Minimal supersymmetric left-right model

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We construct the minimal left-right symmetric model by utilizing only the fields dictated by supersymmetry and automatic R-parity conservation. Allowing for nonrenormalizable operators in the superpotential, we show that parity can be spontaneously broken while preserving electromagnetic gauge invariance. The scale of parity breakdown is predicted in the intermediate region $M_R > 10^{10} - 10^{11}$ GeV, and R-parity remains exact. The theory contains a number of charged and doubly charged Higgs scalars with a low mass of order M_R^2/M_{Planck}, accessible to experiment. [S0556-2821(98)01507-0]

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I. INTRODUCTION

Certainly the most popular extension of the minimal standard model is its supersymmetric counterpart. Other very popular ones are left-right symmetric theories [1], which attribute the observed parity asymmetry in the weak interactions to the spontaneous breakdown of left-right symmetry, i.e. generalized parity transformations. Furthermore, left-right symmetry plays an important role in attempting to understand the smallness of strong CP violation [2], and in this sense provides an alternative to Peccei-Quinn symmetry.

Recently it has been pointed out that a particularly simple solution to the strong CP problem results from the merging of these two proposals [3]. Another, maybe more important *raison d'etre* for supersymmetric left-right models is the fact that they lead naturally to R-parity conservation. Namely, left-right models contain a $B-L$ gauge symmetry, which allows for this possibility [4]. All that is needed is that one uses a version of the theory that incorporates a seesaw mechanism [5] at the renormalizable level. More precisely, R-parity (which keeps particles invariant, and changes the sign of sparticles) can be written as

$$R = (-1)^{3(B-L)+2S} \tag{1}$$

where S is the spin of the particle. It can be shown that in these kind of theories, invariance under $B-L$ implies R-parity conservation [4].

However, the construction of specific models turned out to be unexpectedly nontrivial. Namely, in the minimal version of the theory, at the renormalizable level, symmetry breaking is not possible [6]. This may be cured by adding more fields to the theory [7] and/or assuming that the scale (M_R) of left-right symmetry breaking is not greater than the scale of supersymmetry breaking [6]. We should mention that phenomenological aspects of the supersymmetric left-right theories were also studied in [8], without worrying about the problem of symmetry breaking. We address our attention precisely to this, the central issue of the theory.

Although the experiments still allow for a light M_R, we take seriously the possibility of a large M_R scale, as hinted by both the phenomenological success of the standard model and neutrino physics. In such case, the only hope for a realistic theory lies in considering higher-dimensional operators. This is the scope of this paper.

Using nonrenormalizable operators, we construct the minimal supersymmetric left-right model, and show that it naturally can account for spontaneous breakdown of parity. Furthermore, the electromagnetic charge and color-preserving minimum also automatically leads to an exact R-parity, even after integrating the large scale M_R out. As is well known, preserving R-parity implies the stability of the lightest supersymmetric particle, which has well defined phenomenological implications and provides a dark matter candidate.

It is interesting to compare the theory with the minimal renormalizable supersymmetric left-right model [7]. First, in this case naturalness demands that M_R be bigger than about 10^{10} GeV. Furthermore, there is an important difference in the implementation of the seesaw mechanism, since here, as much as in the minimal nonsupersymmetric models, the mechanism does not stay in its minimal form. We discuss this in detail below.

A main feature of these models is the presence of a small scale $m \sim M_R^2/M_{Planck}$. A number of Higgs particles, specially charged and doubly-charged ones have their mass proportional to m. This provides the central phenomenological implication of the theory, since for M_R in the intermediate regime $10^{10} - 10^{12}$ GeV, relevant for neutrino physics, these particles became accessible to new accelerators. This is perhaps the most appealing aspect of the theory. We proceed now with the construction of the model.

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II. THE MINIMAL RENORMALIZABLE MODEL

The minimal left-right extension to the standard model [1] is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. Its supersymmetric version contains three generations of quark and lepton chiral superfields transforming as

$$Q=(3,2,1,1/3), \quad Q_c=(3^*,1,2,-1/3),$$
$$L=(1,2,1,-1), \quad L_c=(1,1,2,1) \tag{2}$$

where the numbers in the brackets denote the quantum numbers under $SU(3)_c$, $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$ respectively.

The Higgs sector consists of two left-handed and two right-handed triplets

$$\Delta=(1,3,1,2), \quad \overline{\Delta}=(1,3,1,-2),$$
$$\Delta_c=(1,1,3,-2), \quad \overline{\Delta}_c=(1,1,3,2) \tag{3}$$

in charge of breaking $SU(2)_R$ symmetry at a large scale M_R; the choice of the adjoint representation is the minimal necessary to achieve a seesaw mechanism for the neutrino mass, and the number of fields is doubled with respect to the nonsupersymmetric version to ensure anomaly cancellations. Of course, one could achieve a seesaw mechanism through nonrenormalizable operators even if one uses doublets instead of triplets. However, in this case just as in the minimal supersymmetric standard model (MSSM) one loses R-parity.

Likewise, to break the remaining standard model symmetry two bidoublets are necessary,

$$\Phi_i=(1,2,2,0) \quad (i=1,2) \tag{4}$$

with $i=1,2$, in order to achieve a nonvanishing Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix.

The gauge symmetry is augmented by a discrete parity or left-right (L-R) symmetry under which the fields transform as

$$Q \leftrightarrow Q_c^*, \quad L \leftrightarrow L_c^*, \quad \Phi_i \leftrightarrow \Phi_i^\dagger, \quad \Delta \leftrightarrow \Delta_c^*, \quad \overline{\Delta} \leftrightarrow \overline{\Delta}_c^*.$$

With this Higgs content, the most general *renormalizable* superpotential is given by

$$W_0 = m\mathrm{Tr}\Delta\overline{\Delta} + m^*\mathrm{Tr}\Delta_c\overline{\Delta}_c + \mu_{ij}\mathrm{Tr}\tau_2\Phi_i^T\tau_2\Phi_j + i\mathbf{f}L^T\tau_2\Delta L$$
$$+ i\mathbf{f}^*L_c^T\tau_2\Delta_c L_c + \mathbf{h}_l^{(i)}L^T\tau_2\Phi_i\tau_2 L_c + \mathbf{h}_q^{(i)}Q^T\tau_2\Phi_i\tau_2 Q_c \tag{5}$$

where $\mathbf{h}_{q,l}^{(i)} = \mathbf{h}_{q,l}^{(i)\dagger}$, $\mu_{ij}=\mu_{ji}=\mu_{ij}^*$, $\mathbf{f}$ is a symmetric matrix, and generation and color indices are understood.

It can be seen at once from the first two terms in (5) that it is impossible to break L-R symmetry with such a simple superpotential. The minimum will occur for vanishing vacuum expectation values (VEVs) of Δ_c, $\overline{\Delta}_c$, Δ and $\overline{\Delta}$. It is clear that the D-term potential vanishes too for the vanishing VEVs. The addition of soft terms is easily shown to be of no help, since the self-couplings of the triplet fields have fixed values given by the gauge couplings. Parity cannot be broken in the minimal renormalizable model.

One can think of two ways out of this problem. The first is to enlarge the Higgs sector. It was suggested by Kuchimanchi and Mohapatra [6] to introduce a parity-odd singlet, coupled appropriately to the triplet fields so as to ensure symmetry breaking. However, it was noticed immediately that the theory has a set of degenerate minima connected by a flat direction, all of them breaking parity. The problem appears when soft supersymmetry breaking terms are switched on: the degeneracy is lifted, but the global minimum that results happens to break electromagnetic charge. Because of the flat direction connecting the minima, there is no hope that the field remains in the phenomenologically acceptable vacuum, it simply rolls down to the global minimum after supersymmetry is softly broken. The only way to save the model is to *assume a low* $SU(2)_R$ *breaking scale*, and the price one has to pay is to break R-parity spontaneously.

In a recent paper [7], two of the authors (C.S.A. and G.S.) with Benakli, have proved that the minimal extension of the Higgs sector consists on the addition of a couple of triplet fields, Ω (1,3,1,0) and Ω_c (1,1,3,0), instead of the singlet. In this model the breaking of $SU(2)_R$ is achieved in two stages, passing through an intermediate phase $SU(2)_L \times U(1)_R \times U(1)_{B-L}$, and breaking $U(1)_R \times U(1)_{B-l}$ at a lower scale. This type of low $B-L$ models are interesting in their own right, and considered a number of times in the literature. It turns out that this theory contains in fact only one parity-breaking minimum, that also preserves electromagnetic charge, and reduces to the minimal supersymmetric standard model (MSSM) with R-parity.

The second possible way of saving the minimal model is to add nonrenormalizable terms, while keeping the minimal Higgs content. This possibility was suggested in [9] where nonrenormalizable soft terms were used to favor the charge-preserving minimum. However, no systematic study of the effects of the nonrenormalizable interactions in the superpotential was carried out. Another example of the use of nonrenormalizable terms in $B-L$ models was given in [10], although not in a manifestly left-right symmetric model. We show in the next section how the addition of nonrenormalizable terms suppressed by a high scale $M \sim M_{Planck}$, with the field content given by (2), (3), (4) suffices to ensure the correct pattern of symmetry breaking.

III. THE MINIMAL NONRENORMALIZABLE MODEL

Consider the superpotential (5). At a first stage, one can ignore the terms involving the bidoublet fields Φ_i, that is, we can take a $SU(2)_R$-breaking scale $M_R \gg M_W, M_S$. The most general superpotential including nonrenormalizable dimension four operators that one can write becomes

$$W_{nr} = m(\mathrm{Tr}\Delta\overline{\Delta} + \mathrm{Tr}\Delta_c\overline{\Delta}_c) + i\mathbf{f}(L^T\tau_2\Delta L + L_c^T\tau_2\Delta_c L_c)$$
$$+ \frac{a}{2M}[(\mathrm{Tr}\Delta\overline{\Delta})^2 + (\mathrm{Tr}\Delta_c\overline{\Delta}_c)^2] + \frac{c}{M}\mathrm{Tr}\Delta\overline{\Delta}\mathrm{Tr}\Delta_c\overline{\Delta}_c$$
$$+ \frac{b}{2M}[\mathrm{Tr}\Delta^2\mathrm{Tr}\overline{\Delta}^2 + \mathrm{Tr}\Delta_c^2\mathrm{Tr}\overline{\Delta}_c^2] + \frac{1}{M}[d_1\mathrm{Tr}\Delta^2\mathrm{Tr}\Delta_c^2$$
$$+ d_2\mathrm{Tr}\overline{\Delta}^2\mathrm{Tr}\overline{\Delta}_c^2] \tag{6}$$

where we assume $M \sim M_{Planck} \simeq 10^{19}$ GeV and for simplicity we have taken the couplings to be real. In the above, we keep the left-handed fields since we have to show that parity can be broken spontaneously and at the same time we wish to know whether R-parity is broken or not.

The set of minima of the theory are to be determined by imposing the vanishing of both F and D terms. Our first concern is to make sure that these minima are isolated, i.e. that there are no flat directions connecting the phenomenologically allowed minimum with any other nonphysical one. Then it would be an easier task to prove that the desired minimum exists. Above the scale of supersymmetry breaking all the minima are degenerate, therefore we will be concerned with potentially dangerous tunneling to physically unacceptable minima only at scales below M_S. We will finally argue that tunneling at this scale is highly suppressed.

The basic result governing the minimization of potentials in globally supersymmetric theories [11] is that the space of D-flat VEVs may be coordinatized by the set of holomorphic gauge invariants formed from the chiral multiplets. The space of flat directions will be spanned by the subset of these holomorphic invariants that cannot be determined by imposing the F-flat conditions. To find this subset in our case, we start by considering the F equations for the left-handed fields Δ, $\overline{\Delta}$ and L:

$$F_{\overline{\Delta}} = \left(m + \frac{a}{M}\,\mathrm{Tr}\Delta\overline{\Delta} + \frac{c}{M}\,\mathrm{Tr}\Delta_c\overline{\Delta_c} \right)\Delta + \left(\frac{b}{M}\,\mathrm{Tr}\Delta^2 + \frac{d_1}{M}\,\mathrm{Tr}\Delta_c^2 \right)\overline{\Delta} = 0,$$

$$F_{\Delta} = \left(m + \frac{a}{M}\,\mathrm{Tr}\Delta\overline{\Delta} + \frac{c}{M}\,\mathrm{Tr}\Delta_c\overline{\Delta_c} \right)\overline{\Delta} + \left(\frac{b}{M}\,\mathrm{Tr}\overline{\Delta}^2 + \frac{d_2}{M}\,\mathrm{Tr}\overline{\Delta_c}^2 \right)\Delta + iF\tau_2 L L^T = 0,$$

$$F_L = 2i\mathbf{f}\tau_2 \Delta L = 0. \tag{7}$$

Here, we consider for simplicity the case of only one generation of leptons. The extension to the realistic multigeneration case is straightforward.

Clearly, there exists a solution $\langle\Delta\rangle = \langle\overline{\Delta}\rangle = \langle L\rangle = 0$. Imposing this condition, we are left only with the following holomorphic invariants

$$x_1 = \mathrm{Tr}\Delta_c\overline{\Delta_c}, \quad x_2 = \mathrm{Tr}\Delta_c^2\mathrm{Tr}\overline{\Delta_c}^2,$$

$$x_3 = L_c^T\tau_2\Delta_c L_c, \quad x_4 = L_c^T\tau_2\overline{\Delta_c}L_c\mathrm{Tr}\Delta_c^2 \tag{8}$$

and the F-flat conditions

$$F_{\overline{\Delta_c}} = \left(m + \frac{a}{M}\,\mathrm{Tr}\Delta_c\overline{\Delta_c} \right)\Delta_c + \left(\frac{b}{M}\,\mathrm{Tr}\Delta_c^2 \right)\overline{\Delta_c} = 0,$$

$$F_{\Delta_c} = \left(m + \frac{a}{M}\,\mathrm{Tr}\Delta_c\overline{\Delta_c} \right)\overline{\Delta_c} + \left(\frac{b}{M}\,\mathrm{Tr}\overline{\Delta_c}^2 \right)\Delta_c + iF\tau_2 L_c L_c^T = 0,$$

$$F_{L_c} = 2i\mathbf{f}\tau_2\Delta_c L_c = 0. \tag{9}$$

As can be seen immediately, x_3, x_4 are made to vanish using F_{L_c}. It is also straightforward to convince oneself that using $\mathrm{Tr}\overline{\Delta_c}F_{\overline{\Delta_c}} = 0$ and $\mathrm{Tr}\Delta_c F_{\overline{\Delta_c}} = 0$ the remaining invariants x_1, x_2 are determined.

It can be shown that Eq. (9) admits in fact two solutions. With a definition of electric charge

$$Q = T_{3L} + T_{3R} + \frac{1}{2}(B-L) \tag{10}$$

the one of interest is

$$\langle\Delta^c\rangle = d\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad \langle\overline{\Delta^c}\rangle = d\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \tag{11}$$

with $d = \sqrt{-mM/a}$. These VEVs break $B-L$ by two units, and from (1) we see that R-parity remains unbroken at this stage.

It is an easy task to demonstrate, using D and F terms, that this solution, the only one that breaks parity while preserving electromagnetic charge, necessarily implies

$$\langle L_c\rangle = 0 \tag{12}$$

so that R-parity is preserved in the supersymmetric limit.

Thus we have succeeded in breaking parity while preserving R-parity. One can worry that the procedure above may not be sufficiently general to ensure that the minimum is indeed isolated, since we have first set the VEVs of the left-handed fields to zero and then required that the minimum be isolated in the restricted space of VEVs parametrized by the right handed gauge invariants.

To ensure that flat directions do not run through the parity breaking minimum, we perturb the VEVs of all fields $\Delta, \overline{\Delta}, \Delta_c, \overline{\Delta_c}, L_i, L_i^c$ (generically denoted ψ) by an arbitrary small perturbation $\psi = \langle\psi\rangle + \epsilon\hat{\psi}$. We then demand that the conditions for a supersymmetric vacuum $F = D = 0$ are satisfied order by order in an expansion in powers of $\epsilon = 0$. If the resulting equations have nontrivial solutions for the ''flat directions'' $\hat{\psi}$, our minimum is not isolated.

For the restricted set of fields kept here it is easy to show that the parity-breaking minimum is indeed isolated, i.e., $\hat{\psi} = 0$. For instance the $F = 0$ equations for Δ and $\overline{\Delta}$ at next to lowest order in ϵ immediately ensure that $\hat{\Delta}$ and $\hat{\overline{\Delta}}$ are exactly zero, and continuing one finds that in fact $\hat{\psi} = 0$. When the bidoublet and quark fields are included the analysis is more challenging. Although it is easy to show that even in their presence the left handed triplets do not participate in any flat direction through the parity-breaking minimum, there may well be flat directions through the minimum involving the bidoublet and quark fields. Details of the analysis in the both cases will be presented elsewhere.

The scale of $SU(2)_R$ breaking M_R is of order $\sqrt{mM}$. We leave the discussion of the phenomenological implications for the next session. We conclude this one with some words on the stability of the vacuum. The degenerated minima are separated by barriers of order M_R. After soft terms become relevant, the degeneracy is lifted up to an order M_S. It is therefore enough to have $M_R \gg M_S$ to get a negligible tunneling probability. This is precisely what happens in this model, as we discuss now.

IV. MASS SPECTRUM

We have seen that parity is broken at a scale M_R of order $\sqrt{mM}$. Now, it is natural to assume m bigger than the electroweak scale, for otherwise the soft-breaking terms will effectively mimic its role [10]. With $m \gtrsim 100$ GeV and $M \sim M_{Planck}$, we get a right-handed scale $M_R \gtrsim 10^{10}$ GeV.

After symmetry breaking, the Higgs fields get masses through the VEV of the right-handed triplets in the usual way. However, in some cases the mass terms arise form the nonrenormalizable terms, thus some particles get only a small mass of order m. This is the case with the left-handed triplets Δ and $\overline{\Delta}$, and with the two double-charged fields and one of the neutral combinations in Δ_c and $\overline{\Delta}_c$. The remaining fields in Δ_c and $\overline{\Delta}_c$ will have masses of order the M_R scale.

The bidoublet deserves a particular attention. Namely, the nonrenormalizable superpotential including Φ will have terms of the form

$$W(m_\Phi) = \mu_{ij} \mathrm{Tr} \tau_2 \Phi_i^T \tau_2 \Phi_j + + \frac{\alpha_{ij}}{M} \mathrm{Tr} \tau_2 \Phi_i^T \overline{\Delta}_c \Delta_c \tau_2 \Phi_j + \frac{\beta_{ij}}{M} \mathrm{Tr} \tau_2 \Phi_i^T \tau_2 \Phi_j \mathrm{Tr} \Delta_c \overline{\Delta}_c. \tag{13}$$

When $\Delta_c, \overline{\Delta}_c$ get the VEVs (11), the mass terms for Φ read

$$W(m_\Phi) = \mu'_{ij} \mathrm{Tr} \tau_2 \Phi_i^T \tau_2 \Phi_j + \frac{m}{2a} \alpha_{ij} \mathrm{Tr} \tau_2 \Phi_i^T \tau_3 \tau_2 \Phi_j \tag{14}$$

with $\mu'_{ij} = \mu_{ij} + m(\alpha_{ij} + 2\beta_{ij})/2a$. Thus the two left-handed doublets in each bidoublet get split, one of them acquiring a mass of order m, and the other (after the usual fine-tuning of the MSSM) a mass of the order of the electroweak scale.

In other words, the minimal L-R model will reduce to the MSSM only below the scale m.

V. SEESAW MECHANISM

In the supersymmetric version of left-right theories, the seesaw mechanism can have novel features. This has been noticed in Ref. [7], for the model with a low $B-L$ scale cited above. In that case, the Δ field coupling to the left-handed neutrino does not acquire a VEV, in sharp contrast with the nonsupersymmetric case [12]. The seesaw mechanism is then said to be ''clean,'' in the sense that it takes its canonical form.

This is not the case however in the nonrenormalizable minimal model. Namely, the bidoublet superpotential will have terms like

$$W_{NR}(\Phi) = \cdots + \frac{\eta_{ij}}{M} \mathrm{Tr} \tau_2 \Phi_i^T \Delta_c \tau_2 \Phi_j \Delta + \cdots, \tag{15}$$

which will give rise to terms linear in Δ after parity breaking, of the order $\sqrt{m/M}$. Such tadpole term will force Δ to get a VEV after electroweak breaking $\langle \Delta \rangle \sim M_W^2/M_R$, which is precisely the situation one encounters on the nonsupersymmetric version of the theory. This has an impact on neutrino masses, and provides an important distinction from the renormalizable version of supersymmetric left-right models [7].

VI. R-PARITY CONSERVATION

As we have seen, at the large scale, charge conservation demands also conservation of R-parity. The question is what happens after the heavy fields are integrated out and the soft supersymmetry breaking terms are switched on. Here the analysis proceeds completely along the lines of Ref. [7]. Since M_R is very large, the breakdown of R-parity at low energies would imply an almost-massless majoron coupled to the Z-boson, which is ruled out experimentally. This is one of the central aspects of supersymmetric left-right theories with large M_R: R-parity is an exact symmetry of the low-energy effective theory. This has well-known important phenomenological and cosmological implications. In particular, the lightest supersymmetric partner must be stable, becoming a natural dark matter candidate.

VII. SUMMARY AND OUTLOOK

Left-right symmetry (or $B-L$) provides a natural gauge principle rationale for R-parity, and thus offers a framework for the study of the predictivity of its breaking. It also plays an important role in understanding the smallness of strong CP violation. On the other hand, it turned out surprisingly hard to construct a realistic supersymmetric left-right model and it was claimed that the minimal such theory cannot work (unless $M_R \leqslant M_S$).

However, we find out that the simple inclusion of nonrenormalizable $d=4$ terms in the superpotential, even if cut off by M_{Planck}, leads to a perfectly consistent model with the spontaneous breakdown of parity.

Our predictions are

(1) A number of charged and doubly-charged Higgs scalars with a mass $m \simeq M_R^2/M_{Planck}$. Thus, even for a large M_R in the intermediate scale $10^{10}-10^{12}$ GeV, interesting for neutrino physics, these new particles can be found in the near future experiments. This is the crucial prediction.

(2) R-parity remains an exact symmetry of the low-energy theory.

(3) The seesaw mechanism takes a similar form as in the nonsupersymmetric models, and this is in sharp contrast with the renormalizable version.

We leave the last word to experiment.

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