Renormalization of two-Higgs-doublet models

R. Santos* and A. Barroso
Departamento de Física, Faculdade de Ciências, Universidade de Lisboa, Campo Grande, C1, 1700 Lisboa, Portugal
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In this paper we perform the complete one-loop renormalization of a general two-Higgs-doublet model. We present all the vertices for this model including the ones in the scalar sector and calculate all the counterterms of the theory. [S0556-2821(97)03819-8]

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I. INTRODUCTION

When the LEP accelerator at CERN enters the second phase of its program, the SU(2)⊗U (1) standard model does not need any more praise. The theory has been successfully scrutinized and the agreement between its predictions and the experimental results is impressive (e.g., Ref. [1]). Besides the effort of large teams of devoted experimenters, this endeavor also required a number of detailed calculations beyond the lowest order of perturbation theory. Hence, one can say that the renormalization of the SU(2)⊗U(1) theory has passed from the formal stage of its establishment [2] into the world of practical calculations. For this purpose it is very useful to have the review article of Aoki *et al.* [3] which can be considered as a good SU(2)⊗U(1) practitioner guide. So far, it seems that such a guide does not exist for the two-Higgs-doublet models (2HDM's). This is the aim of this article.

Several reasons can be given to justify the study of the standard model with two doublets. In our opinion, the best reason is the fact that there is no information about the Higgs sector. Hence, given the crucial role that the scalar sector plays in the theory, it is at least prudent to explore reasonable extensions of the minimal Higgs sector.

Over the last few years, a great deal of work has been invested in the study of several production and decay mechanisms associated with the Higgs bosons of the 2HDM. Fortunately, this large amount of work is beautifully and systematically presented in the *Higgs Hunter's Guide* [4], which we shall consider as our basic reference for the work done until the end of 1989.

Several authors have performed one-loop calculations in the 2HDM. After the experimental evidence for a top quark mass [5], Méndez and Pomarol [6] have computed, in the unitary gauge, the $O(m_t^2/M_W^2)$ corrections to the hadronic width of the Higgs bosons. In the minimal supersymmetric standard model (MSSM) several authors [7] have estimated the process $H^+ \to W^+ \gamma$ which is forbidden to occur at the tree level. Because of this fact, the calculation can be done, including all reducible and irreducible three-point functions, which do not require the specification of the renormalization scheme and the calculation of the counterterms. Another relevant work with a great deal of details about the renormalization of the MSSM is the article by Pierce and Papadopoulos [8] where they have considered one-loop corrections to the decay $H \to ZZ$. However, to preserve the mass sum rule for the renormalized masses of the neutral Higgs bosons, they introduce a modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme to renormalize the angle β. Clearly, this is not entirely consistent with the on-shell scheme and furthermore it is not valid in the general 2HDM. A systematic on-shell renormalization study for the Higgs and gauge boson sectors of the MSSM was carried out by Chankowski, Pokorski, and Rosiek [9]. Here we present a similar work for a general 2HDM. The potential depends on seven real parameters rather than three as is the case for the MSSM. On the other hand, instead of renormalizing the parameters of the potential, as was done by Chankowski *et al.* [9], we renormalize the masses m_H, m_h, m_A, and m_H^+ and the angles $\beta = \tan(v_2/v_1)$ and α.

II. HIGGS POTENTIAL

To define our notation we start with a brief review of the two-Higgs-doublet potential. Let ϕ_i, with $i=1,2$, denote two complex scalar doublets with hypercharge $Y=1$. Introducing the complete set of invariants $x_1 = \phi_1^\dagger \phi_1$, $x_2 = \phi_2^\dagger \phi_2$, $x_3 = \mathrm{Re}\{\phi_1^\dagger \phi_2\}$, and $x_4 = \mathrm{Im}\{\phi_1^\dagger \phi_2\}$, it is clear that the most general SU(2)⊗U(1)-invariant renormalizable potential depends on 14 real parameters and can be written in the form

$$V = -\sum_{i=1}^{4} \mu_i^2 x_i + \sum_{i \leq j=1}^{4} b_{ij} x_i x_j . \tag{1}$$

Under CP the fields transform as

$$\phi_i \to e^{i\alpha_i} \phi_i^* , \tag{2}$$

with arbitrary phase α_i. Choosing these phases to be zero, it is immediate to conclude that an explicit CP-conserving potential V_{CP} has $\mu_4^2 = b_{14} = b_{24} = b_{34} = 0$. Hence, V_{CP} depends on ten real arbitrary parameters. However, such a potential could still break CP spontaneously [10]. In a previous paper [11] we have shown that there are two possibilities to impose in a natural way that the potential has only CP-invariant minima. These require $b_{13} = b_{23} = 0$ and either $\mu_3^2 = 0$ and $b_{33} \neq b_{44}$ or $\mu_3^2 \neq 0$ and $b_{33} = b_{44}$. Here we shall use the first version of the potential which we rewrite in the form

$$V = -\mu_1^2 x_1 - \mu_2^2 x_2 + \lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_3 x_3^2 + \lambda_4 x_4^2 + \lambda_5 x_1 x_2 . \tag{3}$$

*Electronic address: fsantos@skull.cc.fc.ul.pt

Notice that this seven-parameter potential obeys the discrete symmetry $\phi_1 \to -\phi_1$ which is usually introduced to guarantee the absence of flavor-changing neutral currents (FCNC's) in the tree-level Yukawa couplings. It is interesting to point out [11] that potentials with only CP-invariant minima are consistent with the absence of FCNC's in the fermionic sector. Now, denoting by $v_i/\sqrt{2}$ the vacuum expectation value of each of the two doublets, we can write ϕ_i in the form

$$\phi_i = \begin{bmatrix} a_i^+ \\ (v_i + b_i + i c_i)/\sqrt{2} \end{bmatrix}, \tag{4}$$

where a_i^+ are complex fields, and b_i and c_i are real fields. This, in turn, enables us to rewrite the potential (3) as

$$V = -\frac{1}{2}\lambda_3 \begin{bmatrix} a_1^+ & a_2^+ \end{bmatrix} M_\beta \begin{bmatrix} a_1^- \\ a_2^- \end{bmatrix} + \frac{1}{4}(\lambda_4 - \lambda_3)\begin{bmatrix} c_1 & c_2 \end{bmatrix} M_\beta \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} b_1 & b_2 \end{bmatrix} M_\alpha \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + \begin{bmatrix} a_1^+ & a_2^+ \end{bmatrix} T \begin{bmatrix} a_1^- \\ a_2^- \end{bmatrix}$$
$$+ \frac{1}{2}\begin{bmatrix} c_1 & c_2 \end{bmatrix} T \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} b_1 & b_2 \end{bmatrix} T \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} + T_1 b_1 + T_2 b_2 + \text{cubic and quartic terms}, \tag{5}$$

with the matrices M_β, M_α, and T defined as

$$M_\beta = \begin{bmatrix} v_2^2 & -v_1 v_2 \\ -v_1 v_2 & v_1^2 \end{bmatrix}, \tag{6a}$$

$$M_\alpha = \begin{bmatrix} 2v_1^2\lambda_1 & v_1 v_2(\lambda_3+\lambda_5) \\ v_1 v_2(\lambda_3+\lambda_5) & 2v_2^2\lambda_2 \end{bmatrix}, \tag{6b}$$

$$T = \begin{bmatrix} \frac{T_1}{v_1} & 0 \\ 0 & \frac{T_2}{v_2} \end{bmatrix}, \tag{6c}$$

with

$$T_1 = v_1\left(-\mu_1^2 + \lambda_1 v_1^2 + \frac{\lambda_3+\lambda_5}{2} v_2^2 \right), \tag{7a}$$

$$T_2 = v_2\left(-\mu_2^2 + \lambda_2 v_2^2 + \frac{\lambda_3+\lambda_5}{2} v_1^2 \right). \tag{7b}$$

The conditions for a local extreme of the potential are $T_1 = T_2 = 0$. Diagonalizing the quadratic terms of V one obtains the mass eigenstates: two neutral CP-even scalar particles H and h, a neutral CP-odd scalar particle A, and the would-be Goldstone boson partner of the Z, G_0, a charged Higgs field H^+, and the Goldstone associated with the W boson, G^+. The relations between the mass eigenstates and the SU(2)⊗U(1) eigenstates are

$$\begin{bmatrix} H \\ h \end{bmatrix} = R_\alpha \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \tag{8a}$$

$$\begin{bmatrix} H^+ \\ G^+ \end{bmatrix} = R_\beta \begin{bmatrix} a_1^+ \\ a_2^+ \end{bmatrix}, \tag{8b}$$

$$\begin{bmatrix} A \\ G_0 \end{bmatrix} = R_\beta \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}, \tag{8c}$$

with

$$R_\alpha = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}, \tag{9a}$$

$$R_\beta = \begin{bmatrix} -\sin\beta & \cos\beta \\ \cos\beta & \sin\beta \end{bmatrix}, \tag{9b}$$

$$\tan\beta = \frac{v_2}{v_1}, \quad \tan 2\alpha = \frac{v_1 v_2(\lambda_3+\lambda_5)}{\lambda_2 v_2^2 - \lambda_1 v_1^2}. \tag{9c}$$

For the renormalization program it is convenient to rewrite V in terms of the mass eigenstates. After some straightforward algebra one obtains

$$\mathcal{L} = -T_H H - T_h h - H^2\left\{ \frac{M_H^2}{2} + \frac{T_{HH}}{2} \right\} - h^2\left\{ \frac{M_h^2}{2} + \frac{T_{hh}}{2} \right\} - Hh\{T_{Hh}\} - A^2\left\{ \frac{M_A^2}{2} + \frac{T_{AA}}{2} \right\} - G_0^2\left\{ \frac{T_{G_0G_0}}{2} \right\} - AG_0\{T_{AG_0}\}$$
$$- H^+H^-\{M_{H^+}^2 + T_{H^+H^-}\} - (H^+G^- + G^+H^-)\{T_{H^+G^-}\} - G^+G^-\{T_{G^+G^-}\} + \text{cubic and quartic terms}, \tag{10}$$

with

$$T_{HH}=\frac{g}{M_W}\,\frac{T_{\alpha\beta}+T_\delta \sin^2\alpha}{\sin 2\beta}, \tag{11a}$$

$$T_{hh}=\frac{g}{M_W}\,\frac{T_{\alpha\beta}+T_\delta \cos^2\alpha}{\sin 2\beta}, \tag{11b}$$

$$T_{Hh}=\frac{g}{2M_W}\,\frac{T_\delta \sin 2\alpha}{\sin 2\beta}, \tag{11c}$$

$$T_{H^+H^-}=T_{AA}=\frac{g}{M_W}\,\frac{T_{\alpha\beta}+T_\delta \cos^2\beta}{\sin 2\beta}, \tag{12a}$$

$$T_{G^+G^-}=T_{G_0G_0}=\frac{g}{M_W}\,\frac{T_{\alpha\beta}+T_\delta \sin^2\beta}{\sin 2\beta}, \tag{12b}$$

$$T_{H^+G^-}=T_{AG_0}=\frac{g}{2M_W}T_\delta, \tag{12c}$$

$$\begin{bmatrix} T_H \\ T_h \end{bmatrix}=R_\alpha \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}, \tag{13a}$$

$$T_\delta = T_H \sin\delta + T_h \cos\delta, \tag{13b}$$

$$T_{\alpha\beta}=\sin\beta(T_H\cos\alpha - T_h \sin\alpha), \tag{13c}$$

and $\delta=\alpha-\beta$. As we have already pointed out, at the tree level, all T terms are zero. So, at the tree level, the linear terms and the mixed terms vanish and the coefficients of the terms with quadratic fields are, as they should be, their mass squared. However, at one-loop order these statements are no longer true, and this particular form of writing V will be useful in the derivation of the counterterms to renormalize some scalar particles Green's functions.

III. LAGRANGIAN

A. Classical Lagrangian

For completeness let us write the classical Lagrangian of the standard model in the form

$$\mathcal{L}_C=\mathcal{L}_{\mathrm{YM}}+\mathcal{L}_F+\mathcal{L}_S+\mathcal{L}_Y, \tag{14}$$

where $\mathcal{L}_{\mathrm{YM}}$ is the gauge boson sector of the model, $\mathcal{L}_F$ denotes the fermionic kinetic term and their couplings to the gauge bosons, $\mathcal{L}_S$ stands for the scalar sector of the theory, and $\mathcal{L}_Y$ denotes the Yukawa couplings of fermion and scalar particles. The first two terms of Eq. (14) are the same for the standard model and for the 2HDM and so there is no need to write them explicitly here. The scalar Lagrangian is given by

$$\mathcal{L}_S=\sum_{i=1}^{2}\,(D_\mu\phi_i)^\dagger D^\mu\phi_i - V(\phi_1,\phi_2), \tag{15}$$

where

$$D_\mu=\partial_\mu - i g_1 I^a W_\mu^a + i g_2 \frac{Y}{2}B_\mu \tag{16}$$

is the covariant derivative and $V(\phi_1,\phi_2)$ is the potential that we have discussed in the previous paragraph. The Yukawa Lagrangian is, again, a straightforward generalization of the similar form in the standard model. In principle we could write all terms in $\mathcal{L}_Y$ in the form

$$g_{ij}^k [u \quad d]_L^i \phi^k d_R^j, \tag{17}$$

where the g_{ij}^k are arbitrary Yukawa constants and i and j are quark generation indices. However, to avoid the existence of tree-level FCNC's, one should impose the condition that the same scalar doublet ϕ^k does not couple to both up and down quarks. There are essentially four ways of doing this and so there are four variations of the model. A further discussion of this point, which is not relevant for the renormalization discussion, can be found in the *Higgs Hunter's Guide* [4]. The four different models will be presented in Appendix A.

B. Gauge fixing and ghost Lagrangians

At the quantum level the action involves another contribution to the Lagrangian called the gauge-fixing term $\mathcal{L}_{\mathrm{GF}}$. The existence of such a term is by now a textbook subject. So we can simply state that calculations are easily done in the so-called linear R_ξ gauges given by

$$\mathcal{L}_{\mathrm{GF}}=-\frac{1}{2\xi_A}(\partial\cdot A)^2-\frac{1}{2\xi_Z}(\partial\cdot Z-\xi_Z M_Z G_0)^2 - \frac{1}{\xi_W}|\partial\cdot W^+ + i\xi_W M_W G^+|^2, \tag{18}$$

where ξ_W, ξ_A, ξ_Z are arbitrary parameters and the Z and the photon field A are expressed in terms of the original gauge fields by the equations

$$Z_\mu=\cos\theta_W W_\mu^3+\sin\theta_W B_\mu, \tag{19a}$$

$$A_\mu=-\sin\theta_W W_\mu^3+\cos\theta_W B_\mu. \tag{19b}$$

Just for completeness let us recall that

$$M_W=\frac{1}{2}v g_1, \tag{20a}$$

$$M_Z=\frac{1}{2}v\sqrt{g_1^2+g_2^2}, \tag{20b}$$

and the electric charge e is given in terms of the SU(2) and U(1) gauge couplings g_1 and g_2, respectively, by the relation

$$e=\frac{g_1 g_2}{\sqrt{g_1^2+g_2^2}}. \tag{20c}$$

We perform our calculations in the on-shell renormalization scheme and the physical parameters of the theory are the fermion masses, the Higgs masses, the gauge bosons masses, the angles α and β, the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and the electric charge e. In this scheme, the Weinberg angle is not an independent parameter but just a shorthand notation for the ratio of the W and Z masses, i.e., $\cos\theta_W=M_W/M_Z$. As was stated and explained by several authors [12] an alternative scheme, which takes advantage of the good precision of the measurements of the Fermi coupling constant G_F, is obtained replacing M_W by G_F.

The introduction of $\mathcal{L}_{\rm GF}$, which essentially removes the contribution of equivalent orbits in the Feynman path integral, induces the existence of ghost fields. After Becchi-Rouet-Stora-Tyutin (BRST) [13] symmetry was discovered the best way to introduce the ghost contribution is to follow the method advocated by Baulieu [14], where this symmetry is promoted to the role of replacing at the quantum level the classical gauge symmetry. In this way, one can be sure to obtain all ghost interaction terms and in particular the four-point interactions.[1] However, with our choice of gauge fixings, one could also use the better known Faddeev-Popov prescription [15]. In any way we obtain

$$\begin{aligned}\mathcal{L}_{\rm FP}=&-\overline{C^+}[\partial^2+M_W^2]C^- -\overline{C^-}[\partial^2+M_W^2]C^+\\&-\overline{C^Z}[\partial^2+M_Z^2]\overline{C^Z}-C^A\partial^2C^A\\&+\text{cubic and quartic terms.}\end{aligned}\tag{21}$$

The cubic and quartic terms are similar to the ones in the standard model with the replacement $H(\mathrm{SM})\to H\cos\delta-h\sin\delta$.

IV. RENORMALIZATION PROGRAM

A. Renormalization of the fields and parameters

So far, the fields and parameters in the quantum Lagrangian are bare. When this Lagrangian is used to calculate the Green's functions in perturbation theory, renormalized fields and couplings have to be introduced. In fact, the calculations of some Feynman diagrams give divergent results. The use of a regularization prescription, in our case dimensional regularization, isolates the divergences in a well prescribed way. Furthermore, the proof of renormalizability, already obtained in 1971 [2], shows that these ultraviolet divergences can be absorbed by a suitable scaling of the fields and parameters of the theory. Deciding on a renormalization scheme, in our case the on-shell scheme, fixes the relation between renormalized and unrenormalized Green's functions. This is the general framework for the renormalization of the 2HDM that we use. However, even in the simpler standard one-Higgs-doublet model, the same on-shell renormalization scheme can be implemented essentially in two ways. In the first one, followed by Böhm *et al.* [16], the gauge boson field renormalization respects the original gauge symmetry; i.e., the scaling is

$$W_\mu^a\to Z_W^{1/2}W_\mu^a,$$

$$B_\mu\to Z_B^{1/2}B_\mu.$$

The second alternative followed by Aoki *et al.* [3] introduces the scaling at the level of the physical fields W, Z, and A. Then, since Z and A have the same quantum numbers, they get mixed under renormalization, i.e.,

[1]See Ref. [14] for a further discussion of this point.

TABLE I. Renormalization schemes of Böhm *et al.* and Aoki *et al.*

	Böhm *et al.*	Aoki *et al.*
$\mathcal{L}_{YM}$	$Z_W, Z_B, \delta e$	$Z_W, Z_{ZZ}, Z_{ZA}, Z_{AZ}, Z_{AA}$ $\delta M_W^2, \delta M_Z^2, \delta e$
$\mathcal{L}_{GF}$	$\delta\xi_i^W, \delta\xi_i^3, \delta\xi_i^B, i=1,2$	0
$\mathcal{L}_S$	$Z_\phi, \delta v, \delta\mu^2, \delta\lambda$	$Z_H, Z_{G_0}, Z_{G^+}, \delta M_H^2, T$
Total	13	13

$$\begin{bmatrix}Z_\mu\\A_\mu\end{bmatrix}_0=\begin{bmatrix}Z_{ZZ}^{1/2} & Z_{ZA}^{1/2}\\Z_{AZ}^{1/2} & Z_{AA}^{1/2}\end{bmatrix}\begin{bmatrix}Z_\mu\\A_\mu\end{bmatrix}\tag{22a}$$

and

$$W_{\mu0}^\pm=Z_W^{1/2}W_\mu^\pm,\tag{22b}$$

where the bare fields are denoted by a zero subscript. At first glance it looks as though the first alternative is more economical. However, this is misleading since in this scheme the gauge fixing involves 6 renormalization parameters, whereas in the second, the $\mathcal{L}_{\rm GF}$ is, essentially, unrenormalized. Leaving aside the fermionic sector, the comparison between the renormalization parameters in the two schemes is shown in Table I.

In our extension to the 2HDM we found that the second scheme turned out to be the most convenient one. This we will explain in the following paragraph. To close this section let us define some of the entries in Table I, in particular the ones that will be used later. The mass counterterms are introduced in the renormalized Lagrangian via the scaling

$$M_W^2\to M_W^2+\delta M_W^2,\tag{23a}$$

$$M_Z^2\to M_Z^2+\delta M_Z^2,\tag{23b}$$

$$M_H^2\to M_H^2+\delta M_H^2.\tag{23c}$$

The scaling of the Higgs field and of the would-be Goldstone bosons, i.e.,

$$H\to Z_H^{1/2}H,\tag{24a}$$

$$G_0\to Z_{G_0}^{1/2}G_0,\tag{24b}$$

$$G^+\to Z_{G^+}^{1/2}G^+,\tag{24c}$$

introduces the remaining wave function renormalization parameters. The counterterm T, which stands for tadpoles, is needed to cancel the one-particle irreducible Green's functions. Later on we will come back to this point.

B. Renormalization of the gauge fixing

We start this discussion with the standard one-Higgs-doublet model. In the scalar part of the Lagrangian, $\mathcal{L}_S$, after symmetry breaking, two-particle mixed terms of the form

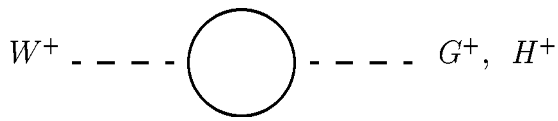

FIG. 1. WG and WH mixing.

$iM_W\partial^\mu W_\mu^- G^+$ are generated. To define the propagators of the theory those terms have to be eliminated. This is obvious in the unitary gauge where the would-be Goldstone bosons disappear, but it is also true in the R_ξ gauges where the last term in Eq. (18) gives a contribution with the opposite sign to the term that we have considered. Clearly, if the gauge fixing is renormalized, the introduction of the same relations between bare and renormalized fields both in $\mathcal{L}_S$ and $\mathcal{L}_{\rm GF}$ makes this cancellation true to all orders in perturbation theory. Then one is left with no counterterm to renormalize the mixed $W_\mu^- G^+$ two-particle Green's functions, represented in Fig. 1.

For illustrative purpose let us write a linear $\mathcal{L}_{\rm GF}$ in the general form

$$\mathcal{L}_{\rm GF}=-\frac{1}{\xi}(\partial^\mu W_\mu^+ + \xi X^+ G^+)(\partial^\mu W_\mu^- - \xi X^- G^-)+\cdots, \tag{25}$$

where X^+G^+ is defined by the integral

$$X^+G^+=\int d^4y X^+(x-y)G^+(y) \tag{26}$$

and $X^+(x-y)$ is a distribution.

The renormalization implies

$$\xi_W \to Z_\xi \xi_W, \tag{27a}$$

$$W_\mu \to Z_W^{1/2} W_\mu, \tag{27b}$$

$$G^+ \to Z_{G^+}^{1/2} G^+. \tag{27c}$$

Thus, if one renormalizes the function X^+ such that

$$X^+ = Z_W^{-1/2} Z_{G^+}^{-1/2} X_R^+, \tag{27d}$$

it is clear that the mixed terms remain unrenormalized. Furthermore, with the condition $Z_\xi = Z_W$, all the terms in the Lagrangian given by Eq. (25) remain unchanged. However, if one tries to apply the same recipe for the 2HDM, we end up with the following counterterms generated by $\mathcal{L}_{\rm GF}$:

$$\mathcal{L}_{\rm GF}^{\rm ct}=\cdots+(iM_W Z_G^{-1/2} Z_{GH}^{1/2}\partial^\mu W_\mu^+ H^- + \text{H.c.}). \tag{28}$$

Such a counterterm with opposite sign is generated by the scalar piece of the classical Lagrangian, $\mathcal{L}_S$, which means that, now, the two-particle WH Green's function is left without a counterterm. Fortunately, Baulieu [14] has proved within the BRST framework that a linear gauge-fixing term is not affected by radiative corrections. So rather than struggling with gauge-fixing Lagrangians with extra ξ parameters, we will follow Ross and Taylor [17] in their celebrated paper and do not renormalize $\mathcal{L}_{GF}$ given by Eq. (18). In other words, the fields and parameters in this equation are already assumed to be the renormalized ones. Furthermore, in the calculation we choose $\xi_A=\xi_Z=\xi_W=1$, which corresponds to the usual Feynman–'t Hooft gauge.

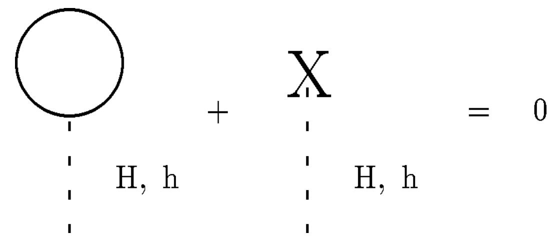

FIG. 2. The tadpole condition.

C. One-particle irreducible Green's functions

After the discovery of BRST symmetry, the renormalization of gauge theories is proved using BRST Ward identities. In the one-doublet standard model, these identities are independent of the sign of the μ^2 term in the Higgs potential. Then, the proof of the renormalizability of the spontaneously broken standard theory follows immediately.

Recently [18], Schilling and van Nieuwenhuizen have explicitly proved the multiplicative renormalization of an SU(2) gauge model. In this case, both the vacuum expectation value v and the scalar field are multiplicatively renormalized by a different Z factor. Hence, it is clear that, in this case, the tree-level condition $-\mu^2+\lambda v^2=0$ is not mantained in higher orders. In the potential, $-\mu^2+\lambda v^2$ is the coefficient of the term linear in the Higgs field. So, in this multiplicative renormalization scheme, there will be renormalized linear terms in H.

An alternative is to introduce an additive renormalization scheme for the scalar fields. In other words, we shift the fields by an additive constant such that their vacuum expectation value vanishes order by order. This is the scheme that we follow here.

In Fig. 2 we show these so-called tadpole diagrams together with their counterterms chosen in such a way that the renormalized Green's functions vanish. These conditions, namely,

$$\Sigma_H + T_H Z_{HH}^{1/2} + T_h Z_{hH}^{1/2} = 0, \tag{29a}$$

$$\Sigma_h + T_h Z_{hh}^{1/2} + T_H Z_{Hh}^{1/2} = 0, \tag{29b}$$

fix, order by order, the values of $T_{H,h}$. Notice that, because of CP conservation, there is no tadpole diagram for the pseudoscalar field. Furthermore, beyond one loop the tadpoles are mixed by the wave function renormalization.

Naively one could assume that this corresponds simply to forgetting about the tadpole diagrams. Indeed, this is the case, for any diagram that differs from a lower order one by a simple addition of a tadpole subgraph. However, we still have to evaluate the counterterms given by Eqs. (29a) and (29b) because those counterterms are going to influence the results for two-point renormalized Green's functions. This is already seen in Eq. (10) and it will be shown in the next paragraph.

D. Two-particle irreducible Green's functions

In this section we discuss the renormalization of the two-point Green's functions. The only differences from the standard model are in the scalar sector and in the mixing between the scalar and gauge boson sectors. Hence, we only discuss those cases and refer to Aoki *et al.* [3] for the remaining two-point functions.

We can start the renormalization program from the tree-level Lagrangian. The renormalized fields and masses are defined by the relations

$$\begin{bmatrix} H^\pm \\ G^\pm \end{bmatrix}_0 = \begin{bmatrix} Z^{1/2}_{H^+H^+} & Z^{1/2}_{H^+G^+} \\ Z^{1/2}_{G^+H^+} & Z^{1/2}_{G^+G^+} \end{bmatrix} \begin{bmatrix} H^\pm \\ G^\pm \end{bmatrix},$$

$$M^2_{H^+0} = M^2_{H^+} + \delta M^2_{H^+}. \tag{30}$$

We also define the renormalized angles and the renormalized SU(2) gauge coupling by the relations $\alpha_0 = \alpha + \delta\alpha$, $\beta_0 = \beta + \delta\beta$ and $g_0 = g + \delta g$. The renormalization of the angles will be discussed in a later section. The renormalized tadpole functions can be written as

$$T_{H^+H^-} = \frac{g+\delta g}{(M_W^2 + \delta M_W^2)^{1/2}} \frac{T_{\alpha\beta} + T_\delta \cos^2(\beta+\delta\beta)}{\sin[2(\beta+\delta\beta)]}, \tag{31a}$$

$$T_{G^+G^-} = \frac{g+\delta g}{(M_W^2 + \delta M_W^2)^{1/2}} \frac{T_{\alpha\beta} + T_\delta \sin^2(\beta+\delta\beta)}{\sin[2(\beta+\delta\beta)]}, \tag{31b}$$

$$T_{H^+G^-} = \frac{g+\delta g}{2(M_W^2 + \delta M_W^2)^{1/2}} T_\delta, \tag{31c}$$

with

$$T_\delta = T_H \sin[\alpha+\delta\alpha-(\beta+\delta\beta)] + T_h \cos[\alpha+\delta\alpha-(\beta+\delta\beta)], \tag{32a}$$

$$T_{\alpha\beta} = \sin(\beta+\delta\beta)[T_H\cos(\alpha+\delta\alpha) - T_h \sin(\alpha+\delta\alpha)]. \tag{32b}$$

Now we have to find the counterterms for the two-point functions. The bilinear terms in the Lagrangian for the charged Higgs sector are

$$\begin{aligned}\mathcal{L} = & -H^+[Z_{H^+H^+}(\partial^2 + M^2_{H^+} + \delta M^2_{H^+}) + Z_{G^+H^+}\partial^2 + Z_{H^+H^+}T_{H^+H^-} + Z_{G^+H^+}T_{G^+G^-} + 2Z^{1/2}_{H^+H^+}Z^{1/2}_{G^+H^+}T_{H^+G^-}]H^- \\ & -G^+[Z_{H^+G^+}(\partial^2 + M^2_{H^+} + \delta M^2_{H^+}) + Z_{G^+G^+}\partial^2 + Z_{G^+G^+}T_{G^+G^-} + Z_{H^+G^+}T_{H^+H^-} + 2Z^{1/2}_{G^+G^+}Z^{1/2}_{H^+G^+}T_{H^+G^-}]G^- \\ & -H^+[Z^{1/2}_{H^+H^+}Z^{1/2}_{H^+G^+}(\partial^2 + M^2_{H^+} + \delta M^2_{H^+}) + Z^{1/2}_{G^+G^+}Z^{1/2}_{G^+H^+}\partial^2 + Z^{1/2}_{H^+H^+}Z^{1/2}_{H^+G^+}T_{H^+H^-} + Z^{1/2}_{G^+G^+}Z^{1/2}_{G^+H^+}T_{G^+G^-} \\ & +(Z^{1/2}_{G^+G^+}Z^{1/2}_{H^+H^+} + Z^{1/2}_{G^+H^+}Z^{1/2}_{H^+G^+})T_{H^+G^-}]G^- + \text{H.c.}\end{aligned} \tag{33}$$

Using the usual recipe for on-shell renormalization, that is, demanding that the pole stay at the physical mass and that the residue be one, we arrive at the following set of renormalization conditions:

$$\begin{aligned}&\Sigma_{H^+H^+}(M^2_{H^+}) - Z_{H^+H^+}\delta M^2_{H^+} + Z_{G^+H^+}M^2_{H^+} \\ &\quad - Z_{H^+H^+}T_{H^+H^-} - Z_{G^+H^+}T_{G^+G^-} \\ &\quad - 2Z^{1/2}_{H^+H^+}Z^{1/2}_{G^+H^+}T_{H^+G^-} = 0,\end{aligned} \tag{34a}$$

$$\frac{d}{dq^2}\Sigma_{H^+H^+}(M^2_{H^+}) + Z_{H^+H^+} + Z_{G^+H^+} = 0, \tag{34b}$$

$$\begin{aligned}&\Sigma_{G^+G^+}(0) - Z_{H^+G^+}(M^2_{H^+} + \delta M^2_{H^+}) - Z_{G^+G^+}T_{G^+G^-} \\ &\quad - Z_{H^+G^+}T_{H^+H^-} - 2Z^{1/2}_{G^+G^+}Z^{1/2}_{H^+G^+}T_{H^+G^-} = 0,\end{aligned} \tag{35a}$$

$$\frac{d}{dq^2}\Sigma_{G^+G^+}(0) + Z_{G^+G^+} + Z_{H^+G^+} = 0, \tag{35b}$$

$$\begin{aligned}&\Sigma_{H^+G^+}(0) - Z^{1/2}_{H^+H^+}Z^{1/2}_{H^+G^+}(M^2_{H^+} + \delta M^2_{H^+}) \\ &\quad - Z^{1/2}_{H^+H^+}Z^{1/2}_{H^+G^+}T_{H^+H^-} - Z^{1/2}_{G^+G^+}Z^{1/2}_{G^+H^+}T_{G^+G^-} \\ &\quad - (Z^{1/2}_{G^+G^+}Z^{1/2}_{H^+H^+} + Z^{1/2}_{G^+H^+}Z^{1/2}_{H^+G^+})T_{H^+G^-} = 0,\end{aligned} \tag{36a}$$

$$\begin{aligned}&\Sigma_{G^+H^+}(M^2_{H^+}) - Z^{1/2}_{H^+H^+}Z^{1/2}_{H^+G^+}\delta M^2_{H^+} + Z^{1/2}_{G^+G^+}Z^{1/2}_{G^+H^+}M^2_{H^+} \\ &\quad - Z^{1/2}_{H^+H^+}Z^{1/2}_{H^+G^+}T_{H^+H^-} - Z^{1/2}_{G^+G^+}Z^{1/2}_{G^+H^+}T_{G^+G^-} \\ &\quad - (Z^{1/2}_{G^+G^+}Z^{1/2}_{H^+H^+} + Z^{1/2}_{G^+H^+}Z^{1/2}_{H^+G^+})T_{H^+G^-} = 0.\end{aligned} \tag{36b}$$

With these six equations we can determine the five renormalization constants. Notice the explicit appearance of the tadpole counterterms. There is one dependent equation due to a Ward identity in the charged sector, which is

$$\begin{aligned}&\langle 0|T\partial^\mu W^+_\mu \partial^\nu W^-_\nu|0\rangle - iM_W\langle 0|TG^+\partial^\nu W^-_\nu|0\rangle \\ &\quad + iM_W\langle 0|T\partial^\mu W^+_\mu G^-|0\rangle + M_W^2\langle 0|TG^+G^-|0\rangle = 0.\end{aligned} \tag{37}$$

Finally let us discuss the mixed terms in the charged sector. Bearing in mind the discussion about the gauge-fixing Lagrangian in the previous section, the counterterms can be taken from

$$\mathcal{L}=i(M_W^2+\delta M_W^2)^{1/2}Z_W^{1/2}Z_{G^+G^+}^{1/2}W_\mu^-\partial^\mu G^+ + \text{H.c.} + i(M_W^2+\delta M_W^2)^{1/2}Z_W^{1/2}Z_{G^+H^+}^{1/2}W_\mu^-\partial^\mu H^+ + \text{H.c.} \tag{38}$$

The gauge-fixing Lagrangian (25) will cancel the tree-level terms in Eq. (38) and so the final mixed Lagrangian is, in fact, a counterterm Lagrangian for the self-energies WG and WH. Notice that we did not explicitly introduce any counterterms for the Green's functions WG and WH. So we end up this section by writing simbolically $Z_{W^+G^+}$ and $Z_{W^+H^+}$ as

$$Z_{W^+G^+}^{1/2}=ik_\mu(M_W^2+\delta M_W^2)^{1/2}Z_W^{1/2}Z_{G^+G^+}^{1/2}, \tag{39}$$

$$Z_{W^+H^+}^{1/2}=ik_\mu(M_W^2+\delta M_W^2)^{1/2}Z_W^{1/2}Z_{G^+H^+}^{1/2}. \tag{40}$$

The complete set of counterterms for the scalar and mixed sectors can be found in Appendix B.

E. Three-particle irreducible Green's functions

In the on-shell renormalization scheme that we have adopted, the gauge couplings g_1 and g_2 are not independent parameters. In fact, they are both related to the gauge boson masses and to the electric charge e, i.e.,

$$g_1=e\frac{M_Z}{M_W},$$

$$g_2=e\frac{M_Z}{(M_W^2-M_Z^2)^{1/2}}.$$

Then, in the one-Higgs-doublet model, only one further renormalization constant $Y=\delta e/e$ remains to be fixed. This is simply done by imposing the condition

$$\overline{u(m_f)}\Gamma_R^\mu u(m_f)|_{k^\mu\to 0}=\overline{u}\,\gamma^\mu u \tag{41}$$

for any charged fermion, where Γ_R^μ is the renormalized three-point photon fermion vertex. Usually, following the traditional QED prescription, where the Thompson limit was introduced to define $\alpha=e^2/(4\pi)$, one uses the electron as the charged fermion. However, the universality of the on-shell charge guarantees that one can use any charged fermion. Since the theory is by itself well defined, one could alternatively fix Y by using the renormalized $W^+W^-\gamma$ three-point function, namely,

$$\begin{aligned}[\epsilon_\beta(p)\epsilon_\gamma(q)\Gamma_R^{\beta\gamma\mu}]_{k^\mu\to 0 p^2=q^2=M_W^2} &= \lim_{k^\mu\to 0}[\epsilon(q)\cdot(k-p)\epsilon^\mu(p)+(p-q)^\mu\epsilon(p)\cdot\epsilon(q) \\ &\quad +(q-k).p\epsilon^\mu(q)]=0.\end{aligned} \tag{42}$$

Besides the gauge-coupling renormalization, fixed by the photon coupling, the W quark-quark vertex requires the additional renormalization of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. For the standard one-Higgs-doublet model this renormalization of the CKM matrix was evaluated by Denner and Sack [19]. In this article we sketch their analysis in order to point out its extensions to the 2HDM.

Let us consider the decay $W^+\to u_I\overline{d_j}$, where $I, j=1, 2, 3$ are the generation indices (upper case for up quarks). At the tree level the decay amplitude is

$$T=V_{Ij}T_0, \tag{43}$$

with

$$T_0=-\frac{g}{\sqrt{2}}\overline{u_I}(m_I)\not\epsilon\gamma_L v(m_j). \tag{44}$$

At one loop, in the on-shell renormalization scheme the self-energy corrections to the external legs vanish and the proper vertex diagrams give an amplitude T_1^v that can be written in the form

$$T_1^v=V_{Ij}T_0\Delta, \tag{45}$$

where Δ stands for the result of the loop calculation. To obtain the full one-loop amplitude one has to add the counterterms, i.e.,

$$T_1=T_1^v+T_1^c, \tag{46}$$

with

$$T_1^c=V_{Ij}T_0\left[\frac{\delta g}{g}-\frac{1}{2}\delta Z_W\right]+\frac{1}{2}T_0\left[\sum_J \delta Z_{JI}^{*L}V_{Jj}+\sum_i V_{Ii}\delta Z_{ij}^L\right] + T_0\delta V_{Ij}. \tag{47}$$

Now we have to face the problem of imposing some conditions to fix the CKM counterterms δV_{Ij}. Denner and Sack [19] have split the quark wave function renormalization parameters δZ^L into their Hermitian and antihermitian contributions, namely,

$$\delta Z^L=\frac{1}{2}(\delta Z^L+\delta Z^{*L})+\frac{1}{2}(\delta Z^L-\delta Z^{*L}), \tag{48}$$

and then they have fixed δV_{Ij} by the condition

$$\delta V_{Ij}=-\frac{1}{4}\left[\sum_J (\delta Z_{IJ}^{*L}-\delta Z_{JI}^L)V_{Jj}+\sum_i V_{Ii}(\delta Z_{ij}^L-\delta Z_{ji}^{*L})\right]. \tag{49}$$

It is possible to prove [19] that δV_{Ij} is needed precisely to cancel the divergent contribution to the right-hand side of Eq. (49). Hence, the use of Eq. (49) to fix also the finite piece of δV_{Ij} is the choice made by Denner and Sack. Alternatively, one could select four physical $Wq\overline{q}$ decay processes and impose the vanishing of T_1 for these decays. In this case, the transitions $W^+\to u\overline{d}$, $W^+\to u\overline{s}$, $W^+\to u\overline{b}$, and $t\to bW^+$

TABLE II. Coupling constants for the fermion-scalar interactions.

	Model I	Model II	Model III	Model IV
α_{eh}	$-\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\cos\beta}$	$-\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\cos\beta}$
α_{dh}	$-\frac{\cos\alpha}{\sin\beta}$	$-\frac{\cos\alpha}{\sin\beta}$	$\frac{\sin\alpha}{\cos\beta}$	$\frac{\sin\alpha}{\cos\beta}$
α_{eH}	$-\frac{\sin\alpha}{\sin\beta}$	$-\frac{\cos\alpha}{\cos\beta}$	$-\frac{\sin\alpha}{\sin\beta}$	$-\frac{\cos\alpha}{\cos\beta}$
α_{dH}	$-\frac{\sin\alpha}{\sin\beta}$	$-\frac{\sin\alpha}{\sin\beta}$	$-\frac{\cos\alpha}{\cos\beta}$	$-\frac{\cos\alpha}{\cos\beta}$
β_e	$-\cot\beta$	$\tan\beta$	$-\cot\beta$	$\tan\beta$
β_d	$-\cot\beta$	$-\cot\beta$	$\tan\beta$	$\tan\beta$

could form an interesting set. However, this process has the disavantage of shifting all one-loop correction to some amplitudes.

Since the renormalization of the CKM matrix vanishes in the limit of degenerate down quark[2] masses, most loop corrections to the W decay process are done in this approximation. This is equivalent to dropping the last term in Eq. (47) and, in the same term, to replace the sum over J and i simply by the $J=I$ and $j=i$ contributions. Hence, in this approximation T_1 is directly proportional to a single CKM element V_{Ij}. As far as we know all standard model analyses of the values of the CKM matrix elements are done in this approximation. In fact, the work of Denner and Sack has shown that the error of this approximation is of the order 10^{-6}, far smaller than any other theoretical and experimental uncertainties.

In the 2HDM, one can do a similar analysis with the difference that there are further contributions to the irreducible vertex and to δZ_L coming from diagrams with neutral and charged Higgs bosons. Because some of these vertices could be enhanced by the factor $\tan\beta(\cot\beta)$, one could expect to see such enhancement in the result.

In the 2HDM there are two further couplings α and β that need to be renormalized. This can be done imposing some physical conditions on the renormalized three-point or four-point scalar vertex functions. There are in this model 8 cubic and 14 quartic vertices among the neutral and charged Higgs bosons and any two of those can be selected. However, most of these vertices have a complicated dependence on the angles and, furthermore, without knowing the Higgs boson masses it is difficult to select a physical process such as, for instance, $H\rightarrow hh$. Luckily, the vertices $\overline{e}eh$ and $H^{\pm}e\nu$, which induce the tree-level decays $h\rightarrow e^{+}e^{-}$ and $H^{-}\rightarrow e^{-}\overline{\nu}$, have a simple dependence on the angles (see Table II) and, at the same time, we already know that the present bounds on the Higgs boson masses allow these decays to occur.

In a recent calculation [20] of the top-loop contribution to the decay $H^{+}\rightarrow hW^{+}$, where the vertex depends only on the combination $\beta-\alpha$, we renormalize $(\beta-\alpha)$ using the corresponding process $H^{+}\rightarrow HW^{+}$. In the absence of any information on Higgs boson scattering and Higgs leptonic decays this is perhaps the only consistent way to proceed.

[2]Obviously, the same is true in the limit of degenerate up quark mass althought this case is less realistic.

V. CONCLUSION

In this article we discuss the renormalization program of the CP-conserving 2HDM. We summarize our main points as follows.

(1) In a general 2HDM, the condition that the potential has only CP-invariant minima leads to a seven-parameter potential [11] which is consistent with the absence of FCNC's in the fermionic sector.

(2) The scalar part of the Lagrangian, written in terms of the mass eigenstates, shows the existence of linear terms in the neutral scalar fields H and h [cf. Eq. (10)]. Clearly, those terms are zero at the tree level due to the minimum conditions. However, at one loop, the requirement of vanishing renormalized linear terms induces, by itself, bilinear mixed terms of the type Hh. Furthermore, the renormalization condition for the tadpoles [cf. Eqs. (29a) and (29b)] will induce an additional mixing of the same type. Hence, in general, the renormalization of the one-, two-, and three-particle irreducible Green's functions cannot be implemented sequentially. So one is faced with the task of solving a coupled system of equations. Fortunetely, for one-loop calculations, the equations decouple.

(3) In Sec. IV B we discuss the problems connected with the renormalization of the gauge fixing. Working with a gauge-fixing Lagrangian where all parameters are already renormalized [17], we have shown that the counterterms for the mixed two-point functions with one gauge boson and one scalar arise naturally without any further conditions [cf. Eqs. (B22)–(B27)].

(4) In Sec. IV E we discuss the renormalization of the angles α and β. Contrary to the MSSM, these angles are now physically independent parameters of the model. Hence, we propose to use the decays $h\rightarrow e^{+}e^{-}$ and $H^{-}\rightarrow e^{-}\overline{\nu}$ to renormalize α and β.

(5) Finally, in a set of appendixes we list the vertices and the renormalization constants of the 2HDM in order to make this information readily available to other users.

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APPENDIX A: FEYNMAN RULES

In this appendix we present the Feynman rules for the interactions involving scalar fields. All other interactions are standard and can be found in [3]. We have chosen the Feynman–t Hooft gauge and followed the convention that all the momenta in the vertices are incoming.

We start by defining the quantities

$$A_{\alpha\beta} \equiv \cos^3\beta \sin\alpha + \sin^3\beta \cos\alpha,$$

$$B_{\alpha\beta} \equiv \cos^3\beta \cos\alpha + \sin^3\beta \sin\alpha,$$

$$C_{\alpha\beta} \equiv \sin^3\alpha \cos\beta + \cos^3\alpha \sin\beta,$$

$$D_{\alpha\beta} \equiv \cos^3\alpha \cos\beta - \sin^3\alpha \sin\beta.$$

In the Yukawa Lagrangian, the fermions can couple with the scalars in four different and independent ways, with no flavor changing. The couplings for those models are shown in Table II. In model I only ϕ_2 couples to all fermions; in model II ϕ_2 couples to the quarks and ϕ_1 coulpes to the leptons; in model III ϕ_2 couples to the up quarks and to the leptons and ϕ_1 couples to the down quarks; finally in Model IV ϕ_2 couples to the up quarks and ϕ_1 couples to the down quarks and the leptons. The two Higgs doublets ϕ_1 and ϕ_2 are defined in the expression (4). These couplings will be used in Appendix A 3.

1. Triple scalar vertices

$$H^+H^-h \quad -\frac{ig}{M_W}\left(\frac{M_h^2}{\sin 2\beta}B_{\alpha\beta} - M_{H^+}^2 \sin\delta\right),$$

$$H^+H^-H \quad -\frac{ig}{M_W}\left(\frac{M_H^2}{\sin 2\beta}A_{\alpha\beta} + M_{H^+}^2 \cos\delta\right),$$

$$AAh \quad -\frac{ig}{M_W}\left(\frac{M_h^2}{\sin 2\beta}B_{\alpha\beta} - M_A^2 \sin\delta\right),$$

$$AAH \quad -\frac{ig}{M_W}\left(\frac{M_H^2}{\sin 2\beta}A_{\alpha\beta} + M_A^2 \cos\delta\right),$$

$$hhh \quad \frac{3ig}{M_W}\frac{M_h^2}{\sin 2\beta}D_{\alpha\beta},$$

$$HHH \quad -\frac{3ig}{M_W}\frac{M_H^2}{\sin 2\beta}C_{\alpha\beta},$$

$$hHH \quad \frac{ig}{2M_W}\frac{\sin 2\alpha \sin\delta(2M_H^2 + M_h^2)}{\sin 2\beta},$$

$$hhH \quad -\frac{ig}{2M_W}\frac{\sin 2\alpha \cos\delta(M_H^2 + 2M_h^2)}{\sin 2\beta},$$

$$hH^\mp G^\pm \quad -\frac{ig}{2M_W}\cos\delta(M_h^2 - M_{H^+}^2),$$

$$HH^\mp G^\pm \quad \frac{ig}{2M_W}\sin\delta(M_H^2 - M_{H^+}^2),$$

$$hAG_0 \quad -\frac{ig}{2M_W}\cos\delta(M_h^2 - M_A^2),$$

$$HAG_0 \quad \frac{ig}{2M_W}\sin\delta(M_H^2 - M_A^2),$$

$$hG_0G_0 \quad \frac{ig}{2M_W}\sin\delta M_h^2,$$

$$hG^+G^+ \quad \frac{ig}{2M_W}\sin\delta M_h^2,$$

$$HG_0G_0 \quad -\frac{ig}{2M_W}\cos\delta M_H^2,$$

$$HG^+G^+ \quad -\frac{ig}{2M_W}\cos\delta M_H^2,$$

$$AH^\mp G^\pm \quad \pm\frac{g}{2M_W}(M_A^2 - M_{H^+}^2).$$

2. Quartic scalar vertices

$$H^+H^-H^+H^- \quad -\frac{ig^2}{\sin^2 2\beta M_W^2}(M_H^2 A_{\alpha\beta}^2 + M_h^2 B_{\alpha\beta}^2),$$

$$AAAA \quad -\frac{3ig^2}{\sin^2 2\beta M_W^2}(M_H^2 A_{\alpha\beta}^2 + M_h^2 B_{\alpha\beta}^2),$$

$$AAH^+H^- \quad -\frac{ig}{\sin^2 2\beta M_W^2}(M_H^2 A_{\alpha\beta}^2 + M_h^2 B_{\alpha\beta}^2),$$

$$H^+H^-hh \quad -\frac{ig^2}{2M_W^2}\left[\frac{1}{\sin^2 2\beta}(M_H^2 A_{\alpha\beta}\sin 2\alpha\cos\delta - 2M_h^2 B_{\alpha\beta}D_{\alpha\beta}) + 2M_{H^+}^2\sin^2\delta\right],$$

$$H^+H^-HH \quad -\frac{ig^2}{2M_W^2}\left[\frac{1}{\sin^2 2\beta}(2M_H^2 A_{\alpha\beta}C_{\alpha\beta} + M_h^2 B_{\alpha\beta}\sin 2\alpha\sin\delta) + 2M_{H^+}^2\cos^2\delta\right],$$

$$AAhh \quad -\frac{ig^2}{2M_W^2}\left[\frac{1}{\sin^2 2\beta}(M_H^2 A_{\alpha\beta}\sin 2\alpha\cos\delta - 2M_h^2 B_{\alpha\beta}D_{\alpha\beta}) + 2M_A^2\sin^2\delta\right],$$

$$AAHH \quad -\frac{ig^2}{2M_W^2}\left[\frac{1}{\sin^2 2\beta}(2M_H^2 A_{\alpha\beta}C_{\alpha\beta} + M_h^2 B_{\alpha\beta}\sin 2\alpha\sin\delta) + 2M_A^2\cos^2\delta\right],$$

$$H^+H^-Hh \quad -\frac{ig^2}{2M_W^2}\left[\frac{1}{\sin^2 2\beta}(M_H^2 A_{\alpha\beta}\sin 2\alpha\sin\delta + M_h^2 B_{\alpha\beta}\sin 2\alpha\cos\delta) - M_{H^+}^2\sin 2\delta\right],$$

$$AAHh \quad -\frac{ig^2}{2M_W^2}\left[\frac{1}{\sin^2 2\beta}(M_H^2 A_{\alpha\beta}\sin 2\alpha \sin\delta + M_h^2 B_{\alpha\beta}\sin 2\alpha\cos\delta) - M_A^2 \sin 2\delta\right],$$

$$hhhh \quad -\frac{3ig^2}{4\sin^2 2\beta M_w^2}(4M_h^2 D_{\alpha\beta}^2 + M_H^2 \sin^2 2\alpha \cos^2\delta),$$

$$HHHH \quad -\frac{3ig^2}{4\sin^2 2\beta M_W^2}(M_h^2 \sin^2 2\alpha \sin^2\delta + 4M_H^2 C_{\alpha\beta}^2),$$

$$hhhH \quad -\frac{3ig^2}{8\sin^2 2\beta M_W^2}(4M_h^2 D_{\alpha\beta}\sin 2\alpha\cos\delta + M_H^2 \sin^2 2\alpha \sin 2\delta),$$

$$HHHh \quad -\frac{3ig^2}{8\sin^2 2\beta M_W^2}(M_h^2 \sin^2 2\alpha \sin 2\delta + 4M_H^2 C_{\alpha\beta}\sin 2\alpha \sin\delta),$$

$$hhHH \quad -\frac{ig^2 \sin 2\alpha}{4\sin 2\beta M_W^2}\left[M_H^2 - M_h^2 + \frac{3\sin 2\alpha}{\sin 2\beta}(\sin^2\delta M_H^2 + \cos^2\delta M_h^2)\right],$$

$$AAG_0G_0 \quad -\frac{ig^2}{4M_W^2}\left[\frac{\sin 2\alpha}{\sin 2\beta}(M_H^2 - M_h^2) + 3(\sin^2\delta M_H^2 + \cos^2\delta M_h^2)\right],$$

$$H^+H^-G^+G^- \quad -\frac{ig^2}{4M_W^2}\left[M_A^2 + \frac{\sin 2\alpha}{\sin 2\beta}(M_H^2 - M_h^2) + 3(\sin^2\delta M_H^2 + \cos^2\delta M_h^2)\right],$$

$$H^\pm H^\pm G^\mp G^\mp \quad -\frac{ig^2}{2M_W^2}(-M_A^2 + \sin^2\delta M_H^2 + \cos^2\delta M_h^2),$$

$$H^\mp H^\pm AG_0 \quad -\frac{ig^2}{4M_W^2}(-M_{H^+}^2 + \sin^2\delta M_H^2 + \cos^2\delta M_h^2),$$

$$G^+G^-AA \quad -\frac{ig^2}{2M_W^2}\left[M_{H^+}^2 + \frac{1}{\sin 2\beta}(\cos\delta A_{\alpha\beta}M_H^2 - \sin\delta B_{\alpha\beta}M_h^2)\right],$$

$$H^+H^-G_0G_0 \quad -\frac{ig^2}{2M_W^2}\left[M_{H^+}^2 + \frac{1}{\sin 2\beta}(\cos\delta A_{\alpha\beta}M_H^2 - \sin\delta B_{\alpha\beta}M_h^2)\right],$$

$$H^+H^-H^\mp G^\pm \quad -\frac{ig^2}{M_W^2}\left[\frac{\sin\delta}{\sin 2\beta}(A_{\alpha\beta}M_H^2 + B_{\alpha\beta}M_h^2)\right],$$

$$H^+H^-G_0A \quad -\frac{ig^2}{2M_W^2}\left[\frac{\sin\delta}{\sin 2\beta}(A_{\alpha\beta}M_H^2 + B_{\alpha\beta}M_h^2)\right],$$

$$AAAG_0 \quad -\frac{3ig^2}{2M_W^2}\left[\frac{\sin\delta}{\sin 2\beta}(A_{\alpha\beta}M_H^2 + B_{\alpha\beta}M_h^2)\right],$$

$$AAH^\mp G^\pm \quad -\frac{ig^2}{2M_W^2}\left[\frac{\sin\delta}{\sin 2\beta}(A_{\alpha\beta}M_H^2 + B_{\alpha\beta}M_h^2)\right],$$

$$G^+G^-G_0A \quad -\frac{ig^2}{8M_W^2}\sin 2\delta(M_H^2 - M_h^2),$$

$$G^+G^-H^\mp G^\pm \quad -\frac{ig^2}{4M_W^2}\sin 2\delta(M_H^2 - M_h^2),$$

$$G_0G_0G_0A \quad -\frac{3ig^2}{8M_W^2}\sin 2\delta(M_H^2 - M_h^2),$$

$$G_0G_0H^\mp G^\pm \quad -\frac{ig^2}{8M_W^2}\sin 2\delta(M_H^2 - M_h^2),$$

$$G^+G^-hh \quad -\frac{ig^2}{4M_W^2}\left[\frac{1}{\sin 2\beta}(\sin 2\alpha\cos^2\delta M_H^2 - 2\sin\delta D_{\alpha\beta}M_h^2) + 2\cos^2\delta M_{H^+}^2\right],$$

$$G_0G_0hh \quad -\frac{ig^2}{4M_W^2}\left[\frac{1}{\sin 2\beta}(\sin 2\alpha\cos^2\delta M_H^2 - 2\sin\delta D_{\alpha\beta}M_h^2) + 2\cos^2\delta M_A^2\right],$$

$$G^+G^-HH \quad -\frac{ig^2}{4M_W^2}\left[\frac{1}{\sin 2\beta}(2\cos\delta C_{\alpha\beta}M_H^2 - \sin^2\delta \sin 2\alpha M_h^2) + 2\sin^2\delta M_{H^+}^2\right],$$

$$G_0G_0HH \quad -\frac{ig^2}{4M_W^2}\left[\frac{1}{\sin 2\beta}(2\cos\delta C_{\alpha\beta}M_H^2 - \sin^2\delta \sin 2\alpha M_h^2) + 2\sin^2\delta M_A^2\right],$$

$$H^\mp G^\pm HH \quad -\frac{ig^2}{8M_W^2}\left[\frac{1}{\sin 2\beta}(4\cos\delta C_{\alpha\beta}M_H^2 + \sin 2\delta \sin 2\alpha M_h^2) - 2\sin 2\delta M_{H^+}^2\right],$$

$$H^{\mp}G^{\pm}hh \quad -\frac{ig^2}{8M_W^2}\left[\frac{1}{\sin 2\beta}(\sin 2\delta \sin 2\alpha M_H^2 + 4\cos\delta D_{\alpha\beta}M_h^2) + 2\sin 2\delta M_{H^+}^2\right],$$

$$AG_0HH \quad -\frac{ig^2}{8M_W^2}\left[\frac{1}{\sin 2\beta}(4\cos\delta C_{\alpha\beta}M_H^2 + \sin 2\delta \sin 2\alpha M_h^2) - 2\sin 2\delta M_A^2\right],$$

$$AG_0hh \quad -\frac{ig^2}{8M_W^2}\left[\frac{1}{\sin 2\beta}(\sin 2\alpha \sin 2\delta M_H^2 + 4\cos\delta D_{\alpha\beta}M_h^2) + 2\sin 2\delta M_A^2\right],$$

$$G^{\mp}H^{\pm}hA \quad \pm\frac{g^2}{4M_W^2}\sin\delta(M_A^2 - M_{H^+}^2),$$

$$G^{\mp}H^{\pm}HG_0 \quad \pm\frac{g^2}{4M_W^2}\sin\delta(M_A^2 - M_{H^+}^2),$$

$$G^{\mp}H^{\pm}hG_0 \quad \pm\frac{g^2}{4M_W^2}\cos\delta(M_A^2 - M_{H^+}^2),$$

$$G^{\mp}H^{\pm}HA \quad \mp\frac{g^2}{4M_W^2}\cos\delta(M_A^2 - M_{H^+}^2),$$

$$G^+G^-hH \quad -\frac{ig^2}{8M_W^2}\left[\frac{\sin 2\alpha}{\sin 2\beta}(M_H^2 - M_h^2) + 2\sin 2\delta M_{H^+}^2\right],$$

$$G_0G_0hH \quad -\frac{ig^2}{8M_W^2}\left[\frac{\sin 2\alpha}{\sin 2\beta}(M_H^2 - M_h^2) + 2\sin 2\delta M_A^2\right],$$

$$G^{\mp}H^{\pm}hH \quad -\frac{ig^2}{4M_W^2}\left[\frac{\sin 2\alpha}{\sin 2\beta}(\sin^2\delta M_H^2 + \cos^2\delta M_h^2) - \cos 2\delta M_{H^+}^2\right],$$

$$AG_0hH \quad -\frac{ig^2}{4M_W^2}\left[\frac{\sin 2\alpha}{\sin 2\beta}(\sin^2\delta M_H^2 + \cos^2\delta M_h^2) - \cos 2\delta M_A^2\right],$$

$$G^+G^-G^+G^- \quad -\frac{ig^2}{4M_W^2}(\sin^2\delta M_h^2 + \cos^2\delta M_H^2),$$

$$G_0G_0G_0G_0 \quad -\frac{3ig^2}{4M_W^2}(\sin^2\delta M_h^2 + \cos^2\delta M_H^2),$$

$$G^+G^-G_0G_0 \quad -\frac{ig^2}{4M_W^2}(\sin^2\delta M_h^2 + \cos^2\delta M_H^2).$$

3. Fermion-scalar vertices

$$\overline{e}_ie_ih \quad \frac{ig}{2M_W}\alpha_{eh}m_{e_i},$$

$$\overline{u}_iu_ih \quad -\frac{ig}{2M_W}\frac{\cos\alpha}{\sin\beta}m_{u_i},$$

$$\overline{d}_id_ih \quad \frac{ig}{2M_W}\alpha_{dh}m_{d_i},$$

$$\overline{e}_ie_iH \quad \frac{ig}{2M_W}\alpha_{eH}m_{e_i},$$

$$\overline{u}_iu_iH \quad -\frac{ig}{2M_W}\frac{\sin\alpha}{\sin\beta}m_{u_i},$$

$$\overline{d}_id_iH \quad \frac{ig}{2M_W}\alpha_{dH}m_{d_i},$$

$$\overline{e}_ie_iA \quad -\frac{g}{2M_W}\beta_e m_{e_i}\gamma_5,$$

$$\overline{u}_iu_iA \quad -\frac{g}{2M_W}\cot\beta m_{u_i}\gamma_5,$$

$$\overline{d}_id_iA \quad -\frac{g}{2M_W}\beta_d m_{d_i}\gamma_5,$$

$$\overline{e}_ie_iG_0 \quad \frac{g}{2M_W}m_{e_i}\gamma_5,$$

$$\overline{u}_iu_iG_0 \quad -\frac{g}{2M_W}m_{u_i}\gamma_5,$$

$$\overline{d}_id_iG_0 \quad \frac{g}{2M_W}m_{d_i}\gamma_5,$$

$$\overline{e}_i\nu_iH^+ \quad \frac{ig}{2\sqrt{2}M_W}\beta_e m_{e_i}(1+\gamma_5),$$

$$\overline{u}_id_jH^+ \quad \frac{ig}{2\sqrt{2}M_W}V_{ij}[\beta_d m_{d_j}(1+\gamma_5) + \cot\beta m_{u_i}(1-\gamma_5)],$$

$$\overline{\nu}_ie_iH^- \quad \frac{ig}{2\sqrt{2}M_W}\beta_e m_{e_i}(1-\gamma_5),$$

$$\overline{d}_iu_jH^- \quad \frac{ig}{2\sqrt{2}M_W}V_{ij}^*[\beta_d m_{d_i}(1-\gamma_5) + \cot\beta m_{u_j}(1+\gamma_5)],$$

$$\overline{e}_i\nu_iG^+ \quad -\frac{ig}{2\sqrt{2}M_W}m_{e_i}(1+\gamma_5),$$

$$\overline{u}_id_jG^+ \quad \frac{ig}{2\sqrt{2}M_W}V_{ij}[-m_{d_j}(1+\gamma_5) + m_{u_i}(1-\gamma_5)],$$

$\overline{\nu_i} e_i G^-$ $\quad -\frac{ig}{2\sqrt{2}M_W} m_{e_i}(1-\gamma_5)$,

$\overline{d_i} u_j G^-$ $\quad \frac{ig}{2\sqrt{2}M_W} V^*_{ij}[-m_{d_i}(1-\gamma_5)+m_{u_j}(1+\gamma_5)]$.

4. Gauge-boson−scalar vertices

$hZ_\mu Z_\nu$ $\quad -ig\frac{M_Z}{\cos\theta_W}\sin\delta g_{\mu\nu}$,

$hW^+_\mu W^-_\nu$ $\quad -igM_W\sin\delta g_{\mu\nu}$,

$HZ_\mu Z_\nu$ $\quad ig\frac{M_Z}{\cos\theta_W}\cos\delta g_{\mu\nu}$,

$HW^+_\mu W^-_\nu$ $\quad igM_W\cos\delta g_{\mu\nu}$,

$hhZ_\mu Z_\nu$ $\quad \frac{ig^2}{2\cos^2\theta_W} g_{\mu\nu}$,

$hhW^+_\mu W^-_\nu$ $\quad \frac{ig^2}{2} g_{\mu\nu}$,

$HHZ_\mu Z_\nu$ $\quad \frac{ig^2}{2\cos^2\theta_W} g_{\mu\nu}$,

$HHW^+_\mu W^-_\nu$ $\quad \frac{ig^2}{2} g_{\mu\nu}$,

$AAZ_\mu Z_\nu$ $\quad \frac{ig^2}{2\cos^2\theta_W} g_{\mu\nu}$,

$AAW^+_\mu W^-_\nu$ $\quad \frac{ig^2}{2} g_{\mu\nu}$,

$G_0G_0Z_\mu Z_\nu$ $\quad \frac{ig^2}{2\cos^2\theta_W} g_{\mu\nu}$,

$G_0G_0W^+_\mu W^-_\nu$ $\quad \frac{ig^2}{2} g_{\mu\nu}$,

$H^+H^-Z_\mu Z_\nu$ $\quad 2ie^2\cot^2(2\theta_W)g_{\mu\nu}$,

$H^+H^-W^+_\mu W^-_\nu$ $\quad \frac{ig^2}{2} g_{\mu\nu}$,

$G^+G^-Z_\mu Z_\nu$ $\quad 2ie^2\cot^2(2\theta_W)g_{\mu\nu}$,

$G^+G^-W^+_\mu W^-_\nu$ $\quad \frac{ig^2}{2} g_{\mu\nu}$,

$H^+H^-A_\mu A_\nu$ $\quad 2ie^2 g_{\mu\nu}$,

$G^+G^-A_\mu A_\nu$ $\quad 2ie^2 g_{\mu\nu}$,

$H^+H^-A_\mu Z_\nu$ $\quad -2ie^2\cot(2\theta_W)g_{\mu\nu}$,

$G^+G^-A_\mu Z_\nu$ $\quad -2ie^2\cot(2\theta_W)g_{\mu\nu}$,

$H^\pm AW^\mp_\mu A_\nu$ $\quad \pm\frac{g^2\sin(\theta_W)}{2} g_{\mu\nu}$,

$H^\pm AW^\mp_\mu Z_\nu$ $\quad \pm\frac{g^2\sin(\theta_W)}{2}\tan(\theta_W) g_{\mu\nu}$,

$G^\pm G_0W^\mp_\mu A_\nu$ $\quad \pm\frac{g^2\sin(\theta_W)}{2} g_{\mu\nu}$,

$G^\pm G_0W^\mp_\mu Z_\nu$ $\quad \pm\frac{g^2\sin(\theta_W)}{2}\tan(\theta_W) g_{\mu\nu}$,

$H^\pm hW^\mp_\mu A_\nu$ $\quad \cos\delta\frac{ig^2\sin(\theta_W)}{2} g_{\mu\nu}$,

$H^\pm hW^\mp_\mu Z_\nu$ $\quad \cos\delta\frac{ig^2\sin(\theta_W)}{2}\tan(\theta_W) g_{\mu\nu}$,

$G^\pm HW^\mp_\mu A_\nu$ $\quad \cos\delta\frac{ig^2\sin(\theta_W)}{2} g_{\mu\nu}$,

$G^\pm HW^\mp_\mu Z_\nu$ $\quad \cos\delta\frac{ig^2\sin(\theta_W)}{2}\tan(\theta_W) g_{\mu\nu}$,

$H^\pm HW^\mp_\mu A_\nu$ $\quad \sin\delta\frac{ig^2\sin(\theta_W)}{2} g_{\mu\nu}$,

$H^\pm HW^\mp_\mu Z_\nu$ $\quad \sin\delta\frac{ig^2\sin(\theta_W)}{2}\tan(\theta_W) g_{\mu\nu}$,

$G^\pm hW^\mp_\mu A_\nu$ $\quad -\sin\delta\frac{ig^2\sin(\theta_W)}{2} g_{\mu\nu}$,

$G^\pm hW^\mp_\mu Z_\nu$ $\quad -\sin\delta\frac{ig^2\sin(\theta_W)}{2}\tan(\theta_W) g_{\mu\nu}$,

$G^\pm W^\mp_\mu A_\nu$ $\quad iM_W e g_{\mu\nu}$,

$G^\pm W^\mp_\mu Z_\nu$ $\quad iM_W e\tan\theta_W g_{\mu\nu}$,

$A_\mu H^+H^-$ $\quad ie(p_{H^+}-p_{H^-})_\mu$,

$Z_\mu H^+H^-$ $\quad -ie\cot(2\theta_W)(p_{H^+}-p_{H^-})_\mu$,

$A_\mu G^+G^-$ $\quad ie(p_{G^+}-p_{G^-})_\mu$,

$Z_\mu G^+G^-$ $\quad -ie\cot(2\theta_W)(p_{G^+}-p_{G^-})_\mu$,

$Z_\mu HG_0$ $\quad \frac{gM_Z}{2M_W}\cos\delta(p_H-p_{G_0})_\mu$,

$Z_\mu hA$ $\quad \frac{gM_Z}{2M_W}\cos\delta(p_h-p_A)_\mu$,

$$Z_\mu HA \quad \frac{gM_Z}{2M_W}\sin\delta(p_H - p_A)_\mu,$$

$$Z_\mu hG_0 \quad \frac{gM_Z}{2M_W}\sin\delta(p_{G_0} - p_h)_\mu,$$

$$W^\mp_\mu H^\pm A \quad \frac{g}{2}(p_{H^\pm} - p_A)_\mu,$$

$$W^\mp_\mu G^\pm G_0 \quad \frac{g}{2}(p_{G^\pm} - p_{G_0})_\mu,$$

$$W^\mp_\mu H^\pm h \pm \quad i\cos\delta\frac{g}{2}(p_{H^\pm} - p_h)_\mu,$$

$$W^\mp_\mu G^\pm H \quad i\cos\delta\frac{g}{2}(p_{G^\pm} - p_H)_\mu,$$

$$W^\mp_\mu H^\pm H \quad \pm i\sin\delta\frac{g}{2}(p_{H^\pm} - p_H)_\mu,$$

$$W^\mp_\mu G^\pm h \quad \mp i\sin\delta\frac{g}{2}(p_{G^\pm} - p_h)_\mu.$$

5. Ghost-scalar vertices

$$\overline{C^\pm}C^\mp H \quad -\frac{igM_W}{2}\cos\delta,$$

$$\overline{C^\pm}C^\mp h \quad \frac{igM_W}{2}\sin\delta,$$

$$\overline{C^\pm}C^\mp G_0 \quad \pm\frac{igM_W}{2},$$

$$\overline{C_Z}C_Z H \quad -\frac{igM_Z}{2\cos\theta_W}\cos\delta,$$

$$\overline{C_Z}C_Z h \quad \frac{igM_Z}{2\cos\theta_W}\sin\delta,$$

$$\overline{C^\pm}C_Z G^\mp \quad \pm\frac{igM_Z}{2}\cos(2\theta_W),$$

$$\overline{C^\pm}C_A G^\mp \quad \mp ieM_W,$$

$$\overline{C_Z}C^\pm G^\mp \quad -\frac{igM_Z}{2}.$$

APPENDIX B: RENORMALIZATION CONSTANTS

1. Two-point functions

a. Scalar counterterms

In the CP-even scalar sector the six renormalization constants Z_{HH}, Z_{hh}, Z_{Hh}, Z_{hH}, δM_H^2, and δM_h^2 are determined by solving the set of equations

$$\Sigma_{HH}(M_H^2) - Z_{HH}\delta M_H^2 - Z_{hH}(M_h^2 - M_H^2 + \delta M_h^2) - T_{HH}Z_{HH} - T_{hh}Z_{hH} - 2T_{Hh}Z_{HH}^{1/2}Z_{hH}^{1/2} = 0, \tag{B1}$$

$$\frac{d}{dq^2}\Sigma_{HH}(M_H^2) + Z_{HH} + Z_{hH} = 0, \tag{B2}$$

$$\Sigma_{hh}(M_h^2) - Z_{hh}\delta M_h^2 - Z_{Hh}(M_H^2 - M_h^2 + \delta M_H^2) - T_{hh}Z_{hh} - T_{HH}Z_{Hh} - 2T_{Hh}Z_{hh}^{1/2}Z_{Hh}^{1/2} = 0, \tag{B3}$$

$$\frac{d}{dq^2}\Sigma_{hh}(M_h^2) + Z_{hh} + Z_{Hh} = 0, \tag{B4}$$

$$\Sigma_{Hh}(M_H^2) - Z_{HH}^{1/2}Z_{Hh}^{1/2}\delta M_H^2 - Z_{hh}^{1/2}Z_{hH}^{1/2}(M_h^2 - M_H^2 + \delta M_h^2) - T_{HH}Z_{HH}^{1/2}Z_{Hh}^{1/2} - T_{hh}Z_{hh}^{1/2}Z_{hH}^{1/2} - T_{Hh}(Z_{HH}^{1/2}Z_{hh}^{1/2} + Z_{Hh}^{1/2}Z_{hH}^{1/2}) = 0, \tag{B5}$$

$$\Sigma_{Hh}(M_h^2) - Z_{hh}^{1/2}Z_{hH}^{1/2}\delta M_h^2 - Z_{HH}^{1/2}Z_{Hh}^{1/2}(M_H^2 - M_h^2 + \delta M_H^2) - T_{HH}Z_{HH}^{1/2}Z_{Hh}^{1/2} - T_{hh}Z_{hh}^{1/2}Z_{hH}^{1/2} - T_{Hh}(Z_{HH}^{1/2}Z_{hh}^{1/2} + Z_{Hh}^{1/2}Z_{hH}^{1/2}) = 0, \tag{B6}$$

with the renormalized tadpole functions defined as

$$T_{HH} = \frac{g + \delta g}{(M_W^2 + \delta M_W^2)^{1/2}}\,\frac{T_{\alpha\beta} + T_\delta \sin^2(\alpha + \delta\alpha)}{\sin[2(\beta + \delta\beta)]}, \tag{B7}$$

$$T_{hh} = \frac{g + \delta g}{(M_W^2 + \delta M_W^2)^{1/2}}\,\frac{T_{\alpha\beta} + T_\delta \cos^2(\alpha + \delta\alpha)}{\sin[2(\beta + \delta\beta)]}, \tag{B8}$$

$$T_{Hh} = \frac{g + \delta g}{2(M_W^2 + \delta M_W^2)^{1/2}}\,\frac{T_\delta \sin[2(\alpha + \delta\alpha)]}{\sin[2(\beta + \delta\beta)]}. \tag{B9}$$

The CP-odd scalar sector has five renormalization constants to be determined, Z_{AA}, $Z_{G_0G_0}$, Z_{AG_0}, Z_{G_0A}, and δM_A^2, because the Goldstone boson G_0 is massless. Because of the tree-level relations $T_{AA} = T_{H^+H^-}$, $T_{G_0G_0} = T_{G^+G^-}$, and $T_{AG_0} = T_{H^+G^-}$, the renormalized tadpole functions for this sector are defined in Eqs. (31) and (32). From the following set of six equations only five are independent due to the Ward identity equivalent to Eq. (37) but for the neutral sector:

$$\Sigma_{AA}(M_A^2) - Z_{AA}\delta M_A^2 - Z_{G_0A}M_A^2 - T_{AA}Z_{AA} - T_{G_0G_0}Z_{G_0A} - 2T_{AG_0}Z_{AA}^{1/2}Z_{G_0A}^{1/2} = 0, \tag{B10}$$

$$\frac{d}{dq^2}\Sigma_{AA}(M_A^2) + Z_{AA} + Z_{G_0A} = 0, \tag{B11}$$

$$\Sigma_{G_0G_0}(0) - Z_{AG_0}(M_A^2 + \delta M_A^2) - T_{G_0G_0}Z_{G_0G_0} - T_{AA}Z_{AG_0} - 2T_{AG_0}Z_{G_0G_0}^{1/2}Z_{AG_0}^{1/2} = 0, \tag{B12}$$

$$\frac{d}{dq^2}\Sigma_{G_0G_0}(0)+Z_{G_0G_0}+Z_{AG_0}=0, \tag{B13}$$

$$\Sigma_{AG_0}(M_A^2)-Z_{AA}^{1/2}Z_{AG_0}^{1/2}\delta M_A^2+Z_{G_0G_0}^{1/2}Z_{G_0A}^{1/2}M_A^2-T_{AA}Z_{AA}^{1/2}Z_{AG_0}^{1/2} -T_{G_0G_0}Z_{G_0G_0}^{1/2}Z_{G_0A}^{1/2}-T_{AG_0}(Z_{AA}^{1/2}Z_{G_0G_0}^{1/2}+Z_{AG_0}^{1/2}Z_{G_0A}^{1/2})=0, \tag{B14}$$

$$\Sigma_{AG_0}(0)-Z_{AA}^{1/2}Z_{AG_0}^{1/2}(M_A^2+\delta M_A^2)-T_{AA}Z_{AA}^{1/2}Z_{AG_0}^{1/2} -T_{G_0G_0}Z_{G_0G_0}^{1/2}Z_{G_0A}^{1/2}-T_{AG_0}(Z_{AA}^{1/2}Z_{G_0G_0}^{1/2}+Z_{AG_0}^{1/2}Z_{G_0A}^{1/2})=0. \tag{B15}$$

Finally, the charged sector behaves like the CP-even one. The five renormalization constants to be determined are, in this case, $Z_{H^+H^+}$, $Z_{G^+G^+}$, $Z_{H^+G^+}$, $Z_{G^+H^+}$, and $\delta M_{H^+}^2$. The equations are

$$\Sigma_{H^+H^+}(M_{H^+}^2)-Z_{H^+H^+}\delta M_{H^+}^2-Z_{G^+H^+}M_{H^+}^2 -T_{H^+H^-}Z_{H^+H^+}-T_{G^+G^-}Z_{G^+H^+} -2T_{H^+G^-}Z_{H^+H^+}^{1/2}Z_{G^+H^+}^{1/2}=0, \tag{B16}$$

$$\frac{d}{dq^2}\Sigma_{H^+H^+}(M_{H^+}^2)+Z_{H^+H^+}+Z_{G^+H^+}=0, \tag{B17}$$

$$\Sigma_{G^+G^+}(0)-Z_{H^+G^+}(M_{H^+}^2+\delta M_{H^+}^2)-T_{G^+G^-}Z_{G^+G^+} -T_{H^+H^-}Z_{H^+G^+}-2T_{H^+G^-}Z_{G^+G^+}^{1/2}Z_{H^+G^+}^{1/2}=0, \tag{B18}$$

$$\frac{d}{dq^2}\Sigma_{G^+G^+}(0)+Z_{G^+G^+}+Z_{H^+G^+}=0, \tag{B19}$$

$$\Sigma_{H^+G^+}(M_{H^+}^2)-Z_{H^+H^+}^{1/2}Z_{H^+G^+}^{1/2}\delta M_{H^+}^2+Z_{G^+G^+}^{1/2}Z_{G^+H^+}^{1/2}M_{H^+}^2 -T_{H^+H^-}Z_{H^+H^+}^{1/2}Z_{H^+G^+}^{1/2}-T_{G^+G^-}Z_{G^+G^+}^{1/2}Z_{G^+H^+}^{1/2} -T_{H^+G^-}(Z_{H^+H^+}^{1/2}Z_{G^+G^+}^{1/2}+Z_{H^+G^+}^{1/2}Z_{G^+H^+}^{1/2})=0, \tag{B20}$$

$$\Sigma_{H^+G^+}(0)-Z_{H^+H^+}^{1/2}Z_{H^+G^+}^{1/2}(M_{H^+}^2+\delta M_{H^+}^2) -T_{H^+H^-}Z_{H^+H^+}^{1/2}Z_{H^+G^+}^{1/2}-T_{G^+G^-}Z_{G^+G^+}^{1/2}Z_{G^+H^+}^{1/2} -T_{H^+G^-}(Z_{H^+H^+}^{1/2}Z_{G^+G^+}^{1/2}+Z_{H^+G^+}^{1/2}Z_{G^+H^+}^{1/2})=0. \tag{B21}$$

The renormalized quantities T_δ and $T_{\alpha\beta}$ are defined in Eqs. (32a) and (32b). The renormalized tadpole functions for the charged sector are defined in Eqs. (31a), (31b), and (31c).

b. Mixed counterterms

The complete set of counterterms for the mixed gauge-scalar sector can be written as

$$Z_{W^+G^+}^{1/2}=ik_\mu(M_W^2+\delta M_W^2)^{1/2}Z_W^{1/2}Z_{G^+G^+}^{1/2}, \tag{B22}$$

$$Z_{W^+H^+}^{1/2}=ik_\mu(M_W^2+\delta M_W^2)^{1/2}Z_W^{1/2}Z_{G^+H^+}^{1/2}, \tag{B23}$$

$$Z_{ZG_0}^{1/2}=ik_\mu(M_Z^2+\delta M_Z^2)^{1/2}Z_{ZZ}^{1/2}Z_{G_0G_0}^{1/2}, \tag{B24}$$

$$Z_{ZA}^{1/2}=ik_\mu(M_Z^2+\delta M_Z^2)^{1/2}Z_{ZZ}^{1/2}Z_{G_0A}^{1/2}, \tag{B25}$$

$$Z_{\gamma G_0}^{1/2}=ik_\mu(M_Z^2+\delta M_Z^2)^{1/2}Z_{Z\gamma}^{1/2}Z_{G_0G_0}^{1/2}, \tag{B26}$$

$$Z_{\gamma A}^{1/2}=ik_\mu(M_Z^2+\delta M_Z^2)^{1/2}Z_{Z\gamma}^{1/2}Z_{G_0A}^{1/2}. \tag{B27}$$

2. Three- and four-point functions

In this section we present the counterterms for the three- and four-point functions involving scalar and other fields. The scalar-scalar couterterms will not be shown since Higgs boson scattering and Higgs boson decay involving scalar particles only, in both initial and final states, are already calculated at the tree level and were never observed experimentally. So there is no point in doing loop corrections to processes not yet observed. However, it is staightforward to deduce any of those counterterms: First rewritte the scalar Lagrangian as a function of the renormalized fields; then group all terms with the same number and type of fields; the factor that multipies those fields is the field renormalization factor; finally renormalize the coupling.

All along this section we concentrate on the field renormalization. We use γ_μ instead of A_μ to represent the photon field so that it will not be confused with the pseudoscalar field A. The parameter renormalization is written symbolically as δg_{ijk} and δg_{ijkl} where i, j, k, and l are the fields in the vertex. These quantities are determined by a simple variation of the independent parameters in the vertex. We have chosen as free parameters the particle masses, the electric charge, the two angles in the scalar sector (α and β), and the four independent angles in the CKM matrix. We use several constants as a bookeeping to make the vertex expressions simpler. Among them are the SU(2) gauge constant g, the Weinberg angle θ_W, the angle $\delta=\alpha-\beta$, and the couplings expressed in Table II. The first three can be written in terms of the independent parameters as

$$\frac{\delta g}{g}=\frac{\delta e}{e}+\frac{M_W^2}{2(M_Z^2-M_W^2)}\left[\frac{\delta M_W^2}{M_W^2}-\frac{\delta M_Z^2}{M_Z^2}\right], \tag{B28}$$

$$\delta\theta_W=-\frac{\delta M_W}{(M_Z^2-M_W^2)^{1/2}}+\frac{M_W\delta M_Z}{M_Z(M_Z^2-M_W^2)^{1/2}}, \tag{B29}$$

$$\delta(\delta)=\delta\alpha-\delta\beta. \tag{B30}$$

The parameter renormalization in the vertices is easily calculated and so we will just give an example of how it is done. In the example we will use the vertex $g_{\overline{e_i}e_ih}$, for models I and III,

$$\delta g_{\bar{e}_i e_i h} = \delta g \frac{i\alpha_{eh} m_{e_i}}{2M_W} - \delta M_W \frac{ig\alpha_{eh} m_{e_i}}{2M_W^2} + (\delta\alpha_{eh} m_{e_i} + \delta m_{e_i}\alpha_{eh})\frac{ig}{2M_W}, \tag{B31}$$

with

$$\delta\alpha_{eh} = \frac{\sin\alpha}{\sin\beta}\delta\alpha - \frac{\cos\alpha\cos\beta}{\sin^2\beta}\delta\beta. \tag{B32}$$

a. 1 scalar + 2 gauge

$$hZ_\mu Z_\nu \quad g_{hZZ}Z_{hh}^{1/2}Z_{ZZ} + g_{HZZ}Z_{Hh}^{1/2}Z_{ZZ} + \delta g_{hZZ},$$

$$HZ_\mu Z_\nu \quad g_{HZZ}Z_{HH}^{1/2}Z_{ZZ} + g_{hZZ}Z_{hH}^{1/2}Z_{ZZ} + \delta g_{HZZ},$$

$$h\gamma_\mu\gamma_\nu \quad g_{hZZ}Z_{hh}^{1/2}Z_{Z\gamma} + g_{HZZ}Z_{Hh}^{1/2}Z_{Z\gamma},$$

$$H\gamma_\mu\gamma_\nu \quad g_{HZZ}Z_{HH}^{1/2}Z_{Z\gamma} + g_{hZZ}Z_{hH}^{1/2}Z_{Z\gamma},$$

$$hZ_\mu\gamma_\nu \quad 2g_{hZZ}Z_{hh}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2} + 2g_{HZZ}Z_{Hh}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2},$$

$$HZ_\mu\gamma_\nu \quad 2g_{HZZ}Z_{HH}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2} + 2g_{hZZ}Z_{hH}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2},$$

$$hW_\mu^+W_\nu^- \quad g_{hWW}Z_{hh}^{1/2}Z_W + g_{HWW}Z_{Hh}^{1/2}Z_W + \delta g_{hWW},$$

$$HW_\mu^+W_\nu^- \quad g_{HWW}Z_{HH}^{1/2}Z_W + g_{hWW}Z_{hH}^{1/2}Z_W + \delta g_{HWW},$$

$$G^\pm W_\mu^\mp\gamma_\nu \quad g_{G^+W\gamma}Z_{G^+G^+}^{1/2}Z_W^{1/2}Z_{\gamma\gamma}^{1/2} + g_{G^+WZ}Z_{G^+G^+}^{1/2}Z_W^{1/2}Z_{Z\gamma}^{1/2} + \delta g_{G^+W\gamma},$$

$$G^\pm W_\mu^\mp Z_\nu \quad g_{G^+WZ}Z_{G^+G^+}^{1/2}Z_W^{1/2}Z_{ZZ}^{1/2} + g_{G^+W\gamma}Z_{G^+G^+}^{1/2}Z_W^{1/2}Z_{\gamma Z}^{1/2} + \delta g_{G^+WZ},$$

$$H^\pm W_\mu^\mp\gamma_\nu \quad g_{G^+W\gamma}Z_{G^+H^+}^{1/2}Z_W^{1/2}Z_{\gamma\gamma}^{1/2} + g_{G^+WZ}Z_{G^+H^+}^{1/2}Z_W^{1/2}Z_{Z\gamma}^{1/2},$$

$$H^\pm W_\mu^\mp Z_\nu \quad g_{G^+WZ}Z_{G^+H^+}^{1/2}Z_W^{1/2}Z_{ZZ}^{1/2} + g_{G^+W\gamma}Z_{G^+H^+}^{1/2}Z_W^{1/2}Z_{\gamma Z}^{1/2}.$$

b. 2 scalar + 2 gauge

$$hhZ_\mu Z_\nu \quad g_{hhZZ}Z_{hh}Z_{ZZ} + g_{HHZZ}Z_{Hh}Z_{ZZ} + \delta g_{hhZZ},$$

$$HHZ_\mu Z_\nu \quad g_{HHZZ}Z_{HH}Z_{ZZ} + g_{hhZZ}Z_{hH}Z_{ZZ} + \delta g_{HHZZ},$$

$$hh\gamma_\mu\gamma_\nu \quad g_{hhZZ}Z_{hh}Z_{Z\gamma} + g_{HHZZ}Z_{Hh}Z_{Z\gamma},$$

$$HH\gamma_\mu\gamma_\nu \quad g_{HHZZ}Z_{HH}Z_{Z\gamma} + g_{hhZZ}Z_{hH}Z_{Z\gamma},$$

$$hhZ_\mu\gamma_\nu \quad 2g_{hhZZ}Z_{hh}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2} + 2g_{HHZZ}Z_{Hh}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2},$$

$$HHZ_\mu\gamma_\nu \quad 2g_{HHZZ}Z_{HH}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2} + 2g_{hhZZ}Z_{hH}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2},$$

$$hHZ_\mu Z_\nu \quad 2g_{HHZZ}Z_{HH}^{1/2}Z_{Hh}^{1/2}Z_{ZZ} + 2g_{hhZZ}Z_{hh}^{1/2}Z_{hH}^{1/2}Z_{ZZ},$$

$$hH\gamma_\mu\gamma_\nu \quad 2g_{hhZZ}Z_{hh}^{1/2}Z_{hH}^{1/2}Z_{Z\gamma} + 2g_{HHZZ}Z_{HH}^{1/2}Z_{Hh}^{1/2}Z_{Z\gamma},$$

$$hHZ_\mu\gamma_\nu 4 \quad g_{hhZZ}Z_{hh}^{1/2}Z_{hH}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2} + 4g_{HHZZ}Z_{HH}^{1/2}Z_{Hh}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2},$$

$$hhW_\mu^+W_\nu^- \quad g_{hhWW}Z_{hh}Z_W + g_{HHWW}Z_{Hh}Z_W + \delta g_{hhWW},$$

$$HHW_\mu^+W_\nu^- \quad g_{HHWW}Z_{HH}Z_W + g_{hhWW}Z_{hH}Z_W + \delta g_{HHWW},$$

$$hHW_\mu^+W_\nu^- \quad 2g_{hhWW}Z_{hh}^{1/2}Z_{hH}^{1/2}Z_W + 2g_{HHWW}Z_{HH}^{1/2}Z_{Hh}^{1/2}Z_W,$$

$$AAZ_\mu Z_\nu \quad g_{AAZZ}Z_{AA}Z_{ZZ} + g_{G_0G_0ZZ}Z_{G_0A}Z_{ZZ} + \delta g_{AAZZ},$$

$$G_0G_0Z_\mu Z_\nu \quad g_{G_0G_0ZZ}Z_{G_0G_0}Z_{ZZ}+g_{AAZZ}Z_{AG_0}Z_{ZZ}+\delta g_{G_0G_0ZZ},$$

$$AA\gamma_\mu\gamma_\nu \quad g_{AAZZ}Z_{AA}Z_{Z\gamma}+g_{G_0G_0ZZ}Z_{G_0A}Z_{Z\gamma},$$

$$G_0G_0\gamma_\mu\gamma_\nu \quad g_{G_0G_0ZZ}Z_{G_0G_0}Z_{Z\gamma}+g_{AAZZ}Z_{AG_0}Z_{Z\gamma},$$

$$AAZ_\mu\gamma_\nu \quad 2g_{AAZZ}Z_{AA}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2}+2g_{G_0G_0ZZ}Z_{G_0A}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2},$$

$$G_0G_0Z_\mu\gamma_\nu \quad 2g_{AAZZ}Z_{AG_0}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2}+2g_{G_0G_0ZZ}Z_{G_0G_0}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2},$$

$$AG_0Z_\mu Z_\nu \quad 2g_{AAZZ}Z_{AA}^{1/2}Z_{AG_0}^{1/2}Z_{ZZ}+2g_{G_0G_0ZZ}Z_{G_0G_0}^{1/2}Z_{G_0A}^{1/2}Z_{ZZ},$$

$$AG_0\gamma_\mu\gamma_\nu \quad 2g_{AAZZ}Z_{AA}^{1/2}Z_{AG_0}^{1/2}Z_{Z\gamma}+2g_{G_0G_0ZZ}Z_{G_0G_0}^{1/2}Z_{G_0A}^{1/2}Z_{Z\gamma},$$

$$AG_0Z_\mu\gamma_\nu 4 \quad g_{AAZZ}Z_{AA}^{1/2}Z_{AG_0}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2}+4g_{G_0G_0ZZ}Z_{G_0G_0}^{1/2}Z_{G_0A}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2},$$

$$AAW_\mu^+W_\nu^- \quad g_{AAWW}Z_{AA}Z_W+g_{G_0G_0WW}Z_{G_0A}Z_W+\delta g_{AAWW},$$

$$G_0G_0W_\mu^+W_\nu^- \quad g_{G_0G_0WW}Z_{G_0G_0}Z_W+g_{AAWW}Z_{AG_0}Z_W+\delta g_{G_0G_0WW},$$

$$AG_0W_\mu^+W_\nu^- \quad 2g_{AAWW}Z_{AA}^{1/2}Z_{AG_0}^{1/2}Z_W+2g_{G_0G_0WW}Z_{G_0G_0}^{1/2}Z_{G_0A}^{1/2}Z_W,$$

$$H^+H^-Z_\mu Z_\nu \quad g_{H^+H^-ZZ}Z_{H^+H^-}Z_{ZZ}+g_{G^+G^-ZZ}Z_{G^+H^-}Z_{ZZ}+g_{H^+H^-\gamma\gamma}Z_{H^+H^-}Z_{\gamma Z}+g_{G^+G^-\gamma\gamma}Z_{G^+H^-}Z_{\gamma Z}$$
$$+g_{H^+H^-\gamma Z}Z_{H^+H^-}Z_{\gamma Z}^{1/2}Z_{ZZ}^{1/2}+g_{G^+G^-\gamma Z}Z_{G^+H^-}Z_{\gamma Z}^{1/2}Z_{ZZ}^{1/2}+\delta g_{H^+H^-ZZ},$$

$$G^+G^-Z_\mu Z_\nu \quad g_{G^+G^-ZZ}Z_{G^+G^-}Z_{ZZ}+g_{H^+H^-ZZ}Z_{H^+G^-}Z_{ZZ}+g_{H^+H^-\gamma\gamma}Z_{H^+G^-}Z_{\gamma Z}+g_{G^+G^-\gamma\gamma}Z_{G^+G^-}Z_{\gamma Z}$$
$$+g_{H^+H^-\gamma Z}Z_{H^+G^-}Z_{\gamma Z}^{1/2}Z_{ZZ}^{1/2}+g_{G^+G^-\gamma Z}Z_{G^+G^-}Z_{\gamma Z}^{1/2}Z_{ZZ}^{1/2}+\delta g_{G^+G^-ZZ},$$

$$H^+H^-\gamma_\mu\gamma_\nu \quad g_{H^+H^-\gamma\gamma}Z_{H^+H^-}Z_{\gamma\gamma}+g_{H^+H^-ZZ}Z_{H^+H^-}Z_{Z\gamma}+g_{G^+G^-ZZ}Z_{G^+H^-}Z_{Z\gamma}+g_{G^+G^-\gamma\gamma}Z_{G^+H^-}Z_{\gamma\gamma}$$
$$+g_{H^+H^-\gamma Z}Z_{H^+H^-}Z_{\gamma\gamma}^{1/2}Z_{Z\gamma}^{1/2}+g_{G^+G^-\gamma Z}Z_{G^+H^-}Z_{\gamma\gamma}^{1/2}Z_{Z\gamma}^{1/2}+\delta g_{H^+H^-\gamma\gamma},$$

$$G^+G^-\gamma_\mu\gamma_\nu \quad g_{G^+G^-\gamma\gamma}Z_{G^+G^-}Z_{\gamma\gamma}+g_{H^+H^-ZZ}Z_{H^+G^-}Z_{Z\gamma}+g_{G^+G^-ZZ}Z_{G^+G^-}Z_{Z\gamma}+g_{H^+H^-\gamma\gamma}Z_{H^+G^-}Z_{\gamma\gamma}$$
$$+g_{H^+H^-\gamma Z}Z_{H^+G^-}Z_{\gamma\gamma}^{1/2}Z_{Z\gamma}^{1/2}+g_{G^+G^-\gamma Z}Z_{G^+G^-}Z_{\gamma\gamma}^{1/2}Z_{Z\gamma}^{1/2}+\delta g_{G^+G^-\gamma\gamma},$$

$$H^+H^-\gamma_\mu Z_\nu \quad g_{H^+H^-\gamma Z}Z_{H^+H^-}(Z_{\gamma\gamma}^{1/2}Z_{ZZ}^{1/2}+Z_{\gamma Z}^{1/2}Z_{Z\gamma}^{1/2})+2g_{H^+H^-ZZ}Z_{H^+H^-}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2}+g_{G^+G^-\gamma Z}Z_{G^+H^-}(Z_{\gamma\gamma}^{1/2}Z_{ZZ}^{1/2}+Z_{\gamma Z}^{1/2}Z_{Z\gamma}^{1/2})$$
$$+2g_{G^+G^-ZZ}Z_{G^+H^-}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2}+2g_{G^+G^-\gamma\gamma}Z_{G^+H^-}Z_{\gamma\gamma}^{1/2}Z_{\gamma Z}^{1/2}+2g_{H^+H^-\gamma\gamma}Z_{H^+H^-}Z_{\gamma\gamma}^{1/2}Z_{\gamma Z}^{1/2}+\delta g_{H^+H^-\gamma Z},$$

$$G^+G^-\gamma_\mu Z_\nu \quad g_{G^+G^-\gamma Z}Z_{G^+G^-}(Z_{\gamma\gamma}^{1/2}Z_{ZZ}^{1/2}+Z_{\gamma Z}^{1/2}Z_{Z\gamma}^{1/2})+2g_{H^+H^-ZZ}Z_{H^+G^-}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2}+g_{H^+H^-\gamma Z}Z_{H^+G^-}(Z_{\gamma\gamma}^{1/2}Z_{ZZ}^{1/2}+Z_{\gamma Z}^{1/2}Z_{Z\gamma}^{1/2})$$
$$+2g_{G^+G^-ZZ}Z_{G^+G^-}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2}+2g_{H^+H^-\gamma\gamma}Z_{H^+G^-}Z_{\gamma\gamma}^{1/2}Z_{\gamma Z}^{1/2}+2g_{G^+G^-\gamma\gamma}Z_{G^+G^-}Z_{\gamma\gamma}^{1/2}Z_{\gamma Z}^{1/2}+\delta g_{G^+G^-\gamma Z},$$

$$H^\pm G^\mp Z_\mu Z_\nu \quad g_{H^+H^-ZZ}Z_{H^+H^-}^{1/2}Z_{H^+G^-}^{1/2}Z_{ZZ}+g_{G^+G^-ZZ}Z_{G^+G^-}^{1/2}Z_{G^+H^-}^{1/2}Z_{ZZ}+g_{H^+H^-\gamma\gamma}Z_{H^+H^-}^{1/2}Z_{H^+G^-}^{1/2}Z_{\gamma Z}$$
$$+g_{G^+G^-\gamma\gamma}Z_{G^+G^-}^{1/2}Z_{G^+H^-}^{1/2}Z_{\gamma Z}+g_{H^+H^-\gamma Z}Z_{H^+H^-}^{1/2}Z_{H^+G^-}^{1/2}Z_{\gamma Z}^{1/2}Z_{ZZ}^{1/2}+g_{G^+G^-\gamma Z}Z_{G^+G^-}^{1/2}Z_{G^+H^-}^{1/2}Z_{\gamma Z}^{1/2}Z_{ZZ}^{1/2},$$

$$H^\pm G^\mp\gamma_\mu\gamma_\nu \quad g_{H^+H^-ZZ}Z_{H^+H^-}^{1/2}Z_{H^+G^-}^{1/2}Z_{Z\gamma}+g_{G^+G^-ZZ}Z_{G^+G^-}^{1/2}Z_{G^+H^-}^{1/2}Z_{Z\gamma}+g_{H^+H^-\gamma\gamma}Z_{H^+H^-}^{1/2}Z_{H^+G^-}^{1/2}Z_{\gamma\gamma}$$
$$+g_{G^+G^-\gamma\gamma}Z_{G^+G^-}^{1/2}Z_{G^+H^-}^{1/2}Z_{\gamma\gamma}+g_{H^+H^-\gamma Z}Z_{H^+H^-}^{1/2}Z_{H^+G^-}^{1/2}Z_{\gamma\gamma}^{1/2}Z_{Z\gamma}^{1/2}+g_{G^+G^-\gamma Z}Z_{G^+G^-}^{1/2}Z_{G^+H^-}^{1/2}Z_{\gamma\gamma}^{1/2}Z_{Z\gamma}^{1/2},$$

$$H^\pm G^\mp\gamma_\mu Z_\nu \quad 2g_{H^+H^-ZZ}Z_{H^+H^-}^{1/2}Z_{H^+G^-}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2}+2g_{G^+G^-\gamma\gamma}Z_{G^+G^-}^{1/2}Z_{G^+H^-}^{1/2}Z_{\gamma\gamma}^{1/2}Z_{\gamma Z}^{1/2}+2g_{G^+G^-ZZ}Z_{G^+G^-}^{1/2}Z_{G^+H^-}^{1/2}Z_{ZZ}^{1/2}Z_{Z\gamma}^{1/2}$$
$$+2g_{H^+H^-\gamma\gamma}Z_{H^+H^-}^{1/2}Z_{H^+G^-}^{1/2}Z_{\gamma\gamma}^{1/2}Z_{\gamma Z}^{1/2}+g_{G^+G^-\gamma Z}Z_{G^+G^-}^{1/2}Z_{G^+H^-}^{1/2}(Z_{\gamma\gamma}^{1/2}Z_{ZZ}^{1/2}+Z_{\gamma Z}^{1/2}Z_{Z\gamma}^{1/2})$$
$$+g_{H^+H^-\gamma Z}Z_{H^+H^-}^{1/2}Z_{H^+G^-}^{1/2}(Z_{\gamma\gamma}^{1/2}Z_{ZZ}^{1/2}+Z_{\gamma Z}^{1/2}Z_{Z\gamma}^{1/2}),$$

$$
\begin{aligned}
&H^+H^-W^+_\mu W^-_\nu \quad && g_{H^+H^-WW}Z_{H^+H^-}Z_W+g_{G^+G^-WW}Z_{G^+H^-}Z_W+\delta g_{H^+H^-WW}\,,\\
&G^+G^-W^+_\mu W^-_\nu \quad && g_{G^+G^-WW}Z_{G^+G^-}Z_W+g_{H^+H^-WW}Z_{H^+G^-}Z_W+\delta g_{G^+G^-WW}\,,\\
&H^\pm G^\mp W^+_\mu W^-_\nu \quad && g_{H^+H^-WW}Z^{1/2}_{H^+H^-}Z^{1/2}_{H^+G^-}Z_W+g_{G^+G^-WW}Z^{1/2}_{G^+G^-}Z^{1/2}_{G^+H^-}Z_W\,,\\
&H^\pm AW^\mp_\mu\gamma_\nu \quad && g_{H^+AW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{AA}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+AWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{AA}Z^{1/2}_W Z^{1/2}_{Z\gamma}+g_{G^+G_0W\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{G_0A}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}\\
& && +g_{G^+G_0WZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{G_0A}Z^{1/2}_W Z^{1/2}_{Z\gamma}+\delta g_{H^+AW\gamma}\,,\\
&H^\pm AW^\mp_\mu Z_\nu \quad && g_{H^+AWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{AA}Z^{1/2}_W Z^{1/2}_{ZZ}+g_{H^+AW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{AA}Z^{1/2}_W Z^{1/2}_{\gamma Z}+g_{G^+G_0W\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{G_0A}Z^{1/2}_W Z^{1/2}_{\gamma Z}\\
& && +g_{G^+G_0WZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{G_0A}Z^{1/2}_W Z^{1/2}_{ZZ}+\delta g_{H^+AWZ}\,,\\
&G^\pm G_0W^\mp_\mu\gamma_\nu \quad && g_{G^+G_0W\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0G_0}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+AW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{AG_0}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+AWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{AG_0}Z^{1/2}_W Z^{1/2}_{Z\gamma}\\
& && +g_{G^+G_0WZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0G_0}Z^{1/2}_W Z^{1/2}_{Z\gamma}+\delta g_{G^+G_0W\gamma}\,,\\
&G^\pm G_0W^\mp_\mu Z_\nu \quad && g_{G^+G_0WZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0G_0}Z^{1/2}_W Z^{1/2}_{ZZ}+g_{H^+AW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{AG_0}Z^{1/2}_W Z^{1/2}_{\gamma Z}+g_{H^+AWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{AG_0}Z^{1/2}_W Z^{1/2}_{ZZ}\\
& && +g_{G^+G_0W\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0G_0}Z^{1/2}_W Z^{1/2}_{\gamma Z}+\delta g_{G^+G_0WZ}\,,\\
&H^\pm G_0W^\mp_\mu\gamma_\nu \quad && g_{G^+G_0W\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0G_0}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+AW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{AG_0}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+AWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{AG_0}Z^{1/2}_W Z^{1/2}_{Z\gamma}\\
& && +g_{G^+G_0WZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0G_0}Z^{1/2}_W Z^{1/2}_{Z\gamma}\,,\\
&H^\pm G_0W^\mp_\mu Z_\nu \quad && g_{H^+AW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{AG_0}Z^{1/2}_W Z^{1/2}_{\gamma Z}+g_{H^+AWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{AG_0}Z^{1/2}_W Z^{1/2}_{ZZ}+g_{G^+G_0W\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{G_0G_0}Z^{1/2}_W Z^{1/2}_{\gamma Z}\\
& && +g_{G^+G_0WZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{G_0G_0}Z^{1/2}_W Z^{1/2}_{ZZ}\,,\\
&G^\pm AW^\mp_\mu\gamma_\nu \quad && g_{H^+AW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{AA}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+AWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{AA}Z^{1/2}_W Z^{1/2}_{Z\gamma}+g_{G^+G_0W\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0A}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}\\
& && +g_{G^+G_0WZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0A}Z^{1/2}_W Z^{1/2}_{Z\gamma}\,,\\
&G^\pm AW^\mp_\mu Z_\nu \quad && g_{H^+AW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{AA}Z^{1/2}_W Z^{1/2}_{\gamma Z}+g_{H^+AWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{AA}Z^{1/2}_W Z^{1/2}_{ZZ}+g_{G^+G_0W\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0A}Z^{1/2}_W Z^{1/2}_{\gamma Z}\\
& && +g_{G^+G_0WZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0A}Z^{1/2}_W Z^{1/2}_{ZZ}\,,\\
&H^\pm hW^\mp_\mu\gamma_\nu \quad && g_{H^+hW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{hh}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{G^+hW\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{hh}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{G^+hWZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{hh}Z^{1/2}_W Z^{1/2}_{Z\gamma}\\
& && +g_{G^+HWZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{Hh}Z^{1/2}_W Z^{1/2}_{Z\gamma}\\
& && +g_{G^+HW\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{Hh}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+HW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{Hh}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+HWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{Hh}Z^{1/2}_W Z^{1/2}_{Z\gamma}\\
& && +g_{H^+hWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{hh}Z^{1/2}_W Z^{1/2}_{Z\gamma}+\delta g_{H^+hW\gamma}\,,\\
&G^\pm hW^\mp_\mu\gamma_\nu \quad && g_{G^+hW\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{hh}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+hW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{hh}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{G^+hWZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{hh}Z^{1/2}_W Z^{1/2}_{Z\gamma}\\
& && +g_{G^+HWZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{Hh}Z^{1/2}_W Z^{1/2}_{Z\gamma}\\
& && +g_{G^+HW\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{Hh}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+HW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{Hh}Z^{1/2}_W Z^{1/2}_{\gamma\gamma}+g_{H^+HWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{Hh}Z^{1/2}_W Z^{1/2}_{Z\gamma}\\
& && +g_{H^+hWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{hh}Z^{1/2}_W Z^{1/2}_{Z\gamma}+\delta g_{G^+hW\gamma}\,,
\end{aligned}
$$

$$\begin{aligned} G^{\pm}hW^{\mp}_{\mu}Z_{\nu} \quad & g_{G^+hWZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{hh}Z^{1/2}_{W}Z^{1/2}_{ZZ}+g_{H^+hW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{hh}Z^{1/2}_{W}Z^{1/2}_{\gamma Z}+g_{G^+hW\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{hh}Z^{1/2}_{W}Z^{1/2}_{\gamma Z} \\ & +g_{G^+HWZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{Hh}Z^{1/2}_{W}Z^{1/2}_{ZZ} \\ & +g_{G^+HW\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{Hh}Z^{1/2}_{W}Z^{1/2}_{\gamma Z}+g_{H^+HWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{Hh}Z^{1/2}_{W}Z^{1/2}_{ZZ} \\ & +g_{H^+hWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{hh}Z^{1/2}_{W}Z^{1/2}_{ZZ}+g_{H^+HW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{Hh}Z^{1/2}_{W}Z^{1/2}_{\gamma Z}+\delta g_{G^+hWZ}, \end{aligned}$$

$$\begin{aligned} G^{\pm}HW^{\mp}_{\mu}Z_{\nu} \quad & g_{G^+HWZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{ZZ}+g_{H^+hW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{\gamma Z}+g_{G^+hW\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{\gamma Z} \\ & +g_{G^+hWZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{ZZ} \\ & +g_{G^+HW\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{\gamma Z}+g_{H^+HW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{\gamma Z} \\ & +g_{H^+HWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{ZZ}+g_{H^+hWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{ZZ}+\delta g_{G^+HWZ}, \end{aligned}$$

$$\begin{aligned} G^{\pm}HW^{\mp}_{\mu}\gamma_{\nu} \quad & g_{G^+HW\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{\gamma\gamma}+g_{H^+hW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{\gamma\gamma}+g_{G^+hW\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{\gamma\gamma} \\ & +g_{G^+hWZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{Z\gamma}+g_{G^+HWZ}Z^{1/2}_{G^+G^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{Z\gamma}+g_{H^+HW\gamma}Z^{1/2}_{H^+G^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{\gamma\gamma} \\ & +g_{H^+HWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{Z\gamma}+g_{H^+hWZ}Z^{1/2}_{H^+G^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{Z\gamma}+\delta g_{G^+HW\gamma}, \end{aligned}$$

$$\begin{aligned} H^{\pm}HW^{\mp}_{\mu}\gamma_{\nu} \quad & g_{H^+HW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{\gamma\gamma}+g_{H^+hW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{\gamma\gamma}+g_{G^+hW\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{\gamma\gamma} \\ & +g_{G^+hWZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{Z\gamma} \\ & +g_{G^+HWZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{Z\gamma}+g_{G^+HW\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{\gamma\gamma}+g_{H^+HWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{Z\gamma} \\ & +g_{H^+hWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{Z\gamma}+\delta g_{H^+HW\gamma}, \end{aligned}$$

$$\begin{aligned} H^{\pm}HW^{\mp}_{\mu}Z_{\nu} \quad & g_{H^+HWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{ZZ}+g_{H^+hW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{\gamma Z}+g_{G^+hW\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{\gamma Z} \\ & +g_{G^+hWZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{ZZ} \\ & +g_{G^+HWZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{ZZ}+g_{G^+HW\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{\gamma Z}+g_{H^+HW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{HH}Z^{1/2}_{W}Z^{1/2}_{\gamma Z} \\ & +g_{H^+hWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{hH}Z^{1/2}_{W}Z^{1/2}_{ZZ}+\delta g_{H^+HWZ}, \end{aligned}$$

$$\begin{aligned} H^{\pm}hW^{\mp}_{\mu}Z_{\nu} \quad & g_{H^+hWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{hh}Z^{1/2}_{W}Z^{1/2}_{ZZ}+g_{H^+hW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{hh}Z^{1/2}_{W}Z^{1/2}_{\gamma Z}+g_{G^+hW\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{hh}Z^{1/2}_{W}Z^{1/2}_{\gamma Z} \\ & +g_{G^+hWZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{hh}Z^{1/2}_{W}Z^{1/2}_{ZZ} \\ & +g_{G^+HWZ}Z^{1/2}_{G^+H^-}Z^{1/2}_{Hh}Z^{1/2}_{W}Z^{1/2}_{ZZ}+g_{G^+HW\gamma}Z^{1/2}_{G^+H^-}Z^{1/2}_{Hh}Z^{1/2}_{W}Z^{1/2}_{\gamma Z} \\ & +g_{H^+HW\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{Hh}Z^{1/2}_{W}Z^{1/2}_{\gamma Z}+g_{H^+HWZ}Z^{1/2}_{H^+H^-}Z^{1/2}_{Hh}Z^{1/2}_{W}Z^{1/2}_{ZZ}+\delta g_{H^+hWZ}. \end{aligned}$$

c. 2 scalar + 1 gauge

$$H^+H^-\gamma_{\mu} \quad (p_{H^+}-p_{H^-})_{\mu}[g_{H^+H^-\gamma}Z_{H^+H^-}Z^{1/2}_{\gamma\gamma}+g_{H^+H^-Z}Z_{H^+H^-}Z^{1/2}_{Z\gamma}+g_{G^+G^-\gamma}Z_{G^+H^-}Z^{1/2}_{\gamma\gamma}+g_{G^+G^-Z}Z_{G^+H^-}Z^{1/2}_{Z\gamma}+\delta g_{H^+H^-\gamma}],$$

$$H^+H^-Z_{\mu} \quad (p_{H^+}-p_{H^-})_{\mu}[g_{H^+H^-Z}Z_{H^+H^-}Z^{1/2}_{ZZ}+g_{H^+H^-\gamma}Z_{H^+H^-}Z^{1/2}_{\gamma Z}+g_{G^+G^-\gamma}Z_{G^+H^-}Z^{1/2}_{\gamma Z}+g_{G^+G^-Z}Z_{G^+H^-}Z^{1/2}_{ZZ}+\delta g_{H^+H^-Z}],$$

$$G^+G^-\gamma_{\mu} \quad (p_{G^+}-p_{G^-})_{\mu}[g_{G^+G^-\gamma}Z_{G^+G^-}Z^{1/2}_{\gamma\gamma}+g_{H^+H^-Z}Z_{H^+G^-}Z^{1/2}_{Z\gamma}+g_{H^+H^-\gamma}Z_{H^+G^-}Z^{1/2}_{\gamma\gamma}+g_{G^+G^-Z}Z_{G^+G^-}Z^{1/2}_{Z\gamma}+\delta g_{G^+G^-\gamma}],$$

$$G^+G^-Z_{\mu} \quad (p_{G^+}-p_{G^-})_{\mu}[g_{G^+G^-Z}Z_{G^+G^-}Z^{1/2}_{ZZ}+g_{H^+H^-Z}Z_{H^+G^-}Z^{1/2}_{ZZ}+g_{H^+H^-\gamma}Z_{H^+G^-}Z^{1/2}_{\gamma Z}+g_{G^+G^-\gamma}Z_{G^+G^-}Z^{1/2}_{\gamma Z}+\delta g_{G^+G^-Z}],$$

$$\begin{aligned} H^{\pm}G^{\mp}\gamma_{\mu} \quad & \pm(p_{H^{\pm}}-p_{G^{\mp}})_{\mu}[g_{H^+H^-\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{H^+G^-}Z^{1/2}_{\gamma\gamma}+g_{H^+H^-Z}Z^{1/2}_{H^+H^-}Z^{1/2}_{H^+G^-}Z^{1/2}_{Z\gamma}+g_{G^+G^-\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{G^+H^-}Z^{1/2}_{\gamma\gamma} \\ & +g_{G^+G^-Z}Z^{1/2}_{G^+G^-}Z^{1/2}_{G^+H^-}Z^{1/2}_{Z\gamma}], \end{aligned}$$

$$H^{\pm}G^{\mp}Z_{\mu} \quad \pm(p_{H^{\pm}}-p_{G^{\mp}})_{\mu}[g_{H^+H^-\gamma}Z^{1/2}_{H^+H^-}Z^{1/2}_{H^+G^-}Z^{1/2}_{\gamma Z}+g_{H^+H^-Z}Z^{1/2}_{H^+H^-}Z^{1/2}_{H^+G^-}Z^{1/2}_{ZZ}+g_{G^+G^-\gamma}Z^{1/2}_{G^+G^-}Z^{1/2}_{G^+H^-}Z^{1/2}_{\gamma Z}$$
$$+g_{G^+G^-Z}Z^{1/2}_{G^+G^-}Z^{1/2}_{G^+H^-}Z^{1/2}_{ZZ}],$$

$$HG_0Z_{\mu} \quad (p_H-p_{G_0})_{\mu}Z^{1/2}_{ZZ}[g_{HG_0Z}Z^{1/2}_{HH}Z^{1/2}_{G_0G_0}+g_{HAZ}Z^{1/2}_{HH}Z^{1/2}_{AG_0}+g_{hAZ}Z^{1/2}_{hH}Z^{1/2}_{AG_0}+g_{hG_0Z}Z^{1/2}_{hH}Z^{1/2}_{G_0G_0}+\delta g_{HG_0Z}],$$

$$HAZ_{\mu} \quad (p_H-p_A)_{\mu}Z^{1/2}_{ZZ}[g_{HAZ}Z^{1/2}_{HH}Z^{1/2}_{AA}+g_{HG_0Z}Z^{1/2}_{HH}Z^{1/2}_{G_0A}+g_{hAZ}Z^{1/2}_{hH}Z^{1/2}_{AA}+g_{hG_0Z}Z^{1/2}_{hH}Z^{1/2}_{G_0A}+\delta g_{HAZ}],$$

$$hAZ_{\mu} \quad (p_h-p_A)_{\mu}Z^{1/2}_{ZZ}[g_{hAZ}Z^{1/2}_{hh}Z^{1/2}_{AA}+g_{HG_0Z}Z^{1/2}_{Hh}Z^{1/2}_{G_0A}+g_{HAZ}Z^{1/2}_{Hh}Z^{1/2}_{AA}+g_{hG_0Z}Z^{1/2}_{hh}Z^{1/2}_{G_0A}+\delta g_{hAZ}],$$

$$hG_0Z_{\mu} \quad (p_h-p_{G_0})_{\mu}Z^{1/2}_{ZZ}[g_{hG_0Z}Z^{1/2}_{hh}Z^{1/2}_{G_0G_0}+g_{HG_0Z}Z^{1/2}_{Hh}Z^{1/2}_{G_0G_0}+g_{HAZ}Z^{1/2}_{Hh}Z^{1/2}_{AG_0}+g_{hAZ}Z^{1/2}_{hh}Z^{1/2}_{AG_0}+\delta g_{hG_0Z}],$$

$$HG_0\gamma_{\mu} \quad (p_H-p_{G_0})_{\mu}Z^{1/2}_{Z\gamma}[g_{HG_0Z}Z^{1/2}_{HH}Z^{1/2}_{G_0G_0}+g_{HAZ}Z^{1/2}_{HH}Z^{1/2}_{AG_0}+g_{hAZ}Z^{1/2}_{hH}Z^{1/2}_{AG_0}+g_{hG_0Z}Z^{1/2}_{hH}Z^{1/2}_{G_0G_0}],$$

$$HA\gamma_{\mu} \quad (p_H-p_A)_{\mu}Z^{1/2}_{Z\gamma}[g_{HAZ}Z^{1/2}_{HH}Z^{1/2}_{AA}+g_{HG_0Z}Z^{1/2}_{HH}Z^{1/2}_{G_0A}+g_{hAZ}Z^{1/2}_{hH}Z^{1/2}_{AA}+g_{hG_0Z}Z^{1/2}_{hH}Z^{1/2}_{G_0A}],$$

$$hA\gamma_{\mu} \quad (p_h-p_A)_{\mu}Z^{1/2}_{Z\gamma}[g_{hAZ}Z^{1/2}_{hh}Z^{1/2}_{AA}+g_{HG_0Z}Z^{1/2}_{Hh}Z^{1/2}_{G_0A}+g_{HAZ}Z^{1/2}_{Hh}Z^{1/2}_{AA}+g_{hG_0Z}Z^{1/2}_{hh}Z^{1/2}_{G_0A}],$$

$$hG_0\gamma_{\mu} \quad (p_h-p_{G_0})_{\mu}Z^{1/2}_{Z\gamma}[g_{hG_0Z}Z^{1/2}_{hh}Z^{1/2}_{G_0G_0}+g_{HG_0Z}Z^{1/2}_{Hh}Z^{1/2}_{G_0G_0}+g_{HAZ}Z^{1/2}_{Hh}Z^{1/2}_{AG_0}+g_{hAZ}Z^{1/2}_{hh}Z^{1/2}_{AG_0}],$$

$$H^{\pm}AW^{\mp}_{\mu} \quad (p_{H^{\pm}}-p_A)_{\mu}Z^{1/2}_{W}[g_{H^+AW}Z^{1/2}_{H^+H^-}Z^{1/2}_{AA}+g_{G^+G_0W}Z^{1/2}_{G^+H^-}Z^{1/2}_{G_0A}+\delta g_{H^+AW}],$$

$$G^{\pm}G_0W^{\mp}_{\mu} \quad (p_{G^{\pm}}-p_{G_0})_{\mu}Z^{1/2}_{W}[g_{G^+G_0W}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0G_0}+g_{H^+AW}Z^{1/2}_{H^+G^-}Z^{1/2}_{AG_0}+\delta g_{G^+G_0W}],$$

$$H^{\pm}G_0W^{\mp}_{\mu} \quad (p_{H^{\pm}}-p_{G_0})_{\mu}Z^{1/2}_{W}[g_{H^+AW}Z^{1/2}_{H^+H^-}Z^{1/2}_{AG_0}+g_{G^+G_0W}Z^{1/2}_{G^+H^-}Z^{1/2}_{G_0G_0}],$$

$$G^{\pm}AW^{\mp}_{\mu} \quad (p_{G^{\pm}}-p_A)_{\mu}Z^{1/2}_{W}[g_{G^+G_0W}Z^{1/2}_{G^+G^-}Z^{1/2}_{G_0A}+g_{H^+AW}Z^{1/2}_{H^+G^-}Z^{1/2}_{AA}],$$

$$H^{\pm}hW^{\mp}_{\mu} \quad (p_{H^+}-p_h)_{\mu}Z^{1/2}_{W}[g_{H^+hW}Z^{1/2}_{H^+H^-}Z^{1/2}_{hh}+g_{G^+hW}Z^{1/2}_{G^+H^-}Z^{1/2}_{hh}+g_{G^+HW}Z^{1/2}_{G^+H^-}Z^{1/2}_{Hh}+g_{H^+HW}Z^{1/2}_{H^+H^-}Z^{1/2}_{Hh}+\delta g_{H^+hW}],$$

$$G^{\pm}hW^{\mp}_{\mu} \quad (p_{G^+}-p_h)_{\mu}Z^{1/2}_{W}[g_{G^+hW}Z^{1/2}_{G^+G^-}Z^{1/2}_{hh}+g_{H^+hW}Z^{1/2}_{H^+G^-}Z^{1/2}_{hh}+g_{G^+HW}Z^{1/2}_{G^+G^-}Z^{1/2}_{Hh}+g_{H^+HW}Z^{1/2}_{H^+G^-}Z^{1/2}_{Hh}+\delta g_{G^+hW}],$$

$$G^{\pm}HW^{\mp}_{\mu} \quad (p_{G^+}-p_H)_{\mu}Z^{1/2}_{W}[g_{G^+HW}Z^{1/2}_{G^+G^-}Z^{1/2}_{HH}+g_{H^+hW}Z^{1/2}_{H^+G^-}Z^{1/2}_{hH}+g_{G^+hW}Z^{1/2}_{G^+G^-}Z^{1/2}_{hH}+g_{H^+HW}Z^{1/2}_{H^+G^-}Z^{1/2}_{HH}+\delta g_{G^+HW}],$$

$$H^{\pm}HW^{\mp}_{\mu} \quad (p_{H^+}-p_H)_{\mu}Z^{1/2}_{W}[g_{H^+HW}Z^{1/2}_{H^+H^-}Z^{1/2}_{HH}+g_{G^+hW}Z^{1/2}_{G^+H^-}Z^{1/2}_{hH}+g_{G^+HW}Z^{1/2}_{G^+H^-}Z^{1/2}_{HH}+g_{H^+hW}Z^{1/2}_{H^+H^-}Z^{1/2}_{hH}+\delta g_{H^+HW}].$$

d. 1 scalar + 2 fermions

In this section we present the counterterms for the scalar-fermion interactions. For the interactions with the neutral particles, ψ_i will stand for up and down quarks, and charged leptons. For the interactions with the charged scalar particles we will use uppercase letters for fermions with $I_3=-1/2$ and lowercase for $I_3=1/2$. To simplify the form of the counterterms $[g_{ijk}]_L$ will stand for the left part of the coupling (proportional to γ_L) and $[g_{ijk}]_R$ will stand for the right part of the same coupling. For the leptons, one of the couplings has to be set to zero by the reader. We also define the quantities

$$Z^{1/2}_{\psi}=Z^{1/2}_{L}\gamma_L+Z^{1/2}_{R}\gamma_R, \tag{B33}$$

$$Z^{1/2}_{\bar{\psi}}=[Z^{\dagger}_{L}]^{1/2}\gamma_R+[Z^{\dagger}_{R}]^{1/2}\gamma_L. \tag{B34}$$

The counterterms are

$$\overline{\psi_k}\psi_l h \quad g_{\overline{\psi_i}\psi_i h}[Z^{1/2}_{\bar{\psi}}]_{ki}[Z^{1/2}_{\psi}]_{il}Z^{1/2}_{hh}+g_{\overline{\psi_i}\psi_i H}[Z^{1/2}_{\bar{\psi}}]_{ki}[Z^{1/2}_{\psi}]_{il}Z^{1/2}_{Hh}+\delta g_{\overline{\psi_i}\psi_i h},$$

$$\overline{\psi_k}\psi_l H \quad g_{\overline{\psi_i}\psi_i H}[Z^{1/2}_{\bar{\psi}}]_{ki}[Z^{1/2}_{\psi}]_{il}Z^{1/2}_{HH}+g_{\overline{\psi_i}\psi_i h}[Z^{1/2}_{\bar{\psi}}]_{ki}[Z^{1/2}_{\psi}]_{il}Z^{1/2}_{hH}+\delta g_{\overline{\psi_i}\psi_i H},$$

$$\overline{\psi_k}\psi_l \quad A g_{\overline{\psi_i}\psi_i A}[Z^{1/2}_{\bar{\psi}}]_{ki}[Z^{1/2}_{\psi}]_{il}Z^{1/2}_{AA}+g_{\overline{\psi_i}\psi_i G_0}[Z^{1/2}_{\bar{\psi}}]_{ki}[Z^{1/2}_{\psi}]_{il}Z^{1/2}_{G_0A}+\delta g_{\overline{\psi_i}\psi_i A},$$

$$\overline{\psi_k}\psi_l G_0 \quad g_{\overline{\psi_i}\psi_i G_0}[Z_{\psi}^{1/2}]_{ki}[Z_{\psi}^{1/2}]_{il}Z_{G_0G_0}^{1/2}+g_{\overline{\psi_i}\psi_i A}[Z_{\psi}^{1/2}]_{ki}[Z_{\psi}^{1/2}]_{il}Z_{AG_0}^{1/2}+\delta g_{\overline{\psi_i}\psi_i G_0},$$

$$\overline{\psi_I}\psi_i H^- \quad [Z_R^{\dagger}]_{IJ}^{1/2}[Z_L]_{ji}^{1/2}\{[g_{\overline{\psi_J}\psi_j H^-}]_L Z_{H^+H^-}^{1/2}+[g_{\overline{\psi_J}\psi_j G^-}]_L Z_{G^+H^-}^{1/2}\}+[Z_L^{\dagger}]_{IJ}^{1/2}[Z_R]_{ji}^{1/2}\{[g_{\overline{\psi_J}\psi_j H^-}]_R Z_{H^+H^-}^{1/2}+[g_{\overline{\psi_J}\psi_j G^-}]_R Z_{G^+H^-}^{1/2}\}$$
$$+\delta g_{\overline{\psi_J}\psi_j H^-},$$

$$\overline{\psi_i}\psi_I H^+ \quad [Z_R^{\dagger}]_{ij}^{1/2}[Z_L]_{JI}^{1/2}\{[g_{\overline{\psi_j}\psi_J H^+}]_L Z_{H^+H^-}^{1/2}+[g_{\overline{\psi_j}\psi_J G^+}]_L Z_{G^+H^-}^{1/2}\}+[Z_L^{\dagger}]_{ij}^{1/2}[Z_R]_{JI}^{1/2}\{[g_{\overline{\psi_j}\psi_J H^+}]_R Z_{H^+H^-}^{1/2}+[g_{\overline{\psi_j}\psi_J G^+}]_R Z_{G^+H^-}^{1/2}\}$$
$$+\delta g_{\overline{\psi_j}\psi_J H^+},$$

$$\overline{\psi_I}\psi_i G^- \quad [Z_R^{\dagger}]_{IJ}^{1/2}[Z_L]_{ji}^{1/2}\{[g_{\overline{\psi_J}\psi_j G^-}]_L Z_{G^+G^-}^{1/2}+[g_{\overline{\psi_J}\psi_j H^-}]_L Z_{H^+G^-}^{1/2}\}+[Z_L^{\dagger}]_{IJ}^{1/2}[Z_R]_{ji}^{1/2}\{[g_{\overline{\psi_J}\psi_j G^-}]_R Z_{G^+G^-}^{1/2}+[g_{\overline{\psi_J}\psi_j H^-}]_R Z_{H^+G^-}^{1/2}\}$$
$$+\delta g_{\overline{\psi_J}\psi_j G^-},$$

$$\overline{\psi_i}\psi_I G^+ \quad [Z_R^{\dagger}]_{ij}^{1/2}[Z_L]_{JI}^{1/2}\{[g_{\overline{\psi_j}\psi_J G^+}]_L Z_{G^+G^-}^{1/2}+[g_{\overline{\psi_j}\psi_J H^+}]_L Z_{H^+G^-}^{1/2}\}+[Z_L^{\dagger}]_{ij}^{1/2}[Z_R]_{JI}^{1/2}\{[g_{\overline{\psi_j}\psi_J G^+}]_R Z_{G^+G^-}^{1/2}+[g_{\overline{\psi_j}\psi_J H^+}]_R Z_{H^+G^-}^{1/2}\}$$
$$+\delta g_{\overline{\psi_j}\psi_J G^+}.$$

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