CP violation and the muon anomaly in $N=1$ supergravity

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The one loop supersymmetric electroweak correction to the anomalous magnetic moment of the muon is derived in the minimal $N=1$ supergravity unification with two CP violating phases. A numerical analysis of the CP violating effects on $g_\mu-2$ is carried out with the cancellation mechanism to guarantee the satisfaction of the experimental limits on the electric dipole moments of the electron and on the neutron. It is found that the effects of the CP phases can significantly affect the supersymmetric electroweak correction to $g_\mu-2$, and that the numerical size of such a correction can be as large or larger than the standard model electroweak correction to $g_\mu-2$. These results are of import for the new Brookhaven experiment which is expected to increase the sensitivity of the $g_\mu-2$ measurements by a factor of 20 and allow for a test of the electroweak correction in the near future.

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I. INTRODUCTION

Supersymmetric theories contain a large number of CP violating phases which arise from the soft supersymmetry (SUSY) breaking sector of the theory and contribute to the electric dipole moment (EDM) of the quarks and the leptons. Currently there exist stringent limits on the neutron [1] and on the electron [2] EDM. Thus CP violation in supersymmetric theories is severely constrained by experiment. To satisfy these constraints it has generally been assumed that the CP violating phases are small [3,4]. However, small phases constitute a fine tuning and an alternative possibility suggested is that the CP violating phases can be large O(1) and the EDM constraints could be satisfied by the choice of a heavy spectrum [5]. However, for CP phases O(1) the satisfaction of the EDM constraints may require the SUSY spectrum to lie in the several TeV region, thus putting the spectrum even beyond the reach of the CERN Large Hadron Collider (LHC). More recently a third possibility has been proposed, and that is of internal cancellations among various contributions to the electron and the neutron EDMs [6], and there have been further developments of this idea [7–11]. Since the cancellation mechanism allows for the possibility of large CP violating phases, it is of considerable interest to explore the effects of such large phases on low energy physics and several studies exploring the effects of large phases have recently been reported. These include the effects of large CP phases on dark matter [12,13] and on low energy phenomena [14–17], as well as other SUSY phenomena [18–23].

In this paper we investigate the effects of CP violation on the supersymmetric electroweak contributions to $g_\mu-2$. This analysis extends the previous analyses of supersymmetric electroweak contributions without the inclusion of the CP violating effects [24,25]. This investigation is timely since the Brookhaven experiment E821 has started collecting data and in the near future will improve the sensitivity of the $g_\mu-2$ measurements to allow a test of the standard model electroweak contribution [26]. It is already known that the supersymmetric electroweak contributions to $g_\mu-2$ can be as large or larger [25,27,28] than the standard model electroweak contribution [29] and it is thus of interest to investigate the effects of large CP violating phases on the supersymmetric muon anomaly.

We begin by exhibiting the SUSY breaking sector of the CP violating phases relevant for our case. It is given by

$$\begin{aligned} V_{SB}=&m_1^2|H_1|^2+m_2^2|H_2|^2-[B\mu\epsilon_{ij}H_1^iH_2^j+\text{H.c.}]\\ &+m_{\tilde L}^2[\tilde\nu_\mu^*\tilde\nu_\mu+\tilde\mu_L^*\tilde\mu_L]+m_{\tilde R}^2\tilde\mu_R^*\tilde\mu_R\\ &+\frac{gm_0}{\sqrt{2}m_W}\epsilon_{ij}\left[\frac{m_\mu A_\mu}{\cos\beta}H_1^i\tilde l_L^j\tilde\mu_R^*+\text{H.c.}\right]\\ &+\frac{1}{2}[\tilde m_2\bar{\tilde W}^a\tilde W^a+\tilde m_1\bar{\tilde B}\tilde B]+\Delta V_{SB} \end{aligned} \tag{1}$$

where $\tilde l_L$ is the $SU(2)_L$ smuon doublet, $\tan\beta=|\langle H_2\rangle/\langle H_1\rangle|$ where H_1 gives mass to the muon. The quantities A_μ, μ, and B are in general complex.

In this analysis we shall limit ourselves to the framework of the minimal supergravity model [30]. In the minimal supergravity (MSUGRA) framework the soft SUSY breaking is characterized by the parameters m_0, $m_{1/2}$, A_0, $\tan\beta$, $\theta_{\mu0}$, and α_{A0}, where m_0 is the universal scalar mass at the grand unified theory (GUT) scale, $m_{1/2}$ is the universal gauginos mass at the GUT scale, A_0 is the universal trilinear coupling at the GUT scale, $\theta_{\mu0}$ is the phase of μ_0 at the GUT scale, and α_{A0} is the phase of A_0. In the analysis we use one-loop renormalization group equations (RGEs) for the evolution of the soft SUSY breaking parameters and for the parameter μ, and two-loop RGEs for the gauge and Yukawa couplings. The phase of μ does not run because it cancels out of the one loop renormalization group equation of μ. However, the magnitude and the phase of A_μ do evolve. Thus while the phase of A_μ is modified from $\alpha_{A_{\mu0}}$ at the GUT scale to its

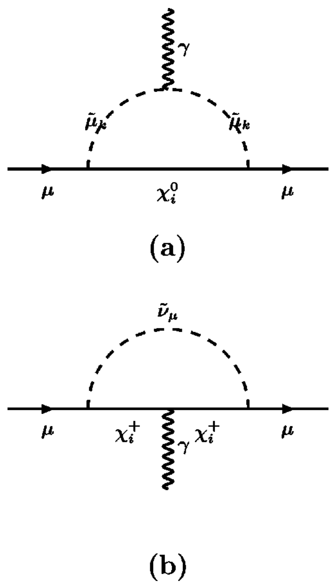

FIG. 1. The one loop contribution to $g_\mu - 2$ from (a) neutralino exchange and (b) chargino exchange diagrams.

value α_{A_μ} at the electroweak scale, the phase of μ is unaffected at the one loop level, i.e., $\theta_\mu = \theta_{\mu 0}$.

The outline of the rest of the paper is as follows: In Sec. II we derive a general formula for the contribution to $a_f = (g_f - 2)/2$ in the presence of CP violating phases. In Sec. III we compute the supersymmetric electroweak corrections to a_μ from the chargino exchange and in Sec. IV we compute the supersymmetric electroweak corrections to a_μ from the neutralino exchange. A discussion of these results is given in Sec. V and a numerical analysis of the effects of CP violating phases is given in Sec. VI. We summarize our results in Sec. VII.

II. $g-2$ CALCULATION WITH CP VIOLATION IN SUSY

In this section we derive a general formula for the contribution to a_μ for an interaction with CP violating phases which would be typical of the interactions that we will encounter in Secs. III and IV. For a theory of a fermion ψ_f of mass m_f interacting with other heavy fermions ψ_i's and heavy scalars ϕ_k's with masses m_i and m_k, the interaction that contains CP violation is in general given by

$$-\mathcal{L}_{int} = \sum_{ik} \bar{\psi}_f \left(K_{ik} \frac{1-\gamma_5}{2} + L_{ik} \frac{1+\gamma_5}{2} \right) \psi_i \phi_k + \text{H.c.} \quad (2)$$

Here $\mathcal{L}$ violates CP invariance iff $\text{Im}(K_{ik} L_{ik}^*)$ is different from zero. The one loop contribution to a_f is given by

$$a_f = a_f^1 + a_f^2 \quad (3)$$

where a_f^1 and a_f^2 come from Fig. 1(a) and Fig. 1(b) respectively. a_f^1 is a sum of two terms, $a_f^1 = a_f^{11} + a_f^{12}$, where

$$a_f^{11} = \sum_{ik} \frac{m_f}{16\pi^2 m_i} \text{Re}(K_{ik} L_{ik}^*) F_1\left(\frac{m_k^2}{m_i^2} \right) \quad (4)$$

and

$$F_1(x) = \frac{1}{(x-1)^3} (1 - x^2 + 2x \ln x) \quad (5)$$

and where

$$a_f^{12} = \sum_{ik} \frac{m_f^2}{96\pi^2 m_i^2} (|K_{ik}|^2 + |L_{ik}|^2) F_2\left(\frac{m_k^2}{m_i^2} \right) \quad (6)$$

and

$$F_2(x) = \frac{1}{(x-1)^4} (-x^3 + 6x^2 - 3x - 2 - 6x \ln x). \quad (7)$$

Similarly, a_f^2 also consists of two terms $a_f^2 = a_f^{21} + a_f^{22}$, where

$$a_f^{21} = -\sum_{ik} \frac{m_f}{16\pi^2 m_i} \text{Re}(K_{ik} L_{ik}^*) F_3\left(\frac{m_k^2}{m_i^2} \right) \quad (8)$$

and

$$F_3(x) = \frac{1}{(x-1)^3} (3x^2 - 4x + 1 - 2x^2 \ln x) \quad (9)$$

and where

$$a_f^{22} = \sum_{ik} \frac{m_f^2}{96\pi^2 m_i^2} (|K_{ik}|^2 + |L_{ik}|^2) F_4\left(\frac{m_k^2}{m_i^2} \right) \quad (10)$$

and

$$F_4(x) = \frac{1}{(x-1)^4} (2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x). \quad (11)$$

III. CHARGINO CONTRIBUTIONS WITH CP VIOLATING PHASES

The chargino matrix with CP violating phases is given by

$$M_C = \begin{pmatrix} \tilde{m}_2 & \sqrt{2} m_W \sin\beta \\ \sqrt{2} m_W \cos\beta & |\mu| e^{i\theta_\mu} \end{pmatrix}. \quad (12)$$

This matrix can be diagonalized by a biunitary transformation $U^* M_C V^{-1} = \text{diag}(\tilde{m}_{\chi_1^+}, \tilde{m}_{\chi_2^+})$. By looking at the muon-sneutrino-chargino interaction

$$-\mathcal{L}_{\mu-\tilde{\nu}-\tilde{\chi}^+} = g \bar{\mu} [V_{11} P_R - \kappa_\mu U_{12}^* P_L] \tilde{\chi}_1^+ \tilde{\nu} + g \bar{\mu} [V_{21} P_R - \kappa_\mu U_{22}^* P_L] \tilde{\chi}_2^+ \tilde{\nu} + \text{H.c.}, \quad (13)$$

where $\kappa_\mu = m_\mu / \sqrt{2} M_W \cos\beta$, we find that the chargino exchange to a_μ is given by

$$a_\mu^{\chi^+} = a_\mu^{21} + a_\mu^{22} \quad (14)$$

where

$$a_\mu^{21}=\frac{m_\mu\alpha_{EM}}{4\pi\sin^2\theta_W}\frac{m_\mu}{\sqrt{2}m_W\cos\beta}\sum_{i=1}^{2}\frac{1}{M_{\chi_i^+}}\mathrm{Re}(U_{i2}^*V_{i1}^*)F_3\left(\frac{M_{\tilde{\nu}}^2}{M_{\chi_i^+}^2}\right) \tag{15}$$

and

$$a_\mu^{22}=\frac{m_\mu^2\alpha_{EM}}{24\pi\sin^2\theta_W}\sum_{i=1}^{2}\frac{1}{M_{\chi_i^+}^2}\left(\frac{m_\mu^2}{2m_W^2\cos^2\beta}|U_{i2}|^2+|V_{i1}|^2\right)F_4\left(\frac{M_{\tilde{\nu}}^2}{M_{\chi_i^+}^2}\right). \tag{16}$$

The phase which enters here is θ_μ through the matrix elements of U and V.

IV. NEUTRALINO CONTRIBUTIONS WITH *CP* VIOLATING PHASES

The neutralino mass matrix M_{χ^0} is a complex symmetric matrix and is given by

$$\begin{pmatrix} \tilde{m}_1 & 0 & -M_z\sin\theta_W\cos\beta & M_z\sin\theta_W\sin\beta \\ 0 & \tilde{m}_2 & M_z\cos\theta_W\cos\beta & -M_z\cos\theta_W\sin\beta \\ -M_z\sin\theta_W\cos\beta & M_z\cos\theta_W\cos\beta & 0 & -|\mu|e^{i\theta_\mu} \\ M_z\sin\theta_W\sin\beta & -M_z\cos\theta_W\sin\beta & -|\mu|e^{i\theta_\mu} & 0 \end{pmatrix}. \tag{17}$$

The matrix M_{χ^0} can be diagonalized by the unitary transformation

$$X^TM_{\chi^0}X=\mathrm{diag}(\tilde{m}_{\chi_1^0},\tilde{m}_{\chi_2^0},\tilde{m}_{\chi_3^0},\tilde{m}_{\chi_4^0}). \tag{18}$$

The smuon (mass)^2 matrix is given by

$$M_{\tilde{\mu}}^2=\begin{pmatrix} M_{\tilde{L}}^2+m_\mu^2-M_z^2\left(\frac{1}{2}-\sin^2\theta_W\right)\cos 2\beta & m_\mu(A_\mu^*m_0-\mu\tan\beta) \\ m_\mu(A_\mu m_0-\mu^*\tan\beta) & M_{\tilde{R}}^2+m_\mu^2-M_Z^2\sin^2\theta_W\cos 2\beta \end{pmatrix}. \tag{19}$$

This matrix is Hermitian and can be diagonalized by the unitary transformation

$$D_\mu^\dagger M_{\tilde{\mu}}^2D_\mu=\mathrm{diag}(M_{\tilde{\mu}1}^2,M_{\tilde{\mu}2}^2). \tag{20}$$

The muon-smuon-neutralino interaction in the mass diagonal basis is defined by

$$\begin{aligned}-\mathcal{L}_{\mu-\tilde{\mu}-\tilde{\chi}^0}=\sum_{j=1}^{4}&\sqrt{2}\bar{\mu}[(\alpha_{\mu j}D_{\mu 11}-\gamma_{\mu j}D_{\mu 21})P_L\\&+(\beta_{\mu j}D_{\mu 11}-\delta_{\mu j}D_{\mu 21})P_R]\tilde{\chi}_j^0\tilde{\mu}_1\\&+\sqrt{2}\bar{\mu}[(\alpha_{\mu j}D_{\mu 12}-\gamma_{\mu j}D_{\mu 22})P_L\\&+(\beta_{\mu j}D_{\mu 12}-\delta_{\mu j}D_{\mu 22})P_R]\tilde{\chi}_j^0\tilde{\mu}_2+\mathrm{H.c.}\end{aligned} \tag{21}$$

where α, β, γ and δ are given by

$$\alpha_{\mu j}=\frac{gm_\mu X_{3,j}}{2m_W\cos\beta} \tag{22}$$

$$\beta_{\mu j}=eQ_\mu X_{1j}'^*+\frac{g}{\cos\theta_W}X_{2j}'^*(T_{3\mu}-Q_\mu\sin^2\theta_W) \tag{23}$$

$$\gamma_{\mu j}=eQ_\mu X_{1j}'-\frac{gQ_\mu\sin^2\theta_W}{\cos\theta_W}X_{2j}' \tag{24}$$

$$\delta_{\mu j}=-\frac{gm_\mu X_{3,j}^*}{2m_W\cos\beta} \tag{25}$$

and where

$$X_{1j}'=X_{1j}\cos\theta_W+X_{2j}\sin\theta_W \tag{26}$$

$$X_{2j}'=-X_{1j}\sin\theta_W+X_{2j}\cos\theta_W. \tag{27}$$

The neutralino exchange contribution to a_μ is given by

$$a_\mu^{\chi^0}=a_\mu^{11}+a_\mu^{12} \tag{28}$$

where

$$a_\mu^{11}=\frac{m_\mu\alpha_{EM}}{4\pi\sin^2\theta_W}\sum_{j=1}^{4}\sum_{k=1}^{2}\frac{1}{M_{\chi_j^0}}\eta_{\mu j}^kF_1\left(\frac{M_{\tilde{\mu}_k}^2}{M_{\chi_j^0}^2}\right) \tag{29}$$

and

$$a_\mu^{12}=\frac{m_\mu^2\alpha_{EM}}{24\pi\sin^2\theta_W}\sum_{j=1}^{4}\sum_{k=1}^{2}\frac{1}{M_{\chi_j^0}^2}X_{\mu j}^k F_2\left(\frac{M_{\tilde{\mu}_k}^2}{M_{\chi_j^0}^2}\right). \tag{30}$$

Here

$$\eta_{\mu j}^k=-\tan^2\theta_W\ \mathrm{Re}(X_{1j}^2D_{1k}^*D_{2k})-\tan\theta_W\ \mathrm{Re}(X_{2j}X_{1j}D_{1k}^*D_{2k})+\frac{m_\mu\tan\theta_W}{M_W\cos\beta}|D_{2k}|^2\ \mathrm{Re}(X_{3j}X_{1j})$$
$$-\frac{m_\mu\tan\theta_W}{2M_W\cos\beta}|D_{1k}|^2\ \mathrm{Re}(X_{3j}X_{1j})-\frac{m_\mu}{2M_W\cos\beta}|D_{1k}|^2\ \mathrm{Re}(X_{3j}X_{2j})+\frac{m_\mu^2}{2M_W^2\cos^2\beta}\mathrm{Re}(X_{3j}^2D_{2k}^*D_{1k}) \tag{31}$$

and

$$X_{\mu j}^k=\frac{m_\mu^2}{2M_W^2\cos^2\beta}|X_{3j}|^2+\frac{1}{2}\tan^2\theta_W|X_{1j}|^2(|D_{1k}|^2+4|D_{2k}|^2)+\frac{1}{2}|X_{2j}|^2|D_{1k}|^2+\tan\theta_W|D_{1k}|^2\ \mathrm{Re}(X_{1j}X_{2j}^*)$$
$$+\frac{m_\mu\tan\theta_W}{M_W\cos\beta}\mathrm{Re}(X_{3j}X_{1j}^*D_{1k}D_{2k}^*)-\frac{m_\mu}{M_W\cos\beta}\mathrm{Re}(X_{3j}X_{2j}^*D_{1k}D_{2k}^*). \tag{32}$$

The matrix elements of X carry the phase of μ and the matrix elements of D carry both the phase of μ and the phase of the trilinear parameter A_μ where A_μ is the renormalization group evolved value of A_{μ_0} at the Z scale.

V. DISCUSSION OF RESULTS

It is interesting to consider the supersymmetric limit of our results when the soft SUSY breaking terms vanish. In this limit Eq. (14) which arises from the chargino exchange gives a contribution which is equal in magnitude and opposite in sign to the contribution from the W exchange. Thus we find that, in the supersymmetric limit,

$$a_\mu^W+a_\mu^{\chi^+}=0. \tag{33}$$

Similarly taking the supersymmetric limit of Eq. (28) we find that the massive modes neutralino exchange contribution is equal in magnitude and opposite in sign to the Z boson exchange contribution so that

$$a_\mu^Z+a_\mu^{\chi^0}(\text{massive modes})=0. \tag{34}$$

This is what one expects on general grounds [31,32] and our explicit evaluations satisfy Eqs. (33) and (34). The proof of Eqs. (33) and (34) is given in the Appendix. A result similar to Eq. (34) holds for the massless modes but its proof requires extension of the results of Sec. II to include m_f corrections in the loop integrals. This extension will be discussed elsewhere.

Next we discuss the limit of vanishing CP violating phases. In this limit the unitary matrices U and V become orthogonal matrices. Using the notation

$$V^{-1}\rightarrow O_1,\quad U^*\rightarrow O_2^T \tag{35}$$

where O_1 and O_2 are orthogonal matrices, the chargino contributions take the form

$$a_\mu^{21}=\frac{m_\mu\alpha_{EM}}{4\pi\sin^2\theta_W}\frac{m_\mu}{\sqrt{2}m_W\cos\beta}\sum_{i=1}^{2}\frac{1}{M_{\chi_i^+}}O_{22i}O_{1i1}^T F_3(\xi_{\nu i}) \tag{36}$$

and

$$a_\mu^{22}=\frac{m_\mu^2\alpha_{EM}}{24\pi\sin^2\theta_W}\sum_{i=1}^{2}\frac{1}{M_{\chi_i^+}^2}\left(\frac{m_\mu^2}{2m_W^2\cos^2\beta}(O_{22i})^2+(O_{1i1}^T)^2\right)\times F_4(\xi_{\nu i}) \tag{37}$$

where $\xi_{\nu i}=M_{\tilde{\nu}}^2/M_{\chi_i^+}^2$. The neutralino exchange contributions in the CP violating limit can similarly be obtained from Eqs. (28)–(32) by the replacement

$$X\rightarrow O,\quad D\rightarrow S \tag{38}$$

where O and S are orthogonal matrices. Our results for the chargino and neutralino contributions go to the result of the previous works in [25] in the vanishing CP phase limit considered above.

VI. ANALYSIS OF *CP* VIOLATING EFFECTS

Before discussing the effects of CP violating phases on the supersymmetric contributions to a_μ we summarize briefly the current experimental and theoretical situation regarding a_μ. The most accurate determination of a_μ is from the CERN experiment [33] which gives a value of $a_\mu^{expt}=11659230(84)\times10^{-10}$ while the standard model determination including α^5 QED contributions [34], hadronic vacuum polarization [35] and light by light hadronic contributions [36], and the complete two loop standard model electroweak contributions [37], is $a_\mu^{SM}=11659162(6.5)\times 10^{-10}$. Here essentially the entire error shown in parentheses comes from the hadronic sector. It is expected that the new Brookhaven g_μ experiment [26,38] will improve by a factor of 20 the determination of a_μ over the previous a_μ measurement [33]; i.e., the error in the experimental determination of a_μ is expected to go down to 4×10^{-10}. This means that even with no further reduction in the hadronic error the new g_μ experiment will be able to test the standard model electroweak corrections which contribute an amount [37] $a_\mu^{EW}(\mathrm{SM})=15.1(0.4)\times 10^{-10}$. However, it has been pointed out that the supersymmetric electroweak corrections can be as large or larger than the standard model electroweak corrections, and thus the new g_μ experiment will also probe supersymmetry [25,27,28]. In this context it is important to know how large the CP violating effects are on the supersymmetric electroweak anomaly.

Previous analyses of g_μ in supersymmetry did not consider the effects of CP violating phases because the effects of such phases were expected to be generally small due to the electric dipole moment constraints. As mentioned in Sec. I in the conventional scenarios the current experimental constraints on the electron EDM (d_e) and on the neutron EDM (d_n) are satisfied either by the choice of small CP violating phases [3,4] or by the choice of a heavy mass spectrum of supersymmetric particles [5]. For the first case, the CP violating effects are small because of the smallness of the CP violating phases, while for the second the supersymmetric contribution to $g_\mu-2$ will itself be small compared to the standard model result to be of relevance. However, as also pointed out in Sec. I with the cancellation mechanism [6] one can satisfy the EDM constraints with large CP violating phases and not too massive a SUSY spectrum and thus it is of relevance to examine the effects of CP violating phases on a_μ.

For the case of the electron EDM the cancellations occur between the chargino and the neutralino exchange contributions while for the case of the neutron EDM the cancellations can occur in a two step process. Thus, for the neutron case, the EDM receives contributions from the electric dipole, the chromoelectric dipole and the purely gluonic dimension-6 operators. For the electric and the chromoelectric dipole operators cancellations can occur between the chargino, the gluino and the neutralino exchange contributions. There is, however, the possibility of a further cancellation, and that is among the electric dipole, the chromoelectric dipole and the purely gluonic contributions. Recently, it has been pointed out that in addition to the above contributions certain two loop graphs may also contribute significantly in some regions of the parameter space [39]. In our analysis we have included the effects of these contributions as well. However, we find that the effect of these terms is relatively small compared to the other contributions. The regions of interest in the parameter space are those where the cancellations among different components happen simultaneously for the case of the electron EDM and of the neutron EDM so as to satisfy the experimental lower limits, which for the neutron is [1]

$$|d_n|<6.3\times 10^{-26}\ e\ \mathrm{cm} \tag{39}$$

and for the electron is [2]

$$|d_e|<4.3\times 10^{-27}\ e\ \mathrm{cm}. \tag{40}$$

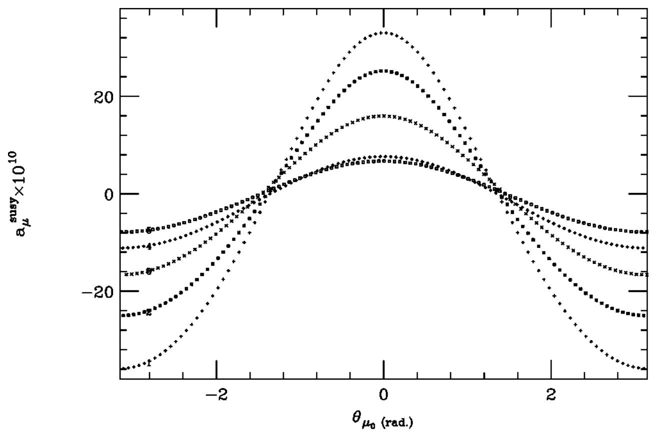

FIG. 2. Plot of a_μ^{SUSY} as a function of the CP violating phase θ_{μ_0}. The values of the other parameters for curves (1)–(5) correspond to the cases (1)–(5) in Table I.

We discuss now the size of the CP violating effects on a_μ^{SUSY}. In Fig. 2 we exhibit the effect of the variation of the CP violating phase θ_{μ_0} on a_μ^{SUSY}, without the imposition of the EDM constraint, as a function of θ_{μ_0}. The values of the other parameters (m_0, $m_{\tilde g}$, $\tan\beta$, α_{A_0}) for curves (1)–(5) can be read off from Table I for the cases (1)–(5). We find

TABLE I. Parameters corresponding to cases (1)–(5). Other parameters are (1) $m_0=60$, $m_{1/2}=123$, $\tan\beta=3.5$, $|A_0|=5.45$, (2) $m_0=65$, $m_{1/2}=119$, $\tan\beta=2.6$, $|A_0|=2.93$, (3) $m_0=80$, $m_{1/2}=147$, $\tan\beta=2.6$, $|A_0|=2.93$, (4) $m_0=120$, $m_{1/2}=228$, $\tan\beta=3.5$, $|A_0|=5.47$, (5) $m_0=120$, $m_{1/2}=220$, $\tan\beta=2.6$, $|A_0|=2.93$, where all masses are in GeV units.

	θ_{μ_0}	α_{A_0}	$d_n(10^{-26}\ e\ \mathrm{cm})$	$d_e(10^{-27}\ e\ \mathrm{cm})$	$a_\mu(\theta_{\mu_0},\alpha_A)\ (10^{-9})$
(1)	3.108	−0.2	5.4	−2.7	−3.6
(2)	3.08	−0.45	4.86	−4.26	−2.5
(3)	3.02	−1.0	−3.6	−3.1	−1.7
(4)	3.1	−0.2	−4.9	−0.93	−1.1
(5)	3.02	−1.0	−5.0	1.1	−.78

TABLE II. The values of a_μ for four CP conserving cases with all other parameters the same as in the corresponding cases in Table I.

	$a_\mu(0,0)$ (10^{-9})	$a_\mu(0,\pi)$ (10^{-9})	$a_\mu(\pi,0)$ (10^{-9})	$a_\mu(\pi,\pi)$ (10^{-9})
(1)	3.25	4.18	−3.5	−2.6
(2)	2.49	3.1	−2.6	−1.98
(3)	1.5	1.86	−1.9	−1.34
(4)	.75	1.12	−1.13	−.75
(5)	.62	.82	−.89	−.6

that the effect of the CP violating phase is very substantial. A similar analysis of the effects of the variation of the CP violating phase α_{A_0} on a_μ^{SUSY}, also without the imposition of the EDM constraint, is given in Fig. 3. Again the value of the parameters other than α_{A_0} for the curves labeled (1)–(5) can be read off from Table I. Here again the effects of the CP violating phase α_{A_0} are found to be quite substantial although not as large as those from θ_{μ_0}. The reason for this discrepancy is easily understood. In the region of the parameter space considered the chargino contribution is large and this contribution is independent of α_{A_0} since α_{A_0} does not enter in the chargino mass matrix. Thus the α_{A_0} dependence enters only via the smuon mass matrix, while θ_{μ_0} enters via all mass matrices.

Inclusion of the EDM constraint puts stringent constraints on the parameter space of MSUGRA. As an illustration we give in Fig. 4 the plot of the EDM of the electron and for the neutron as a function of α_{A0} for curves (1) and (3) of Fig. 3. The figure illustrates the simultaneous cancellation occurring for the electron and neutron EDMs in narrow regions of α_{A0} and in these regions the experimental EDM constraints can be satisfied. We note the appearance of two cancellation minima in the cases considered. These double minima reveal the strong dependence on the phases of the various terms that contribute to the EDMs. The effects of CP violating phases on a_μ can be significant in these domains. In Table I we give a set of illustrative points where a simultaneous cancellation in the electron and the neutron EDM occurs. The size of the effects of CP violating phases on a_μ can be seen from the values of a_μ at these points and the corresponding four CP conserving cases in Table II. A comparison of the results of Tables I and II with those of Figs. 2 and 3 shows that with the inclusion of the EDM constraints the CP violating effects are much reduced for the points chosen here. However, even with the inclusion of the EDM constraints the CP effects on a_μ can still be quite substantial as a comparison of Tables I and II exhibits. Inclusion of more than two phases makes the satisfaction of the EDM constraints much easier and detailed analyses show that there are significant regions of the parameter space where the CP violating phases are large and cancellations occur to render the electron and the neutron EDM in conformity with experiment [7,9]. Such regions are of considerable interest in the investigations of SUSY phenomena at low energy. The effects of CP violating phases in these regions could be substantial. However, a quantitative discussion of these effects requires inclusion of non-universal effects which are outside the framework of the MSUGRA model discussed here.

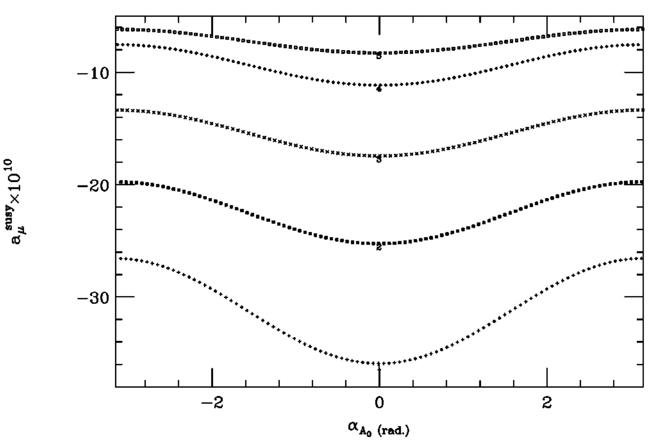

FIG. 3. Plot of a_μ^{SUSY} as a function of the CP violating phase α_{A_0}. The values of the other parameters for curves (1)–(5) correspond to the cases (1)–(5) in Table I.

VII. CONCLUSIONS

In this paper we have derived the general one loop formula for the effects of CP violating phases on the anoma-

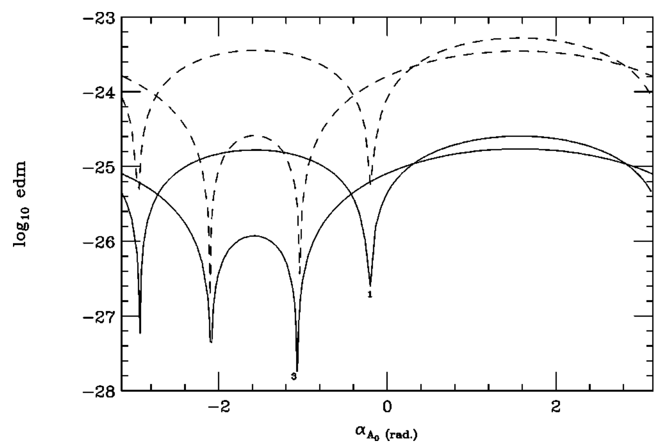

FIG. 4. Exhibition of the dependence of the |EDM| of the electron (solid line) and the neutron (dashed line) and the cancellation as a function of α_{A_0}. The curves with minima to the extreme left and the extreme right have other parameters corresponding to case (1) of Table I, while the curves with two minima in the middle have other parameters corresponding to case (3) of Table I.

lous magnetic moment of a fermion. We then specialized our analysis to the case of the calculation of CP violating effects on the supersymmetric muon anomaly. Here the contributions arise from the one loop chargino and neutralino exchange diagrams. The numerical analysis of the CP violating effects is strongly constrained by the experimental EDM constraints on the electron [2] and on the neutron [1]. Our analysis including these constraints shows that the size of the CP violating effects is strongly dependent on the region of the parameter space one is in and that the CP violating phases can produce substantial affects on the supersymmetric electroweak contribution. We also find that the supersymmetric contribution to the muon anomaly in the presence of large CP violating phases consistent with the EDM constraints can be as large or larger than the standard model electroweak contribution. These results are of interest in view of the new BNL muon $g-2$ experiment which will improve the accuracy of the muon $g-2$ measurement by a factor of 20 and test the electroweak correction to $g_\mu-2$.

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APPENDIX: THE SUPERSYMMETRIC LIMIT

In this appendix we exhibit the supersymmetric limit of the chargino exchange contribution. The supersymmetric limit corresponds to $M_{\tilde{\nu}}=0$, $\tilde{m}_i=0$ (1,2), $\tan\beta=1$ and $\mu=0$. In this limit $F_3(0)=-1$, $F_4(0)=1$, and the unitary matrices U and V take the values

$$U=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1\\ -1 & 1\end{pmatrix},\quad V=\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1\\ 1 & -1\end{pmatrix} \tag{A1}$$

where $U^*M_CV^{-1}=\mathrm{diag}(M_W,M_W)$. In this limit a_μ^{21}, a_μ^{22} and the total chargino contribution $a_\mu^{\chi^+}$ are given by

$$a_\mu^{21}=-\frac{\alpha_{EM}}{4\pi\sin^2\theta_W}\frac{m_\mu^2}{M_W^2},\quad a_\mu^{22}=\frac{\alpha_{EM}}{24\pi\sin^2\theta_W}\frac{m_\mu^2}{M_W^2} \tag{A2}$$

$$a_\mu^{\chi^+}=-\frac{5\alpha_{EM}}{24\pi\sin^2\theta_W}\frac{m_\mu^2}{M_W^2}. \tag{A3}$$

The result of Eq. (A3) is to be compared with the contribution arising from the exchange of the W boson [29]:

$$a_\mu^W=\frac{5m_\mu^2G_F}{12\pi^2\sqrt{2}}. \tag{A4}$$

Using $G_F=\pi\alpha_{em}/(M_W^2\sqrt{2}\sin^2\theta_W)$ we then find that the sum of the chargino and the W exchange contributions vanishes in the supersymmetric limit.

Next we consider the neutralino exchange contribution to $a_\mu^{\chi^0}$ in the supersymmetric limit. In this limit two of the eigenvalues of the neutralino mass matrix are zero and the other two are $\pm M_Z$. However, we choose the unitary transformation X so that the non-vanishing eigenvalues are all positive definite: i.e.,

$$X^TM_{\chi^0}X=\mathrm{diag}(0,0,M_Z,M_Z). \tag{A5}$$

In this case the unitary matrix X takes on the form

$$\begin{pmatrix} \alpha & \beta & \frac{\sin\theta_W}{\sqrt{2}} & i\frac{\sin\theta_W}{\sqrt{2}}\\ \alpha\tan\theta_W & \beta\tan\theta_W & -\frac{\cos\theta_W}{\sqrt{2}} & -i\frac{\cos\theta_W}{\sqrt{2}}\\ \alpha & -\frac{1}{2}\beta\sec^2\theta_W & -\frac{1}{2} & \frac{i}{2}\\ \alpha & -\frac{1}{2}\beta\sec^2\theta_W & \frac{1}{2} & -\frac{i}{2}\end{pmatrix} \tag{A6}$$

where

$$\alpha=\frac{1}{\sqrt{3+\tan^2\theta_W}},\quad \beta=\frac{1}{\sqrt{1+\tan^2\theta_W+\frac{1}{2}\sec^4\theta_W}}. \tag{A7}$$

The appearance of i $(=\sqrt{-1})$ in the last column in X is to guarantee that the eigenvalues are all positive definite. In the supersymmetric limit $\eta_{\mu j}^k$ take the following form:

$$\eta_{\mu j}^1=-\frac{m_\mu}{\sqrt{2}M_W}\mathrm{Re}(X_{3j}X_{2j})-\frac{m_\mu}{\sqrt{2}M_W}\tan\theta_W\,\mathrm{Re}(X_{3j}X_{1j}) \tag{A8}$$

and

$$\eta_{\mu j}^2=\frac{\sqrt{2}m_\mu}{M_W}\tan\theta_W\,\mathrm{Re}(X_{3j}X_{1j}) \tag{A9}$$

while $X_{\mu j}^k$ take the form

$$X_{\mu j}^1=\frac{m_\mu^2}{M_W^2}|X_{3j}|^2+\frac{1}{2}\tan^2\theta_W|X_{1j}|^2+\frac{1}{2}|X_{2j}|^2 +\tan\theta_W\,\mathrm{Re}(X_{1j}X_{2j}^*) \tag{A10}$$

$$X_{\mu j}^2=\frac{m_\mu^2}{M_W^2}|X_{3j}|^2+2\tan^2\theta_W|X_{1j}|^2. \tag{A11}$$

Using the above and the limit $F_1(0)=-1$, $F_2(0)=-2$ and by ignoring the terms of higher order of m_μ, one finds that a_μ^{11} and a_μ^{12} simplify as follows:

$$a_\mu^{11}=\sum_{j=3}^{4}\frac{m_\mu^2\alpha_{EM}}{4\sqrt{2}\pi\sin^2\theta_W M_W M_Z}\times[\mathrm{Re}(X_{3j}X_{2j})+\tan\theta_W\,\mathrm{Re}(X_{3j}X_{1j})-2\tan\theta_W\,\mathrm{Re}(X_{3j}X_{1j})] \quad \text{(A12)}$$

$$a_\mu^{12}=-2\sum_{j=3}^{4}\frac{m_\mu^2\alpha_{EM}}{48\pi\sin^2\theta_W M_Z^2}[5\tan\theta_W|X_{1j}|^2+|X_{2j}|^2+2\tan\theta_W\,\mathrm{Re}(X_{1j}X_{2j}^*)]. \quad \text{(A13)}$$

Substitution of the explicit form of X from Eq. (A6) into Eqs. (A12) and (A13) gives

$$a_\mu^{11}=\frac{m_\mu^2 G_F}{2\sqrt{2}\pi^2}\left(\frac{1}{2}\right) \quad \text{(A14)}$$

$$a_\mu^{12}=-\frac{m_\mu^2 G_F}{2\sqrt{2}\pi^2}\left(\frac{4}{3}\sin^4\theta_W-\frac{2}{3}\sin^2\theta_W+\frac{1}{6}\right) \quad \text{(A15)}$$

and

$$a_\mu^{\chi^0}=-\frac{m_\mu^2 G_F}{2\sqrt{2}\pi^2}\left(\frac{4}{3}\sin^4\theta_W-\frac{2}{3}\sin^2\theta_W-\frac{1}{3}\right). \quad \text{(A16)}$$

The result of Eq. (A16) is to be compared to the standard model Z exchange contribution [29]

$$a_\mu^{Z}=\frac{m_\mu^2 G_F}{2\sqrt{2}\pi^2}\left[-\frac{5}{12}+\frac{4}{3}\left(\sin^2\theta_W-\frac{1}{4}\right)^2\right]. \quad \text{(A17)}$$

Thus one finds that in the supersymmetric limit the sum of the neutralino and the Z boson exchange contributions vanishes.

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