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Models of neutrino masses and mixings

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Abstract. We review theoretical ideas, problems and implications of neutrino masses and mixing angles. We give a general discussion of schemes with three light neutrinos. Several specific examples are analysed in some detail, particularly those that can be embedded into grand unified theories.

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1. Introduction

There is by now convincing evidence from the experimental study of atmospheric and solar neutrinos [1]–[3] (the results of the HOMESTAKE experiment are reported in [4]) for the existence of at least two distinct frequencies of neutrino oscillations. This in turn implies non-vanishing neutrino masses and a mixing matrix, in analogy with the quark sector and the CKM matrix. So *a priori* the study of masses and mixings in the lepton sector should be considered at least as important as that in the quark sector. However, actually there are a number of features that make neutrinos especially interesting. In fact, the smallness of neutrino masses is probably related to the fact that νs are completely neutral (i.e. they carry no charge which is exactly conserved) and are Majorana particles with masses inversely proportional to the large scale where lepton number (L) conservation is violated. Majorana masses can arise from the see-saw mechanism [5], in which case there is some relation with the Dirac masses, or from higher-dimensional non-renormalizable operators which come from a different sector of the Lagrangian density than any other fermion mass terms. The relation with L non-conservation and the fact that the observed neutrino oscillation frequencies are well compatible with a large scale for L non-conservation, points to a tantalizing connection with grand unified theories (GUTs). So neutrino masses and mixings can represent a probe into the physics at GUT energy scales and offer a different perspective on the problem of flavour and the origin of fermion masses. There are also direct connections with important issues in astrophysics and cosmology as, for example, baryogenesis through leptogenesis [6] and the possibly non-negligible contribution of neutrinos to hot dark matter in the Universe.

Recently, there have been new important experimental results that have considerably improved our knowledge. The SNO experiment has confirmed that the solar neutrino deficit is due to neutrino oscillations and not to a flaw in our modelling of the Sun [2]: the total neutrino flux is in agreement with the solar model but only about one-third arrives on Earth as ν_e while the remaining part consists of other kinds of active neutrinos, presumably ν_μ and ν_τ. The allowed amount of sterile neutrinos is strongly constrained. The KamLAND experiment has established that $\overline{\nu_e}$ from reactors show oscillations over an average distance of about 180 km which are perfectly compatible with the frequency and mixing angle corresponding to one of the solutions of the solar neutrino problem (the large angle (LA) solution) [7]. Thus, the results from solar neutrinos have been reproduced and improved by a terrestrial experiment. Also the coincidence of the frequency for neutrinos from the Sun and for antineutrinos from reactors is consistent with the validity of CPT invariance. The validity of this symmetry had been questioned because of the puzzling LSND claim of a signal that could indicate a third distinct oscillation frequency (hence implying either more than three light neutrinos or CPT violation). In September 2003 new results have been published by the SNO Collaboration [3], obtained after adding salt to their heavy-water detector in order to increase the sensitivity to the neutral current channels. The previous results have been confirmed with increased accuracy. The allowed region for the LA solution has been further restricted with the elimination of the upper region in Δm^2_{12}. Of great importance have also been the first results from WMAP [8] on the cosmic radiation background. The related determination of cosmological parameters, in combination with other measurements, leads to an upper limit on the cosmological neutrino density $\Omega_\nu \lesssim 0.015$. This is a very important result that indicates that neutrinos are not a major component of the dark matter in the Universe. For three degenerate neutrinos the WMAP limit implies an upper bound on the common mass given by $m_\nu < 0.23$ eV [8]. Given the priors that are assumed for this determination, i.e. a definite cosmological model, a two-digit value for the bound is not to be taken too seriously [9]. Still the quoted value is about an order of magnitude smaller than the bound from tritium beta decay and of the same order of the upper bound on the Majorana mass that fixes the rate of neutrinoless double beta decay.

In spite of this progress there are many alternative models of neutrino masses [10]. This variety is mostly due to the considerable experimental ambiguities that still exist. One first missing input is the absolute scale of neutrino masses: neutrino oscillations only determine mass-squared differences. For atmospheric neutrinos $\Delta m^2_{atm} \sim 2.6 \times 10^{-3}\,\mathrm{eV}^2$, whereas for solar neutrinos $\Delta m^2_{sol} \sim 7 \times 10^{-5}\,\mathrm{eV}^2$. Another key missing quantity is the value of the third mixing angle s_{13} on which only a bound is known, $s_{13} < 0.22$. Then it is essential to know whether the LSND signal [11], which has not been confirmed by KARMEN [12] and is currently being double-checked by MiniBoone [13], will be confirmed or will be excluded. If LSND is right, we probably need at least four light neutrinos; if not we can do with only the three known ones.

Here we will briefly summarize the main categories of neutrino mass models, discuss their respective advantages and difficulties and give a number of examples. We illustrate how forthcoming experiments can discriminate among the various alternatives. We will devote special attention to a comprehensive discussion in a GUT framework of neutrino masses together with all other fermion masses. This is, for example, possible in models based on SU(5)$\times$ U(1)$_F$ or on SO(10) (we always consider SUSY GUTs) [14, 15].

2. Basic formulae and data for three-neutrino mixing

We assume in the following that the LSND signal [11] will not be confirmed so that there are only two distinct neutrino oscillation frequencies, the atmospheric and the solar frequencies. These two can be reproduced with the known three light neutrino species (see e.g. [16] for more than three neutrinos).

Neutrino oscillations are due to a misalignment between the flavour basis, $\nu' \equiv (\nu_e, \nu_\mu, \nu_\tau)$, where ν_e is the partner of the mass and flavour eigenstate e^- in a left-handed (LH) weak isospin SU(2) doublet (similarly for ν_μ and ν_τ) and the mass eigenstates $\nu \equiv (\nu_1, \nu_2, \nu_3)$ [17, 18]:

$$\nu' = U\nu, \tag{1}$$

where U is the unitary 3×3 mixing matrix. Given the definition of U and the transformation properties of the effective light neutrino mass matrix m_ν,

$$\nu'^{\mathrm{T}} m_\nu \nu' = \nu^{\mathrm{T}} U^{\mathrm{T}} m_\nu U \nu, \qquad U^{\mathrm{T}} m_\nu U = \mathrm{Diag}(m_1, m_2, m_3) \equiv m_{diag}, \tag{2}$$

we obtain the general form of m_ν (i.e. of the light ν mass matrix in the basis where the charged lepton mass is a diagonal matrix):

$$m_\nu = U^* m_{diag} U^\dagger. \tag{3}$$

The matrix U can be parametrized in terms of three mixing angles θ_{12}, θ_{23} and θ_{13} $(0 \leqslant \theta_{ij} \leqslant \pi/2)$ and one phase φ $(0 \leqslant \varphi \leqslant 2\pi)$ [19], exactly as for the quark mixing matrix V_{CKM}. The following definition of mixing angles can be adopted:

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\mathrm{e}^{\mathrm{i}\varphi} \\ 0 & 1 & 0 \\ -s_{13}\mathrm{e}^{-\mathrm{i}\varphi} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

Table 1. Square mass differences and mixing angles.

	Lower limit (3σ)	Best value	Upper limit (3σ)
$(\Delta m^2_{sun})_{LA}$ ($10^{-5}\,\mathrm{eV}^2$)	5.4	6.9	9.5
Δm^2_{atm} ($10^{-3}\,\mathrm{eV}^2$)	1.4	2.6	3.7
$\sin^2\theta_{12}$	0.23	0.30	0.39
$\sin^2\theta_{23}$	0.31	0.52	0.72
$\sin^2\theta_{13}$	0	0.006	0.054

where $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$. In addition, if ν are Majorana particles, we have the relative phases among the Majorana masses m_1, m_2 and m_3. If we choose m_3 real and positive, these phases are carried by $m_{1,2} \equiv |m_{1,2}|\mathrm{e}^{\mathrm{i}\phi_{1,2}}$ [20].[3] Thus, in general, nine parameters are added to the SM when non-vanishing neutrino masses are included: three eigenvalues, three mixing angles and three CP violating phases.

In our notation the two frequencies, $\Delta m^2_I/4E$ $(I = sun, atm)$, are parametrized in terms of the ν mass eigenvalues by

$$\Delta m^2_{sun} \equiv |\Delta m^2_{12}|, \qquad \Delta m^2_{atm} \equiv |\Delta m^2_{23}|, \tag{5}$$

where $\Delta m^2_{12} = |m_2|^2 - |m_1|^2 > 0$ and $\Delta m^2_{23} = m_3^2 - |m_2|^2$. The numbering 1, 2, 3 corresponds to our definition of the frequencies and, in principle, may not coincide with the ordering from the lightest to the heaviest state. From experiment, see table 1 [25], we know that s_{13} is small, according to CHOOZ, $s_{13} < 0.22$ (3σ) [22]. Atmospheric neutrino oscillations mainly depend on $(\Delta m^2_{atm}, \theta_{23}, \theta_{13})$, whereas solar oscillations are controlled by $(\Delta m^2_{sol}, \theta_{12}, \theta_{13})$. Therefore, in the ideal limit of exactly vanishing s_{13}, the solar and atmospheric oscillations decouple and depend on two separate sets of two-flavour parameters. For atmospheric neutrinos we have $c_{23} \sim s_{23} \sim 1/\sqrt{2}$, corresponding to nearly maximal mixing. Oscillations of muon neutrinos into tau neutrinos are favoured over oscillations into sterile neutrinos (ν_s). The conversion probability and the zenith angular distribution of high-energy muon neutrinos are sensitive to matter effects, which distinguish ν_τ from ν_s. Moreover, for conversion of ν_μ into pure ν_s, neutral current events would become up/down asymmetric. In both cases data strongly disfavour the pure sterile case. Oscillations into ν_τ are also indirectly supported by a SK data sample that can be interpreted in terms of enriched τ-like charged-current events. The sterile component of the neutrino participating in atmospheric oscillations should amount to less than 0.25 at 90% CL. Disappearance of laboratory-produced muon neutrinos has also been confirmed within expectations by the K2K experiment.

The only surviving solution to the solar neutrino problem after KamLAND and SNO-salt results is LA [23]–[25], with $\Delta m^2_{sol} \approx 7 \times 10^{-5}\,\mathrm{eV}^2$ and $\sin^2\theta_{12} \approx 0.3$. Before KamLAND the interpretation of solar neutrino data in terms of oscillations required the knowledge of the Boron neutrino flux, f_B. For instance, charged and neutral current data from SNO are sensitive, respectively, to $f_B\langle P_{ee}\rangle$ and to $f_B\langle\sum_a P_{ea}\rangle$ $(a = e, \nu, \tau)$, where $\langle P_{ef}\rangle$ denotes the appropriately averaged conversion probability from ν_e to ν_f. KamLAND [7] provides a direct measurement of $\langle P_{ee}\rangle$. Beyond the impact on the oscillation parameters and a check that the solar standard

[3] Mass matrices with a general dependence on ϕ, $\phi_{1,2}$ have been analysed in [21].

model works well ($f_B = (1.00 \pm 0.06) \times 5.05 \times 10^6\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$), the comparison among these experiments shows that the conversion of boron solar neutrinos into sterile neutrinos is compatible with zero. Therefore, the LSND indication for a third oscillation frequency associated with one or more sterile neutrinos is not supported by any other experiment, at the moment. Now, after KamLAND, also the possibility that such a frequency originates from a CPT violating neutrino spectrum [26] has no independent support. Data from solar neutrino experiments and KamLAND, involving, respectively, electron neutrinos and electron antineutrinos, are compatible with a CPT invariant spectrum.

If we take maximal s_{23} and keep only linear terms in $u = s_{13}\mathrm{e}^{\mathrm{i}\varphi}$, from experiment we find the following structure of the U_{fi} ($f = e,\mu,\tau$, $i = 1, 2, 3$) mixing matrix, apart from sign convention redefinitions:

$$U_{fi} = \begin{pmatrix} c_{12} & s_{12} & u \\ -(s_{12}+c_{12}u^*)/\sqrt{2} & (c_{12}-s_{12}u^*)/\sqrt{2} & 1/\sqrt{2} \\ (s_{12}-c_{12}u^*)/\sqrt{2} & -(c_{12}+s_{12}u^*)/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}, \tag{6}$$

where θ_{12} is close to $\pi/6$ (for $s_{12} = 1/\sqrt{3}$ and $u = 0$ we have the so-called tri-bimaximal mixing pattern (for models based on discrete symmetries see also [27]), with the entries in the second column all equal to $1/\sqrt{3}$ in absolute value). Given the observed frequencies and our notation in equation (5), there are three possible patterns of mass eigenvalues:

$$\begin{aligned} &\text{Degenerate:} && |m_1| \sim |m_2| \sim |m_3| \gg |m_i - m_j|, \\ &\text{Inverted hierarchy:} && |m_1| \sim |m_2| \gg |m_3|, \\ &\text{Normal hierarchy:} && |m_3| \gg |m_{2,1}|. \end{aligned} \tag{7}$$

Models based on all these patterns have been proposed and studied and all are in fact viable at present. In the following we will first discuss neutrino masses in general and, in particular, Majorana neutrinos. Then we recall the existing constraints on the absolute scale of neutrino masses. We then discuss the importance of neutrinoless double beta decay that, if observed, would confirm the Majorana nature of neutrinos. Also the knowledge of the rate of this process could discriminate among the possible patterns of neutrino masses in (7). The possible importance of heavy Majorana neutrinos for the explanation of baryogenesis through leptogenesis in the early Universe will be briefly discussed. We finally review the phenomenology of neutrino mass models based on the three spectral patterns in (7) and the respective advantages and problems.

3. Neutrino masses and lepton number violation

Neutrino oscillations imply neutrino masses which in turn demand either the existence of right-handed (RH) neutrinos (Dirac masses) or lepton number L violation (Majorana masses) or both. Given that neutrino masses are certainly extremely small, it is really difficult from the theory point of view to avoid the conclusion that L conservation must be violated. In fact, in terms of lepton number violation the smallness of neutrino masses can be explained as inversely proportional to the very large scale where L is violated, of order M_{GUT} or even M_{Pl}.

Once we accept L non-conservation, we gain an elegant explanation for the smallness of neutrino masses. If L is not conserved, even in the absence of heavy RH neutrinos, Majorana masses for neutrinos can be generated by dimension-five operators [28] of the form

$$O_5 = \frac{(Hl)_i^{\mathrm{T}} \lambda_{ij} (Hl)_j}{\Lambda} + \mathrm{h.c.}, \tag{8}$$

with H being the ordinary Higgs doublet, l_i the SU(2) lepton doublets, λ a matrix in flavour space, Λ a large scale of mass of order M_{GUT} or M_{Pl} and a charge conjugation matrix C between the lepton fields is understood. Neutrino masses generated by O_5 are of the order $m_\nu \approx v^2/\Lambda$ for $\lambda_{ij} \approx \mathrm{O}(1)$, where $v \sim \mathrm{O}(100\,\mathrm{GeV})$ is the vacuum expectation value of the ordinary Higgs.

We consider that the existence of RH neutrinos ν^c is quite plausible because all GUT groups larger than SU(5) require them. In particular, the fact that ν^c completes the representation 16 of SO(10): $16 = \bar{5} + 10 + 1$, so that all fermions of each family are contained in a single representation of the unifying group is too impressive not to be significant. At least as a classification group SO(10) must be of some relevance. Thus, in the following, we assume that there are both ν^c and L non-conservation. With these assumptions the see-saw mechanism [5] is possible. Also to fix notations we recall that in its simplest form it arises as follows. Consider the $\mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)}$ invariant Lagrangian giving rise to Dirac and ν^c Majorana masses (for the time being we consider the ν (versus ν^c) Majorana mass terms as comparatively negligible):

$$\mathcal{L} = -\nu^{c\mathrm{T}} y_\nu (Hl) + \tfrac{1}{2} \nu^{c\mathrm{T}} M \nu^c + \mathrm{h.c.} \tag{9}$$

The Dirac mass matrix $m_D \equiv y_\nu v/\sqrt{2}$, originating from electroweak symmetry breaking, is, in general, non-hermitian and non-symmetric, whereas the Majorana mass matrix M is symmetric, $M = M^{\mathrm{T}}$. We expect the eigenvalues of M to be of order M_{GUT} or more because ν^c Majorana masses are $\mathrm{SU(3)} \times \mathrm{SU(2)} \times \mathrm{U(1)}$ invariant, hence unprotected and naturally of the order of the cutoff of the low-energy theory. Since all ν^c are very heavy we can integrate them away. For this purpose we write down the equations of motion for ν^c in the static limit, i.e. neglecting their kinetic terms,

$$-\frac{\partial \mathcal{L}}{\partial \nu^c} = y_\nu (Hl) - M\nu^c = 0. \tag{10}$$

From this, by solving for ν^c, we obtain

$$\nu^c = M^{-1} y_\nu (Hl). \tag{11}$$

We now replace in the Lagrangian, equation (9), this expression for ν^c and we get the operator O_5 of equation (8) with

$$\frac{2\lambda}{\Lambda} = -y_\nu^{\mathrm{T}} M^{-1} y_\nu, \tag{12}$$

and the resulting neutrino mass matrix reads

$$m_\nu = m_D^{\mathrm{T}} M^{-1} m_D. \tag{13}$$

This is the well-known see-saw mechanism result [5]: the light neutrino masses are quadratic in the Dirac masses and inversely proportional to the large Majorana mass. If some ν^c are massless or light they would not be integrated away but simply added to the light neutrinos. Notice that the above results hold true for any number n of heavy neutral fermions R coupled to the three known neutrinos. In this more general case, M is an $n \times n$ symmetric matrix and the coupling between heavy and light fields is described by the rectangular $n \times 3$ matrix m_D. Note that for $m_\nu \approx \sqrt{\Delta m^2_{atm}} \approx 0.05\,\mathrm{eV}$ and $m_\nu \approx m_D^2/M$ with $m_D \approx v \approx 200\,\mathrm{GeV}$ we find $M \approx 10^{15}\,\mathrm{GeV}$ which indeed is an impressive indication for M_{GUT}.

If additional non-renormalizable contributions to O_5, equation (8), are comparatively non-negligible, they should simply be added. For instance, in SO(10) or in left–right extensions of the SM, an $SU(2)_L$ triplet can couple to lepton doublets and may induce a sizeable contribution to neutrino masses. At the level of the low-energy effective theory, such contribution is still described by the operator O_5 of equation (8), obtained by integrating out the heavy $SU(2)_L$ triplet (which also acquires a VEV due to its coupling to the Higgs doublets). This contribution is called type II to be distinguished from that obtained by the exchange of RH neutrinos (type I). After elimination of the heavy fields, at the level of the effective low-energy theory, the two types of terms are equivalent. In particular, they have identical transformation properties under a chiral change of basis in flavour space. The difference is, however, that in the see-saw mechanism, the Dirac matrix m_D is presumably related to ordinary fermion masses because they are both generated by the Higgs mechanism and both must obey GUT-induced constraints. Thus, if we assume the see-saw mechanism in its simplest type I version, more constraints are implied.

4. Importance of neutrinoless double beta decay

Oscillation experiments do not provide information about the absolute neutrino spectrum and cannot distinguish between pure Dirac and Majorana neutrinos. From the endpoint of tritium beta decay spectrum we have an absolute upper limit of 2.2 eV (at 95% CL) on the mass of electron antineutrino [29], which, combined with the observed oscillation frequencies under the assumption of three CPT-invariant light neutrinos, represents also an upper bound on the masses of the other active neutrinos. Complementary information on the sum of neutrino masses is also provided by the galaxy power spectrum combined with measurements of the cosmic microwave background anisotropies. According to the recent analysis of the WMAP collaboration [8], $\sum_i |m_i| < 0.69\,\mathrm{eV}$ (at 95% CL). More conservative analyses [9] give $\sum_i |m_i| < 1.01\,\mathrm{eV}$, still much more restrictive than the laboratory bound.

The discovery of $0\nu\beta\beta$ decay would be very important because it would establish lepton number violation and the Majorana nature of νs, and provide direct information on the absolute scale of neutrino masses. As already mentioned, the present limit from $0\nu\beta\beta$ is $|m_{ee}| < 0.2\,\mathrm{eV}$ or to be more conservative $|m_{ee}| < 0.3$–$0.5\,\mathrm{eV}$ [30, 31]. Note, however, that the WMAP limit implies for three degenerate νs $|m| < 0.23\,\mathrm{eV}$ and, taken at face value, this limit would pose a direct constraint on m_{ee}.

It is interesting to see what is the level at which a signal can be expected or at least not excluded in the different classes of models in (7) [32, 33]. The quantity which is bound by experiments is the 11 entry of the ν mass matrix, which in general, from equations (2) and (4), is given by

$$|m_{ee}| = |(1 - s_{13}^2)(m_1 c_{12}^2 + m_2 s_{12}^2) + m_3 e^{2i\phi} s_{13}^2|. \tag{14}$$

For three-neutrino models with degenerate, inverse hierarchy or normal hierarchy mass patterns, starting from this general formula it is simple to derive the following bounds.

(a) *Degenerate case.* If $|m|$ is the common mass and we take $s_{13} = 0$, which is a safe approximation in this case, because $|m_3|$ cannot compensate for the smallness of s_{13}, we have $m_{ee} \sim |m|(c_{12}^2 \pm s_{12}^2)$. Here the phase ambiguity has been reduced to a sign ambiguity which is sufficient for deriving bounds. So, depending on the sign we have $m_{ee} = |m|$ or $m_{ee} = |m| \cos 2\theta_{12}$. We conclude that in this case m_{ee} could be as large as the present experimental limit but should be at least of order $\mathrm{O}(\sqrt{\Delta m_{atm}^2}) \sim \mathrm{O}(10^{-2}\,\mathrm{eV})$ unless the solar angle is practically maximal, in which case the minus sign option can be arbitrarily small. However, the experimental 2σ range of the solar angle does not favour a cancellation by more than a factor of 3.

(b) *Inverse hierarchy case.* In this case, the same approximate formula $m_{ee} = |m|(c_{12}^2 \pm s_{12}^2)$ holds because m_3 is small and s_{13} can be neglected. The difference is that here we know that $|m| \approx \sqrt{\Delta m_{atm}^2}$ so that $|m_{ee}| < \sqrt{\Delta m_{atm}^2} \sim 0.05\,\mathrm{eV}$. At the same time, since a full cancellation between the two contributions cannot take place, we expect $|m_{ee}| > 0.01\,\mathrm{eV}$.

(c) *Normal hierarchy case.* Here we cannot in general neglect the m_3 term. However, in this case, $|m_{ee}| \sim |\sqrt{\Delta m_{sun}^2} s_{12}^2 \pm \sqrt{\Delta m_{atm}^2} s_{13}^2|$ and we have the bound $|m_{ee}| <$ a few $10^{-3}\,\mathrm{eV}$.

Recently, evidence for $0\nu\beta\beta$ was claimed in [34] at the 4.2σ level corresponding to $|m_{ee}| \sim 0.2$–$0.6\,\mathrm{eV}$ (0.1–$0.9\,\mathrm{eV}$ in a more conservative estimate of the involved nuclear matrix elements). If confirmed this would rule out cases (b) and (c) and point to case (a) or to models with more than three neutrinos. We recall that further contributions to $0\nu\beta\beta$ transition amplitudes might occur in models with additional L violating interactions, such as R-parity breaking supersymmetry.

5. Baryogenesis via leptogenesis from heavy ν^c decay

In the Universe, we observe an apparent excess of baryons over antibaryons. It is appealing that one can explain the observed baryon asymmetry by dynamical evolution (baryogenesis) starting from an initial state of the Universe with zero baryon number. For baryogenesis, one needs the three famous Sakharov conditions: B violation, CP violation and no thermal equilibrium. In the history of the Universe, these necessary requirements could have occurred at different epochs. Note however that the asymmetry generated by one epoch could be erased at following epochs if not protected by some dynamical reason. In principle, these conditions could be verified in the SM at the electroweak phase transition. B is violated by instantons when kT is of the order of the weak scale (but $\mathrm{B} - \mathrm{L}$ is conserved), CP is violated by the CKM phase and sufficiently marked out-of-equilibrium conditions could be realized during the electroweak phase transition. So the conditions for baryogenesis at the weak scale in the SM superficially appear to be present. However, a more quantitative analysis [35] shows that baryogenesis is not possible in the SM because there is not enough CP violation and the phase transition is not sufficiently strong first-order, unless $m_H < 80\,\mathrm{GeV}$, which is by now completely excluded by LEP. In SUSY extensions of the SM, in particular in the MSSM, there are additional sources of CP violation and the bound on m_H is modified by a sufficient amount by the presence of scalars with large couplings to the Higgs sector, typically the s-top. What is required is that $m_h \sim 80$–$110\,\mathrm{GeV}$, a s-top not heavier than the top quark and, preferentially, a small $\tan\beta$. However, also this possibility has by now become at best marginal with the results from LEP2.

If baryogenesis at the weak scale is excluded by the data it can occur at or just below the GUT scale, after inflation. However, only that part with $|\mathrm{B}-\mathrm{L}|>0$ would survive and not be erased at the weak scale by instanton effects. Thus baryogenesis at $kT \sim 10^{10}$–10^{15} GeV needs $\mathrm{B}-\mathrm{L}$ violation at some stage like for m_ν if neutrinos are Majorana particles. The two effects could be related if baryogenesis arises from leptogenesis then converted into baryogenesis by instantons [6]. Recent results on neutrino masses are compatible with this elegant possibility [36]. Thus the case of baryogenesis through leptogenesis has been boosted by the recent results on neutrinos [37].

In leptogenesis the baryon asymmetry is produced by the out of equilibrium, CP and L-violating decays of heavy right-handed neutrinos ν^c. In the simplest cases the spectrum of RH neutrinos is assumed to be hierarchical, $M_1 \ll M_{2,3}$ and the mechanism is dominated by the lightest state, ν_1^c. Thus, the expected baryon asymmetry is proportional to the product $\epsilon_1\delta$ between the CP decay asymmetry

$$\epsilon_1 = \frac{\Gamma(\nu_1^c \to l) - \Gamma(\nu_1^c \to \bar{l})}{\Gamma(\nu_1^c \to l) + \Gamma(\nu_1^c \to \bar{l})}, \tag{15}$$

and the efficiency factor $\delta \leqslant 1$, the fraction of the produced asymmetry that survives after ν_1^c decay. To keep δ close to 1 and avoid washing out the developed lepton asymmetry, L-violating interactions should be sufficiently weak to stay out of equilibrium when ν_1^c decays. This in turn happens when the lifetime of ν_1^c exceeds the age of the Universe, which is expressed by the condition [38]

$$\tilde{m}_1 \equiv \frac{(m_D m_D^\dagger)_{11}}{M_1} \leqslant 10^{-3}\,\mathrm{eV}, \tag{16}$$

with $\min\{|m1|, |m2|, |m3|\} \leqslant \tilde{m}_1$. This condition does not provide yet an absolute bound on neutrino masses since, in realistic simulations, the observed baryon asymmetry is achieved for $\delta < 1$. For hierarchical RH neutrino masses the asymmetry ϵ_1 is given by

$$\epsilon_1 \approx -\frac{3\sigma}{8\pi}\frac{1}{(y_\nu y_\nu^\dagger)_{11}} \sum_{j=2,3} \mathrm{Im}[(y_\nu y_\nu^\dagger)_{1j}^2]\frac{M_1}{M_j} \tag{17}$$

($\sigma = 1$ in the supersymmetric case and 1/2 in the non-supersymmetric one) which gives rise to the Davidson–Ibarra (DI) bound [39]:

$$|\epsilon_1| \leqslant \frac{3\sigma}{8\pi}\frac{M_1}{\langle H^0\rangle^2}(\max\{|m1|, |m2|, |m3|\} - \min\{|m1|, |m2|, |m3|\}), \tag{18}$$

where $\langle H^0\rangle$ denotes the VEV of the Higgs doublet giving mass to the up-quarks. Several interesting constraints on the neutrino spectrum emerge from the above discussion. First of all, in the assumed limit $M_{2,3} \gg M_1$, the CP asymmetry vanishes for a completely degenerate light neutrino spectrum (of course, this limit is rather unnatural in the see-saw context). Moreover, depending on the details of the assumed cosmological scenario which fixes the initial abundance of RH neutrinos, the DI bound translates into a lower bound on the lightest RH neutrino mass M_1, ranging from approximately 10^7 to 10^9 GeV [39, 40]. Finally, an absolute upper bound on light neutrino masses can be derived. Indeed, if we enhance the absolute scale of light neutrino masses, on the one hand we depart from the out-of-equilibrium condition of equation (16) and, on the other, the DI bound, proportional to $\Delta m^2_{atm}/m_3$, becomes more and more stringent. Quantitative studies [40] show that

$$|m_i| < 0.12\text{–}0.15\,\text{eV}. \tag{19}$$

It should be stressed that these bounds hold under the assumption of strict hierarchy in the RH neutrino sector. Indeed, by relaxing the assumption $M_{2,3} \gg M_1$, there are important corrections to the expression of ϵ_1 in equation (17), which may receive a resonant enhancement and which do not vanish any longer for degenerate light neutrinos. For a moderate degeneracy among RH neutrinos, the DI bound is violated and M_1 can be considerably lower than 10^7–10^9 GeV [41]. At the same time, the enhanced CP-violating asymmetry allows a successful leptogenesis even for light neutrino masses around the eV scale [41]. In any case, it is impressive that the resulting range of neutrino masses is fully consistent with the results on neutrino oscillations.

6. Degenerate neutrinos

For degenerate neutrinos the average m^2 is much larger than the splittings. At first sight the degenerate case is the most appealing: the observation of nearly maximal atmospheric neutrino mixing and the more recent result that also the solar mixing is large suggests that all ν masses are nearly degenerate. We shall see that this possibility has become less attractive with the recent new experimental information.

It is clear that in the degenerate case the most probable origin of ν masses is from some dimension 5 operators $(Hl)_i^{\mathrm{T}}\lambda_{ij}(Hl)_j/\Lambda$ not related to the see-saw mechanism $m_\nu = m_D^{\mathrm{T}}M^{-1}m_D$. In fact, we expect the ν Dirac mass m_D not to be degenerate as for all other fermions and a conspiracy to reinstate a nearly perfect degeneracy between m_D and M, which arise from completely different physics, looks very implausible (see, however, [42]). Thus, in degenerate models, in general, there is no direct relation with Dirac masses of quarks and leptons and the possibility of a simultaneous description of all fermion masses within a grand unified theory is more remote [43] (examples of degenerate models are described in [44]).

The degeneracy of neutrinos should be guaranteed by some slightly broken symmetry. Models based on discrete or continuous symmetries have been proposed. For example, in the models of [45, 46], the symmetry is SO(3): in the unbroken limit neutrinos are degenerate and charged leptons are massless. When the symmetry is broken the charged lepton masses are much larger than neutrino splittings because the former are first-order, whereas the latter are second-order in the electroweak symmetry breaking. In this kind of model, the mixing angles are completely undetermined in the symmetric phase and they originate only in the spontaneously broken phase from a misalignment between the symmetry breaking terms for neutrinos and charged leptons.

In principle, when considering models with degenerate masses we must keep in mind that radiative corrections can modify mass splittings and mixing angles in the running from the high scale where neutrino masses are determined at the fundamental level (i.e. the heavy Majorana mass M or M_{GUT}) down to the electroweak scale [47]. These running effects can be evaluated by renormalization group techniques. The effects depend on the parameters

$A_{ab} = (m_a + m_b)/(m_a - m_b)$ and are, with good accuracy, proportional to $\epsilon = y_\tau^2/(16\pi^2)\log(M/m_Z)$, where y_τ is the τ-lepton Yukawa coupling. The value of ϵ is around 10^{-5} in the SM and larger by a factor $1 + \tan^2\beta$ in the MSSM. The corrections are negligible for $|A_{ab}\epsilon| \lesssim 0(1)$. When some of these quantities are large a rapid transition takes place towards a fixed point configuration of mixing angles that does not correspond to the observed pattern. In practice, given the present upper bounds on the degenerate neutrino common mass m_0 and the LA value of Δm^2_{sol}, the corrections are always negligible in the SM and can only become sizable in the MSSM if $\tan^2\beta$ is large, m_0 is close to its absolute upper bound and m_1 and m_2 (those entering in the smallest difference Δm^2_{sol}) are nearly equal in absolute value and sign. Note that these remarks do not include the effects from thresholds that could affect running in a significant way, depending on the details of the heavy and/or light particle spectrum.

The upper limit on the common value $|m|$ becomes particularly stringent if one adopts the cosmological WMAP bound $|m| < 0.23$ eV [8] (or the more conservative one $|m| < 0.34$ eV [9]). The more direct laboratory limit from tritium beta decay is $|m| < 2.2$ eV [29]. In past years degenerate models with ν masses as large as $|m| \sim 1$–2 eV were considered with the perspective of a large fraction of hot dark matter in the universe. In this case, however, the existing limit [30] on the absence of $0\nu\beta\beta$ ($|m_{ee}| < 0.2$ eV or to be more conservative $|m_{ee}| < 0.3$–0.5 eV) implies [31, 48] approximate double maximal mixing (bimixing) for solar and atmospheric neutrinos. As discussed in section 4, for $|m| \gg m_{ee}$, one needs $m_1 \approx -m_2$ and, to a good accuracy, $c_{12}^2 \approx s_{12}^2$, to satisfy the bound on m_{ee}. This is exemplified by the following texture:

$$m_\nu = m\begin{pmatrix} 0 & -1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & (1+\eta)/2 & (1+\eta)/2 \\ 1/\sqrt{2} & (1+\eta)/2 & (1+\eta)/2 \end{pmatrix}, \tag{20}$$

where $\eta \ll 1$, corresponding to an exact bimaximal mixing, $s_{13} = 0$ and the eigenvalues are $m_1 = m$, $m_2 = -m$ and $m_3 = (1+\eta)m$. This texture has been proposed in the context of a spontaneously broken SO(3) flavour symmetry and it has been studied to analyse the stability of the degenerate spectrum against radiative corrections [47, 49]. A more realistic mass matrix can be obtained by adding small perturbations to m_ν in equation (20):

$$m_\nu = m\begin{pmatrix} \delta & -1/\sqrt{2} & (1-\epsilon)/\sqrt{2} \\ -1/\sqrt{2} & (1+\eta)/2 & (1+\eta-\epsilon)/2 \\ (1-\epsilon)/\sqrt{2} & (1+\eta-\epsilon)/2 & (1+\eta-2\epsilon)/2 \end{pmatrix}, \tag{21}$$

where ϵ parametrizes the leading flavour-dependent radiative corrections (mainly induced by the τ Yukawa coupling) and δ controls m_{ee}. Consider first the case $\delta \ll \epsilon$. To first approximation θ_{12} remains maximal. We get $\Delta m^2_{sun} \approx m^2\epsilon^2/\eta$ and

$$\theta_{13} \approx \left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right)^{1/2}, \qquad m_{ee} \ll m\left(\frac{\Delta m^2_{atm}\Delta m^2_{sun}}{m^4}\right)^{1/2}. \tag{22}$$

If we instead assume $\delta \gg \epsilon$, we find $\Delta m^2_{sun} \approx 2m^2\delta$, $\theta_{23} \approx \pi/4$, $\sin^2 2\theta_{12} \approx 1 - \delta^2/4$. Also in this case the solar mixing angle remains unacceptably close to $\pi/4$, unless further contributions to the mixing matrix are induced from the diagonalization of the charged lepton sector. We get

$$\theta_{13} \approx 0, \qquad m_{ee} \approx \frac{\Delta m^2_{sun}}{2m}, \tag{23}$$

too small for detection if the average neutrino mass m is around the eV scale. We see that with increasing $|m|$ more and more fine tuning is needed to reproduce the LA solution values of Δm^2_{sun} and θ_{12}. In conclusion, even without invoking the WMAP limit, large ν masses, $|m| \sim 1$–2 eV, are disfavoured by the limit on $0\nu\beta\beta$ decay and by the emerging of the LA solution with the solar angle definitely not maximal. From these considerations, it is possible to derive the bound $|m| < 0.9h$ eV (90% CL) [33], where $h \approx 1$ parametrizes the uncertainties in the nuclear matrix elements. Also, we have seen in section 5 that the attractive mechanism of baryogenesis through leptogenesis appears to disfavour $|m| \gtrsim 0.1$ eV, at least in the simplest realizations. All together, after WMAP and KamLAND, among degenerate models those with $|m| \lesssim 0.23$–1 eV are favoured by converging evidence from different points of view.

For $|m|$ not larger than the $0\nu\beta\beta$ bound, one does not need a cancellation in m_{ee} and m_1 and m_2 can be approximately equal in magnitude and phase. For example, in the limit $s_{13} = 0$, the matrix

$$m_\nu = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} \tag{24}$$

corresponds to maximal θ_{23} (pseudo-Dirac 23 submatrix) with $\mathrm{Diag}[m_\nu] = m(1, 1, -1)$. The angle θ_{12} is unstable and a small perturbation can give any value to it. Note, however, that in this case the non-vanishing matrix elements must be of equal absolute value and not just of order 1. So either this is guaranteed by a symmetry (as, for example, in [45]) or the model is unnaturally fine-tuned.

As a different example (also with no cancellation between m_1 and m_2), a model, which is simple to describe but difficult to derive in a natural way, is one [27, 50] where up-quarks, down-quarks and charged leptons have 'democratic' mass matrices, with all entries equal (in first approximation):

$$m_f = \hat{m}_f \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \delta m_f, \tag{25}$$

where $\hat{m}_f$ ($f = u, d, e$) are three overall mass parameters and δm_f denote small perturbations. If we neglect δm_f, the eigenvalues of m_f are given by $(0, 0, 3\hat{m}_f)$. The mass matrix m_f is diagonalized by a unitary matrix U_f which is in part determined by the small term δm_f. If $\delta m_u \approx \delta m_d$, the CKM matrix, given by $V_{CKM} = U_u^\dagger U_d$, is nearly diagonal, due to a compensation between the large mixings contained in U_u and U_d. When the small terms δm_f are diagonal and of the form $\delta m_f = \mathrm{Diag}(-\epsilon_f, \epsilon_f, \delta_f)$ with $\delta_f \gg \epsilon_f$, the matrices U_f are approximately given by (note the analogy with the quark model eigenvalues π^0, η and η'):

$$U_f^\dagger \approx \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \end{pmatrix}. \tag{26}$$

Note that, due to the degeneracy in the 1, 2 sector in the unperturbed limit, any superposition of the first two rows would also be an eigenvector of zero mass. Thus, the choice $(U_f^\dagger)_{13} = 0$ is unjustified in the unperturbed limit and is only determined by a particular form of the perturbation.

At the same time, the lightest quarks and charged leptons acquire a non-vanishing mass. The leading part of the mass matrix in equation (25) is invariant under a discrete $S_{3L} \times S_{3R}$ permutation symmetry. The same requirement leads to the general neutrino mass matrix:

$$m_\nu = m\left[\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} + r\begin{pmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}\right] + \delta m_\nu, \tag{27}$$

where δm_ν is a small symmetry breaking term and the two independent invariants are allowed by the Majorana nature of the light neutrinos. If r vanishes the neutrinos are almost degenerate. In the presence of δm_ν the permutation symmetry is broken and the degeneracy is removed. If, for example, we choose $\delta m_\nu = \mathrm{Diag}(0, \epsilon, \eta)$ with $\epsilon < \eta \ll 1$ and $r \ll \epsilon$, the solar and the atmospheric oscillation frequencies are determined by ϵ and η, respectively. The problems with this class of models are that the solar angle should be close to maximal (typically more than the atmospheric angle) and that the neutrino spectrum and mixing angles are not determined by the symmetric limit (this applies, for example, to $s_{13} \sim 0$) but only by a specific choice of the parameter r and of the perturbations that cannot be easily justified on theoretical grounds. Notice also that the simplest choice of parameters leads to $\sin^2 2\theta_{23}$ very close to 8/9, value which is now excluded at the 2σ level.

In this model, the mixing angles are almost entirely due to the charged lepton sector. Recently, the question of whether observed neutrino mixings can dominantly arise from the charged lepton sector in a natural way was studied in general in [51]. Of course, one can always choose an *ad hoc* basis where this is true: the point is to decide whether this formal choice can be naturally justified in the physical basis where the symmetries of the Lagrangian are specified. The conclusion is that in presence of two large mixing angles θ_{12} and θ_{23} with the third angle θ_{13} being small, the construction of a natural model with dominance of U_e is made much more difficult than in the case of only the atmospheric angle θ_{23} large. Examples of natural models of this sort can be given [46], [51]–[53] and the stated difficulty is reflected in the relatively complicated symmetry structure required.

In conclusion, the parameter space for degenerate models has recently become smaller because of the indications from WMAP (and also, to some extent, from leptogenesis) that tend to lower the maximum common mass allowed for light neutrinos. It is also rather difficult to reproduce the observed pattern of frequencies and mixing angles, in particular two large and one small mixing angle, with the solar angle large but not maximal. Degenerate models that fit can only arise from a very special dynamics or a non-abelian flavour symmetry with suitable breakings.

6.1. Anarchy

Anarchical models [54] can be considered as particular cases of degenerate models with $m^2 \sim \Delta m^2_{atm}$. In this class of models mass degeneracy is replaced by the principle that all mass matrices are structureless in the neutrino sector (including the LH charged fermions and possibly the RH neutrinos). For the LA solution the ratio of the solar and atmospheric frequencies is not so small: $r = (\Delta m^2_{sun})_{LA}/\Delta m^2_{atm} \sim 1/40$ and two out of three mixing angles are large. The key observation is that the see-saw mechanism tends to enhance the ratio of eigenvalues: it is quadratic in m_D so that a hierarchy factor f in m_D becomes f^2 in m_ν and the presence of the Majorana matrix M results in a further widening of the distribution. Another squaring takes place in going from the masses to the oscillation frequencies which are quadratic. As a result, a random generation of the m_D and M matrix elements leads to a distribution of r that peaks around 0.1. At the same time, the distribution of $\sin^2\theta_{ij}$ is rather flat for all three mixing angles. Clearly, the smallness of θ_{13} is problematic for anarchy. This can be turned into the prediction that in anarchical models θ_{13} must be near the present bound (after all, the value 0.2 for $\sin\theta_{13}$ is not that smaller than the maximal value 0.707). In conclusion, if θ_{13} is near the present bound, one can argue that the neutrino masses and mixings, interpreted by the see-saw mechanism, can just arise from structureless underlying Dirac and Majorana matrices.

7. Inverted hierarchy

The inverted hierarchy configuration $|m_1| \sim |m_2| \gg |m_3|$ consists of two levels m_1 and m_2 with small splitting $\Delta m^2_{12} = \Delta m^2_{sun}$ and a common mass given by $|m^2_{1,2}| \sim |\Delta m^2_{atm}| \sim 2.6 \times 10^{-3}\,\mathrm{eV}^2$. One particularly interesting example of this sort [55], which leads to double maximal mixing, is obtained with the phase choice $m_1 = -m_2$ so that, approximately,

$$m_{diag} = \mathrm{Diag}(\sqrt{2}m, -\sqrt{2}m, 0). \tag{28}$$

The effective light neutrino mass matrix

$$m_\nu = U^* m_{diag} U^\dagger, \tag{29}$$

which corresponds to the mixing matrix of double maximal mixing $c_{12} = s_{12} = 1/\sqrt{2}$ and $s_{13} = u = 0$ in equation (6):

$$U_{fi} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/2 & 1/2 & 1/\sqrt{2} \\ 1/2 & -1/2 & 1/\sqrt{2} \end{pmatrix}, \tag{30}$$

is given by

$$m_\nu = m \begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \tag{31}$$

This texture for m_ν can be reproduced by imposing a U(1) flavour symmetry with charge $L_e - L_\mu - L_\tau$ starting either from $(Hl)_i^{\mathrm{T}}\lambda_{ij}(Hl)_j/\Lambda$ or from RH neutrinos via the see-saw mechanism. However, the absolute values of the 12 and 13 terms would be in general different in this case. As a consequence, while the vanishing of s_{13} and the maximal value of θ_{12} are still valid, the atmospheric angle deviates from the maximal value with $\tan\theta_{23} = x$ where x is the absolute value of the ratio $m_{\nu 13}/m_{\nu 12}$. We also note that the 1–2 degeneracy remains stable under radiative corrections [47, 49].

The leading texture in (31) can be perturbed by adding small terms:

$$m_\nu = m \begin{pmatrix} \delta & -1 & 1 \\ -1 & \eta & \eta \\ 1 & \eta & \eta \end{pmatrix}, \tag{32}$$

where δ and η are small ($\ll 1$), real parameters defined up to coefficients of order 1 that can differ in the various matrix elements. One could also make the absolute values of the 12, 13 terms different by terms of order $s_{13}\delta$. The perturbations leave Δm^2_{atm} and θ_{23} unchanged, in first approximation. We obtain $\tan^2\theta_{12} \approx 1+\delta+\eta$ and $\Delta m^2_{sun}/\Delta m^2_{atm} \approx \eta+\delta$, where coefficients of order one have been neglected. Moreover $\theta_{13} \approx \eta$. If $\eta \gg \delta$, we have

$$\theta_{13} \approx \frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}, \qquad m_{ee} \ll \sqrt{\Delta m^2_{sun}} \left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right)^{1/2}. \tag{33}$$

In the other case, $\eta \ll \delta$ we obtain

$$\theta_{13} \ll \frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}, \qquad m_{ee} \approx \frac{1}{2}\sqrt{\Delta m^2_{sun}} \left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right)^{1/2}. \tag{34}$$

There is a well-known difficulty of this scenario to fit the LA solution [55]–[57]. Indeed, barring cancellation between the perturbations, in order to obtain a Δm^2_{sun} close to the best-fit LA value, η and δ should be smaller than about 0.1 and this keeps the value of $\sin^2 2\theta_{12}$ very close to 1, in disagreement with global fits of solar data [23]. Notice that the required deviation of the solar mixing angle from the maximal value is of the order of the Cabibbo angle θ_C and indeed the empirical relation $\theta_{12}+\theta_C = \pi/4$ holds within the experimental errors [58]. However, even starting from exact bimixing the pattern of parameters needed to bring the solar angle down from the maximal value can be obtained in a natural way as an effect of the charged lepton matrix diagonalization (see e.g. [56, 57]). This possibility, studied in detail in [51, 59], is not excluded but is strongly constrained by the observed smallness of s_{13}, as in general the amount of deviation from maximal solar angle is typically of order s_{13} (whereas the deviation from maximal atmospheric mixing are of second order).

With the phase choice $m_1 = m_2$, i.e. for $\mathrm{Diag}[m_\nu] = m(1, 1, 0)$, in the limit $s_{13} = 0$, one obtains the matrix

$$m_\nu = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & -1/2 \\ 0 & -1/2 & 1/2 \end{pmatrix}, \tag{35}$$

which corresponds to large θ_{23} with the solar angle unstable to small perturbations. In this case, fine-tuning or a symmetry is necessary to fix the ratios of the matrix elements as indicated.

In conclusion, also for inverse hierarchy some special dynamics or symmetry is needed to reproduce in detail the observed features of the data.

8. Normal hierarchy

We now discuss the class of models which we consider the simplest approach to neutrino masses and mixings. In particular, in this context, one can formulate the most constrained framework which allows a comprehensive combined study of all fermion masses in GUTs. We start by assuming three widely split νs and the existence of a RH neutrino for each generation, as required to complete a 16-dimensional representation of SO(10) for each generation. We then assume dominance of the see-saw mechanism $m_\nu = m_D^T M^{-1} m_D$. We know that the third-generation eigenvalue of the Dirac mass matrices of up- and down-quarks and of charged leptons is systematically the largest one. It is natural (although not necessary) to imagine that this property could also be true for the Dirac mass of νs: $m_D^{diag} \sim \mathrm{Diag}(0, 0, m_{D3})$. After see-saw we expect m_ν to be even more hierarchical being quadratic in m_D (barring fine-tuned compensations between m_D and M). Note however that for the LA solution, $r \sim 1/40$, so that the required amount of hierarchy, $r = \Delta m^2_{atm}/\Delta m^2_{sun} = m_3^2/m_2^2$ is quite moderate.

A possible difficulty for hierarchical models is that one is used to expect that large splittings correspond to small mixings because normally only close-by states are strongly mixed. In a 2×2 matrix context the requirement of large splitting and large mixings leads to a condition of vanishing determinant and large off-diagonal elements. For example, the matrix

$$\begin{pmatrix} x^2 & x \\ x & 1 \end{pmatrix} \tag{36}$$

has eigenvalues 0 and $1 + x^2$ and for x of O(1) the mixing is large. Thus, in the limit of neglecting small mass terms of order $m_{1,2}$, the demands of large atmospheric neutrino mixing and dominance of m_3 translate into the condition that the 2×2 subdeterminant 23 of the 3×3 mixing matrix approximately vanishes. The problem is to show that this vanishing can be arranged in a natural way without fine-tuning. Once near-maximal atmospheric neutrino mixing is reproduced the solar neutrino mixing can be arranged to be either small or large without difficulty by implementing suitable relations among the small mass terms.

It is not difficult to imagine mechanisms that naturally lead to the approximate vanishing of the 23 subdeterminant. For example, in [60, 61], it is assumed that one ν^c is particularly light and coupled to μ and τ. In a 2×2 simplified context if we have

$$M \propto \begin{pmatrix} \epsilon & 0 \\ 0 & 1 \end{pmatrix}, \qquad M^{-1} \approx \begin{pmatrix} 1/\epsilon & 0 \\ 0 & 0 \end{pmatrix}, \qquad m_D = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \tag{37}$$

then for a generic m_D we find

$$m_\nu = m_D^{\mathrm{T}} M^{-1} m_D \approx \frac{1}{\epsilon} \begin{pmatrix} a^2 & ab \\ ab & b^2 \end{pmatrix}. \tag{38}$$

A different possibility that we find attractive is that, in the limit of neglecting terms of order $m_{1,2}$ and, in the basis where charged leptons are diagonal, the Dirac matrix m_D, defined by $\nu^c m_D \nu$, takes the approximate form, called 'lopsided' [62]–[64]:

$$m_D \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & x & 1 \end{pmatrix}. \tag{39}$$

This matrix has the property that for a generic Majorana matrix M one finds

$$m_\nu = m_D^{\mathrm{T}} M^{-1} m_D \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & x^2 & x \\ 0 & x & 1 \end{pmatrix}. \tag{40}$$

The only condition on M^{-1} is that the 33 entry is non-zero. However, when the approximately vanishing matrix elements are replaced by small terms, one must also assume that no new O(1) terms are generated in m_ν by a compensation between small terms in m_D and large terms in M^{-1}.

It is important for the following discussion to observe that m_D given by equation (39) under a change of basis transforms as $m_D \to V^\dagger m_D U$ where V and U rotate the right and left fields, respectively. It is easy to check that to make m_D diagonal we need large left mixings (i.e. large off-diagonal terms in the matrix that rotates LH fields). Thus, the question is how to reconcile large LH mixings in the leptonic sector with the observed near-diagonal form of V_{CKM}, the quark mixing matrix. Strictly speaking, since $V_{CKM} = U_u^\dagger U_d$, the individual matrices U_u and U_d need not be near-diagonal, but V_{CKM} does, whereas the analogue for leptons apparently cannot be near-diagonal. However, for quarks nothing forbids that, in the basis where m_u is diagonal, the d quark matrix has large non-diagonal terms that can be rotated away by a pure RH rotation. We suggest that this is so and that in some way RH mixings for quarks correspond to LH mixings for leptons.

In the context of (SUSY) SU(5), there is a very attractive hint of how this sort of mechanism can be realized [65, 66]. In the $\bar{5}$ of SU(5) the d^c singlet appears together with the lepton doublet (ν, e). The (u, d) doublet and e^c belong to the 10 and ν^c to the 1 and similarly for the other families. As a consequence, in the simplest model with mass terms arising from only Higgs pentaplets, the Dirac matrix of down-quarks is the transpose of the charged lepton matrix: $m_d = (m_l)^{\mathrm{T}}$. Thus, indeed, a large mixing for RH down-quarks corresponds to a large LH mixing for charged leptons. At leading order we may have the lopsided texture:

$$m_d = (m_l)^{\mathrm{T}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} v_d. \tag{41}$$

In the same simplest approximation with 5 or $\bar{5}$ Higgs, the up-quark mass matrix is symmetric, so that left and right mixing matrices are equal in this case. Then small mixings for up-quarks and small LH mixings for down-quarks are sufficient to guarantee small V_{CKM} mixing angles even for large d quark RH mixings. It is well known that a model where the down and the charged lepton matrices are exactly the transpose of one another cannot be exactly true because of the e/d and μ/s mass ratios. It is also known that one remedy to this problem is to add some Higgs component in the 45 representation of SU(5) [67]. However, the symmetry under transposition can still be a good guideline if we are only interested in the order of magnitude of the matrix entries and not in their exact values. In models with $U(1)_F$ flavour symmetry, equal second- and third-generation charges for the $\bar{5}$ could induce a lopsided form for both the charged leptons and the Dirac neutrino mass matrices. Otherwise, in the very crude model where the Higgs pentaplets come from a pure 10 representation of SO(10) one has $m_D = m_u$, i.e. the Dirac neutrino mass matrix m_D is the same as the up-quark mass matrix . For m_D the dominance of the third family eigenvalue as well as a near-diagonal form could be an order of magnitude remnant of this broken symmetry.

To get a realistic mass matrix, we allow for deviations from the symmetric limit of (40) with $x \sim \mathrm{o}(1)$. For instance, we can consider those models where the neutrino mass matrix elements are dominated, via the see-saw mechanism, by the exchange of two right-handed neutrinos [61]. Since the exchange of a single RH neutrino gives a successful zeroth-order texture, we are encouraged to continue along this line. Thus, we add a sub-dominant contribution of a second RH neutrino, assuming that the third one gives a negligible contribution to the neutrino mass matrix, because it has much smaller Yukawa couplings or is much heavier than the first two. The Lagrangian that describes this plausible subset of see-saw models, written in the mass eigenstate basis of RH neutrinos and charged leptons, is

$$\mathcal{L} = y_i \nu^c H l_i + y_i' \nu^{c\prime} H l_i + \tfrac{1}{2} M \nu^{c2} + \tfrac{1}{2} M' \nu^{c\prime 2}, \tag{42}$$

leading to

$$(m_\nu)_{ij} \propto \frac{y_i y_j}{M} + \frac{y_i' y_j'}{M'}, \tag{43}$$

where $i, j = \{e, \mu, \tau\}$. In particular, if $y_e \ll y_\mu \approx y_\tau$ and $y_\mu' \approx y_\tau'$, we obtain

$$m_\nu = m \begin{pmatrix} \delta & \epsilon & \epsilon \\ \epsilon & 1+\eta & 1+\eta \\ \epsilon & 1+\eta & 1+\eta \end{pmatrix}, \tag{44}$$

where coefficients of order 1 multiplying the small quantities δ, ϵ and η have been omitted. The 23 subdeterminant is generically of order η. The mass matrix in (44) does not describe the most general perturbation of the zeroth-order texture (40). We have implicitly assumed a symmetry between ν_μ and ν_τ which is preserved by the perturbations, at least at the level of the order of magnitudes. The perturbed texture (44) can also arise when the zeros of the lopsided Dirac matrix in (39) are replaced by small quantities. It is possible to construct models along this line based on a spontaneously broken $\mathrm{U}(1)_F$ flavour symmetry, where δ, ϵ and η are given by positive powers of one or more symmetry-breaking parameters. Moreover, by playing with the $\mathrm{U}(1)_F$ charges, we can adjust, to certain extent, the relative hierarchy between η, ϵ and δ [60, 61], [63]–[66], as we will see in section 8. The texture (44) can also be generated in SUSY models with R-parity violation [68].

Let us come back to the mass matrix m_ν of equation (44). After a first rotation by an angle θ_{23} close to $\pi/4$ and a second rotation with $\theta_{13} \approx \epsilon$, we get

$$m_\nu \approx m \begin{pmatrix} \delta+\epsilon^2 & \epsilon & 0 \\ \epsilon & \eta & 0 \\ 0 & 0 & 2 \end{pmatrix}, \tag{45}$$

up to order 1 coefficients in the small entries. To obtain a large solar mixing angle, we need $|\eta - \delta| < \epsilon$. In realistic models, there is no reason for a cancellation between independent perturbations and thus we assume both $\delta \leqslant \epsilon$ and $\eta \leqslant \epsilon$.

Consider first the case $\delta \approx \epsilon$ and $\eta < \epsilon$. The solar mixing angle θ_{12} is large but not maximal, as indicated by the LA solution. We also have $\Delta m^2_{atm} \approx 4m^2$, $\Delta m^2_{sun} \approx \Delta m^2_{atm}\epsilon^2$ and

$$m_{ee} \approx \sqrt{\Delta m^2_{sun}}. \tag{46}$$

If $\eta \approx \epsilon$ and $\delta \ll \epsilon$, we still have a large solar mixing angle and $\Delta m^2_{sun} \approx \epsilon^2 \Delta m^2_{atm}$, as before. However, m_{ee} will be much smaller than the estimate in (46). This is the case of the models based on the above mentioned $\mathrm{U}(1)_F$ flavour symmetry that, at least in its simplest realization, tends to predict $\delta \approx \epsilon^2$. In this class of models we find

$$m_{ee} \approx \sqrt{\Delta m^2_{sun}} \left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}} \right)^{1/2}, \tag{47}$$

below the sensitivity of the next generation of planned experiments. It is worth mentioning that in both cases discussed above, we have

$$\theta_{13} \approx \left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right)^{1/2}, \tag{48}$$

which might be close to the present experimental limit if the oscillation frequency of the LA solution for solar neutrinos is in the upper part of its allowed range.

If both δ and η are much smaller than ϵ, the 12 block of m_ν has an approximate pseudo-Dirac structure and the angle θ_{12} becomes maximal. This situation is typical of some models where leptons have $U(1)_F$ charges of both signs, whereas the order parameters of $U(1)_F$ breaking have all charges of the same sign [65]. We have two eigenvalues approximately given by $\pm m\epsilon$. As an example, we consider the case where $\eta = 0$ and $\delta \approx \epsilon^2$. We find $\sin^2 2\theta_{12} \approx 1 - \epsilon^2/4$, $\Delta m^2_{sun} \approx m^2\epsilon^3$ and

$$\theta_{13} \approx \left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right)^{1/3}, \qquad m_{ee} \approx \sqrt{\Delta m^2_{sun}}\left(\frac{\Delta m^2_{sun}}{\Delta m^2_{atm}}\right)^{1/6}. \tag{49}$$

To recover the LA solution we would need a relatively large value of ϵ. However, this is in general problematic because, on the one hand the presence of a large perturbation raises doubts about the consistency of the whole approach and, on the other, in existing models where all fermion sectors are related to each other, ϵ is never larger than the Cabibbo angle.

Summarizing, within normal hierachical models, there is enough flexibility to reproduce in a natural way the experimental frequencies and mixing angles. In particular, the lopsided matrix solution of the large atmospheric mixing, inspired by SU(5), where the large atmospheric mixing arises from the charged lepton sector, can be extended rather naturally to also account for the solar sector and for the small θ_{13} mixing angle.

8.1. Semi-anarchy

We have seen that anarchy is the absence of structure in the neutrino sector. Here we consider an attenuation of anarchy where the absence of structure is limited to the 23 sector. The typical texture is in this case,

$$m_\nu \approx m \begin{pmatrix} \delta & \epsilon & \epsilon \\ \epsilon & 1 & 1 \\ \epsilon & 1 & 1 \end{pmatrix}, \tag{50}$$

where δ and ϵ are small and by 1 we mean entries of O(1) and also the 23 determinant is of O(1). We see that this texture is similar to equation (44) when $\eta \sim O(1)$. Clearly, in general, we would expect two mass eigenvalues of order 1, in units of m, and one small, of order δ or ϵ^2. This pattern does not fit the observed solar and atmospheric observed frequencies. However, given that the ratio $r = (\Delta m^2_{sun})_{LA}/\Delta m^2_{atm} \sim 1/40$ is not too small, we can assume that the small value of r is generated accidentally, as for anarchy. We see that, if we proceed with the same change of basis as from equation (44) to equation (45), it is sufficient that by chance $\eta \sim \delta + \epsilon^2$ in order to obtain the correct value of r with large θ_{23} and θ_{12} and small $\theta_{13} \sim \epsilon$. The natural smallness of θ_{13} is the main advantage over anarchy. We will come back to this class of models in a following section.

9. Grand unified models of fermion masses

We have seen that the smallness of neutrino masses interpreted via the see-saw mechanism directly leads to a scale Λ for L non-conservation which is remarkably close to M_{GUT}. Thus, neutrino masses and mixings should find a natural context in a GUT treatment of all fermion masses. The hierarchical pattern of quark and lepton masses, within a generation and across generations, requires some dynamical suppression mechanism that acts differently on the various particles. This hierarchy can be generated by a number of operators of different dimensions suppressed by inverse powers of the cut-off Λ_c of the theory. In some realizations, the different powers of $1/\Lambda_c$ correspond to different orders in some symmetry-breaking parameter v_f arising from the spontaneous breaking of a flavour symmetry. In the next subsections, we describe some simplest models based on $SU(5) \times U(1)_F$ and on SO(10) which illustrate these possibilities (for models based on non-abelian flavour symmetries see [69]). It is notoriously difficult to turn these models into fully realistic theories, due to well-known problems such as the doublet–triplet splitting, the proton lifetime, the gauge coupling unification beyond leading order and the wrong mass relations for charged fermions of the first two generations. Some of these problems can be solved by adopting the elegant idea of GUTs in extra dimensions [70]. Here we adopt the GUT framework simply as a convenient testing ground for different neutrino mass scenarios.

9.1. Models based on horizontal abelian charges

We discuss here some explicit examples of grand unified models in the framework of a unified SUSY SU(5) theory with an additional $U(1)_F$ flavour symmetry. The SU(5) generators act 'vertically' inside one generation, whereas the $U(1)_F$ charges are different 'horizontally' from one generation to the other. If, for a given interaction vertex, the $U(1)_F$ charges do not add to zero, the vertex is forbidden in the symmetric limit. However, the symmetry is spontaneously broken by the VEVs v_f of a number of 'flavon' fields with non-vanishing charge. Then a forbidden coupling is rescued but is suppressed by powers of the small parameters v_f/Λ_c with the exponents larger for larger charge mismatch [71]. We expect $M_{GUT} \lesssim v_f \lesssim \Lambda_c \lesssim M_{Pl}$. Here we discuss some aspects of the description of fermion masses in this framework.

In these models, the known generations of quarks and leptons are contained in triplets Ψ_i^{10} and $\Psi_i^{\bar{5}}$ ($i = 1, 2, 3$) corresponding to the three generations, transforming as 10 and $\bar{5}$ of SU(5), respectively. Three more SU(5) singlets Ψ_i^1 describe the RH neutrinos. In SUSY models, we have two Higgs multiplets H_u and H_d, which transform as 5 and $\bar{5}$ in the minimal model. The two Higgs multiplets may have the same or different charges. In all the models that we discuss the large atmospheric mixing angle is described by assigning equal flavour charge to muon and tau neutrinos and their weak SU(2) partners (all belonging to the $\bar{5} \equiv (l, d^c)$ representation of SU(5)). Instead, the solar neutrino oscillations can be obtained with different, inequivalent charge assignments. There are many variants of these models: fermion charges can all be non-negative with only negatively charged flavons, or there can be fermion charges of different signs with either flavons of both charges or only flavons of one charge. We can have that only the top quark mass is allowed in the symmetric limit, or that also other third-generation fermion masses are allowed. The Higgs charges can be equal, in particular both vanishing or can be different. We can arrange that all the structure is in charged fermion masses while neutrinos are anarchical.

9.1.1. F(fermions) $\geqslant$ 0. Consider, for example, a simple model with all charges of matter fields being non-negative and containing one single flavon $\bar{\theta}$ of charge F$= -1$. For a maximum of simplicity we also assume that all the third-generation masses are directly allowed in the symmetric limit. This is realized by taking vanishing charges for the Higgses and for the third-generation components Ψ_3^{10}, $\Psi_3^{\bar{5}}$ and Ψ_3^1. If we define $\mathrm{F}(\Psi_i^R) \equiv q_i^R$ $(R = 10, \bar{5}, 1; i = 1, 2, 3)$, then the generic mass matrix m has the form

$$m = \begin{pmatrix} y_{11}\lambda^{q_1^R+q_1^{R'}} & y_{12}\lambda^{q_1^R+q_2^{R'}} & y_{13}\lambda^{q_1^R+q_3^{R'}} \\ y_{21}\lambda^{q_2^R+q_1^{R'}} & y_{22}\lambda^{q_2^R+q_2^{R'}} & y_{23}\lambda^{q_2^R+q_3^{R'}} \\ y_{31}\lambda^{q_3^R+q_1^{R'}} & y_{32}\lambda^{q_3^R+q_2^{R'}} & y_{33}\lambda^{q_3^R+q_3^{R'}} \end{pmatrix} v, \tag{51}$$

where all the y_{ij} are dimensionless complex coefficients of order 1 and $m_u, m_d = m_l^{\mathrm{T}}, m_D$ and M arise by choosing $(R, R') = (10, 10), (\bar{5}, 10), (1, \bar{5})$ and $(1, 1)$, respectively. We have $\lambda \equiv \langle\bar{\theta}\rangle/\Lambda_c$ and the quantity v represents the appropriate VEV or mass parameter. The models with all non-negative charges and one single flavon have particularly simple factorization properties. For instance, in the see-saw expression for $m_\nu = m_D^{\mathrm{T}} M^{-1} m_D$, the dependence on the q_i^1 charges drops out and only that from $q_i^{\bar{5}}$ remains. In addition, for the neutrino mixing matrix U_{ij}, which is determined by m_ν in the basis where the charged leptons are diagonal, one can prove that $U_{ij} \approx \lambda^{|q_i^{\bar{5}} - q_j^{\bar{5}}|}$, in terms of the differences of the $\bar{5}$ charges, when terms that are down by powers of the small parameter λ are neglected. Similarly, the CKM matrix elements are approximately determined by only the 10 charges [71]: $V_{ij}^{CKM} \approx \lambda^{|q_i^{10} - q_j^{10}|}$. If the symmetry-breaking parameter λ is numerically close to the Cabibbo angle, we can choose

$$(q_1^{10}, q_2^{10}, q_3^{10}) = (3, 2, 0), \tag{52}$$

thus reproducing $V_{us} \sim \lambda$, $V_{cb} \sim \lambda^2$ and $V_{ub} \sim \lambda^3$. The same q_i^{10} charges also fix $m_u : m_c : m_t \sim \lambda^6 : \lambda^4 : 1$. The experimental value of m_u (the relevant mass values are those at the GUT scale: $m = m(M_{GUT})$ [72]) would rather prefer $q_1^{10} = 4$. Taking into account this indication and the presence of the unknown coefficients $y_{ij} \sim \mathrm{O}(1)$ it is difficult to decide between $q_1^{10} = 3$ or 4 and both are acceptable. Of course, the charges $(q_1^{10}, q_2^{10}, q_3^{10}) = (2, 1, 0)$ would represent an equally good choice, provided we appropriately rescale the expansion parameter λ. Turning to the $\bar{5}$ charges, if we take [63]–[65], [73, 74]

$$(q_1^{\bar{5}}, q_2^{\bar{5}}, q_3^{\bar{5}}) = (b, 0, 0), \quad b \geqslant 0, \tag{53}$$

together with equation (52) we get the patterns $m_d : m_s : m_b \sim m_e : m_\mu : m_\tau \sim \lambda^{3+b} : \lambda^2 : 1$. Moreover, the 22, 23, 32, 33 entries of the effective light neutrino mass matrix m_ν are all O(1), thus accommodating the nearly maximal value of s_{23}. The small non-diagonal terms of the charged lepton mass matrix cannot change this. We obtain, where arbitrary O(1) coefficients are omitted:

$$m_\nu = \begin{pmatrix} \lambda^{2b} & \lambda^b & \lambda^b \\ \lambda^b & 1 & 1 \\ \lambda^b & 1 & 1 \end{pmatrix} \frac{v_u^2}{\Lambda} \quad (\mathrm{A}, \mathrm{SA}), \tag{54}$$

where v_u is the VEVs of the Higgs doublet giving mass to the up-quarks and all the entries are specified up to order 1 coefficients. If we take $v_u \sim 250\,\mathrm{GeV}$, the mass scale Λ of the heavy Majorana neutrinos turns out to be close to the unification scale, $\Lambda \sim 10^{15}\,\mathrm{GeV}$.

If b vanishes, then the light neutrino mass matrix will be structureless and we recover the anarchical (A) picture of neutrinos discussed in section 6.1. In a large sample of anarchical models, generated with random coefficients, the resulting neutrino mass spectrum can exhibit either normal or inverse hierarchy. For down-quarks and charged leptons we obtain a weakened hierarchy, essentially the square root than that of up-quarks.

If b is positive, then the light neutrino mass matrix will be structureless only in the (2, 3) subsector and we get semi-anarchical (SA) models, introduced in section 8.1. In this case, the neutrino mass spectrum has normal hierarchy. However, unless the (2, 3) subdeterminant is accidentally suppressed, atmospheric and solar oscillation frequencies are expected to be of the same order and, in addition, the preferred solar mixing angle is small. Nevertheless, such a suppression can occur in a fraction of semi-anarchical models generated with random, order 1 coefficients. The real advantage over the fully anarchical scheme is represented by the suppression in U_{e3}.

Note that in all previous cases we could add a constant to $q_i^{\bar{5}}$, for example, by taking $(q_1^{\bar{5}}, q_2^{\bar{5}}, q_3^{\bar{5}}) = (2+b, 2, 2)$. This would only have the consequence to leave the top quark as the only unsuppressed mass and to decrease the resulting value of $\tan\beta = v_u/v_d$ down to $\lambda^2 m_t/m_b$. A constant shift of the charges q_i^1 might also provide a suppression of the leading ν^c mass eigenvalue, from Λ_c down to the appropriate scale Λ. One can also consider models where the 5 and $\bar{5}$ Higgs charges are different, as in the 'realistic' SU(5) model of Altarelli *et al* [75]. Also in these models, the top mass could be the only one to be non-vanishing in the symmetric limit and the value of $\tan\beta$ can be adjusted.

9.1.2. F(fermions) and F(flavons) of both signs. Models with naturally large 23 splittings are obtained if we allow negative charges and, at the same time, either introduce flavons of opposite charges or stipulate that matrix elements with overall negative charge are put to zero. For example, we can assign to the fermion fields the set of F charges given by

$$\begin{aligned}
&(q_1^{10}, q_2^{10}, q_3^{10}) = (3, 2, 0),\\
&(q_1^{\bar{5}}, q_2^{\bar{5}}, q_3^{\bar{5}}) = (b, 0, 0), \quad b \geqslant 2a > 0,\\
&(q_1^{1}, q_2^{1}, q_3^{1}) = (a, -a, 0).
\end{aligned} \tag{55}$$

We consider the Yukawa coupling allowed by $U(1)_F$-neutral Higgs multiplets in the 5 and $\bar{5}$ SU(5) representations and by a pair θ and $\bar{\theta}$ of SU(5) singlets with F = 1 and F = −1, respectively. If $b = 2$ or 3, the up, down and charged lepton sectors are not essentially different than in the SA case. Also in this case the O(1) off-diagonal entry of m_l, typical of lopsided models, gives rise to a large LH mixing in the 23 block which corresponds to a large RH mixing in the d mass matrix. In the neutrino sector, after diagonalization of the charged lepton sector and after integrating out the heavy RH neutrinos we obtain the following neutrino mass matrix in the low-energy effective theory:

$$m_\nu = \begin{pmatrix} \lambda^{2b} & \lambda^b & \lambda^b \\ \lambda^b & 1+\lambda^a\lambda'^a & 1+\lambda^a\lambda'^a \\ \lambda^b & 1+\lambda^a\lambda'^a & 1+\lambda^a\lambda'^a \end{pmatrix} \frac{v_u^2}{\Lambda} \quad (\mathrm{H}), \tag{56}$$

where λ' is given by $\langle\theta\rangle/\Lambda_c$ and Λ as before denotes the large mass scale associated to the RH neutrinos: $\Lambda \gg v_{u,d}$. The O(1) elements in the 23 block are produced by combining the large LH mixing induced by the charged lepton sector and the large LH mixing in m_D. A crucial property

Table 2. Models and their flavour charges.

Model	Ψ_{10}	$\Psi_{\bar{5}}$	Ψ_1	(H_u, H_d)
Anarchical (A)	(3, 2, 0)	(0, 0, 0)	(0, 0, 0)	(0, 0)
Semi-anarchical (SA)	(2, 1, 0)	(1, 0, 0)	(2, 1, 0)	(0, 0)
Hierarchical ($\mathrm{H_I}$)	(6, 4, 0)	(2, 0, 0)	(1, −1, 0)	(0, 0)
Hierarchical ($\mathrm{H_{II}}$)	(5, 3, 0)	(2, 0, 0)	(1, −1, 0)	(0, 0)
Inversely hierarchical ($\mathrm{IH_I}$)	(3, 2, 0)	(1, −1, −1)	(−1, +1, 0)	(0, +1)
Inversely hierarchical ($\mathrm{IH_{II}}$)	(6, 4, 0)	(1, −1, −1)	(−1, +1, 0)	(0, +1)

of m_ν is that, as a result of the see-saw mechanism and of the specific $\mathrm{U}(1)_F$ charge assignment, the determinant of the 23 block is automatically of $\mathrm{O}(\lambda^a\lambda'^a)$ (for this the presence of negative charge values, leading to the presence of both λ and λ' is essential [64, 65]). The neutrino mass matrix of equation (56) is a particular case of the more general pattern presented in equation (44), for $\delta \approx \lambda^{2b}$, $\epsilon \approx \lambda^b$ and $\eta \approx \lambda^a\lambda'^a$. If we take $\lambda \approx \lambda'$, it is easy to verify that the eigenvalues of m_ν satisfy the relations

$$m_1 : m_2 : m_3 = \lambda^{2(b-a)} : \lambda^{2a} : 1. \tag{57}$$

The atmospheric neutrino oscillations require $m_3^2 \sim 10^{-3}\,\mathrm{eV}^2$. The squared-mass difference between the lightest states is of $\mathrm{O}(\lambda^{4a})m_3^2$, not far from the LA solution to the solar neutrino problem if we choose $a = 1$. In general, U_{e3} is non-vanishing, of $\mathrm{O}(\lambda^b)$. Finally, beyond the large mixing in the 23 sector, m_ν provides a mixing angle $\theta_{12} \sim \lambda^{b-2a}$ in the 12 sector. When $b = 2a$, as for instance in the case $b = 2$ and $a = 1$, the LA solution can be reproduced and the resulting neutrino spectrum is hierarchical (H).

Alternatively, an inversely hierarchical (IH) spectrum can be obtained by choosing

$$\begin{aligned}
(q_1^{10}, q_2^{10}, q_3^{10}) &= (3, 2, 0), \\
(q_1^{\bar{5}}, q_2^{\bar{5}}, q_3^{\bar{5}}) &= (1, -1, -1), \\
(q_1^{1}, q_2^{1}, q_3^{1}) &= (-1, 1, 0), \\
(q_{H_u}, q_{H_d}) &= (0, 1).
\end{aligned} \tag{58}$$

Due to the non-vanishing charge of the H_d Higgs doublet, in the charged lepton sector, we recover the same pattern previously discussed. The light neutrino mass matrix is given by

$$m_\nu = \begin{pmatrix} \lambda^2 & 1 & 1 \\ 1 & \lambda'^2 & \lambda'^2 \\ 1 & \lambda'^2 & \lambda'^2 \end{pmatrix} \quad \text{(IH)}. \tag{59}$$

The ratio between the solar and atmospheric oscillation frequencies is not directly related to the subdeterminant of the block 23, in this case.

A representative set of models is listed in table 2. Note that in some cases the charges for Ψ_{10} have been changed from (3, 2, 0) (our reference values in equations (52), (55) and (58)) to (6, 4, 0) or (5, 3, 0). These values are *a posteriori* better suited to reproduce the moderate level of hierarchy implied by the present neutrino oscillation data. Since the neutrino mixing parameters are completely independent on the 10 charges, this change is only important for a better fit to quark and charged lepton masses and mixings once a rather large value of λ is derived from the neutrino data. The hierarchical and the inversely hierarchical models may come into several varieties depending on the number and the charge of the flavour symmetry-breaking (FSB) parameters. Above we have considered the case of two (II) oppositely charged flavons with symmetry-breaking parameters λ and λ'. It may be noticed that the presence of two multiplets θ and $\bar{\theta}$ with opposite F charges could hardly be reconciled, without adding extra structure to the model, with a large common VEV for these fields, due to possible analytic terms of the kind $(\theta\bar{\theta})^n$ in the superpotential. Therefore, it is instructive to explore the consequences of allowing only the negatively charged $\bar{\theta}$ field in the theory, case I. In case I, it is impossible to compensate negative F charges in the Yukawa couplings and the corresponding entries in the neutrino mass matrices vanish. Eventually, these zeros are filled by small contributions, arising, for instance, from the diagonalization of the charged lepton sector or from the transformations needed to make the kinetic terms canonical.

Another important ingredient is represented by the see-saw mechanism [5]. Hierarchical models and semi-anarchical models have similar charges in the $(10, \bar{5})$ sectors and, in the absence of the see-saw mechanism, they would give rise to similar results. Even when the results are expected to be independent of the charges of the RH neutrinos, as it is the case for the anarchical and semi-anarchical models, the see-saw mechanism can induce some sizeable effect in a statistical analysis. For this reason, for each type of model, but the normal-hierarchical ones (the mechanism for the 23 subdeterminant suppression is in fact based on the see-saw mechanism), it is interesting to study the case where RH neutrinos are present and the see-saw contribution is the dominant one (SS) and the case where they are absent and the mass matrix is saturated by the non-renormalizable contribution (NOSS).

With this classification in mind, we can distinguish the following type of models, all supported by specific choices of U(1) charges: $\mathrm{A_{SS}}$, $\mathrm{A_{NOSS}}$, $\mathrm{SA_{SS}}$, $\mathrm{SA_{NOSS}}$, $\mathrm{H_{(SS,I)}}$, $\mathrm{H_{(SS,II)}}$, $\mathrm{IH_{(SS,I)}}$, $\mathrm{IH_{(SS,II)}}$, $\mathrm{IH_{(NOSS,I)}}$ and $\mathrm{IH_{(NOSS,II)}}$.

It is interesting to quantify the ability of each model in reproducing the observed oscillation parameters. For anarchy, it has been observed that random-generated, order-one entries of the neutrino mass matrices (in appropriate units), correctly fit the experimental data with a success rate of few per cent. It is natural to extend this analysis to include also the other models based on $SU(5) \times U(1)$ [76], which have mass matrix elements defined up to order-one dimensionless coefficients y_{ij} (see equation (51)). For each model, successful points in parameter space are selected by asking that the four observable quantities $O_1 = r \equiv \Delta m^2_{12}/|\Delta m^2_{23}|$, $O_2 = \tan^2\theta_{12}$, $O_3 = |U_{e3}| \equiv |\sin\theta_{13}|$ and $O_4 = \tan^2\theta_{23}$ fall in the approximately 3σ allowed ranges [23, 24]:

$$
\begin{aligned}
&0.018 < r < 0.053, \qquad |U_{e3}| < 0.23,\\
&0.30 < \tan^2\theta_{12} < 0.64, \qquad 0.45 < \tan^2\theta_{23} < 2.57.
\end{aligned} \tag{60}
$$

The coefficients y_{ij} of the neutrino sector are random complex numbers with absolute values and phases uniformly distributed in intervals $\mathcal{I} = [0.5, 2]$ and $[0, 2\pi]$, respectively. The dependence of the results on these choices can be estimated by varying $\mathcal{I}$. For each model, an optimization procedure selects the value of the flavour symmetry-breaking parameter $\lambda = \lambda'$ that maximizes the success rate. The success rates are displayed in figures 1 and 2, separately for the SS and NOSS cases. The two sets of models have been individually normalized to give a total rate 100. From the histograms in figures 1 and 2 we see that normal hierarchy models are neatly

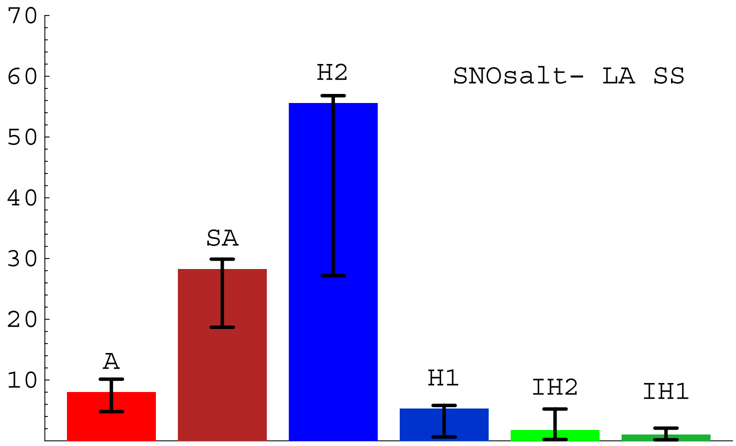

Figure 1. Relative success rates for the LA solution, with see-saw. The sum of the rates has been normalized to 100. The results correspond to the default choice $\mathcal{I} = [0.5, 2]$, and to the following values of $\lambda = \lambda'$: 0.2, 0.25, 0.35, 0.45, 0.45, 0.25 for the models $\mathrm{A_{SS}}$, $\mathrm{SA_{SS}}$, $\mathrm{H_{(SS,II)}}$, $\mathrm{H_{(SS,I)}}$, $\mathrm{IH_{(SS,II)}}$ and $\mathrm{IH_{(SS,I)}}$, respectively. The error bars represent the linear sum of the systematic error due to the choice of $\mathcal{I}$ and the statistical error (see text).

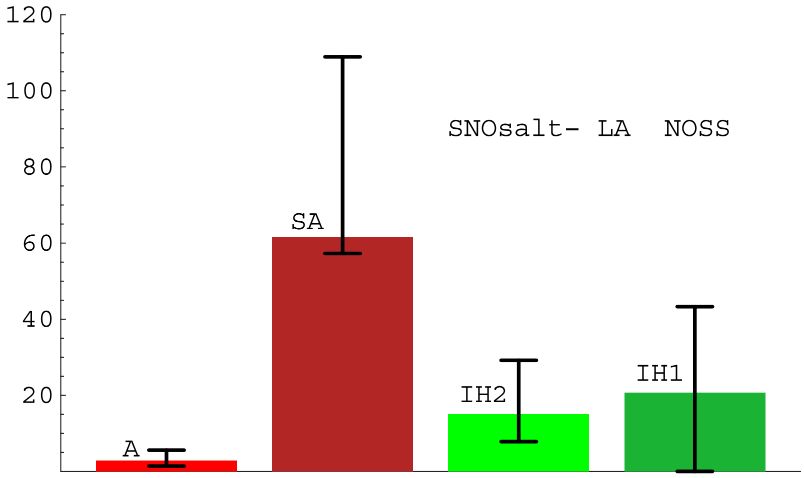

Figure 2. Relative success rates for the LA solution, without see-saw. The sum of the rates has been normalized to 100. The results correspond to the default choice $\mathcal{I} = [0.5, 2]$, and to the following values of $\lambda = \lambda'$: 0.2, 0.2, 0.25, 0.25 for the models $\mathrm{A_{NOSS}}$, $\mathrm{SA_{NOSS}}$, $\mathrm{IH_{(NOSS,II)}}$ and $\mathrm{IH_{(NOSS,I)}}$, respectively (in our notation there are no $\mathrm{H_{(NOSS,I)}}$, $\mathrm{H_{(NOSS,II)}}$ models). The error bars represent the linear sum of the systematic error due to the choice of $\mathcal{I}$ and the statistical error (see text).

preferred over anarchy and inverse hierarchy in the context of these $\mathrm{SU(5)} \times \mathrm{U(1)}$ models. In particular, in the SS case, the $\mathrm{H_{II}}$ models with normal hierarchy, two oppositely charged flavons and suppressed 23 subdeterminant are clearly preferred. Models of the type $\mathrm{H_I}$ are disfavoured. For the relatively large values of the expansion parameter required to fit r, they tend to predict too large $|U_{e3}|$ and $\tan^2\theta_{12} > 1$. We recall that for the chosen charge values the $\mathrm{H_{II}}$ model is of the lopsided type. In the NOSS case the see-saw suppression of the 23 determinant is clearly not operative and all normal hierarchy models coincide with SA.

An interesting question is whether the disfavouring of IH models that we find in our $SU(5) \times U(1)$ framework can be extended to a more general context. In the limit of vanishing λ and λ' the IH texture (see equation (59)) becomes close to that of bimaximal mixing and $\theta_{13} = 0$ (actually with $r = 0$). In our U(1) models $r \approx |U_{e3}| \approx |\tan^2 \theta_{12} - 1| \approx O(\lambda^2)$ (for $\lambda = \lambda'$). In particular, the charged lepton mixings cannot displace too much θ_{12} from its maximal value because the small value of the electron mass forces a sufficiently large value of the relevant charges, which in turn implies that the charged lepton mixing correction to θ_{12} is small. We have already mentioned that corrections from the charged lepton sector can, in principle, bring the predictions of a neutrino matrix of the bimixing type in agreement with the data and that the smallness of s_{13} induces strong constraints. In the particular set-up of $U(1)_F$ models, we have seen that charged lepton corrections are too small to make the solar angle sufficiently different from maximal.

Leptonic mixing in SUSY GUTs with RH neutrinos are potential sources of lepton flavour violating (LFV) processes beyond neutrino oscillations [79]. The observable effects are difficult to estimate since they are sensitive to both the details of the SUSY-breaking mechanism and to the low-energy superparticle spectrum. In models with gravity-mediated SUSY-breaking and universal boundary conditions for the soft breaking terms at the cut-off scale Λ, running effects give rise to off-diagonal slepton masses $\delta m^{2\,LL}_{ij}$ proportional, at leading order, to $C_{ij} = (y_\nu^\dagger \log(\Lambda/M) y_\nu)_{ij}$. Current bounds on LFV transitions $l_i \to l_j \gamma$ translate into an upper bound on the combination $|C_{ij}|$, depending on $\tan\beta$ and soft mass parameters. If, for instance, $\tan\beta = 10$, the present experimental bound on $BR(\mu \to e\gamma)$ already excludes $C_{21} > 1$ for the most plausible values of slepton and gaugino masses. In the models considered in this section C_{21} is dominated by the couplings to the RH neutrino of the third generation and, assuming $(\Lambda/M_3) \approx O(100)$, we roughly expect $|y^*_{32} y_{31}| < 0.2$. This constraint is most easily respected by models with inverse hierarchy. For semi-anarchy and normal hierarchy the constraint is almost saturated, whereas anarchy tends to violate it. Future improvements in the experimental sensitivity could lead to a significant selection of the models.

In conclusion, models based on $SU(5) \times U(1)_F$ are clearly toy models that can only aim at a semiquantitative description of fermion masses. In fact only the order of magnitude of each matrix entry can be specified. However, it is rather impressive that a reasonable description of fermion masses, now also including neutrino masses and mixings, can be obtained in this simple context, which is suggestive of a deeper relation between gauge and flavour quantum numbers. There are 12 mass eigenvalues and six mixing angles that are specified, modulo coefficients of order 1, in terms of a bunch of integer numbers (from half a dozen to a dozen), the charges, plus 1 or more scale parameters. Moreover, all possible type of mass hierarchies can be reproduced within this framework. In a statistically based comparison, the range of r and the small upper limit on U_{e3} are sufficiently constraining to make anarchy neatly disfavoured with respect to models with built-in hierarchy. If only neutrinos are considered, one might counterargue that hierarchical models have at least one more parameter than anarchy, in our case the parameter λ. However, if one looks at quarks and leptons together, as in the GUT models that we consider, then the same parameter that plays the role of an order parameter for the CKM matrix, for example, the Cabibbo angle, can be successfully used to reproduce also the hierarchy implied by the present neutrino data.

9.2. GUT models based on SO(10)

The fermion sector of SO(10) grand unified theories has remarkable properties. It is automatically anomaly-free, independent of the representation content. The RH neutrino provides the completion of a SM family into a 16 representation, thus offering a natural ground for the see-saw mechanism and baryogenesis through leptogenesis. Moreover, the scale of lepton number violation, determined by the gauge symmetry-breaking pattern, can be smaller than the cut-off Λ_c of the theory, and closer to the grand unification scale itself [80], as suggested by the mass scale associated to atmospheric neutrino oscillations.

In their simplest realizations, SO(10) models are left–right symmetric at the GUT scale and we would expect similar mixing angles for quarks and leptons of both chiralities. In left–right symmetric models, smallness of left mixings implies that also right-handed mixings are small, so that all mixings tend to be small, unless non-renormalizable mass operators with a suitable flavour pattern are introduced. In such a context, accommodating the observed mixing properties of quarks and leptons appear more problematic than in SU(5) theories, at first sight.

One possibility is to exploit the see-saw mechanism to enhance the light neutrino mixing angles. As illustrated in equation (38) in a 2×2 context, to have large or even maximal mixing in m_ν, we do not necessarily need large mixing angles in m_D and M. We can obtain a large mixing starting from nearly diagonal mass matrices m_D and M, provided a RH neutrino, equally coupled to ν_μ and ν_τ, is sufficiently light [81, 82] (for an example in the context of SO(10), see e.g. [83]).

Another possibility is to argue that perhaps what appears to be large is not that large after all. The typical small parameter that appears in the mass matrices is $\lambda \sim \sqrt{m_d/m_s} \sim \sqrt{m_\mu/m_\tau} \sim 0.20$–$0.25$. This small parameter is not so small that it cannot become large due to some peculiar accidental enhancement: either a coefficient of order 3, or an exponent of the mass ratio which is less than $1/2$ (due, for example, to a suitable charge assignment), or the addition in phase of an angle from the diagonalization of charged leptons and an angle from neutrino mixing. Typically, by exploiting the freedom in the parameter space, in this set of models a large θ_{23} may be accommodated. The large mixing angle for solar neutrinos requires however the introduction of *ad hoc* terms, such as for instance higher-dimensional operators contributing to the light neutrino masses independent of the see-saw mechanism [84]–[86].

Alternatively, to avoid the introduction of *ad hoc* non-renormalizable operators, it is possible to enlarge the Higgs content and consider, for example, an SO(10) model where all fermion mass matrices originate from renormalizable interactions of matter fields in three 16 representations with two Higgs multiplets, a 10_H and a 126_H [87]:

$$\mathcal{L}_Y = 10_H 16 y_{10} 16 + 126_H 16 y_{126} 16, \tag{61}$$

where y_{10} and y_{126} are two symmetric matrices in flavour space. Both y_{10} and y_{126} contribute to the Dirac mass matrices, with the characteristic factor 3 for the y_{126} between the (u, d) and (ν, e) sectors:

$$m_d = \alpha y_{10} + \beta y_{126}, \qquad m_e = \alpha y_{10} - 3\beta y_{126}, \tag{62}$$

and the correct mass relations for first and second generations can be accommodated. In the most general case, when 126_H acquires VEVs in its $SU(2)_L$ singlet and triplet components, both RH and LH Majorana masses can arise in the neutrino sector. They are both proportional to y_{126} and give rise to type I and type II see-saw, respectively. By assuming dominance of the type

II contribution, we find an interesting link between large atmospheric mixing angle and b–τ unification [88]. In this case, the light neutrino mass matrix is proportional to y_{126} and, from equation (62), it can be directly related to m_e and m_d:

$$m_\nu \propto m_d - m_e. \tag{63}$$

If both m_e and m_d have the approximate pattern

$$\begin{pmatrix} \lambda^2 & \lambda^2 \\ \lambda^2 & 1 \end{pmatrix} \tag{64}$$

in the 23 sector, then b–τ unification forces a cancellation in the 33 entry of m_ν, thus allowing for a large 23 neutrino mixing angle. Most of the Yukawa parameters can be determined by the quark masses and mixing angles and from the charged lepton masses. This allows to predict the atmospheric and solar mixing parameters within a range which is still experimentally allowed and $|U_{e3}| \approx 0.16$, not far from the present upper bound [89]. A drawback of the model is the occurrence of two pairs of Higgs doublets with non-vanishing VEVs, of which only one combination is allowed to remain light. This clearly makes the doublet–triplet splitting problem even more complicated than in minimal models.

Finally, we can abandon the idea that the model is left–right-symmetric at the GUT scale. In this case, the mechanism discussed in section 8, based on asymmetric mass matrices, can be embedded in an SO(10) grand-unified theory in a rather economic way [15, 62, 84, 90, 91]. The 33 entries of the fermion mass matrices can be obtained through the coupling $\mathbf{16}_3\mathbf{16}_3\mathbf{10}_H$ among the fermions in the third generation, $\mathbf{16}_3$, and a Higgs tenplet $\mathbf{10}_H$. The two independent VEVs of the tenplet v_u and v_d give mass, respectively, to t/ν_τ and b/τ. The key point to obtain an asymmetric texture is the introduction of an operator of the kind $\mathbf{16}_2\mathbf{16}_H\mathbf{16}_3\mathbf{16}'_H$. This operator is supposed to arise by integrating out an heavy $\mathbf{10}$ that couples both to $\mathbf{16}_2\mathbf{16}_H$ and to $\mathbf{16}_3\mathbf{16}'_H$. If the $\mathbf{16}_H$ develops a VEV-breaking SO(10) down to SU(5) at a large scale, then, in terms of SU(5) representations, we get an effective coupling of the kind $\bar{\mathbf{5}}_2\mathbf{10}_3\bar{\mathbf{5}}_H$, with a coefficient that can be of order 1. This coupling contributes to the 23 entry of the down-quark mass matrix and to the 32 entry of the charged lepton mass matrix, realizing the desired asymmetry. To distinguish the lepton and quark sectors, one can further introduce an operator of the form $\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H\mathbf{45}_H$ $(i, j = 2, 3)$, with the VEV of the $\mathbf{45}_H$ pointing in the $\mathrm{B-L}$ direction. Additional operators, still of the type $\mathbf{16}_i\mathbf{16}_j\mathbf{16}_H\mathbf{16}'_H$ can contribute to the matrix elements of the first generation. The mass matrices appear as

$$m_u = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & \epsilon/3 \\ 0 & -\epsilon/3 & 1 \end{pmatrix} v_u, \qquad m_d = \begin{pmatrix} 0 & \delta & \delta' \\ \delta & 0 & \sigma+\epsilon/3 \\ \delta' & -\epsilon/3 & 1 \end{pmatrix} v_d, \tag{65}$$

$$m_D = \begin{pmatrix} \eta & 0 & 0 \\ 0 & 0 & -\epsilon \\ 0 & \epsilon & 1 \end{pmatrix} v_u, \qquad m_l = \begin{pmatrix} 0 & \delta & \delta' \\ \delta & 0 & -\epsilon \\ \delta' & \sigma+\epsilon & 1 \end{pmatrix} v_d, \tag{66}$$

$$M = \begin{pmatrix} b^2\eta^2 & -b\epsilon\eta & a\eta \\ -b\epsilon\eta & \epsilon^2 & -\epsilon \\ a\eta & -\epsilon & 1 \end{pmatrix} \Lambda, \tag{67}$$

where $\eta \ll \delta, \delta' \ll \epsilon \ll 1$ and a, b and σ are of order O(1). In the charged fermion sector, the parameters η, δ, δ', ϵ and σ are determined from the lepton masses and from a subset of quark masses and mixing angles, leading to six successful predictions. In the neutrino sector, the lopsidedness of m_l is responsible for the large atmospheric mixing angle, whereas the parameters a and b can be adjusted to obtain the solar mixing angle and the ratio between solar and atmospheric squared mass differences. The model predicts small values for $|U_{e3}|$, in a range accessible only to future neutrino factories [92].

Models based on SO(10) times a flavour symmetry are more difficult to construct because a whole generation is contained in the 16, so that, for example for $\mathrm{U}(1)_F$, one would have the same value of the charge for all quarks and leptons of each generation, which is too rigid. This problem can be circumvented if not all the observed fermions in a given generation belong to a single 16 representation [93].

10. Conclusion

By now there are rather convincing experimental indications for neutrino oscillations. The direct implication of these findings is that neutrino masses are not all vanishing. As a consequence, the phenomenology of neutrino masses and mixings is brought to the forefront. This is a very interesting subject in many respects. It is a window on the physics of GUTs in that the extreme smallness of neutrino masses can only be explained in a natural way if lepton number conservation is violated. If so, neutrino masses are inversely proportional to the large scale where lepton number is violated. Also, the pattern of neutrino masses and mixings interpreted in a GUT framework can provide new clues on the long-standing problem of understanding the origin of the hierarchical structure of quark and lepton mass matrices.

Neutrino oscillations only determine differences of m_i^2 values and the actual scale of neutrino masses remain to be experimentally fixed. The detection of $0\nu\beta\beta$ decay would be extremely important for the determination of the overall scale of neutrino masses, the confirmation of their Majorana nature and the experimental clarification of the ordering of levels in the associated spectrum. The recent results from cosmology indicate that neutrino masses are not a major fraction of the cosmological mass density $\Omega_\nu \lesssim 1.5\%$. The decay of heavy right-handed neutrinos with lepton number non-conservation can provide a viable and attractive model of baryogenesis through leptogenesis. The measured oscillation frequencies and mixings are remarkably consistent with this attractive possibility.

While the existence of oscillations appears to be on a ground of increasing solidity, many important experimental challenges remain. For atmospheric neutrino oscillations the completion of the K2K experiment, which was delayed by the accident that has seriously damaged the Superkamiokande detector, is important for a terrestrial confirmation of the effect and for an independent measurement of the associated parameters. In the near future the experimental study of atmospheric neutrinos will be further pursued with long baseline measurements by MINOS, OPERA, ICARUS. For solar neutrinos the continuation of SNO, KamLAND and the data from Borexino will lead to a more precise determination of the parameters of the LA solution. Finally, a clarification by MINIBOONE of the issue of the LSND alleged signal is necessary to know if 3 light neutrinos are sufficient or additional sterile neutrinos must be introduced, in spite of the apparent lack of independent evidence in the data for such sterile neutrinos and of the fact that attempts of constructing plausible and natural theoretical models have not led so far to compelling results. Further in the future there are projects for neutrino factories and/or superbeams aimed at precision measurements of the oscillation parameters and possibly the detection of CP violation effects in the neutrino sector.

Pending the solution of the existing experimental ambiguities a variety of theoretical models of neutrino masses and mixings are still conceivable. Among three-neutrino models we have described a number of possibilities based on degenerate, inverted hierarchy and normal hierarchy type of spectra. The normal hierarchy option appears to us as the most straightforward and flexible framework. In particular, the large atmospheric mixing can arise from lopsided matrices. Then the observed frequencies and the large solar angle can also be obtained without fine-tuning in models where the 23 subdeterminant is automatically suppressed.

The fact that some neutrino mixing angles are large and even nearly maximal, while surprising at the start, was eventually found to be well compatible with a unified picture of quark and lepton masses within GUTs. The symmetry group at M_{GUT} could be either (SUSY) SU(5) or SO(10) or a larger group. For example, we have seen that models based on anarchy, semianarchy, inverted hierarchy or normal hierarchy can all be naturally implemented by simple assignments of $U(1)_F$ horizontal charges in a semiquantitative unified description of all quark and lepton masses in SUSY $SU(5) \times U(1)_F$. Actually, in this context, if one adopts a statistical criterium, hierarchical models appear to be preferred over anarchy and among them normal hierarchy appears the most probable. Note that in almost all of the existing models of neutrino mixings the atmospheric angle is large but not maximal. If it experimentally turned out that indeed θ_{23} is maximal with good accuracy then very special classes of models would be selected [27, 43, 94].

All we know about neutrino masses is well in harmony with the idea and the mass scale of GUTs. As a consequence, neutrino masses have added phenomenological support to this beautiful idea and to the models of physics beyond the Standard Model that are compatible with it. In particular, if we consider the main classes of new physics that are currently contemplated, like supersymmetry, technicolour, large extra dimensions at the TeV scale, little Higgs models, etc, it is clear that the first one is the most directly related to GUTs. SUSY offers a well-defined model computable up to the GUT scale and is actually supported by the quantitative success of coupling unification in SUSY GUTs. For the other examples quoted all contact with GUTs is lost or at least is much more remote. In this sense, neutrino masses fit particularly well in the SUSY picture that so far remains the standard way beyond the Standard Model.

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References

[1] The SuperKamiokande collaboration 2000 *Phys. Rev. Lett.* **85** 3999
Toshito T [for the SK collaboration] 2001 *Preprint* hep-ex/0105023
The MACRO collaboration 2001 *Phys. Lett.* B **517** 59
Giacomelli G and Giorgini M [for the MACRO collaboration] 2001 *Preprint* hep-ex/0110021

[2] Ahmad Q R *et al* [SNO Collaboration] 2002 *Phys. Rev. Lett.* **89** 011301 (*Preprint* nucl-ex/0204008)
Ahmad Q R *et al* [SNO Collaboration] 2002 *Phys. Rev. Lett.* **89** 011302 (*Preprint* nucl-ex/0204009)

[3] Ahmed S N *et al* [SNO Collaboration] 2003 *Preprint* nucl-ex/0309004
[4] Cleveland B T *et al* 1998 *Astrophys. J.* **496** 505
The GALLEX collaboration 1999 *Phys. Lett.* B **447** 127
The SAGE collaboration 1999 *Phys. Rev.* C **60** 055801
The SuperKamiokande collaboration 2001 *Preprints* hep-ex/0103032 and hep-ex/0103033
The GNO collaboration 2000 *Phys. Lett.* B **490** 16
The SNO collaboration 2001 *Phys. Rev. Lett.* **87** 71301
[5] Yanagida T 1979 *Proc. of the Workshop on Unified Theory and Baryon Number in the Universe* KEK, March
Glashow S L 1980 *Quarks and Leptons* ed M Lévy *et al* (New York: Plenum) p 707
Gell-Mann M, Ramond P and Slansky R 1979 *Supergravity* Stony Brook
Mohapatra R N and Senjanovic G 1980 *Phys. Rev. Lett.* **44** 912
[6] Fukugita M and Yanagida T 1986 *Phys. Lett.* B **174** 45
[7] Eguchi K *et al* [KamLAND Collaboration] 2003 *Phys. Rev. Lett.* **90** 021802 (*Preprint* hep-ex/0212021)
[8] Bennett C L *et al* 2003 *Astrophys. J. Suppl.* **148** 1
Spergel D N *et al* 2003 *Astrophys. J. Suppl.* **148** 175
Kogut A *et al* 2003 *Astrophys. J. Suppl.* **148** 161
Hinshaw G *et al* 2003 *Astrophys. J. Suppl.* **148** 135
Verde L *et al* 2003 *Astrophys. J. Suppl.* **148** 195
Peiris H V *et al* 2003 *Astrophys. J. Suppl.* **148** 213
[9] Hannestad S 2003 *J. Cosmol. Astropart. Phys.* JCAP05(2003)004
Elgaroy O and Lahav O 2003 *J. Cosmol. Astropart. Phys.* JCAP04(2003)004
Hannestad S 2003 *Preprint* hep-ph/0310220
Hannestad S and Raffelt G 2003 *Preprint* hep-ph/0312154
[10] See e.g. Gonzalez-Garcia M C and Nir Y 2003 *Rev. Mod. Phys.* **75** 345 (*Preprint* hep-ph/0202058)
Mohapatra R N 2002 *Preprint* hep-ph/0211252
Grimus W 2003 *Preprint* hep-ph/0307149
King S F 2004 *Rept. Prog. Phys.* **67** 107 (*Preprint* hep-ph/0310204)
[11] The LSND collaboration 2001 *Preprint* hep-ex/0104049
[12] The Karmen collaboration 2000 *Nucl. Phys. Proc. Suppl.* **91** 191
[13] Spentzouris P 2001 *Nucl. Phys. Proc. Suppl.* **100** 163
[14] Altarelli G and Feruglio F 1999 *Phys. Rep.* **320** 295
[15] Barr S M and Dorsner I 2000 *Nucl. Phys.* B **585** 79
Masina I 2001 *Int. J. Mod. Phys.* A **16** 5101
[16] Gaur N, Ghosal A, Ma E and Roy P 1998 *Phys. Rev.* D **58** 071301
Fogli G L, Lisi E and Marrone A 2001 *Phys. Rev.* D **63** 053008
Peres O L and Smirnov A Y 2001 *Nucl. Phys.* B **599** 3
Giunti C 2001 *Nucl. Phys. Proc. Suppl.* **100** 244
Grimus W and Schwetz T 2001 *Eur. Phys. J.* C **20** 1
Gonzalez-Garcia M C, Maltoni M and Pena-Garay C 2001 *Preprint* hep-ph/0108073
Maltoni M, Schwetz T and Valle J W 2001 *Preprint* hep-ph/0112103
Strumia A 2002 *Preprint* hep-ph/0201134
Maltoni M, Schwetz T, Tortola M A and Valle J W 2003 *Preprint* hep-ph/0305312
Cirelli M, Marandella G, Strumia A and Vissani F 2004 *Preprint* hep-ph/0403158
[17] Pontecorvo B 1957 *Sov. Phys. JETP* **6** 429
Pontecorvo B 1957 *Zh. Eksp. Teor. Fiz.* **33** 549
Maki Z, Nakagawa M and Sakata S 1962 *Prog. Theor. Phys.* **28** 870
Pontecorvo B 1968 *Sov. Phys. JETP* **26** 984
Pontecorvo B 1968 *Zh. Eksp. Teor. Fiz.* **53** 1717
Gribov V N and Pontecorvo B 1969 *Phys. Lett.* B **28** 493

[18] Lee B W, Pakvasa S, Shrock R and Sugawara H 1977 *Phys. Rev. Lett.* **38** 937
Lee B W and Shrock R 1977 *Phys. Rev.* D **16** 1444
[19] Cabibbo N 1978 *Phys. Lett.* B **72** 333
[20] Bilenky S M, Hosek J and Petcov S T 1980 *Phys. Lett.* B **94** 495
Schechter J and Valle J W F 1980 *Phys. Rev.* D **22** 2227
Doi M, Kotani T, Nishiura H, Okuda K and Takasugi E 1981 *Phys. Lett.* B **102** 323
[21] Frigerio M and Smirnov A Y 2002 *Nucl. Phys.* B **640** 233
Frigerio M and Smirnov A Y 2003 *Phys. Rev.* D **67** 013007
[22] The CHOOZ collaboration 1999 *Phys. Lett.* B **466** 415
See also The PALO VERDO collaboration 2000 *Phys. Rev. Lett.* **84** 3764
[23] Fogli G L, Lisi E, Montanino D and Palazzo A 2001 *Phys. Rev.* D **64** 093007
Bahcall J N, Gonzalez-Garcia M C and Pena-Garay C 2001 *J. High Energy Phys.* JHEP08(2001)014
Bahcall J N, Gonzalez-Garcia M C and Pena-Garay C 2001 *Preprint* hep-ph/0111150
Barger V, Marfatia D, Whisnant K and Wood B P 2002 *Preprint* hep-ph/0204253
Bandyopadhyay A, Choubey S, Goswami S and Roy D P 2002 *Preprint* hep-ph/0204286
Bahcall J N, Gonzalez-Garcia M C and Pena-Garay C 2002 *Preprint* hep-ph/0204314
Aliani P, Antonelli V, Ferrari R, Picariello M and Torrente-Lujan E 2002 *Preprint* hep-ph/0205053
Fogli G L, Lisi E, Marrone A, Montanino D, Palazzo A and Rotunno A M 2003 *Phys. Rev.* D **67** 073002 (*Preprint* hep-ph/0212127)
Maltoni M, Schwetz T and Valle J W 2003 *Phys. Rev.* D **67** 093003 (*Preprint* hep-ph/0212129)
Bahcall J N, Gonzalez-Garcia M C and Pena-Garay C 2003 *J. High Energy Phys.* JHEP02(2003)009 (*Preprint* hep-ph/0212147)
Bahcall J N and Pena-Garay C 2003 *Preprint* hep-ph/0305159
Fogli G L, Lisi E, Marrone A and Palazzo A 2004 *Phys. Lett.* B **583** 149
Aliani P, Antonelli V, Picariello M and Torrente-Lujan E 2003 *Preprint* hep-ph/0309156
Bandyopadhyay A, Choubey S, Goswami S, Petcov S T and Roy D P 2004 *Phys. Lett.* B **583** 134
de Holanda P C and Smirnov A Y 2003 *Preprint* hep-ph/0309299
[24] Fogli G L, Lisi E and Marrone A 2001 *Phys. Rev.* D **64** 093005
Gonzalez-Garcia M C, Maltoni M and Pena-Garay C 2001 *Preprint* hep-ph/0108073
Gonzalez-Garcia M C and Nir Y 2002 *Preprint* hep-ph/0202058
Fogli G L, Lisi E, Marrone A and Montanino D 2003 *Phys. Rev.* D **67** 093006 (*Preprint* hep-ph/0303064)
[25] Maltoni M, Schwetz T, Tortola M A and Valle J W F 2003 *Phys. Rev.* D **68** 113010
Mena O and Parke S J 2003 *Preprint* hep-ph/0312131
[26] Murayama H and Yanagida T 2001 *Phys. Lett.* B **520** 263
Barenboim G, Borissov L, Lykken J and Smirnov A Y 2001 *Preprint* hep-ph/0108199
Gouvea A D 2002 *Preprint* hep-ph/0204077
Strumia A 2002 *Preprint* hep-ph/0201134
[27] Harrison P F, Perkins D H and Scott W G 2002 *Phys. Lett.* B **530** 167
Harrison P F and Scott W G 2002 *Phys. Lett.* B **535** 163
Harrison P F and Scott W G 2003 *Phys. Lett.* B **557** 76
Harrison P F and Scott W G 2004 *Preprint* hep-ph/0403278
[28] Weinberg S 1979 *Phys. Rev. Lett.* **43** 1566
[29] Weinheimer C, these Proceedings
[30] The HEIDELBERG–MOSCOW collaboration, Klapdor-Kleingrothaus H V *et al* 2001 *Eur. Phys. J.* A **12** 147
Aalseth C E *et al* [16EX Collaboration] 2002 *Preprint* hep-ex/0202026
[31] Bilenky S M 2004 *Preprint* hep-ph/0403245
[32] Petcov S T and Smirnov A Y 1994 *Phys. Lett.* B **322** 109
Bilenky S M, Bottino A, Giunti C and Kim C W 1996 *Phys. Rev.* D **54** 1881
Bilenky S M, Giunti C, Kim C W and Petcov S T 1996 *Phys. Rev.* D **54** 4432
Vissani F 1999 *J. High Energy Phys.* JHEP06(1999)22

Fukuyama T, Matsuda K and Nishiura H 1998 *Phys. Rev.* D **57** 5884
Barger V D and Whisnant K 1999 *Phys. Lett.* B **456** 194
Vissani F 1999 *Preprint* hep-ph/9904349
Giunti C 2000 *Phys. Rev.* D **61** 036002
Czakon M, Gluza J and Zralek M 1999 *Phys. Lett.* B **465** 211
Bilenky S M, Giunti C, Grimus W, Kayser B and Petcov S T 1999 *Phys. Lett.* B **465** 193
Klapdor-Kleingrothaus H V, Pas H and Smirnov A Y 2001 *Phys. Rev.* D **63** 73005
Rodejohann W 2001 *Nucl. Phys.* B **597** 110
Matsuda K, Takeda N, Fukuyama T and Nishiura H 2001 *Phys. Rev.* D **64** 13001
Falcone D and Tramontano F 2001 *Phys. Rev.* D **64** 077302
Bilenky S M, Pascoli S and Petcov S T 2001 *Phys. Rev.* D **64** 53010
Bilenky S M, Pascoli S and Petcov S T 2001 *Phys. Rev.* D **64** 113003
Osland P and Vigdel G 2001 *Phys. Lett.* B **520** 143
Pascoli S, Petcov S T and Wolfenstein L 2002 *Phys. Lett.* B **524** 319
Pascoli S and Petcov S T 2002 *Preprint* hep-ph/0205022
[33] Feruglio F, Strumia A and Vissani F 2002 *Preprint* hep-ph/0201291
[34] Klapdor-Kleingrothaus H V *et al* 2001 *Mod. Phys. Lett.* A **37** 2409
Klapdor-Kleingrothaus H V, Dietz A, Krivosheina I V and Chkvorets O 2004 *Nucl. Instrum. Methods* A **522** 371
Klapdor-Kleingrothaus H V, Dietz A, Krivosheina I V and Chkvorets O 2004 *Phys. Lett.* B **586** 198
[35] For a review see e.g. Trodden M 1999 *Rev. Mod. Phys.* **71** 1463
[36] Kainulainen K and Olive K A *Neutrino Mass (Springer Tracts in Modern Physics)* ed G Altarelli and K Winter (Berlin: Springer)
[37] Luty M A 1992 *Phys. Rev.* D **45** 455
Murayama H and Yanagida T 1994 *Phys. Lett.* B **322** 349
Flanz M, Paschos E A and Sarkar U 1995 *Phys. Lett.* B **345** 248
Flanz M, Paschos E A and Sarkar U 1995 *Phys. Lett.* B **382** 447 (erratum)
Covi L, Roulet E and Vissani F 1996 *Phys. Lett.* B **384** 169
Ma E and Sarkar U 1998 *Phys. Rev. Lett.* **80** 5716
Pilaftsis A 1999 *Int. J. Mod. Phys.* A **14** 1811
Riotto A and Trodden M 1999 *Ann. Rev. Nucl. Part. Sci.* **49** 35
Ellis J R, Lola S and Nanopoulos D V 1999 *Phys. Lett.* B **452** 87
Giudice G F, Peloso M, Riotto A and Tkachev I 1999 *J. High Energy Phys.* JHEP08(1999)014
Barbieri R, Creminelli P, Strumia A and Tetradis N 2000 *Nucl. Phys.* B **575** 61
Falcone D and Tramontano F 2001 *Phys. Lett.* B **506** 1
Nielsen H B and Takanishi Y 2001 *Phys. Lett.* B **507** 241
Joshipura A S, Paschos E A and Rodejohann W 2001 *J. High Energy Phys.* JHEP08(2001)029
Brahmachari B, Ma E and Sarkar U 2001 *Phys. Lett.* B **520** 152
Hirsch M and King S F 2001 *Phys. Rev.* D **64** 113005
Buccella F, Falcone D and Tramontano F 2002 *Phys. Lett.* B **524** 241
Buchmuller W and Wyler D 2001 *Phys. Lett.* B **521** 291
Garcia-Bellido J and Ruiz Morales E 2002 *Phys. Lett.* B **536** 193
Berger M S and Siyeon K 2002 *Phys. Rev.* D **65** 053019
Branco G C, Gonzalez Felipe R, Joaquim F R and Rebelo M N 2002 *Nucl. Phys.* B **640** 202
Fujii M, Hamaguchi K and Yanagida T 2002 *Phys. Rev.* D **65** 115012
Paschos E A 2002 *Nucl. Phys. Proc. Suppl.* **112** 36 (*Preprint* hep-ph/0204137)
Buchmuller W 2001 Lectures given at European School of High-Energy Physics (ESHEP) *Beatenberg, Switzerland, 26 August–8 September 2001* published in Beatenberg 2001, High-Energy Physics, p 205–47
Akhmedov E K, Frigerio M and Smirnov A Y 2003 *J. High Energy Phys.* JHEP09(2003)021

Buchmuller W, Di Bari P and Plumacher M 2003 *Nucl. Phys.* B **665** 445 (*Preprint* hep-ph/0302092)
Chankowski P H and Turzynski K 2003 *Phys. Lett.* B **570** 198
Velasco-Sevilla L 2003 *J. High Energy Phys.* JHEP10(2003)035
Hambye T and Senjanovic G 2004 *Phys. Lett.* B **582** 73
Barger V, Dicus D A, He H J and Li T J 2004 *Phys. Lett.* B **583** 173
Rodejohann W 2004 *Eur. Phys. J.* C **32** 235
Ibe M, Kitano R, Murayama H and Yanagida T 2004 *Preprint* hep-ph/0403198
Cosme N 2004 *Preprint* hep-ph/0403209
[38] Plumacher M 1997 *Z. Phys.* C **74** 549
Buchmuller W and Plumacher M 1999 *Phys. Rep.* **320** 329
Buchmuller W and Plumacher M 2000 *Int. J. Mod. Phys.* A **15** 5047
[39] Davidson S and Ibarra A 2002 *Phys. Lett.* B **535** 25
[40] Giudice G F, Notari A, Raidal M, Riotto A and Strumia A 2004 *Nucl. Phys.* B **685** 89
Buchmuller W, Di Bari P and Plumacher M 2004 *Preprint* hep-ph/0401240
[41] Pilaftsis A and Underwood T E J 2003 *Preprint* hep-ph/0309342
Hambye T, Lin Y, Notari A, Papucci M and Strumia A 2003 *Preprint* hep-ph/0312203
[42] Jezabek M 2002 *Preprint* hep-ph/0205234
[43] Wetterich C 1999 *Phys. Lett.* B **451** 397
[44] Ioannisian A and Valle J W F 1994 *Phys. Lett.* B **332** 93
Mohapatra R N and Nussinov S 1999 *Phys. Rev.* D **60** 013002
Mohapatra R N and Nussinov S 1998 *Phys. Lett.* B **441** 299
Branco G C, Rebelo M N and Silva-Marcos J I 2000 *Phys. Rev.* D **62** 073004
Perez G 2000 *J. High Energy Phys.* JHEP12(2000)027
[45] Barbieri R, Hall L J, Kane G L and Ross G G 1999 *Preprint* hep-ph/9901228
[46] Antusch S and King S F 2004 *Preprint* hep-ph/0402121
[47] Chankowski P H and Pluciennik Z 1993 *Phys. Lett.* B **316** 312
Babu K S, Leung C N and Pantaleone J 1993 *Phys. Lett.* B **319** 191
Tanimoto M 1995 *Phys. Lett.* B **360** 41
Haba N, Okamura N and Sugiura M 2000 *Prog. Theor. Phys.* **103** 367
Ellis J R and Lola S 1999 *Phys. Lett.* B **458** 310
Haba N, Matsui Y, Okamura N and Sugiura M 1999 *Eur. Phys. J.* C **10** 677
Casas J A, Espinosa J R, Ibarra A and Navarro I 1999 *Nucl. Phys.* B **556** 3
Casas J A, Espinosa J R, Ibarra A and Navarro I 2000 *Nucl. Phys.* B **569** 82
Casas J A, Espinosa J R, Ibarra A and Navarro I 1999 *J. High Energy Phys.* JHEP09(1999)015
Barbieri R, Ross G G and Strumia A 1999 *J. High Energy Phys.* JHEP10(1999)020
Haba N and Okamura N 2000 *Eur. Phys. J.* C **14** 347
Haba N, Matsui Y, Okamura N and Sugiura M 2000 *Prog. Theor. Phys.* **103** 145
Chankowski P H, Krolikowski W and Pokorski S 2000 *Phys. Lett.* B **473** 109
Casas J A, Espinosa J R, Ibarra A and Navarro I 2000 *Nucl. Phys.* B **573** 652
Haba N, Matsui Y and Okamura N 2000 *Prog. Theor. Phys.* **103** 807
Kuo T K, Pantaleone J and Wu G H 2001 *Phys. Lett.* B **518** 101
Chankowski P H and Pokorski S 2002 *Int. J. Mod. Phys.* A **17** 575
Antusch S, Kersten J, Lindner M and Ratz M 2002 *Phys. Lett.* B **538** 87
Frigerio M and Smirnov A Y 2003 *J. High Energy Phys.* JHEP02(2003)004
Antusch S, Kersten J, Lindner M and Ratz M 2003 *Nucl. Phys.* B **674** 401
Casas J A, Espinosa J R and Navarro I 2003 *J. High Energy Phys.* JHEP09(2003)048
Broncano A, Gavela M B and Jenkins E 2004 *Preprint* hep-ph/0406019
Hagedorn C, Kersten J and Lindner M 2004 *Preprint* hep-ph/0406103
[48] Vissani F 1997 *Preprint* hep-ph/9708483
Georgi H and Glashow S L 2000 *Phys. Rev.* D **61** 097301

[49] Barbieri R, Ross G G and Strumia A 1999 *J. High Energy Phys.* JHEP10(1999)020
[50] Fritzsch H and Xing Z Z 1996 *Phys. Lett.* B **372** 265
Fritzsch H and Xing Z Z 1998 *Phys. Lett.* B **440** 313
Fritzsch H and Xing Z Z 2000 *Prog. Part. Nucl. Phys.* **45** 1
Fukugita M, Tanimoto M and Yanagida T 1998 *Phys. Rev.* D **57** 4429
Fukugita M, Tanimoto M and Yanagida T 1999 *Phys. Rev.* D **59** 113016
Tanimoto M, Watari T and Yanagida T 1999 *Phys. Lett.* B **461** 345
Kang S K and Kim C S 1999 *Phys. Rev.* D **59** 091302
Tanimoto M 2000 *Phys. Lett.* B **483** 417
Haba N, Matsui Y, Okamura N and Suzuki T 2000 *Phys. Lett.* B **489** 184
Koide Y and Ghosal A 2000 *Phys. Lett.* B **488** 344
Akhmedov E K, Branco G C, Joaquim F R and Silva-Marcos J I 2001 *Phys. Lett.* B **498** 237
Dermisek R 2003 *Preprint* hep-ph/0312206
[51] Altarelli G, Feruglio F and Masina I 2004 *Preprint* hep-ph/0402155
[52] Antusch S and King S F 2004 *Preprint* hep-ph/0403053
[53] Romanino A 2004 *Preprint* hep-ph/0402258
[54] Hall L J, Murayama H and Weiner N 2000 *Phys. Rev. Lett.* **84** 2572
Haba N and Murayama H 2001 *Phys. Rev.* D **63** 053010
Berger M S and Siyeon K 2001 *Phys. Rev.* D **63** 057302
Hirsch M 2001 *Preprint* hep-ph/0102102
Hirsch M and King S F 2001 *Phys. Lett.* B **516** 103 (*Preprint* hep-ph/0102103)
Vissani F 2001 *Phys. Lett.* B **508** 79
de Gouvea A and Murayama H 2003 *Preprint* hep-ph/0301050
Espinosa J R 2003 *Preprint* hep-ph/0306019
[55] Barbieri R, Hall L J, Smith D R, Strumia A and Weiner N 1998 *J. High Energy Phys.* JHEP12(1998)017
Barbieri R, Hall L J and Strumia A 1999 *Phys. Lett.* B **445** 407
See also Zeldovich Ya B 1952 *Dokl. Akad. Nauk. CCCP* **86** 505
Konopinsky E J and Mahmound H 1953 *Phys. Rev.* **92** 1045
Zee A 1980 *Phys. Lett.* B **93** 389
Petcov S 1982 *Phys. Lett.* B **100** 245
Joshipura A S and Rindani S D 2000 *Eur. Phys. J.* C **14** 85
Mohapatra R N, Perez-Lorenzana A and de Sousa Pires C A 2000 *Phys. Lett.* B **474** 355
Shafi Q and Tavartkiladze Z 2000 *Phys. Lett.* B **482** 145 (*Preprint* hep-ph/0101350)
Ghosal A 2000 *Phys. Rev.* D **62** 092001
Lavoura L 2000 *Phys. Rev.* D **62** 093011
Lavoura L and Grimus W 2000 *J. High Energy Phys.* JHEP09(2000)007
Lavoura L and Grimus W 2000 *Phys. Rev.* D **62** 093012
King S F and Singh N N 2001 *Nucl. Phys.* B **596** 81
Oliver J F and Santamaria A 2002 *Phys. Rev.* D **65** 033003
Babu K S and Mohapatra R N 2002 *Preprint* hep-ph/0201176
[56] Dorsner I and Barr S M 2001 *Nucl. Phys.* B **617** 493
[57] He H J, Dicus D A and Ng J N 2002 *Phys. Lett.* B **536** 83
[58] Raidal M 2004 *Preprint* hep-ph/0404046
[59] Frampton P H, Petcov S T and Rodejohann W 2004 *Preprint* hep-ph/0401206
[60] King S F 1998 *Phys. Lett.* B **439** 350
Davidson S and King S F 1998 *Phys. Lett.* B **445** 191
Shafi Q and Tavartkiladze Z 1999 *Phys. Lett.* B **451** 129
[61] King S F 1999 *Nucl. Phys.* B **562** 57
King S F 2000 *Nucl. Phys.* B **576** 85 (*Preprint* hep-ph/0204360)
Frampton P H, Glashow S L and Yanagida T 2002 *Phys. Lett.* B **548** 119 (*Preprint* hep-ph/0208157)

Raidal M and Strumia A 2003 *Phys. Lett.* B **553** 72 (*Preprint* hep-ph/0210021)
Raby S 2003 *Phys. Lett.* B **561** 119 (*Preprint* hep-ph/0302027)
King S F 2003 Invited talk at *10th Int. Workshop on Neutrino Telescopes, Venice, Italy, 11–14 March* (*Preprint* hep-ph/0306095)
[62] Babu K S and Barr S M 1996 *Phys. Lett.* B **381** 202
Albright C H and Barr S M 1998 *Phys. Rev.* D **58** 013002
Albright C H, Babu K S and Barr S M 1998 *Phys. Rev. Lett.* **81** 1167
Frampton P H and Rasin A 2000 *Phys. Lett.* B **478** 424
[63] Irges N, Lavignac S and Ramond P 1998 *Phys. Rev.* D **58** 035003
[64] Altarelli G and Feruglio F 1998 *Phys. Lett.* B **439** 112
Altarelli G and Feruglio F 1998 *J. High Energy Phys.* JHEP11(1998)021
[65] Altarelli G and Feruglio F 1999 *Phys. Lett.* B **451** 388
[66] Sato J and Yanagida T 1998 *Phys. Lett.* B **430** 127
Elwood J K, Irges N and Ramond P 1998 *Phys. Rev. Lett.* **81** 5064
Berezhiani Z and Rossi A 1999 *J. High Energy Phys.* JHEP03(1999)002
Hagiwara K and Okamura N 1999 *Nucl. Phys.* B **548** 60
Bando M and Kugo T 1999 *Prog. Theor. Phys.* **101** 1313
Grimus W and Lavoura L 2001 *J. High Energy Phys.* JHEP07(2001)045 (*Preprint* hep-ph/0204070)
[67] Georgi H and Jarlskog C 1979 *Phys. Lett.* B **86** 297
[68] Drees M, Pakvasa S, Tata X and ter Veldhuis T 1998 *Phys. Rev.* D **57** 5335
Chun E J, Kang S K, Kim C W and Lee U W 1999 *Nucl. Phys.* B **544** 89
Mukhopadhyaya B, Roy S and Vissani F 1998 *Phys. Lett.* B **443** 191
Kong O C 1999 *Mod. Phys. Lett.* A **14** 903
Bisset M, Kong O C, Macesanu C and Orr L H 2000 *Phys. Rev.* D **62** 035001
Joshipura A S and Vempati S K 1999 *Phys. Rev.* D **60** 111303
Datta A, Mukhopadhyaya B and Roy S 2000 *Phys. Rev.* D **61** 055006
Mukhopadhyaya B 2000 *Pramana* **54** 147
Bhattacharyya G, Klapdor-Kleingrothaus H V and Pas H 1999 *Phys. Lett.* B **463** 77
Romao J C, Diaz M A, Hirsch M, Porod W and Valle J W 2000 *Phys. Rev.* D **61** 071703
Abada A and Losada M 2000 *Nucl. Phys.* B **585** 45
Davidson S and Losada M 2000 *J. High Energy Phys.* JHEP05(2000)021
Abada A and Bhattacharyya G 2001 *Phys. Rev.* D **63** 017701
Barger V D, Han T, Hesselbach S and Marfatia D 2001 *Preprint* hep-ph/0108261
Koide Y and Ghosal A 2002 *Preprint* hep-ph/0203113
[69] Kaplan D B and Schmaltz M 1994 *Phys. Rev.* D **49** 3741
Schmaltz M 1995 *Phys. Rev.* D **52** 1643
Frampton P H and Kephart T W 1995 *Int. J. Mod. Phys.* A **10** 4689
Chou K C and Wu Y L 1996 *Phys. Rev.* D **53** 3492
Frampton P H and Kong O C 1996 *Phys. Rev. Lett.* **77** 1699
Babu K S and Barr S M 1996 *Phys. Lett.* B **387** 87
Barbieri R, Hall L J, Raby S and Romanino A 1997 *Nucl. Phys.* B **493** 3
Barbieri R, Hall L J and Romanino A 1997 *Phys. Lett.* B **401** 47
Blazek T, Raby S and Tobe K 1999 *Phys. Rev.* D **60** 113001
Carone C D and Lebed R F 1999 *Phys. Rev.* D **60** 096002
Frampton P H and Rasin A 2000 *Phys. Lett.* B **478** 424
Dermisek R and Raby S 2000 *Phys. Rev.* D **62** 015007
Aranda A, Carone C D and Lebed R F 2000 *Phys. Rev.* D **62** 016009
Aranda A, Carone C D and Lebed R F 2000 *Phys. Lett.* B **474** 170
Falcone D 2001 *Phys. Rev.* D **64** 117302
Aranda A, Carone C D and Meade P 2002 *Phys. Rev.* D **65** 013011

Aranda A, Carone C D and Meade P 2001 *Phys. Rev.* D **64** 117302
Koide Y and Ghosal A 2001 *Phys. Rev.* D **63** 037301
Ross G G and Velasco-Sevilla L 2003 *Nucl. Phys.* B **653** 3 (*Preprint* hep-ph/0208218)
[70] Kawamura Y 2001 *Prog. Theor. Phys.* **105** 999
Altarelli G and Feruglio F 2001 *Phys. Lett.* B **511** 257
Hall L J and Nomura Y 2001 *Phys. Rev.* D **64** 055003
Hebecker A and March-Russell J 2001 *Nucl. Phys.* B **613** 3
Asaka T, Buchmuller W and Covi L 2001 *Phys. Lett.* B **523** 199
Hall L J, Nomura Y, Okui T and Smith D R 2002 *Phys. Rev.* D **65** 035008
Contino R, Pilo L, Rattazzi R and Trincherini E 2002 *Nucl. Phys.* B **622** 227
Dermisek R and Mafi A 2002 *Phys. Rev.* D **65** 055002
Watari T and Yanagida T 2001 *Phys. Lett.* B **519** 164
Hall L J and Nomura Y 2002 *Phys. Rev.* D **65** 125012
Huang C S, Jiang J, Li T J and Liao W 2002 *Phys. Lett.* B **530** 218
Hebecker A and March-Russell J 2002 *Phys. Lett.* B **539** 119
Asaka T, Buchmuller W and Covi L 2002 *Phys. Lett.* B **540** 295
Hall L J and Nomura Y 2002 *Phys. Rev.* D **66** 075004
Barr S M and Dorsner I 2002 *Phys. Rev.* D **66** 065013
Hall L J and Nomura Y 2002 *Preprint* hep-ph/0207079
Hall L J and Nomura Y 2003 *Ann. Phys.* **306** 132
[71] Froggatt C D and Nielsen H B 1979 *Nucl. Phys.* B **147** 277
[72] Fusaoka H and Koide Y 1998 *Phys. Rev.* D **57** 3986
[73] Bijnens J and Wetterich C 1987 *Nucl. Phys.* B **147** 292
Leurer M, Nir Y and Seiberg N 1993 *Nucl. Phys.* B **398** 319
Leurer M, Nir Y and Seiberg N 1994 *Nucl. Phys.* B **420** 468
Ibanez L E and Ross G G 1994 *Phys. Lett.* B **332** 100
Binetruy P and Ramond P 1995 *Phys. Lett.* B **350** 49
Dudas E, Pokorski S and Savoy C A 1995 *Phys. Lett.* B **356** 45
Leontaris G K and Rizos J 2000 *Nucl. Phys.* B **567** 32
Mira J M, Nardi E and Restrepo D A 2000 *Phys. Rev.* D **62** 016002
[74] Binetruy P, Lavignac S and Ramond P 1996 *Nucl. Phys.* B **477** 353
Binetruy P, Lavignac S, Petcov S T and Ramond P 1997 *Nucl. Phys.* B **496** 3
Berezhiani Z and Tavartkiladze Z 1997 *Phys. Lett.* B **396** 150
Jezabek M and Sumino Y 1998 *Phys. Lett.* B **440** 327
Grossman Y, Nir Y and Shadmi Y 1998 *J. High Energy Phys.* JHEP10(1998)007
Buchmuller W and Yanagida T 1999 *Phys. Lett.* B **445** 399
Froggatt C D, Gibson M and Nielsen H B 1999 *Phys. Lett.* B **446** 256
Choi K, Hwang K and Chun E J 1999 *Phys. Rev.* D **60** 031301
Lola S and Ross G G 1999 *Nucl. Phys.* B **553** 81
Nir Y and Shadmi Y 1999 *J. High Energy Phys.* JHEP05(1999)023
Stech B 2000 *Phys. Rev.* D **62** 093019
King S F and Oliveira M 2001 *Phys. Rev.* D **63** 095004
Tanimoto M 2001 *Phys. Lett.* B **501** 231
Bando M and Maekawa N 2001 *Prog. Theor. Phys.* **106** 1255
Nielsen H B and Takanishi Y 2001 *Phys. Scripta T* **93** 44
[75] Altarelli G, Feruglio F and Masina I 2000 *J. High Energy Phys.* JHEP11(2000)040
[76] Altarelli G, Feruglio F and Masina I 2003 *J. High Energy Phys.* JHEP01(2003)035 (*Preprint* hep-ph/0210342)
[77] Barbieri R, Hambye T and Romanino A 2003 *J. High Energy Phys.* JHEP03(2003)017 (*Preprint* hep-ph/0302118)

[78] Lavignac S, Masina I and Savoy C A 2002 *Nucl. Phys.* B **633** 139 (*Preprint* hep-ph/0202086)
[79] Borzumati F and Masiero A 1986 *Phys. Rev. Lett.* **57** 961
Hisano J, Moroi T, Tobe K, Yamaguchi M and Yanagida T 1995 *Phys. Lett.* B **357** 579
Hisano J, Moroi T, Tobe K and Yamaguchi M 1996 *Phys. Rev.* D **53** 2442
Hisano J and Nomura D 1999 *Phys. Rev.* D **59** 116005
Buchmuller W, Delepine D and Vissani F 1999 *Phys. Lett.* B **459** 171
Feng J L, Nir Y and Shadmi Y 2000 *Phys. Rev.* D **61** 113005
Ellis J R, Gomez M E, Leontaris G K, Lola S and Nanopoulos D V 2000 *Eur. Phys. J.* C **14** 319
Buchmuller W, Delepine D and Handoko L T 2000 *Nucl. Phys.* B **576** 445
Sato J, Tobe K and Yanagida T 2001 *Phys. Lett.* B **498** 189
Sato J and Tobe K 2001 *Phys. Rev.* D **63** 116010
Casas J A and Ibarra A 2001 *Nucl. Phys.* B **618** 171
Lavignac S, Masina I and Savoy C A 2001 *Phys. Lett.* B **520** 269
Kageyama A, Kaneko S, Shimoyama N and Tanimoto M 2002 *Phys. Lett.* B **527** 206
Lavignac S, Masina I and Savoy C A 2002 *Nucl. Phys.* B **633** 139
Masiero A, Vempati S K and Vives O 2003 *Nucl. Phys.* B **649** 189
Babu K S, Dutta B and Mohapatra R N 2003 *Phys. Rev.* D **67** 076006
Blazek T and King S F 2003 *Nucl. Phys.* B **662** 359
Petcov S T, Profumo S, Takanishi Y and Yaguna C E 2004 *Nucl. Phys.* B **676** 453
Barr S M 2004 *Phys. Lett.* B **578** 394
[80] See e.g. Buccella F and Savoy C A 2002 *Mod. Phys. Lett.* A **17** 1239
[81] Smirnov A Y 1993 *Phys. Rev.* D **48** 3264
Altarelli G, Feruglio F and Masina I 2000 *Phys. Lett.* B **472** 382
[82] Dorsner I and Smirnov A Y 2004 *Preprint* hep-ph/0403305
[83] Chen M C and Mahanthappa K T 2003 *Phys. Rev.* D **68** 017301
Chen M C and Mahanthappa K T 2003 *Int. J. Mod. Phys.* A **18** 5819
[84] Babu K S, Pati J C and Wilczek F 2000 *Nucl. Phys.* B **566** 33
[85] See e.g. Lola S and Ross G G 1999 *Nucl. Phys.* B **553** 81
[86] Pati J C 2002 *Preprint* hep-ph/0204240
Pati J C 2003 *Int. J. Mod. Phys.* A **18** 4135
[87] Babu K S and Mohapatra R N 1993 *Phys. Rev. Lett.* **70** 2845
Lee D G and Mohapatra R N 1995 *Phys. Rev.* D **51** 1353
Brahmachari B and Mohapatra R N 1998 *Phys. Rev.* D **58** 015001
Matsuda K, Koide Y, Fukuyama T and Nishiura H 2002 *Phys. Rev.* D **65** 033008
Matsuda K, Koide Y, Fukuyama T and Nishiura H 2002 *Phys. Rev.* D **65** 079904 (erratum)
Fukuyama T and Okada N 2002 *J. High Energy Phys.* JHEP11(2002)011
[88] Bajc B, Senjanovic G and Vissani F 2003 *Phys. Rev. Lett.* **90** 051802
[89] Goh H S, Mohapatra R N and Ng S P 2003 *Phys. Lett.* B **570** 215
Goh H S, Mohapatra R N and Ng S P 2003 *Phys. Rev.* D **68** 115008
Goh H S, Mohapatra R N, Nasri S and Ng S P 2004 *Phys. Lett.* B **587** 105
Dutta B, Mimura Y and Mohapatra R N 2004 *Preprint* hep-ph/0402113
Bajc B, Senjanovic G and Vissani F 2004 *Preprint* hep-ph/0402140
[90] Harvey J A, Reiss D B and Ramond P 1982 *Nucl. Phys.* B **199** 223
Berezhiani Z and Tavartkiladze Z 1997 *Phys. Lett.* B **409** 220
Brahmachari B and Mohapatra R N 1998 *Phys. Rev.* D **58** 015001
Nomura Y and Yanagida T 1999 *Phys. Rev.* D **59** 017303
Oda K Y, Takasugi E, Tanaka M and Yoshimura M 1999 *Phys. Rev.* D **59** 055001
Singh N N 2000 *Preprint* hep-ph/0009087
Maekawa N 2001 *Prog. Theor. Phys.* **106** 401
Babu K S and Barr S M 2002 *Phys. Lett.* B **525** 289

[91] Albright C H and Barr S M 2000 *Phys. Rev. Lett.* **85** 244
Albright C H and Barr S M 2000 *Phys. Rev.* D **62** 093008
Albright C H and Barr S M 2001 *Phys. Rev.* D **64** 073010
[92] Albright C H and Geer S 2002 *Phys. Rev.* D **65** 073004
Albright C H and Geer S 2002 *Phys. Lett.* B **532** 311
Albright C H 2003 *Int. J. Mod. Phys.* A **18** 3947
[93] Asaka T 2003 *Phys. Lett.* B **562** 291
[94] Grimus W and Lavoura L 2001 *J. High Energy Phys.* JHEP07(2001)045
Grimus W and Lavoura L 2001 *Acta Phys. Polon.* B **32** 3719
Grimus W and Lavoura L 2003 *Eur. Phys. J.* C **28** 123
Grimus W and Lavoura L 2003 *Phys. Lett.* B **572** 189
Grimus W and Lavoura L 2003 *Preprint* hep-ph/0305309
King S F and Ross G G 2001 *Phys. Lett.* B **520** 243
Ohlsson T and Seidl G 2002 *Phys. Lett.* B **537** 95
Ohlsson T and Seidl G 2002 *Nucl. Phys.* B **643** 247
Babu K S, Ma E and Valle J W F 2003 *Phys. Lett.* B **552** 207
Kubo J, Mondragon A, Mondragon M and Rodriguez-Jauregui E 2003 *Prog. Theor. Phys.* **109** 795
Chen S L G, Frigerio M and Ma E 2004 *Preprint* hep-ph/0404084
Ma E 2004 *Preprint* hep-ph/0404199