mathrm{J}/\psi\mathrm{K_S}}$, namely, about 0.03 for reasons very specific to CKM theory: the leading contributions to $\mathrm{B_s} \Rightarrow \bar{\mathrm{B}}_\mathrm{s}$ and $\mathrm{B_s}/\bar{\mathrm{B}}_\mathrm{s} \to \mathrm{J}/\psi\eta$ involve quarks from the second and third families only; thus **CP** violation can arise on the Cabibbo-suppressed level only [10]. The channel $\mathrm{B_s} \to \mathrm{J}/\psi\phi$ is easier to identify experimentally, yet the final state consisting of two vector mesons is no longer a **CP** eigenstate. While the dominant S-(and the D)-wave configurations are **CP** even, the P-wave one is **CP** odd and thus has to exhibit the opposite asymmetry (in the absence of final state interactions); therefore one has to disentangle these two classes of final states, which can be done through the correlations between the J/ψ and ϕ decay products.

New physics, which in general contains new sources of **CP** violation irrespective of the number of contributing quark flavours, could produce asymmetries of several 10%, i.e. an order of magnitude larger than predicted by the SM and thus clearly distinguished from the latter. Even the apparent fact that $\Delta M_{\mathrm{B_s}}$ is fully consistent with the SM prediction does not rule against such a scenario, since $\Delta M_{\mathrm{B_s}} \simeq 2\mathrm{Re}\frac{q}{p}M_{12}$ while $S_{\mathrm{J}/\psi\eta} = \mathrm{Im}(\frac{q}{p}\frac{T(\bar{\mathrm{B}}_\mathrm{s}\to\phi\eta)}{T(\mathrm{B_s}\to\phi\eta)})$ and new physics could quite conceivably contribute more to the imaginary than the real part [81].

Meson–antimeson oscillations are also characterized by $\Delta\Gamma_\mathrm{B}$, the difference in width for the two mass eigenstates. Unlike for the observed $\Delta\Gamma_\mathrm{K}$ and the searched for $\Delta\Gamma_\mathrm{D}$, which are dominated by long distance dynamics, over which we still have no accurate theoretical control, it is a very reasonable ansatz to calculate $\Delta\Gamma_\mathrm{B}$ using a quark box description. That relies on short distance dynamics apart from one overall hadronic expectation value. However one has to keep in mind that this ansatz could conceivably fail due to the proximity of charm–anticharm thresholds [82].

4.2.3. The semileptonic ***CP*** *asymmetry in* $\mathrm{B_{d,s}}$ *decays.* The **CP** asymmetry defined in equation (93) is proportional to $\Delta\Gamma_\mathrm{B}/\Delta M_\mathrm{B}$, which is a small number irrespective of **CP** violation, namely 0.01 or even less. The theoretical predictions are not very precise, yet certainly small [81]:

$$a_{\mathrm{SL}}(\mathrm{B_d}) \sim 5 \times 10^{-4}, \qquad a_{\mathrm{SL}}(\mathrm{B_s}) \sim 2 \times 10^{-5}. \tag{171}$$

The predicted $a_{SL}(B_s)$ is particularly tiny specifically for CKM dynamics, which cannot generate it on the leading KM level (analogous to the situation for $B_s(t) \to J/\psi\eta/\phi$). New physics could enhance it by even two orders of magnitude. Yet $a_{SL}(B_s) \sim$ few $\times 10^{-3}$ is still a very small number, and it remains to be seen whether the systematic biases in the detection efficiencies for positively and negatively charged particles can be controlled at that level.

4.2.4. The dark horse—$D^0 - \bar{D}^0$ *oscillations and* ***CP*** *violation.* There are three mesons built from *down*-type quarks, namely K^0, B_d and B_s mesons, that can exhibit oscillations, and indeed they have been observed for all three. The landscape is much sparser for *up*-type mesons. Since top quarks decay before they can hadronize [34], neutral T mesons can*not* form, and *a fortiori* oscillations cannot occur. Since π^0 and $\eta^{(\prime)}$ mesons are their own antiparticle, oscillations are not even defined for them. This leaves only D^0 mesons as candidates for exhibiting oscillations with up-type quarks. As described in section 1.5 there is now strong, though not yet compelling evidence for oscillations of D^0 mesons driven by $\Delta M_D \neq 0 \neq \Delta\Gamma_D$. That $D^0 - \bar{D}^0$ oscillations proceed rather slowly is expected in the SM. For the $D^0 \Rightarrow \bar{D}^0$ amplitude is, unlike the decay width Γ_D, doubly-Cabibbo-suppressed and further reduced by the GIM mechanism. There is a long trail of papers in the literature on predicting ΔM_D and $\Delta\Gamma_D$ in the SM. A typical starting point was to compute quark box diagrams, i.e. to mimic what was done for ΔM_K and ΔM_B. This is a reasonable guess for starting, yet not for completing the analysis. For it yielded unnaturally tiny values like $x_D = \Delta M_D/\Gamma_D \sim 10^{-4}$. Two systematic approaches of a complementary nature have been employed for estimating the size of x_D and y_D. (i) A systematic analysis based on a *quark*-level description has been given in [75, 83] yielding $x_D(\mathrm{SM})|_{\mathrm{OPE}}$, $y_D(\mathrm{SM})|_{\mathrm{OPE}} \sim \mathcal{O}(10^{-3})$. (ii) The authors of [76,77] find similar numbers, albeit in a quite different approach: estimating $SU(3)_{Fl}$ breaking for $\Delta\Gamma_D$ from *hadronic* phase space differences for two-, three- and four-body D modes they obtain $y_D(\mathrm{SM}) \sim 0.01$ and inferring x_D from y_D via a dispersion relation they arrive at $0.001 \leqslant |x_D(\mathrm{SM})| \leqslant 0.01$ with x_D and y_D being of opposite signs.

While one predicts similar numbers for x_D and y_D, one should keep in mind that they arise in very different dynamical environments: ΔM_D is generated from *virtual* intermediate states and is thus sensitive to new physics, which could affect it considerably. $\Delta\Gamma_D$, on the other hand, is shaped by *real* intermediate states and is thus hardly sensitive to new physics (for a dissenting opinion, see [78]).

I infer from these considerations that to the best of our present knowledge even values for x_D and y_D as 'high' as 0.01 could be due entirely to SM dynamics of otherwise little interest. It is likewise possible that a large or even dominant part of $x_D \sim 0.01$ in particular is due to new physics. While one should never rule out a theoretical breakthrough, I am less than confident that even the usual panacea, namely lattice QCD, can provide a sufficiently fine instrument in the foreseeable future.

Such agnosticism is particularly frustrating, since the data point to $0.001 \leqslant x_D,\ y_D \leqslant 0.01$ [13–15]. Yet despite this lack of an unequivocal statement from theory one wants to probe these oscillations as accurately as possible even in the absence of the aforementioned breakthrough, since they represent an intriguing quantum mechanical phenomenon. Furthermore—and more importantly—they constitute an important ingredient for **CP** asymmetries arising in D^0 decays due to new physics as explained next. With oscillations on an observable level—and it seems x_D, $y_D \sim 0.005 - 0.01$ satisfy this requirement—the possibilities for **CP** asymmetries proliferate. Those can, as I will illustrate by some examples, provide powerful diagnostic tools for the interpretation of observed effects. For the SM can generate very little **CP** violation in $\mathcal{L}(\Delta C = 2)$, since the third family practically decouples. Even a new physics contribution to $\mathcal{L}(\Delta C = 2)$ that is *sub*-dominant as far as ΔM_D and $\Delta\Gamma_D$ are concerned, can very possibly provide the dominant source for time dependent **CP** asymmetries in D^0 decays. For *a priori* there is no reason why it should not induce a sizable phase relative to the SM contribution.

Consider the transition to a **CP** eigenstate like $D^0 \to K^+K^-, \pi^+\pi^-$. This can be treated in analogy to $B_d \to \pi^+\pi^-$, albeit only a qualitative one, since both x_D and y_D are very small compared with unity. Thus it suffices to give the decay rate evolution to first order in those quantities only (the general expressions can be found in [83]):

$$\Gamma(D^0(t) \to K^+K^-) \propto e^{-\Gamma_1 t}|T(D^0 \to K^+K^-)|^2 \times \left[1 + y_D\frac{t}{\tau_D}\left(1 - \mathrm{Re}\frac{q}{p}\bar{\rho}_{K^+K^-}\right) - x_D\frac{t}{\tau_D}\mathrm{Im}\frac{q}{p}\bar{\rho}_{K^+K^-}\right],$$

$$\Gamma(\bar{D}^0(t) \to K^+K^-) \propto e^{-\Gamma_1 t}|T(\bar{D}^0 \to K^+K^-)|^2 \times \left[1 + y_D\frac{t}{\tau_D}\left(1 - \mathrm{Re}\frac{p}{q}\frac{1}{\rho_{K^+K^-}}\right) - x_D\frac{t}{\tau_D}\mathrm{Im}\frac{p}{q}\frac{1}{\rho_{K^+K^-}}\right]. \tag{172}$$

Some comments might elucidate equations (172).

- **CP** invariance implies (in addition to $|T(D^0 \to K^+K^-)| = |T(\bar{D}^0 \to K^+K^-)|$) $\frac{q}{p}\bar{\rho}_{K^+K^-} = 1$ (and $|q| = |p|$). The transitions $D^0(t) \to K^+K^-$ and $\bar{D}^0(t) \to K^+K^-$ are then described by the same *single* lifetime. That is a consequence of the theorem given by equation (28), since K^+K^- is a **CP** eigenstate.
- The usual three types of **CP** violation can arise, namely the direct and indirect types—$|\bar{\rho}_{K^+K^-}| \neq 0$ and $|q| \neq |p|$, respectively—as well as the one involving the interference between the oscillation and direct decay amplitudes—$\mathrm{Im}\frac{q}{p}\bar{\rho}_{K^+K^-} \neq 0$ leading also to $\mathrm{Re}\frac{q}{p}\bar{\rho}_{K^+K^-} \neq 1$.
- Assuming for simplicity $|T(D^0 \to K^+K^-)| = |T(\bar{D}^0 \to K^+K^-)|$ (CKM dynamics is expected to induce an asymmetry not exceeding 0.1%) and $|q/p| = 1 - \epsilon_D$ one has $(q/p)\bar{\rho}_{K^+K^-} = (1 - \epsilon_D)e^{i\phi_{K\bar{K}}}$ and thus

$$A_\Gamma = \frac{\Gamma(\bar{D}^0(t) \to K^+K^-) - \Gamma(D^0(t) \to K^+K^-)}{\Gamma(\bar{D}^0(t) \to K^+K^-) + \Gamma(D^0(t) \to K^+K^-)} \simeq x_D\frac{t}{\tau_D}\sin\phi_{K\bar{K}} - y_D\frac{t}{\tau_D}\epsilon_D\cos\phi_{K\bar{K}}, \tag{173}$$

  where I have assumed $|\epsilon_D| \ll 1$. BELLE has found [14]

$$A_\Gamma = (0.01 \pm 0.30 \pm 0.15)\%. \tag{174}$$

  While there is no evidence for **CP** violation in the transition, one should also note that the asymmetry is bounded by x_D. For x_D, $y_D \leqslant 0.01$, as indicated by the data, A_Γ could hardly exceed the 1% range, i.e. there is no real bound on ϕ_D or ϵ_D yet. The good news is that if x_D and/or y_D indeed fall into the 0.5–1% range, then any improvement in the experimental sensitivity for a **CP** asymmetry in $D^0(t) \to K^+K^-$ constrains new physics scenarios—or could reveal them [79]. It should be noted that within the SM one predicts $A_\Gamma \leqslant 10^{-5}$ even if allowing for x_D, $y_D = 0.01$. Thus with the present bound on A_Γ of about 0.01 one has a numerical range of close to three orders of magnitude, where a clear signal for new physics could emerge.

Another promising channel for probing for both time dependent and independent **CP** asymmetries is $D^0(t) \to K^+\pi^-$: since it is doubly Cabibbo-suppressed, it should *a priori* exhibit a higher sensitivity to a new physics amplitude. Furthermore it cannot exhibit direct **CP** violation in the SM.

4.2.5. EPR correlations in $D^0 - \bar{D}^0$ *production.* In close analogy to B production in e^+e^- annihilation and the $\Upsilon(4S)$ resonance there is a resonance just above the $D\bar{D}$, yet below the $D^*\bar{D}$ thresholds

$$e^+e^- \to \psi(3770) \to D\bar{D}, \tag{175}$$

where the $D\bar{D}$ pair is produced in a **C** odd configuration. There are two ways to search for such **CP** violation even without measuring the time of decay [83, 85]:

- The discussion given in section 3.2.5 applies here as well, although the numbers are quite different. The reaction

$$e^+e^- \to D^0\bar{D}^0 \to f_a f_b, \tag{176}$$

where $f_{a,b}$ denote **CP** eigenstates of the same parity, can occur only with the help of **CP** violation. Examples are $f_{a,b} = K_S\phi$, K^+K^-, $\pi^+\pi^-$. The charm analogue of equation (153) gets simplified, since $x_D \ll 1$:

$$\mathcal{B}(D^0\bar{D}^0|_{C=-} \to f_a f_b) \simeq \mathcal{B}(D \to f_a)\mathcal{B}(D \to f_b), \\ \cdot \left[2\left|\bar{\rho}_{f_a} - \bar{\rho}_{f_b}\right|^2 + x_D^2 \left| 1 - \frac{q}{p}\bar{\rho}_{f_a}\frac{q}{p}\bar{\rho}_{f_b} \right|^2 \right]. \tag{177}$$

For $f_a = f_b = K^+K^-$ one finds

$$\mathcal{B}(D^0\bar{D}^0|_{C=-} \to [K^+K^-]_D[K^+K^-]_D) \\ \simeq \left[\mathcal{B}(D^0 \to K^+K^-)\right]^2 \cdot x_D^2 \left| 1 - \left[\frac{q}{p}\bar{\rho}_{K^+K^-}\right]^2 \right|^2. \tag{178}$$

For $f_a \neq f_b$, e.g. $f_a = K^+K^-$ and $f_b = K_S\phi$ or $\pi^+\pi^-$, the reaction can proceed even with $x_D = 0$, as can be read off from equation (177).
As for the B_d case equation (177) can also be applied for $f_{a,b}$ not being **CP** eigenstates, yet modes common to D^0 and $\bar{D}^0$. Measuring those rates would provide important information on strong phase shifts the knowledge of which would sharpen our tools for interpreting **CP** asymmetries [83, 85, 86].

- In

$$e^+e^- \to D^0\bar{D}^{0*} + D^{0*}\bar{D}^0 \to D^0\bar{D}^0 + \gamma \tag{179}$$

the $D^0\bar{D}^0$ pair is produced in a **C** even state. The asymmetry in $D^0\bar{D}^0|_{C=+} \to [l^+\nu K^-]_D[K^+K^-]_D$ versus $D^0\bar{D}^0|_{C=+} \to [l^-\nu K^+]_D[K^+K^-]_D$ then depends on the times of decay t_1 and t_2 as $\sin\Delta M_D(t_1 + t_2)$. Integrating it over all t_1 and t_2 yields $2x_D\mathrm{Im}\frac{q}{p}\bar{\rho}_{K^+K^-}$. If one finds such a time integrated **CP** asymmetry one can clarify its origin by searching for its analogue in $e^+e^- \to D^0\bar{D}^0$, where it has to vanish, if it involves $D^0 - \bar{D}^0$ oscillations. The time integrated asymmetry averaged over $D^0\bar{D}^0|_{C=\pm}$ production thus yields $\frac{1}{2}[0 + 2x_D\mathrm{Im}\frac{q}{p}\bar{\rho}_{K^+K^-}] = x_D\mathrm{Im}\frac{q}{p}\bar{\rho}_{K^+K^-}$, which is precisely the result for an *incoherently* produced $D^0\bar{D}^0$ pair.

4.2.6. $e^+e^- \to \Upsilon(5S) \to B_s^{(*)}\bar{B}_s^{(*)}$. $B_s - \bar{B}_s$ oscillations have been resolved experimentally, see equation (13). While they are as rapid as predicted, they are not overly so for the available microvertex detectors. I am confident that LHCb will be able to measure time dependent **CP** asymmetries in B_s decays. Nevertheless it is legitimate and at least of intellectual value to ask, whether they could be probed also in $e^+e^- \to \Upsilon(5S) \to B_s^{(*)}\bar{B}_s^{(*)}$, where the ΔM_{B_s} driven oscillations can*not* be resolved.

There are actually two avenues.

- One searches for

$$e^+e^- \to B_s\bar{B}_s \to [J/\psi\phi]_{\mathbf{CP}=+}[J/\psi\phi]_{\mathbf{CP}=+}, \tag{180}$$

which is described by

$$\mathcal{B}(B_s\bar{B}_s|_{\mathbf{C}=-} \to [J/\psi\phi]_{B_s,\mathbf{CP}=+}[J/\psi\phi]_{B_s,\mathbf{CP}=+})$$
$$\simeq [\mathcal{B}(B_s \to [J/\psi\phi]_{\mathbf{CP}=+})]^2 \cdot \left|1 - \left[\frac{q}{p}\bar{\rho}_{[J/\psi\phi]_{\mathbf{CP}=+}}\right]^2\right|^2. \tag{181}$$

The ‘conditio sine qua non’, namely that oscillations occur, is hidden, since it enters via a factor $x_s^2/(1+x_s^2)$, which is unity for all practical purposes.

- One probes the oscillations driven by $\Delta\Gamma_{B_s}$. Consider the decay into a **CP** eigenstate $f_{\mathbf{CP}}$ without direct **CP** violation; i.e. $|\bar{\rho}_{f_{\mathbf{CP}}}| = 1$ in addition to $|q| \simeq |p|$. Then we have [89]

$$\mathrm{Rate}(B_s(t) \to f_{\mathbf{CP}}) \propto \frac{1}{2}e^{-\Gamma_1 t}|A(f_{\mathbf{CP}})|^2$$
$$\times \cdot \left[1 + \mathrm{Re}\frac{q}{p}\bar{\rho}_{f_{\mathbf{CP}}} + e^{\Delta\Gamma_{B_s}t}\left(1 - \mathrm{Re}\frac{q}{p}\bar{\rho}_{f_{\mathbf{CP}}}\right)\right]. \tag{182}$$

With **CP** symmetry $\frac{q}{p}\bar{\rho}_{f_{\mathbf{CP}}}$ equals the **CP** parity of $f_{\mathbf{CP}}$. This is consistent with the theorem of section 1.2: for $\frac{q}{p}\bar{\rho}_{f_{\mathbf{CP}}} \neq \pm 1$ the decay rate evolution into a **CP** eigenstate is *not* governed by a single exponential in time. On the $\Upsilon(5S)$ resonance the situation is very complex, since in $\Upsilon(5S) \to B_s\bar{B}_s$, $B_s^*\bar{B}_s/B_s\bar{B}_s^* \to B_s\bar{B}_s\gamma$, $B_s^*\bar{B}_s^* \to B_s\bar{B}_s 2\gamma$ the $B_s\bar{B}_s$ pairs are produced both in **C** odd and even configurations.

*4.3. ♠ Probing **CPT** symmetry ♠*

CPT invariance is an almost inescapable consequence of local quantum field theories. Even so one can take a hard-nosed empirical approach to probe its validity. Such projects have gained more attention now, since superstring or M theory introduced to quantize gravity is an intrinsically *non*-local theory. This feature removes one of the assumptions on which the validity of the **CPT** theorem is based; on the other hand nobody has proved that such a theory must lead to **CPT** violation.

Most empirical tests of **CPT** invariance are based on the equality of masses and lifetimes for particles and antiparticles. I do not find them numerically impressive: comparing the bound on the electron and positron mass to the electron mass itself makes dimensional sense, but not much more. Bounds on lifetime differences normalized by the average lifetime are on the 10^{-4} level, and thus have not even reached the 10^{-5} level of direct **CP** violation in $K^0 \to \pi^+\pi^-$ versus $\bar{K}^0 \to \pi^+\pi^-$, see equation (137).

Since meson–antimeson oscillations allow precise measurements of delicate effects it makes eminent sense to employ them in probes of **CPT** symmetry. No violation has been found. Yet even so, these analyses represent beautiful applications of quantum mechanics and experimental acumen.

4.3.1. ♠ Searching with $K_{L,S}$ *beams ♠.* As discussed before, the mass eigenstates $K_{L,S}$ contain both **CP** even and odd components:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\bar{\epsilon}_L|^2}}\left(|K_-\rangle + \bar{\epsilon}_L|K_+\rangle\right),$$

$$|K_S\rangle = \frac{1}{\sqrt{1+|\bar{\epsilon}_S|^2}}\left(|K_+\rangle + \bar{\epsilon}_S|K_-\rangle\right). \tag{183}$$

CPT invariance tells us that the impurity parameter $\bar{\epsilon}_S$ has to coincide with ϵ_L. While $K_L \to 2\pi$ has been well measured now, we have not yet reached the experimental sensitivity to observe **CP** violation in $K_S \to 3\pi$ modes.

The best bounds on **CPT** violation have been obtained in $K \to 2\pi$ decays by measuring the phases of the amplitude ratios η_{+-} and η_{00}, equation (30) [3, 6, 90]. A multistep argument then leads to a bound on $M_{\bar{K}^0} - M_{K^0}$, which is often stated as

$$\frac{|M_{\bar{K}^0} - M_{K^0}|}{M_K} \leqslant 9 \times 10^{-19}, \tag{184}$$

a truly impressive number, yet of obscure meaning, since calibrating by M_K has no more than dimensional justification. A more meaningful yardstick is provided by weak rates

$$\frac{|M_{\bar{K}^0} - M_{K^0}|}{\Delta M_K} = (0.012 \pm 8.0|_{\rm exp} \pm 2.0|_{\rm th}) \times 10^{-5} \qquad \text{or} \tag{185}$$

$$\frac{|M_{\bar{K}^0} - M_{K^0}|}{\Gamma_{K_S}} < 7 \times 10^{-5}. \tag{186}$$

4.4. ♠ Employing entangled pairs in $e^+e^- \to \phi \to K\bar{K}$, $e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d$ ♠

We have already discussed at length how the process

$$e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d \to f_1 f_2 \tag{187}$$

for various channels $B_d/\bar{B}_d \to f_{1,2}$ provides single as well as double interferometry, which allows one to measure delicate quantities like ΔM_B and phases (the latter being inaccessible otherwise). This high sensitivity can also be harnessed in unorthodox ways, namely, to experimentally probe fundamental principles like **CPT** invariance, the superposition principle of quantum mechanics and EPR correlations in a detailed way. The discussion given before can be generalized in a straightforward way by dropping **CPT** constraints like $\bar{\epsilon}_L = \bar{\epsilon}_S$, see [87].

KLOE working at the DAΦNE ring at LNF Frascati near Rome has taken high quality data of the analogous reaction $e^+e^- \to \phi \to K\bar{K}$. One can witness an intriguing interplay between the demands of Bose–Einstein statistics and the linear superposition principle of quantum mechanics with the vastly different lifetimes for K_L and K_S providing an additional experimental handle [91].

It was already stated that a transition like

$$e^+e^- \to \phi \to K_S K_L \to [\pi^+\pi^-]_K[\pi^+\pi^-]_K \tag{188}$$

requires **CP** violation to proceed. This leads to

Problem 2. The two $[\pi^+\pi^-]$ combinations in the final state form a P wave. Yet Bose statistics requiring identical states to be in a *symmetric* configuration would appear to veto this reaction. What is the loophole in this reasoning? The same puzzle can be formulated in terms of

$$e^+e^- \to \Upsilon(4S) \to B_d\bar{B}_d \to [J/\psi K_S]_B[J/\psi K_S]_B. \tag{189}$$

It has even been suggested that one might use $e^+e^- \to \phi \to K\bar{K}$ as a high precision tool to probe Bell's inequality [92], which discriminates between quantum mechanics and local realistic alternatives [91, 93, 94].

4.5. The next ten years and beyond

High energy physics and fundamental science in general is embarking on a fascinating and promising journey into the unknown this year, when the LHC, the huge accelerator at CERN colliding two beams of protons at high energies, will start to operate. Theoretical arguments based on the internal self-consistency of the SM, in particular its Higgs sector, point to the need of new physics to surface directly around the 1 TeV energy or mass scale. ATLAS and CMS, the two gargantuan experiments taking data there, will cover most of the expected parameter space for novel effects to occur. I find the theoretical arguments persuasive and therefore refer to it as 'confidently predicted' new physics (**cpNP**). While various candidates have been suggested for this **cpNP**—SUSY being the most popular choice at present—we do not know what it will be and should not discard nature's ability or even inclination to surprise us with a variant we have not thought of.

While I am confident that novel phenomena will be discovered at the LHC, I am much more skeptical that the ATLAS/CMS programs can fully identify all the salient features of the underlying dynamics. SUSY after all is, at our present level of understanding, more a classification scheme than a class of theories, let alone a specific theory. The HEP community by and large shares this skepticism and has therefore united behind the ILC project—a linear e^+e^- collider to reach the 1 TeV scale—as the next central facility.

As expressed before, I believe we also have to study the impact of this anticipated **cpNP** on heavy flavor dynamics to achieve our goal of inverse theoretical engineering, i.e. to establish which variant of **cpNP** drives electroweak symmetry breaking. To our good fortune LHCb—an experiment dedicated to high statistics studies of beauty transitions—will operate at the LHC as well; CERN deserves great credit for its foresight in approving this experiment more than ten years ago.

Even the absence of an observable signal on a high sensitivity level, while certainly frustrating to the experimental groups searching for any, would be telling. Again as repeatedly stated before, the **cpNP** is likely to affect heavy flavor transitions in less than a numerically massive way. Thus we have to aim for the most sensitive and comprehensive tools. In my considered judgement we will need a Super-Flavor factory [88][7] an e^+e^- machine operating in the $\Upsilon(4S)$ (and even the $\Upsilon(5S)$) region (and hopefully even considerably below it) with a luminosity two orders higher than the very successful B factories. Such a machine with its statistics and clean experimental environment allowing a most comprehensive program would be the optimal (and a rather frugal) complement to the LHC and ILC. This would also allow one to exploit the exemplary surgical precision of meson–antimeson oscillations to the fullest in our quest to understand nature's grand design.

Epilogue

Matter–antimatter oscillations, in particular when embedded in EPR correlations, represent one of the central counter-intuitive features of quantum mechanics emphasizing the intrinsically non-local nature of the latter. Yet they are essential to some of our most profound insights into nature's inner workings, namely the existence of **CP** violation in our universe.

[7] Two projects of a Super-Flavor or Super-B Factory based on different accelerator technologies are being pursued. One is centered at KEK in Japan.

Six instances of the latter have been established experimentally: (i) $\Gamma(K_L \to 2\pi) \neq 0$; (ii) $\Gamma(K^0 \to \pi^+\pi^-) \neq \Gamma(\bar{K}^0 \to \pi^+\pi^-)$; (iii) $\text{rate}(B_d(t) \to J/\psi K_S) \neq \text{rate}(\bar{B}_d(t) \to J/\psi K_S)$; (iv) $\text{rate}(B_d(t) \to \pi^+\pi^-) \neq \text{rate}(\bar{B}_d(t) \to \pi^+\pi^-)$; (v) $\text{rate}(B_d(t) \to \eta' K_S) \neq \text{rate}(\bar{B}_d(t) \to \eta' K_S)$; (vi) $\Gamma(\bar{B}_d \to K^-\pi^+) \neq \Gamma(B_d \to K^+\pi^-)$. Of those only the last asymmetry is unrelated to oscillations; the first one involves oscillations in an essential way and the third through fifth ones also EPR correlations; even the second one due to its minute size probably could never be observed without oscillations, which cause the K_S component to decay away.

Oscillations are not only an essential ingredient, they are also a high precision tool that by their peculiar time pattern validates control over systematic uncertainties. There is every reason to expect that oscillation-related phenomena will yield even deeper insights into nature's fundamental forces.

Appendix A. Glossary for non-particle physicists

I will list here several terms of common use in particle physics that may not be familiar to scientists from other branches.

Baryons. Strongly interacting fermions viewed as bound states of three quarks. Most familiar representatives: protons and neutrons.
Yogi Berra. Founder of the most popular American school of Philosophy; coined the most concise description in layman's terms of quantum mechanics—'When you come to a fork in the road, take it!'—and of baryogenesis 'If the world were perfect, it wouldn't be!'
Colour. An internal quantum number carried by quarks and gluons that gets gauged in QCD.
Confinement. Statement that quarks and antiquarks are permanently bound into hadrons and do not exist as isolated states in nature.
***CPT** theorem*. The combined transformation of charge conjugation **C**, parity **P** and time reversal **T** can always be defined in such a way in quantum field theories that it represents an exact symmetry. This theorem is based on little more than Lorentz invariance and describing the fundamental dynamics through *local* quantum fields. **CPT** symmetry implies equal masses and lifetimes for particles and antiparticles.
Dalitz plot. A three-body final state—such as $B \to 3\pi$—can be completely characterized by two kinematical variables (energy, momenta, two-particle invariant mass). It can thus be represented by a point in a two-dimensional plot with a roughly triangular boundary. Pure kinematics lead to a uniform distribution of such points over the inside of the plot; any non-uniformity reflects non-trivial dynamics like resonance formation, for example $B \to \rho\pi \to 3\pi$. While the parametrisation of the Dalitz plot distribution is not unique, it has to satisfy several internal cross checks, which provide validation. Such studies extract the maximal information on the underlying dynamics.
Direct and indirect evidence for new physics. *Direct*: searches for *quanta* or elementary *particles* not contained in the SM like SUSY partners, exotic quarks, new gauge bosons; *indirect*: probing for non-SM *forces* between SM particles like lepton flavor violating couplings, new gauge interactions.
Gauge bosons. Sometimes also referred to as vector bosons, these are spin one fields that mediate forces through their exchanges. We have one photon, three weak bosons and eight gluons with the latter being the carriers of the strong forces.
Hadrons. Particles subject to the strong force; there are two varieties, 'baryons' and 'mesons' built from three quarks and a quark-antiquark pair, respectively.

Hadronization. Process driven by strong forces that transmogrifies quarks, antiquarks and gluons into hadrons.
HEP. High energy physics.
Higgs mechanism. An apparently very successful feat of theoretical engineering, where spin-zero fields are introduced coupling to both themselves and gauge bosons; through *spontaneous realization of a symmetry*—or spontaneous symmetry breaking for short—one generates masses for gauge bosons without destroying the gauge invariance of the theory by assigning non-vanishing ground state or vacuum expectation values (VEV) to the neutral spin-zero fields. Within the SM these fields can en passant also generate masses for quarks and leptons through their Yukawa couplings. A typical footprint of this mechanism is the emergence of scalar states in the observable particle spectrum—*the* Higgs boson.
Lattice QCD. Algorithms for simulating QCD on a discrete (and finite) space-time lattice to treat nonperturbative dynamics, including hadronization, quantitatively.
Leptons. Particles not subject to strong forces like electrons, muons and neutrinos.
Lepton flavours. Different species of charged and neutral leptons. We know of six lepton flavours: electrons, muons and τ leptons together with their associated neutrinos. The observation of neutrino oscillations $\nu_\mu \Rightarrow \nu_\tau$ & $\nu_e \Rightarrow \nu_\mu$ has shown that lepton flavor violations do occur.
Long distance dynamics. See *QCD*.
Mesons. Strongly interacting bosons viewed as made up from a quark and anti-quark. Most familiar representative: pions. Other examples are kaons, charm and beauty mesons, which are roughly three, ten and thirty times heavier, respectively, than the pion. They carry different quark flavours.
New physics. Dynamical elements (fields and/or forces) outside or beyond the framework of the SM.
Nonleptonic decays. Weak decays of mesons or baryons into final states containing only other hadrons. E.g.: $\mathrm{K} \to 2\pi, 3\pi, \mathrm{B_d} \to \mathrm{J}/\psi \mathrm{K_S}$.
Quark flavours. Different species of quarks. We know of six different quarks: ‘up’ u, ‘down’ d, ‘strange’ s, ‘charm’ c, ‘beauty’ b and ‘top’ t quarks.
QCD. Quantum chromo dynamics, local gauge theory based on the group $SU(3)_C$ describing the strong interactions. It is ‘asymptotically free’, meaning that for high energies or *short distances* the strength of its forces goes to zero and therefore can be treated by perturbation theory. The other side of the coin is ‘infrared slavery’, meaning for low energies or *long distances* its forces become strong and can no longer be treated perturbatively in the usual sense. It qualitatively explains confinement, i.e. why quarks are always bound inside hadrons; yet the nonperturbative dynamics have not been brought fully under quantitative theoretical control.
See-saw mechanism. Dynamical scenario involving very heavy Majorana masses for right-handed neutrinos that explains why the left-handed neutrinos are so unusually light, though not massless.
Semileptonic decays. Weak decays of mesons or baryons into final states containing a charged lepton together with its neutrino in addition to hadrons; e.g. $\mathrm{K_L} \to l^+\nu\pi$, $\mathrm{D}^0 \to l^+\nu\mathrm{K}^-$, $\mathrm{B_d} \to l^+\nu\mathrm{D}^-$.
Short distance dynamics. See *QCD*.
Spontaneous realization of a symmetry. A situation where the dynamics obey some symmetry, be it continuous or discrete, yet the ground state is not invariant under it. Instead there are different, yet equivalent ground states. Such a scenario is also called ‘spontaneous symmetry breaking’ or ‘hidden symmetry’. Examples are superconductivity as described by BCS theory and the ecliptic in our solar system, i.e. the plane in which to a good approximation all

eight planets orbit around the sun. We know how to 'theoretically engineer' a spontaneous realization of a symmetry through the Higgs mechanism (see *Higgs mechanism*), whereby a neutral scalar field develops a non-zero vacuum or ground state expectation value (VEV).
Spontaneous symmetry breaking. See *spontaneous realization of the symmetry*.
Supersymmetry (SUSY). The Coleman–Mandula theorem states that the groups representing the continuous symmetries of nature can be expressed as the direct product of the symmetry groups of the inhomogeneous Lorentz transformations and internal symmetries like colour and isospin (the latter being approximate). This implies that all particles in a symmetry multiplet have to carry the same spin. Supersymmetry is the only known way for going beyond this restriction. Since its algebra contains *anti*commutators in addition to the usual commutators, it combines particles of different spin into supermultiplets. Its simplest implementation brings together particles with spins that differ by half a unit. It cannot represent an exact symmetry, since we know there is no scalar partner to the electron with the same mass. Nevertheless SUSY is seen by many in the HEP community as a most attractive extension of the SM based both on its conceptual and on its phenomenological features.
Standard Model. Local gauge theory based on the group $SU(3)_C \times SU(2)_L \times U(1)$ describing the strong and the electroweak interactions.
VEV. Vacuum or ground state expectation value, see *Higgs mechanism, Spontaneous realization of a symmetry*.

Appendix B. On phase conventions

Equation (63) has two solutions differing in sign. **CP** transformation actually defines antiparticles only up to a complex phase, which implies the same phase ambiguity for $\frac{q}{p}$:

$$|\bar{P}^0\rangle \to \mathrm{e}^{\mathrm{i}\xi}|\bar{P}^0\rangle \Longrightarrow (M_{12}, \Gamma_{12}) \to \mathrm{e}^{\mathrm{i}\xi}(M_{12}, \Gamma_{12})\&\frac{q}{p} \to \mathrm{e}^{-\mathrm{i}\xi}\frac{q}{p}. \tag{B.1}$$

Therefore $\frac{q}{p}$ per se cannot be an observable, nor can M_{12}, Γ_{12}. On the other hand $\frac{q}{p}M_{12}$ and $\frac{q}{p}\Gamma_{12}$ are phase invariant, as it has to be, since the eigenvalues, which are observables, depend on this combination, see equation (62). Also $|\frac{q}{p}|$ is an observable; its *deviation* from unity is one measure of **CP** violation in $\Delta F = 2$ dynamics. It depends on the *relative* phase between M_{12} and Γ_{12}.

Within a *given theory* for $\Delta F = 2$ dynamics one can calculate $\frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12}) = \frac{q}{p}\langle P^0|\mathcal{L}(\Delta F = 2)|\bar{P}^0\rangle$. Then one assigns the labels B and A such that $\Delta M = M_\mathrm{B} - M_A = -2\mathrm{Re}\frac{q}{p}(M_{12} - \frac{i}{2}\Gamma_{12})$ turns out to be *positive*. Then one can compute whether $\Gamma_A > \Gamma_\mathrm{B}$ or $\Gamma_A < \Gamma_\mathrm{B}$.

Appendix C. 'Whose time is it anyway?' (with due apologies to a British comedy series) or: How does one measure time in oscillations?

The short answer is: 'One does not.' Instead one probes—in matter–antimatter oscillations as well as in neutrino oscillations—the flavor of the object under study at a certain point in space. In the case of neutral hadrons this happens through their decay into a flavor specific final state or in the case of neutrinos through their interaction with matter. Knowing their production point (and thus their distance traveled) as well as their momentum one can infer the (proper) time passed between production and decay or interaction.

The longer answer can be given in three parts: (i) the simple approach used by most authors and also in the main text of this review, namely to follow the evolution of the oscillating states having fixed energy as a function of (proper) time yields the correct results. (ii) One can obtain the correct results also by tracking the evolution of oscillating states with fixed momentum, if done properly. (iii) Unfortunately one can also find descriptions that are confusing, misleading or sometimes even erroneous, although all that is needed is elementary quantum mechanics. It is not without interest to point out where some of these unfortunate treatments go wrong. I follow here the discussion given by Lipkin [95] for neutrinos, yet formulate it for meson–antimeson oscillations.

Let us consider mesons B^0 and $\bar{\mathrm{B}}^0$ with mass eigenstates B_H and B_L and $M_{B_H} > M_{B_L}$. Their evolution in proper time is controlled by their masses and can be related to their energies and momenta in the lab frame in terms of the time t and distance D there:

$$|B_{H[L]}(t)\rangle = \mathrm{e}^{-\mathrm{i}M_{B_{H[L]}}\tau_{B_{H[L]}}}|B_{H[L]}(0)\rangle = \mathrm{e}^{-\mathrm{i}(E_{H[L]}t - P_{H[L]}D)}|B_{H[L]}(0)\rangle. \tag{C.1}$$

One usually considers the superposition of states produced with *equal energy* E and thus different momenta $P_{H[L]} = \sqrt{E^2 - M^2_{B_{H[L]}}} \simeq E - M^2_{B_{H[L]}}/2E$. It produces a phase *difference* that leads to oscillations observable in space

$$\mathrm{e}^{-\mathrm{i}\Delta\phi(D)}, \qquad \Delta\phi(D) \simeq \frac{1}{2}\Delta M^2 \cdot \frac{D}{E}, \qquad \Delta M^2 = M^2_{B_H} - M^2_{B_L} \tag{C.2}$$

for the length of their flight path D and relativistic energies $E \gg M$.

One could also consider these mass eigenstates to be produced with common momentum P. Then one has $E_{P_{H[L}} = \sqrt{P^2 + M^2_{B_{H[L]}}} \simeq P + M^2_{B_{H[L]}}/2P$ leading to a phase difference

$$\Delta\phi(D) \simeq \frac{1}{2}\Delta M^2 \cdot \frac{D}{P}, \tag{C.3}$$

which coincides for relativistic particles—$E \simeq P$—with equation (C.2).

This all is simple enough, so what is the problem? A hard-nosed pragmatist can stop reading here. Yet I find it interesting to track the subtleties hidden in these derivations that have caused confusion (or even worse) in the literature, when discussions were given more explicitly in terms of wave packets, arrival times, etc.

Again, first assume the two mass eigenstates to have equal energy E and cover a distance D with velocity $v_{H[L]} = P_{H[L]}/E$. Then we find for the phase factor

$$Et = E\frac{L}{v_{H[L]}} = E^2\frac{L}{P_{H[L]}} \simeq EL\left(1 + \frac{1}{2}\frac{M_{H[L]}/^2}{E^2}\right) \tag{C.4}$$

and thus for the phase difference

$$\Delta\phi(D) \simeq \frac{1}{2}\Delta M^2 \cdot \frac{D}{E} \tag{C.5}$$

in pleasing agreement with equation (C.2).

Now assume instead the two states to have equal momenta and thus different energies. Relating D, the distance traveled, to the flight time one has $t_{H[L]} = D/v_{H[L]} = E_{H[L]}D/P$ and therefore for the phase difference

$$\Delta\phi_{\text{wrong}} = E_H t_H - E_L t_L \simeq (E_H^2 - E_L^2)\frac{D}{P} = \Delta M^2 \cdot \frac{D}{P}, \tag{C.6}$$

which differs from the correct result of equation (C.2) by a factor of 2!

The skeptic will argue that one should not have entered into a description treating the interfering states as classical particles arriving at a given point at different times [96]—and he/she would be correct. Yet sometimes one can learn from tracking even basic mistakes.

One can ask what is so special about assuming *equal energy* for the evolving states rather than *equal momenta*; should those two descriptions not be equivalent? The crucial point is to understand that the nature of the detector cannot be ignored as a matter of principle. An ideal detector in the classical sense would allow to determine the energies as well as the momenta of the mesons precisely and thus their mass $M = \sqrt{E^2 - p^2}$. This would destroy the ability of the P_H and P_L components to interfere, and no oscillations could occur. Quantum mechanics does not allow for such an ideal detector. Realistic detectors are usually stationary. Stodolsky's theorem [97] states there can be no coherence and thus no interference between states of different energies. For its impact is described by a density matrix that is diagonal in energy; thus only states of equal energy can interfere. It is this interaction with the detector that breaks the symmetry between a description in terms of equal energy and equal momentum. The basis of the theorem is quite transparent and actually rather elementary. With purely stationary states one can have observable time dependent effects—if one has two states of different energy interfere. The corollary is that in a stationary system one cannot have interference between different energy states.

One can legitimately be concerned that the discussion has been too crude so far, since it has been phrased in terms of plane waves rather than wave packets. This concern has been addressed [95]. Let us introduce an average group velocity $\bar{v}$ for the wave packet containing the P_H and P_L components:

$$\bar{v} = \frac{p(P_H) + p(P_L)}{E(P_H) + E(P_L)}. \tag{C.7}$$

It controls the time needed for the wave packet to cover a distance L up to a correction term δt reflecting among other things quantum fluctuations:

$$t = \frac{p(P_H) + p(P_L)}{E(P_H) + E(P_L)} \cdot L + \delta t. \tag{C.8}$$

Thus we find

$$\begin{aligned}\Delta\phi &= \frac{p^2(P_H) - p^2(P_L)}{p(P_H) + p(P_L)} \cdot L - \frac{E^2(P_H) - E^2(P_L)}{E(P_H) + E(P_L)} \cdot t \\ &= \frac{\Delta M^2}{p(P_H) + p(P_L)} - \frac{E^2(P_H) - E^2(P_L)}{E(P_H) + E(P_L)} \cdot \delta t.\end{aligned} \tag{C.9}$$

For states of equal energy—$E(P_H) = E(P_L)$—as required by Stodolsky's theorem—we arrive at the usual result obtained in a more hand-waving way before with $p = \frac{1}{2}[p(P_H) + p(P_L)]$. Yet even otherwise—$E(P_H) \neq E(P_L)$—one can show that the last term in equation (C.9) can safely be ignored. Thus one can add the pragmatic answer that both descriptions, namely the one with equal energies or the one with equal momenta can be made to work, as long as one properly follows the prescriptions of basic quantum mechanics. Equation (C.9) can be rewritten for the case of equal momenta $p(P_H) = p(P_L) = p$ as follows:

$$\Delta\phi = \frac{\Delta M^2}{2p} \cdot L \cdot \left(1 + \frac{\bar{v}\delta t}{L}\right). \tag{C.10}$$

As long as the irreducible size of the wave packet, $\bar{v}\delta t$, is small compared with L, the distance traveled, one can safely ignore the second term and thus once again arrive at the usual expression of equation (C.2) even without invoking the stationarity condition.

Appendix D. Counting physical parameters in the CKM matrix

An $N \times N$ CKM matrix (for N families) contains $2N^2$ real parameters; the unitarity constraints reduce it to N^2 independent real parameters. Since the phases of quark fields like other *fermion* fields can be rotated freely, $2N - 1$ phases can be removed from $\mathcal{L}_{CC}$ (a *global* phase rotation of all quark fields has no impact on $\mathcal{L}_{CC}$). Thus we have $(N-1)^2$ *independent physical* parameters. Since an $N \times N$ *orthogonal* matrix has $N(N-1)/2$ angles, we conclude that an $N \times N$ *unitary* matrix also contains $(N-1)(N-2)/2$ *physical phases* [43]. Accordingly:

- for $N = 2$ families we have one angle—the Cabibbo angle—and zero phases,
- for $N = 3$ families we obtain three angles and one irreducible phase; i.e. a three family ansatz can support **CP** violation with a single source—the 'CKM phase',
- for even more families we encounter a proliferation of angles and phases, namely six angles and three phases for $N = 4$.

In section 2.3.1 I gave a geometric interpretation of the orthogonality relations in terms of six triangles in the complex plane. Since V_{CKM} contains only three angles and one phase, there are many relations between the different triangles. One is that these six triangles despite their vastly different shapes all have to possess the same area [6] reflecting the single irreducible phase.

Appendix E. $\mathbf{K^0 - \bar{K}^0}$ oscillations restoring CPT invariance in $\tau^+ \to \mathbf{K}\pi^+\bar{\nu}$ versus $\tau^- \to \mathbf{K}\pi^-\nu$

Let us consider the charged lepton decay channel $\tau^\pm \to \mathrm{K_S}\pi^\pm\nu$, which has been observed with a branching ratio of about 0.5%. The SM predicts for the underlying transition *amplitude*

$$T(\tau^- \to \bar{\mathrm{K}}^0\pi^-\nu) = T(\tau^+ \to \mathrm{K}^0\pi^+\bar{\nu}). \tag{E.1}$$

The observable kaons in the final state are the mass eigenstates $\mathrm{K_{S,L}}$. Ignoring **CP** violation in $\Delta S \neq 0$ dynamics one has

$$\Gamma(\tau^- \to \mathrm{K_S}\pi^-\nu) = \Gamma(\tau^- \to \mathrm{K_L}\pi^-\nu) = \tfrac{1}{2}\Gamma(\tau^- \to \bar{\mathrm{K}}^0\pi^-\nu), \tag{E.2}$$

$$\Gamma(\tau^+ \to \mathrm{K_S}\pi^+\bar{\nu}) = \Gamma(\tau^+ \to \mathrm{K_L}\pi^+\bar{\nu}) = \tfrac{1}{2}\Gamma(\tau^+ \to \mathrm{K}^0\pi^+\bar{\nu}) \tag{E.3}$$

and thus no **CP** asymmetry due to equation (E.1).

The situation becomes considerably more complex and intriguing, once **CP** violation in $\Delta S = 2$ dynamics is included [98]. (For our purposes here we can safely ignore *direct* **CP** violation.) Applying the general formalism of section 2.1 we can write

$$|\mathrm{K_S}\rangle = p|\mathrm{K}^0\rangle + q|\bar{\mathrm{K}}^0\rangle, \qquad |\mathrm{K_L}\rangle = p|\mathrm{K}^0\rangle - q|\bar{\mathrm{K}}^0\rangle, \tag{E.4}$$

$$|\mathrm{K}^0\rangle = \frac{1}{2p}(|\mathrm{K_S}\rangle + |\mathrm{K_L}\rangle), \qquad |\bar{\mathrm{K}}^0\rangle = \frac{1}{2q}(|\mathrm{K_S}\rangle - |\mathrm{K_L}\rangle). \tag{E.5}$$

with $|p|^2 + |q|^2 = 1$. Both $\mathrm{K_S} \to 2\pi$ and $\mathrm{K_L} \to 2\pi$ then occur; i.e. $\mathrm{K_S}$ and $\mathrm{K_L}$ are no longer orthogonal, see equation (61),

$$\langle \mathrm{K_L}|\mathrm{K_S}\rangle = |p|^2 - |q|^2 \simeq 2\mathrm{Re}\,\bar{\epsilon} \simeq (3.32 \pm 0.06) \times 10^{-3} \neq 0 \tag{E.6}$$

as deduced from δ_l, the asymmetry in semileptonic $\mathrm{K_L}$ decays, equation (34). While the 2π final state by itself no longer distinguishes strictly between $\mathrm{K_S}$ and $\mathrm{K_L}$, the difference in the $\mathrm{K_S}$ and $\mathrm{K_L}$ lifetimes still provides a discriminator. When considering the decay rate evolution for $\tau \to [\pi^+\pi^-]_{\mathrm{K}}\pi\nu$ as a function of t_{K}, the proper time of the kaon decay, one has for short times—$t_{\mathrm{K}} \sim \mathcal{O}(1/\Gamma_{\mathrm{S}})$—for all practical purposes only $\mathrm{K_S} \to 2\pi$ decays and finds a **CP** asymmetry:

$$\frac{\Gamma(\tau^+ \to [\pi^+\pi^-]_{''\mathrm{K_S}''}\pi^+\bar{\nu}) - \Gamma(\tau^- \to [\pi^+\pi^-]_{''\mathrm{K_S}''}\pi^-\nu)}{\Gamma(\tau^+ \to [\pi^+\pi^-]_{''\mathrm{K_S}''}\pi^+\bar{\nu}) + \Gamma(\tau^- \to [\pi^+\pi^-]_{''\mathrm{K_S}''}\pi^-\nu)} = |q|^2 - |p|^2. \tag{E.7}$$

For long decay times $t_K \sim \mathcal{O}(1/\Gamma_L)$—one has practically only $K_L \to 2\pi$ and thus obtains:

$$\frac{\Gamma(\tau^+ \to [\pi^+\pi^-]_{"K_L"}\pi^+\bar{\nu}) - \Gamma(\tau^- \to [\pi^+\pi^-]_{"K_L"}\pi^-\nu)}{\Gamma(\tau^+ \to [\pi^+\pi^-]_{"K_L"}\pi^+\bar{\nu}) + \Gamma(\tau^- \to [\pi^+\pi^-]_{"K_L"}\pi^-\nu)} = |q|^2 - |p|^2. \tag{E.8}$$

Measuring the asymmetry of equation (E.8) might be hardly feasible since the K_L decaying mostly outside the detector acts like a second neutrino. Yet it raises a puzzling question: with the asymmetries of equations (E.7) and (E.8) having the same sign, how is the equality between the τ^+ and τ^- lifetimes restored as required by **CPT** invariance? Which other channel can interfere with $\tau \to [\pi^+\pi^-]_K\pi\nu$ and thus provide a compensating **CP** asymmetry?

To resolve this puzzle, we have to note that the **CPT** constraint applies to the sum over all relevant channels with their rates integrated over all times of decay and analyze more carefully the K_S and K_L classification [98]. The asymmetries of equations (E.7) and (E.8) are measured by studying the time elapsed between the τ decay and the moment, when the 2π pair is formed. The first asymmetry is obtained by focussing on short time differences and the second one for large time differences. Exactly because $\langle K_S|K_L\rangle \neq 0$ the decay rate evolution for $\tau \to [f]_K\pi\nu$, where f is an arbitrary final state in $K_{S,L}$ decays, now contains three terms: in addition to the two contributions listed above with time dependences $\propto e^{-\Gamma_S t_K}$ and $\propto e^{-\Gamma_L t_K}$, respectively, we have an *interference* term $\propto e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_K}$ most relevant for intermediate times $1/\Gamma_S \ll t_K \ll 1/\Gamma_L$. I.e., the decay rate evolution of a state born as a K^0 or $\bar{K}^0$ reads as follows:

$$\Gamma(K^0(t_K) \to f) \propto \frac{|T(K_S \to f)|^2}{2|p|^2} \cdot[e^{-\Gamma_S t_K} + |\rho_f|^2 e^{-\Gamma_L t_K} + 2e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_K}\mathrm{Re}(e^{-i\Delta M_K t_K}\rho_f)], \tag{E.9}$$

$$\Gamma(\bar{K}^0(t_K) \to \bar{f}) \propto \frac{|T(K_S \to \bar{f})|^2}{2|q|^2} \cdot[e^{-\Gamma_S t_K} + |\rho_{\bar{f}}|^2 e^{-\Gamma_L t_K} - 2e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_K}\mathrm{Re}(e^{-i\Delta M_K t_K}\rho_{\bar{f}})] \tag{E.10}$$

with $\rho_f = T(K_L \to f)/T(K_S \to f)$. For short times of decay the first term describing K_S decays in equations (E.9) and (E.10) dominates leading to equation (E.7) for $f = \bar{f} = \pi^+\pi^-$. For very long times the second term does producing the same **CP** asymmetry as stated in equation (E.8).

Yet for the intermediate range in times of decay the third term reflecting $K_S - K_L$ interference becomes important. Rewriting the interference term in terms of K^0 and $\bar{K}^0$, integrating over all t_K and finally summing over all possible states f and $\bar{f}$, we arrive at

$$\begin{aligned}\sum_f \int \mathrm{d}t_K e^{-\frac{1}{2}(\Gamma_S+\Gamma_L)t_K}\mathrm{Re}[e^{i\Delta M_K t_K}T(K_S \to f)T(K_L \to f)^*] \\ = \mathrm{Re}\left\{\frac{1}{\frac{\Gamma_L+\Gamma_S}{2} - i\Delta M_K}[(|p|^2 - |q|^2)\Gamma_{11} + 2i\mathrm{Im}(qp^*\Gamma_{12})]\right\} \\ = (|p|^2 - |q|^2) + \mathrm{Re}\left\{\frac{i}{\frac{\Gamma_L+\Gamma_S}{2} - i\Delta M_K}[2\Delta M_K \mathrm{Re}\bar{\epsilon} - \Delta\Gamma_K \mathrm{Im}\bar{\epsilon}]\right\},\end{aligned} \tag{E.11}$$

where we have used relations valid for this problem to first order in the **CP** violating parameters:

$$\Gamma_{11} = \frac{\Gamma_L+\Gamma_S}{2}, \quad p = \frac{1}{\sqrt{2}}(1+\bar{\epsilon}), \quad q = \frac{1}{\sqrt{2}}(1-\bar{\epsilon}), \quad \Delta\Gamma_K = 2\Gamma_{12}. \tag{E.12}$$

Finally using $\arg\epsilon_{\mathrm{K}} = \arctan(\frac{2\Delta M_{\mathrm{K}}}{\Delta\Gamma_{\mathrm{K}}})$ we find that the square brackets in the last line of equation (E.11) vanish; i.e.

$$\sum_f \int \mathrm{d}t_{\mathrm{K}} \mathrm{e}^{-\frac{1}{2}(\Gamma_{\mathrm{S}}+\Gamma_{\mathrm{L}})t_{\mathrm{K}}} \mathrm{Re}[\mathrm{e}^{\mathrm{i}\Delta M_{\mathrm{K}} t_{\mathrm{K}}} T(\mathrm{K_S} \to f) T(\mathrm{K_L} \to f)^*] = (|p|^2 - |q|^2). \tag{E.13}$$

Using equation (E.13) one easily shows that the integrated interference term indeed restores the **CPT** constraint:

$$\sum_f \int \mathrm{d}t_{\mathrm{K}} \Gamma(\tau^+ \to [f]_{\mathrm{K}^0(t_{\mathrm{K}})} \pi^+ \bar{\nu}) = \sum_f \int \mathrm{d}t_{\mathrm{K}} \Gamma(\tau^- \to [\bar{f}]_{\bar{\mathrm{K}}^0(t_{\mathrm{K}})} \pi^- \nu). \tag{E.14}$$

Of course this is as it has to be. Yet it is remarkable how nature achieves it, namely by a savvy use of $\mathrm{K}^0 - \bar{\mathrm{K}}^0$ oscillations.

Appendix F. Solutions to problems

Solution to problem 1. The massive $W^\pm$ vectorboson has two 'ancestors', namely the original massless gauge boson forming the transverse components and the charged scalar component of the Higgs doublet field introduced to drive electroweak symmetry breaking, which re-emerges as the longitudinal $W^\pm$ component. The latter, for which there is no decoupling theorem, generates the m_{t}^2/M_W^2 contribution.

Solution to problem 2. The two $[\pi^+\pi^-]$ combinations are actually not identical with one coming from a $\mathrm{K_S}$ and the other from a $\mathrm{K_L}$ decay; thus they differ in their mass, if ever so slightly.

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