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Flavour-violating charged lepton decays in seesaw-type models

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Abstract

Analytic expressions of lepton-flavour- and lepton-number-violating decays of charged leptons are derived in the context of general $SU(2)_L \otimes U(1)_Y$ seesaw scenarios that are motivated by grand unified theories (GUT's) or superstring models, in which left-handed and/or right-handed neutral singlets are present. Possible constraints imposed by cosmology and low-energy data are briefly discussed. The violation of the decoupling theorem in flavour-dependent graphs due to the presence of heavy neutral leptons of Dirac or Majorana nature is emphasized. Numerical estimates reveal that the decays $\tau^- \to e^- e^- e^+$ or $\tau^- \to e^- \mu^- \mu^+$ can be as large as $\sim 10^{-6}$, which may be observed in LEP experiments or other τ factories.

1. Introduction

The quest for an understanding of the problem of smallness in mass or masslessness of the light known neutrinos, ν_e, ν_μ, and ν_τ, has relied on interesting solutions in the context of extended gauge structures of the minimal Standard Model (SM), such as grand unified theories, e.g. SO(10) models [1], or superstring models with an E_6 symmetry [2]. Among the various solutions, the most attractive one, known as the seesaw mechanism, has been conceived by the authors in Ref. [3] within the framework of SO(10) or left-right symmetric models. In these theories, right-handed neutrinos are

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introduced with the simultaneous inclusion of Majorana masses that violate the lepton-number (L) by $\Delta L = 2$ operators in the Yukawa sector. The neutrino-mass spectrum of a simple seesaw model with one generation of quarks and leptons consists of two massive Majorana neutrinos, ν and N, having masses $m_\nu \simeq m_D^2/m_M$ and $m_N \simeq m_M$. If the Dirac mass term m_D is of the order of a typical charged-lepton or quark mass, as dictated by GUT relations [4], and the Majorana-mass scale m_M is sufficiently large, one can then obtain a very light neutrino ν. The general situation of an interfamily seesaw-type model with a number n_G of weak isodoublets and a number n_R of right-handed neutrinos is more involved [5] and will be discussed in Section 2.

If nature keeps to the pathway of a seesaw-type solution, then heavy Majorana neutrinos at the mass scale of TeV may manifest themselves in L-violating processes at high-energy ee [6,7], ep [8], and pp colliders [9,10], in possible lepton-flavour-violating decays of the Z [11] and Higgs particles (H) [12] or through universality-breaking effects in leptonic diagonal Z-boson decays [13]. Their existence may also influence [14,15] the size of electroweak oblique parameters [16,17], tri-gauge boson WWZ- and ZZZ-couplings [18], or specific Higgs observables considered recently [19,20]. Finally, there are many other places scanned by exhaustive combined analyses of charged-current-universality effects in leptonic π decays, neutral-current interactions in neutrino-nucleon scatterings, τ-polarization asymmetries, neutrino-counting experiments at the CERN Large Electron Positron Collider (LEP), etc. [21,22], in which Majorana neutrinos could also manifest their presence.

Another possible solution of the neutrino-mass problem has been contemplated in the framework of heterotic superstring models [2] or certain scenarios of SO(10) models [23]. The low-energy limit of such theories extend the SM field content by adding new left-handed and right-handed neutral isosinglets, and assuming the absence of $\Delta L = 2$ operators in the Yukawa sector. In a simple one-generation scenario, one obtains three Weyl fermions from which one of them is completely massless to all orders of perturbation theory [24] and the other two are degenerate in mass and thus form a heavy Dirac neutrino which has a mass of the order of the isosinglet Dirac mass M. The Dirac mass M simply connects the right-handed and left-handed chiral singlets in the Yukawa sector. This solution is particularly preferable if the light known neutrinos turn out to be strictly massless. The model could straightforwardly be extended to n_G generations without qualitatively changing its features regarding neutrino masses. In an n_G-generation model, one generally obtains n_G massless neutrinos and n_G heavy Dirac neutral fermions [24,25]. This minimal model is invariant under the gauge group $SU(2)_L \otimes U(1)_Y$ and possesses many attractive features that might be summarized in Ref. [26]. For example, even if the total lepton number is conserved, the model does generally violate the separate leptonic quantum numbers and can hence account for possible L- and/or CP-violating signals at the Z peak [24] or in other high-energy processes [27].

In this paper we carefully study the three-body decays of a charged lepton, l, into other three charged leptons, which we denote hereafter as l', l_1, and $\bar{l}_2$. After detailed calculations, we find that the decay amplitude of $l \to l' l_1 \bar{l}_2$ depends quadratically on the mass of the heavy Dirac or Majorana neutrino, which violates explicitly the decoupling theorem [28]. Such a nondecoupling behaviour has recently been observed to take place in *three-generation* seesaw-type models, where the effective couplings Hll' [12] and Zll' [11,13] show a strong quadratic dependence of the heavy neutrino mass. In past, similar flavour-dependent nondecoupling effects have been found in the one-loop amplitude of the decays $Z \to b\bar{s}$ [29] and $Z \to b\bar{b}$ [30,31], where the top quark plays the rôle of heavy neutrinos.

Among the various decay processes, we find numerically that the decays, $\tau^- \to e^-e^-e^+$ and $\tau^- \to e^-\mu^-\mu^+$ [or complementary the decays $\tau^- \to \mu^-\mu^-\mu^+$ and $\tau^- \to \mu^-e^-e^+$], have the biggest opportunity to be detected at the present or future LEP data. Furthermore, we analyze the effect of genuine Majorana-neutrino contributions to these decays.

The present work is organized as follows: In Section 2 we give a brief description of the basic low-energy structure of the seesaw-type models mentioned above. In Section 3 we discuss general constraints that should be imposed on these models. Analytically, Section 3.1 considers possible constraints based on the assumption that the model should generate a sufficiently large lepton asymmetry via the out-of-equilibrium L-violating decays of a heavy Majorana neutrino which can be converted later on into the observed baryon-number (B) asymmetry in the universe (BAU) due to the sphaleron interactions. In Section 3.2 stringent constraints of possible non-SM mixings are derived by a global analysis of all existing low-energy data. Also, bounds that may be obtained by the non-observation of leptonic non-diagonal Z-boson decays at LEP are discussed in Section 3.3. In Section 4 we analytically calculate the branching ratios of the photonic decays of a lepton (l), $l \to l'\gamma$, and the three-body decay modes of the type $l \to l'l_1\bar{l}_2$ in the context of the models discussed in Section 2. Numerical predictions and discussion of these lepton-flavour-violating decays are summarized in Section 5. We draw our conclusions in Section 6.

2. Theoretical models

In this section we will give a short description of the basic low-energy structure of the two most popular extensions of the SM that can naturally account for very light or strictly massless neutrinos. The field content of these models, which could also be motivated by heterotic superstring models [2] or certain SO(10) GUTs [3,23], is free of anomalies [26]. These two scenarios are: (i) the interfamily seesaw-type model realized in the SM with right-handed neutrinos [3,8,5] and (ii) the SM with left-handed and right-handed neutral singlets [2,24,25].

(i) *The SM with right-handed neutrinos.* In general, the interfamily seesaw-type model, being invariant under the SM gauge group, represents one of the most natural framework to predict heavy Majorana neutrinos. Such a model is obtained by introducing a number $n_{\rm R}$ of right-handed neutrinos, ν^0_{Ri}, in the SM (in addition to $n_{\rm G}$ left-handed ones ν^0_{Li}) and allowing simultaneously the presence of $\Delta L = 2$ operators. The Yukawa sector containing the neutrino masses is then written down as

$$-\mathcal{L}^{\nu}_{Y} = \frac{1}{2}(\bar{\nu}^0_{\rm L},\ \bar{\nu}^{0C}_{\rm R})\ M^{\nu} \begin{pmatrix} \nu^{0C}_{\rm L} \\ \nu^0_{\rm R} \end{pmatrix} + \text{h.c.}, \tag{2.1}$$

where the $(n_G + n_R) \times (n_G + n_R)$-dimensional neutrino-mass matrix

$$M^\nu = \begin{pmatrix} 0 & m_D \\ m_D^T & m_M \end{pmatrix}. \tag{2.2}$$

The matrix M^ν can always be diagonalized by a unitary matrix U^ν of the same dimensionality with the neutrino-mass matrix (i.e. $U^{\nu T} M^\nu U^\nu = \hat{M}^\nu$). One then gets $n_G + n_R$ physical Majorana neutrinos n_i through the unitary transformations

$$\begin{pmatrix} \nu_L^0 \\ \nu_R^{0C} \end{pmatrix}_i = \sum_{j=1}^{n_G+n_R} U_{ij}^{\nu *}\, n_{Lj}\,, \qquad \begin{pmatrix} \nu_L^{0C} \\ \nu_R^0 \end{pmatrix}_i = \sum_{j=1}^{n_G+n_R} U_{ij}^{\nu}\, n_{Rj}. \tag{2.3}$$

The first n_G neutral states, ν_i ($\equiv n_i$ for $i = 1, \dots, n_G$), are identified with the known n_G light neutrinos (i.e. $n_G = 3$), while the remaining n_R mass eigenstates, N_j ($\equiv n_{j+n_G}$ for $j = 1, \dots, n_R$), represent heavy Majorana neutrinos which are novel particles predicted by the model. The quark sector of such an extension can completely be described by the SM.

It is important to notice that the general matrix M^ν of Eq. (2.2) takes the known seesaw form [3] in case $m_M \gg m_D$. Nevertheless, this hierarchical scheme can drastically be relaxed in a two-family seesaw-type model without contradicting experimental upper bounds on light-neutrino masses [5,7,8]. The light-heavy neutrino mixings of such scenarios, $s_L^{\nu_l}$, can, in principle, be scaled as $s_L^{\nu_l} \sim m_D/m_M$ rather than $s_L^{\nu_l} \sim \sqrt{m_{\nu_l}/m_N}$ as usually derived in a one-family seesaw scenario [3,32]. In other words, high Dirac mass terms are allowed to be present in M^ν and only the ratio m_D/m_M ($\sim s_L^{\nu_l}$) gets limited by a global analysis of low-energy and LEP observables. The latter advocates our treatment of originally considering the mixings $s_L^{\nu_l}$ and heavy neutrinos masses m_{N_i} as free phenomenological parameters, being subject later on to the constraints that will be discussed in Section 3.

Adopting the conventions of Ref. [5], the interactions of the Majorana neutrinos, n_i, and charged leptons, l_i, with the gauge bosons, $W^\pm$ and Z, and the unphysical Goldstone bosons, $G^\pm$ and G^0 (in the Feynman–'t Hooft gauge), are correspondingly obtained by the Lagrangians

$$\mathcal{L}_{\rm int}^{W} = -\frac{g_w}{\sqrt{2}} W^{-\mu} \sum_{i=1}^{n_G} \sum_{j=1}^{n_G+n_R} B_{l_i j}\, \bar{l}_i \gamma_\mu P_L\, n_j + \text{h.c.}, \tag{2.4}$$

$$\mathcal{L}_{\rm int}^{Z} = -\frac{g_w}{4c_w} Z^\mu \sum_{i,j=1}^{n_G+n_R} \bar{n}_i \gamma_\mu \left[i\,{\rm Im}\, C_{ij} - \gamma_5 {\rm Re}\, C_{ij} \right] n_j\,, \tag{2.5}$$

$$\mathcal{L}_{\rm int}^{G^\mp} = -\frac{g_w}{\sqrt{2} M_W}\, G^- \sum_{i=1}^{n_G} \sum_{j=1}^{n_G+n_R} B_{l_i j}\, \bar{l}_i \left[m_{l_i} P_L - m_j P_R \right] n_j + \text{h.c.}, \tag{2.6}$$

$$\mathcal{L}_{\rm int}^{G^0} = \frac{i g_w}{4 M_W}\, G^0 \sum_{i,j=1}^{n_G+n_R} \bar{n}_i \left[\gamma_5 (m_i + m_j) {\rm Re}\, C_{ij} + \ i (m_j - m_i) {\rm Im}\, C_{ij} \right] n_j. \tag{2.7}$$

where g_w is the weak coupling constant, $c_w^2 = 1 - s_w^2 = M_W^2/M_Z^2$, $P_L(P_R) = (1 + (-)\gamma_5)/2$, and m_i denotes all the physical neutrino masses. In Eqs. (2.4)-(2.7), B and C are $n_G \times (n_R + n_G)$- and $(n_G + n_R) \times (n_R + n_G)$-dimensional matrices, respectively, which are defined as

$$B_{l_i j} = \sum_{k=1}^{n_G} V^l_{l_i k} U^{\nu *}_{kj}\,, \qquad C_{ij} = \sum_{k=1}^{n_G} U^{\nu}_{ki} U^{\nu *}_{kj}\,, \tag{2.8}$$

where V^l is the leptonic Cabbibo-Kobayashi-Maskawa (CKM) matrix.

Note that the flavour-mixing matrices B and C satisfy a number of identities, which are derived just by using the information of $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ invariance of $\mathcal{L}_Y^\nu$. These identities, which are forced by the renormalizability of the interfamily seesaw-type model, can be summarized as [12,5]

$$\sum_{k=1}^{n_G+n_R} B_{l_1 k} B^*_{l_2 k} = \delta_{l_1 l_2}, \quad \sum_{k=1}^{n_G+n_R} C_{ik} C^*_{jk} = C_{ij}, \quad \sum_{k=1}^{n_G+n_R} B_{lk} C_{ki} = B_{li}, \quad \sum_{k=1}^{n_G} B^*_{l_k i} B_{l_k j} = C_{ij}, \tag{2.9}$$

$$\sum_{k=1}^{n_G+n_R} m_k C_{ik} C_{jk} = 0, \quad \sum_{k=1}^{n_G+n_R} m_k B_{lk} C^*_{ki} = 0, \quad \sum_{k=1}^{n_G+n_R} m_k B_{l_1 k} B_{l_2 k} = 0. \tag{2.10}$$

Consequently, our theoretical analysis should be regarded to be independent of the weak-basis structure of possible neutrino-mass-matrix ansätze [33]. It is now instructive to re-express the Z-boson coupling to the Majorana neutrinos, n_i, as follows:

$$\mathcal{L}^Z_{\mathrm{int}} = -\frac{g_w}{4c_w} Z^\mu \sum_{i,j=1}^{n_G+n_R} \bar{n}_i \gamma_\mu \left[C_{ij} P_L - C^*_{ij} P_R\right] n_j. \tag{2.11}$$

One can thus remark that the coupling $Z n_i n_j$ is generally flavour non-diagonal and has both chiralities in this minimal model.

(ii) *The SM with left-handed and right-handed neutral singlets.* Another attractive scenario predicting for the light neutrinos to be strictly massless serves an extension of the SM, in which left-handed neutral singlets, S_{Li}, in addition to the right-handed neutrinos, $\nu^0_{R_i}$, are introduced. Furthermore, we assume that $\Delta L = 2$ interactions are absent from the model, and the number of right-handed neutrinos, n_R, equals the number of the singlet fields S_{Li}. After the spontaneous break-down of the SM gauge symmetry, the Yukawa sector relevant for the neutrino masses reads [2,23]

$$-\mathcal{L}_Y^\nu = \frac{1}{2}(\bar{\nu}^0_L,\ \bar{\nu}^{0C}_R,\ \bar{S}_L)\ \mathcal{M}^\nu \begin{pmatrix} \nu^{0C}_L \\ \nu^0_R \\ S^C_L \end{pmatrix} + \text{h.c.}, \tag{2.12}$$

where the $(n_G + 2n_R) \times (n_G + 2n_R)$ neutrino-mass matrix is given by

$$\mathcal{M}^\nu = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M^T \\ 0 & M & 0 \end{pmatrix}. \tag{2.13}$$

It is easy to see that the neutrino matrix $\mathcal{M}^\nu$ conserves the total lepton number L by assigning the lepton numbers to the neutrino fields: $L(\nu_L^0) = L(\nu_R^0) = L(S_L) = 1$. Since the rank of the neutrino mass matrix in Eq. (2.13) is $2n_R$, $\mathcal{M}^\nu$ has n_G zero eigenvalues. These n_G massless eigenstates should clearly describe the ordinary light neutrinos, ν_e, ν_μ and ν_τ [2,23]. The remaining $2n_R$ Weyl fermions are degenerate in pairs due to the fact that L is conserved and so form n_R heavy Dirac neutrinos. In general, this viable seesaw-type model can have large Dirac components in $\mathcal{M}^\nu$ and only the ratio m_D/M ($\sim s_L^{\nu_l}$) will again be constrained for $M \gtrsim 100$ GeV (see also our discussion in Section 3). A nice feature of the model is that the individual leptonic quantum numbers may be violated [24,25,34], even if L is conserved. The charged-current Lagrangian can be obtained by Eq. (2.4), while the neutral-current interaction is given by [24]

$$\mathcal{L}_{\rm int}^Z = -\frac{g_w}{2c_w} Z^\mu \sum_{i,j=1}^{n_G+n_R} \bar{n}_i C_{ij} \gamma_\mu P_L n_j. \tag{2.14}$$

The matrices B and C for this specific model obey the sum rules in Eq. (2.9), but not the identities of Eq. (2.10).

At this stage, we must comment on the difference between the Lagrangians (2.5) and (2.14). Since Eq. (2.5) describe Majorana neutrinos contrary to the Lagrangian (2.14) where the massive neutrinos are Dirac, the strength of the $Z\bar{n}_i n_i$ coupling for identical Majorana fermions is two times larger than the one which may naively be red off from $\mathcal{L}_{\rm int}^Z$ in Eqs. (2.5) and (2.11). The off-diagonal coupling $Z\bar{n}_i n_j$ (with $n_i \neq n_j$) is again enhanced by a factor of two, since the charge-conjugate interaction $Z\bar{n}_j n_i$ will equally contribute to the coupling of the Z-boson to Majorana neutrinos. In our forthcoming calculations, we have taken into account all these theoretical differences in treating Majorana and Dirac fields. In fact, we find that taking formally the limit $C_{ij}^* \to 0$ but keeping $C_{ij} \neq 0$ in Eq. (2.11) and considering the afore-mentioned statistical Majorana factors is sufficient to recover the model with additional left-handed neutral singlets.

To make life easier, we ultimately make the following reasonable assumptions: To a good approximation, we assume that possible novel particles related to the above unified theories, such as leptoquarks [35] or extra charged and neutral gauge bosons, $W_R^\pm$ [36] and Z' [37,22], are sufficiently heavy so as to decouple completely from the low-energy processes discussed in Sections 3.3 and 4. For obvious reasons, possible singlet and triplet Majoron fields [38-41] are considered to couple very weakly to matter so that we can safely ignore them in our considerations.

3. General constraints on the models

3.1. Cosmological constraints

Unified theories based on the gauge group SO(10) or E_6 can naturally accommodate right-handed neutrinos in addition to quarks and leptons of the SM. In such theories, the Majorana mass m_M can directly be related to the $B-L$ scale of a local symmetry which is assumed to be spontaneously broken. It is therefore evident that the mass of heavy

Majorana neutrinos will be determined by the scale of $B-L$ breaking. Moreover, out-of-equilibrium lepton-number-violating decays of heavy Majorana neutrinos can generate a non-zero L [42] in the universe through the L-violating interactions of Eqs. (2.4)-(2.7). This excess in L can be converted into a B asymmetry of the universe via the $(B+L)$-violating sphaleron interactions, which are in thermal equilibrium above the critical temperature of the electroweak phase transition [43,44]. Many studies have recently been devoted to constrain the $(B-L)$-violating mass scale by making use of the drastic out-of-equilibrium condition for the $\Delta L = 2$ operators, and so to derive a lower mass bound on the heavy Majorana neutrinos [45–49]. For example, in Ref. [45] conceivable scenarios predicting heavy Majorana neutrinos with $m_N = 1 - 10$ TeV could naturally account for the observed BAU. Subsequently, it was argued [46] that the m_N lower bound of ~ 1 TeV was considerably underestimated and a lower bound on $m_N > 10^5$ TeV should be imposed in a two-generation scenario of right-handed neutrinos with large interfamily mixings so as to be compatible with the existing BAU. This would obviously imply that probing Majorana-neutrino physics at collider energies may not be phenomenologically interesting.

The latter observation can indeed be valid in a two-generation-mixing model with two right-handed neutrinos. In general, in three-generation models with lepton-flavour mixings, a careful inspection of chemical potentials has shown that the stringent mass bound of heavy Majorana neutrinos mentioned above can be weakened dramatically and is quite model dependent [48,49]. In particular, it is sufficient that in equilibrium state one individual lepton number, e.g. L_μ, is conserved in order to generate the BAU via the sphaleron interactions, even if nonvanishing operators with $\Delta L_{l_i} \neq \Delta L_\mu$ were in thermal equilibrium [48]. The reason is that sphalerons generally conserve the quantum numbers $B/3 - L_{l_i}$ [47,48] and thus preserve any BAU generated by an excess, e.g., in L_μ, from being washed out. Similar conclusions have been drawn in Ref. [49].

A viable scenario of heavy Majorana neutrinos with masses $\sim$ TeV can easily be realized in the SM with $n_{\rm R} = 4$. If the BAU is to be generated through an excess in muonic number density, this asymmetry in L_μ can be achieved by considering a neutrino mass matrix, M^ν, similar to the CP-violating scenario given in Ref. [19]. This scenario contains one left-handed neutrino, ν^0_{L1}, to which, for the case at hand, a muonic quantum number should be assigned, and two right-handed neutrinos, denoted as ν^0_{R3} and ν^0_{R4}. The explicit form of M^ν is then given by [19]

$$M^\nu = \begin{pmatrix} 0 & a & b \\ a & A & 0 \\ b & 0 & B \end{pmatrix}, \tag{3.1}$$

where a and b are in general complex numbers, and A and B can be chosen to be real. Out-of-equilibrium conditions for generating a sufficiently large asymmetry in L_μ, which can give rise to the established BAU, lead to the stringent lower bounds on the masses of the corresponding physical heavy neutrinos $N_{3,4}$ as consistently obtained by Ref. [46]. However, the remaining e- and τ-lepton families can strongly mix each other via two *additional* right-handed neutrinos, e.g., ν^0_{R1} and ν^0_{R2}, and form an *individual* 4×4 seesaw-type matrix. Operators $\Delta L_e \neq 0$ and $\Delta L_\tau \neq 0$ are now allowed to be in thermal equilibrium provided that $\Delta(L_e - L_\mu) = 0$ and $\Delta(L_\tau - L_\mu) = 0$. The latter condition is automatically satisfied due to the construction of this specific scenario with $n_R = 4$. As a consequence, the severe lower mass bounds on the physical heavy neutrinos N_1 and N_2 can be evaded completely. A similar analysis in a SM with $n_R = 3$ is more involved due to the flavour-mixing effects in the neutrino-mass matrix and can be given elsewhere.

3.2. Low-energy constraints

There exists a great number of low-energy experiments that could set upper bounds on possible non-SM couplings [21]. The most significant experimental tests giving stringent constraints turn out to be the neutrino counting at the Z peak, the precise measurement of the muon width $\mu \to e\nu_e\nu_\mu$, charged-current universality effects on the observable $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$, non-universality effects on $B(\tau \to e\nu\nu)/B(\tau \to \mu\nu\nu)$, and other nuclear physics effects and experiments. All these constraints, which are derived by the low-energy data mentioned above, depend crucially on the gauge structure of the model under discussion. For example, assuming the supersymmetric (SUSY) nature of the E_6 models or SUSY-SO(10) unified theories [50,51], and R-parity invariance, the neutralino state could then be the lightest supersymmetric particle (LSP) which is stable. If the mass of the LSP is assumed to be in the vicinity of $M_Z/2$, then an additional invisible decay channel for the Z boson will open kinematically and neutrino-counting limits imposed on the couplings $Z\nu_i\nu_j$ may not be applicable. Furthermore, an analysis of decays of the type $Z \to N\nu$, which have been considered in Ref. [27], suggests that $m_N \gtrsim 100$ GeV for $(s_L^{\nu_l})^2 \sim 0.01$.

Thus, identifying the non-SM-mixing angles $(s_L^{\nu_l})^2$ of Ref. [21] as

$$(s_L^{\nu_l})^2 \equiv \sum_{j=1}^{n_R} |B_{lN_j}|^2, \tag{3.2}$$

and in view of the discussion given above, one may tolerate the following upper limits [21]:

$$(s_L^{\nu_e})^2 < 0.015, \quad (s_L^{\nu_\tau})^2 < 0.070, \quad (s_L^{\nu_\mu})^2 < 1 \times 10^{-9}. \tag{3.3}$$

In Eq. (3.3) the tight upper bound on $s_L^{\nu_\mu}$ represents that the muonic quantum number is practically conserved in thermal equilibrium. Note that, without loss of generality, one could equally interchange the upper limit on $(s_L^{\nu_e})^2$ with that of $(s_L^{\nu_\mu})^2$. To be precise, we will assume $(s_L^{\nu_\mu})^2 \simeq 0$ in what follows.

Another limitation to the parameters of our model comes from the requirement of the validity of perturbative unitarity that can be violated in the limit of large heavy-neutrino masses. A qualitative estimate for the latter may be obtained by requiring that the total widths, Γ_{N_i}, and masses of neutrino fields N_i satisfy the inequality

$$\frac{\Gamma_{N_i}}{m_{N_i}} < \frac{1}{2}. \tag{3.4}$$

The total widths of the heavy neutrinos, Γ_{N_i}, can be written down as a sum over all possible decay channels [5], i.e.

$$\Gamma_{N_i} = \sum_{l_j} \Gamma(N_i \to l_j^{\pm} W^{\mp}) + \sum_{\nu_j} \Big(\Gamma(N_i \to \nu_j Z) + \Gamma(N_i \to \nu_j H) \Big). \tag{3.5}$$

In the limit of $m_{N_i} \gg M_W,\ M_Z,\ M_H$, the above expression simplifies to

$$\Gamma_{N_i} = \frac{\alpha_w}{4M_W^2}\ m_{N_i}^3\ |C_{N_iN_i}|^2, \tag{3.6}$$

with $\alpha_w = g_w^2/4\pi$.

3.3. Constraints from leptonic Z-boson decays

Aside from low-energy constraints discussed in Section 3.2, many extensions of the SM derived by unified theories may give rise to lepton-flavour-violating decays of the Z boson [52,53,11]. In particular, it has been found in Ref. [11] that the non-observation of such non-SM signals at LEP may impose combined bounds both on heavy neutrino masses m_{N_i} and mixings $(s_L^{\nu_l})^2$. The reason is that the amplitude of such a decay depends quadratically on the heavy neutrino mass, leading to measurable rates. In a self-explanatory way, the amplitude of the decay $Z \to ll'$ may generally be parametrized as

$$\mathcal{T}(Z \to \bar{l}l') = \frac{ig_w\alpha_w}{8\pi c_w}\ \mathcal{F}_Z^{ll'}\ \varepsilon_Z^{\mu}\ \bar{u}_{l'}\gamma_\mu(1-\gamma_5)v_l, \tag{3.7}$$

where $\alpha_w = g_w^2/4\pi$ and the form factor $\mathcal{F}_Z^{ll'}$, which is induced by the Feynman graphs of Fig. 2 at the one-loop electroweak order, is given in Appendix A. The branching ratio of this decay mode is obtained by

$$B(Z \to \bar{l}l' + \bar{l}'l) = \frac{\alpha_w^3}{48\pi^2 c_w^3}\frac{M_W}{\Gamma_Z}\left|\mathcal{F}_Z^{ll'}\right|^2, \tag{3.8}$$

where $\Gamma_Z = 2.49$ GeV is the experimental value of the total width of the Z boson [56]. We postpone the numerical discussion of possible constraints that might arise due to lepton-flavour-violating decays of the Z boson in Section 5.

4. Flavour-violating decays of charged leptons

In Sections 4.1 and 4.2, we will theoretically analyze the possibility of lepton flavour nonconservation in decays of the form $l \to l'\gamma$ and $l \to l'l_1\bar{l}_2$, respectively. As mentioned in Section 1, l, l', l_1 and l_2 denote usual charged leptons, i.e. the e, μ and τ leptons.

4.1. The decay $l \to l'\gamma$

In the framework of the minimal class of models discussed in Section 2, heavy Majorana or Dirac neutrinos can give rise to the decay $l \to l'\gamma$. The Feynman graphs responsible for such a decay are shown in Fig. 1. Applying electromagnetic gauge

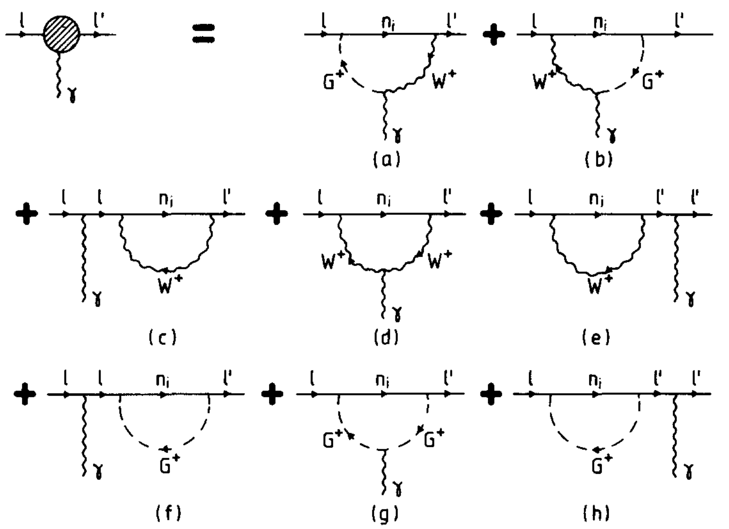

Fig. 1. Feynman graphs responsible for generating the effective vertex $\gamma ll'$ $(l \neq l')$.

invariance to the decay amplitude $l(p) \to l'(p')\gamma(q)$, where the photon, γ, may be off-mass shell, yields [55,57]

$$\mathcal{T}(l \to l'\gamma) = -i\frac{e\alpha_w}{16\pi M_W^2}\varepsilon_\gamma^\mu\, \bar{u}_{l'}\Bigg[\sum_{i=1}^{n_G+n_R} B_{li}^* B_{l'i} F_\gamma(\lambda_i)(q^2\gamma_\mu - \not{q}q_\mu)(1-\gamma_5)$$
$$- \sum_{i=1}^{n_G+n_R} B_{li}^* B_{l'i} G_\gamma(\lambda_i)\; i\sigma_{\mu\nu}q^\nu\Big(m_{l'}(1-\gamma_5) + m_l(1+\gamma_5)\Big)\Bigg]u_l, \quad (4.1)$$

where $\lambda_i = m_i^2/M_W^2$, $q = p - p'$ denotes the outgoing momentum of the photon, and the form factors F_γ and G_γ are given in Appendix B. It is now straightforward to calculate the branching ratio of $l \to l'\gamma$

$$B(l \to l'\gamma) = \frac{\alpha_w^3 s_w^2}{256\pi^2}\frac{m_l^4}{M_W^4}\frac{m_l}{\Gamma_l}\left|G_\gamma^{ll'}\right|^2, \quad (4.2)$$

where Γ_l is the total width of the decaying lepton l, while $G_\gamma^{ll'}$ in Eq. (4.2) represents a composite form factor defined in Appendix B. Specifically, for the total width of the τ lepton, we use the experimental value, $\Gamma_\tau = 2.1581 \times 10^{-12}$ GeV [56], whereas the muon total width may be obtained by [54]

$$\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3}\left(1 - 8\frac{m_e^2}{m_\mu^2}\right)\left[1 + \frac{\alpha_{\rm em}}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right], \quad (4.3)$$

where $\alpha_{\rm em} = e^2/4\pi$. The muon total width given in Eq. (4.3) is in excellent agreement with the experimental value reported in Ref. [56].

The experimental upper bounds arising from the non-observation of decays of the type $l \to l'\gamma$ are [56]

$$
\begin{aligned}
&B(\tau \to e\gamma) < 1.2 \times 10^{-4},\\
&B(\tau \to \mu\gamma) < 4.2 \times 10^{-6},\\
&B(\mu \to e\gamma) < 4.9 \times 10^{-11},
\end{aligned}
\tag{4.4}
$$

at 90% CL. Using the values for the mixing angles $(s_L^{\nu_l})^2$ of Eq. (3.3), one easily finds that the photonic decays involving muons are extremely suppressed in our minimal scenarios. Furthermore, the theoretical prediction $B(\tau \to e\gamma) \lesssim 10^{-7}$ shows that photonic decays of a τ lepton may not be the most favourable place to probe heavy neutrino physics.

4.2. Three-body leptonic decays $l \to l'l_1\bar{l}_2$

In a three-generation model the decaying charged lepton l will either be a muon or a τ lepton. There are seven possible decays of the generic form $l \to l'l_1\bar{l}_2$

$$
\begin{aligned}
&(\mathrm{a}) \quad \tau^- \to \mu^-\mu^-\mu^+,\\
&(\mathrm{b}) \quad \tau^- \to \mu^-\mu^-e^+,\\
&(\mathrm{c}) \quad \tau^- \to e^-\mu^-\mu^+,\\
&(\mathrm{d}) \quad \tau^- \to e^-e^-\mu^+,\\
&(\mathrm{e}) \quad \tau^- \to e^-\mu^-e^+,\\
&(\mathrm{f}) \quad \tau^- \to e^-e^-e^+,\\
&(\mathrm{g}) \quad \mu^- \to e^-e^-e^+.
\end{aligned}
\tag{4.5}
$$

To facilitate our computational task, we divide the decays in Eq. (4.5) into three categories according to the leptonic flavours in the final state: Category (i) contains all the decays where $l' \neq l_2$ and $l_1 = l_2$ or $l' = l_2$ and $l_1 \neq l_2$ (i.e. the decays (c) and (e)). Category (ii) comprises all the decays where $l' = l_1 = l_2$ (i.e. the decays (a), (f) and (g)). And lastly, all the decays with final leptons having $l' \neq l_2$, $l_1 \neq l_2$ belong to the category (iii) (i.e. the decays (b) and (d)).

The transition amplitude of the decay $l(p) \to l'(p')l_1(p_1)\bar{l}_2(p_2)$ receives contributions from γ- and Z-mediated graphs shown in Figs. 1 and 2, respectively, and box diagrams given in Fig. 3. These three different amplitudes are conveniently written down as follows:

$$
\begin{aligned}
\mathcal{T}_\gamma(l \to l'l_1\bar{l}_2) = &-\frac{i\alpha_w^2 s_w^2}{4M_W^2}\delta_{l_1l_2}\sum_{i=1}^{n_G+n_R} B_{li}^* B_{l'i}\; \bar{u}_{l_1}\gamma^\mu v_{l_2}\; \bar{u}_{l'}\Big[F_\gamma(\lambda_i)\left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2}\right)(1-\gamma_5)\\
&-iG_\gamma(\lambda_i)\sigma_{\mu\nu}\frac{q^\nu}{q^2}(m_l(1+\gamma_5) + m_{l'}(1-\gamma_5))\Big]u_l,
\end{aligned}
\tag{4.6}
$$

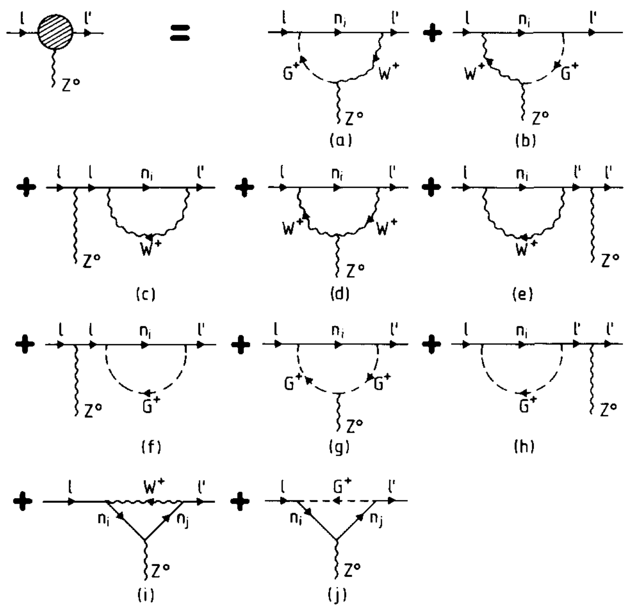

Fig. 2. Feynman graphs responsible for generating the effective vertex Zll' $(l \neq l')$.

$$\mathcal{T}_Z(l \to l' l_1 \bar{l}_2) = -\frac{i\alpha_w^2}{16M_W^2}\, \bar{u}_{l'}\gamma_\mu(1-\gamma_5)u_l\ \bar{u}_{l_1}\gamma^\mu(1-4s_w^2-\gamma_5)v_{l_2}$$
$$\times\, \delta_{l_1 l_2} \sum_{i,j=1}^{n_G+n_R} B^*_{li}B_{l'j}\Big[\delta_{ij}F_Z(\lambda_i) + C_{ij}H_Z(\lambda_i,\lambda_j) + C^*_{ij}G_Z(\lambda_i,\lambda_j)\Big]\,, \tag{4.7}$$

$$\mathcal{T}_{\rm Box}(l \to l' l_1 \bar{l}_2) = -\frac{i\alpha_w^2}{16M_W^2}\, \bar{u}_{l'}\gamma_\mu(1-\gamma_5)u_l\ \bar{u}_{l_1}\gamma^\mu(1-\gamma_5)v_{l_2}$$
$$\times \sum_{i,j=1}^{n_G+n_R}\Big[(B_{l'i}B_{l_1j}+B_{l_1i}B_{l'j})B^*_{li}B^*_{l_2j}\,F_{\rm Box}(\lambda_i,\lambda_j)$$
$$+B_{l'i}B_{l_1i}B^*_{lj}B^*_{l_2j}\,G_{\rm Box}(\lambda_i,\lambda_j)\Big]\,, \tag{4.8}$$

where $q = p_1 + p_2$. In addition to the photonic form factors F_γ and G_γ in Eq. (4.6), the form factors F_Z, H_Z, G_Z, $F_{\rm Box}$, and $G_{\rm Box}$ are given in Appendix B. Note that the term proportional to G_γ in Eq. (4.6) contains a non-local interaction which is singular in the limit $q^2 \to 0$.

Following the classification mentioned above, the branching ratio for all decays belonging to the first category is found to be [1]

[1] In our calculations we have used a notation similar to the authors in Ref. [34]. However, our branching-ratio expressions (4.9) and (4.10) are at variance with their results.

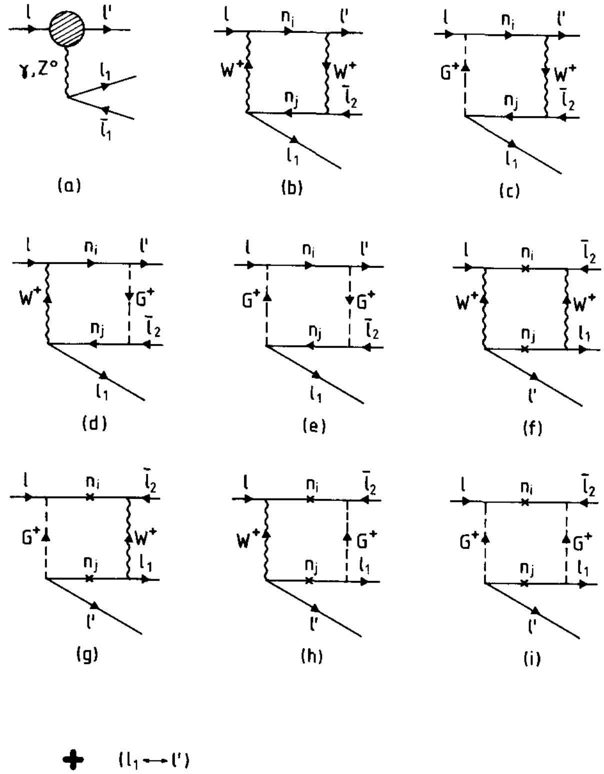

Fig. 3. Feynman diagrams relevant for the leptonic decays $l \to l' l_1 \bar{l}_2$.

$$
\begin{aligned}
B(l^- \to l'^- l_1^- l_2^+, l' \neq l_2, l_1 = l_2) &= \frac{\alpha_w^4}{24576\pi^3} \frac{m_l^4}{M_W^4} \frac{m_l}{\Gamma_l} \\
&\times \Bigg\{ \left| F_{\text{Box}}^{ll'l_1l_1} + F_Z^{ll'} - 2s_w^2 (F_Z^{ll'} - F_\gamma^{ll'}) \right|^2 + 4s_w^4 \left| F_Z^{ll'} - F_\gamma^{ll'} \right|^2 \\
&+ 8s_w^2 \, \text{Re}[(F_Z^{ll'} + F_{\text{Box}}^{ll'l_1l_1}) G_\gamma^{ll'\,*}] - 32 s_w^4 \, \text{Re}[(F_Z^{ll'} - F_\gamma^{ll'}) G_\gamma^{ll'\,*}] \\
&+ 32 s_w^4 \left| G_\gamma^{ll'} \right|^2 \left(\ln \frac{m_l^2}{m_{l_1}^2} - 3 \right) \Bigg\},
\end{aligned}
\tag{4.9}
$$

where $F_\gamma^{ll'}$, $G_\gamma^{ll'}$, $F_Z^{ll'}$, and $F_{\text{Box}}^{ll'l_1l_2}$ are composite form factors defined explicitly in Appendix B. The branching ratios referring to the categories (ii) and (iii) are correspondingly given by

$$
B(l^- \to l'^- l_1^- l_2^+, l' = l_1 = l_2) = \frac{\alpha_w^4}{24576\pi^3} \frac{m_l^4}{M_W^4} \frac{m_l}{\Gamma_l}
$$

$$\times \left\{ 2\left|\tfrac{1}{2}F_{\rm Box}^{ll_1l_1l_1} + F_Z^{ll_1} - 2s_w^2(F_Z^{ll_1} - F_\gamma^{ll_1})\right|^2 + 4s_w^4\left|F_Z^{ll_1} - F_\gamma^{ll_1}\right|^2 \right.$$

$$+16s_w^2\,{\rm Re}[(F_Z^{ll_1} + \tfrac{1}{2}F_{\rm Box}^{ll_1l_1l_1})G_\gamma^{ll_1\,*}] - 48s_w^4\,{\rm Re}[(F_Z^{ll_1} - F_\gamma^{ll_1})G_\gamma^{ll_1\,*}]$$

$$\left. +32s_w^4\left|G_\gamma^{ll_1}\right|^2\left(\ln\frac{m_l^2}{m_{l_1}^2} - \frac{11}{4}\right)\right\}$$

$$B(l^- \to l'^- l_1^- l_2^+, l_1 \neq l_2, l' \neq l_2) = \frac{\alpha_w^4}{49152\pi^3}\frac{m_l^4}{M_W^4}\frac{m_l}{\Gamma_l}\left|F_{\rm Box}^{ll'l_1l_2}\right|^2. \qquad (4.10)$$

Eqs. (4.9) and (4.10) contain a non-local interaction in terms $\propto G_\gamma^{ll'}$ and $G_\gamma^{ll_1}$, which is discussed in detail in Appendix C. In Eq. (4.10), one has to take into account statistical symmetrization factors for the two identical final leptons (i.e. $l' = l_1$), as well as additional Feynman graphs resulting from the interchange of the lepton l' with l_1. The set of decays in (iii) can only be induced by the box graphs shown in Fig. 3. The amplitude of such a class of decays (i.e. decays (b) and (d)) is always proportional to $(s_L^{\nu_l})^2 s_L^{\nu_e} s_L^{\nu_\mu}$ and the corresponding branching ratios are hence expected to be vanishingly small even if one uses the upper value of $s_L^{\nu_\mu}$ in Eq. (3.3). For reasons of mere academic interest, we simply note that $B(\tau^- \to e^-e^-\mu^+), B(\tau^- \to \mu^-\mu^-e^+) \lesssim 10^{-12}$. As a consequence, we find that the decays (c) and (f) in Eq. (4.5) deserve the biggest attention and will hence be analyzed numerically in Section 5.

5. Numerical evaluation and discussion

We will now investigate the phenomenological impact of the two seesaw-type models outlined in Section 2. In order to pin down numerical predictions, we will, for definiteness, assume an extension of the SM by two right-handed neutrinos. The neutrino mass spectrum of such a model consists of three light Majorana neutrinos which have been identified with the three known neutrinos, ν_e, ν_μ, and ν_τ, and two heavy ones denoted by N_1 and N_2. As already mentioned in Section 2, the seesaw-type extension of the SM with one left-handed and one right-handed chiral singlets can effectively be recovered by the SM with two right-handed neutrinos when taking the degenerate mass limit for the two heavy Majorana neutrinos. It is therefore obvious that branching-ratio results for the SM with one left-handed and one right-handed neutral singlets can be red off from the SM with two right-handed neutrinos in the specific case $m_{N_1} = m_{N_2} = m_N$.

Apart from the two heavy Majorana neutrino masses which are free parameters of the theory, the model contains numerous mixing angles, B_{li} and C_{ij}, for which the only restriction comes from a low-energy analysis as discussed in Sections 3.2 and 3.3. However, in our minimal model with two right-handed neutrinos one can derive, with the help of the identities in Eqs. (2.9) and (2.10), the useful relations

$$B_{lN_1} = \frac{\rho^{1/4}s_L^{\nu_l}}{\sqrt{1+\rho^{1/2}}}, \qquad B_{lN_2} = \frac{is_L^{\nu_l}}{\sqrt{1+\rho^{1/2}}}, \qquad (5.1)$$

where $\rho = m_{N_2}^2/m_{N_1}^2$. The mixings $C_{N_iN_j}$ can also be obtained by employing Eq. (2.9). In this way one gets

$$C_{N_1N_1} = \frac{\rho^{1/2}}{1+\rho^{1/2}} \sum_{l=1}^{n_G} (s_L^{\nu_l})^2, \qquad C_{N_2N_2} = \frac{1}{1+\rho^{1/2}} \sum_{l=1}^{n_G} (s_L^{\nu_l})^2,$$
$$C_{N_1N_2} = -C_{N_2N_1} = \frac{i\rho^{1/4}}{1+\rho^{1/2}} \sum_{l=1}^{n_G} (s_L^{\nu_l})^2. \tag{5.2}$$

Evidently, our minimal scenario depends only on the masses of the heavy Majorana neutrinos, m_{N_1} and m_{N_2} (or equivalently on m_{N_1} and ρ), and the mixing angles $(s_L^{\nu_l})^2$, which are directly constrained by a global analysis of low-energy data.

In our illustrative model, with the help of Eq. (5.2) we can easily obtain the maximal heavy neutrino mass allowed by perturbative unitarity. Satisfying Eq. (3.4) for both heavy neutrinos N_1 and N_2, one gets the global relation

$$m_{N_1}^2 \leqslant \frac{2M_W^2}{\alpha_w} \frac{1+\rho^{-1/2}}{\rho^{1/2}} \left[\sum_{l=1}^{n_G} (s_L^{\nu_l})^2 \right]^{-1}, \tag{5.3}$$

with $\rho \geqslant 1$. Condition (5.3) has thoroughly been used in our numerical estimates to impose an upper bound on $m_{N_{1,2}}$.

For reasons mentioned in Section 4.2, we present the branching ratios for the leptonic decays $\tau^- \to e^-e^-e^+$ and $\tau^- \to e^-\mu^-\mu^+$ in Fig. 4. To gauge to which extend our minimal model can predict measurable rates, we have first assumed the maximally allowed values [21] for $(s_L^{\nu_\tau})^2 = 0.07$ and $(s_L^{\nu_e})^2 = 0.015$ $((s_L^{\nu_\mu})^2 \simeq 0)$ given in Eq. (3.3). From Fig. 4 we find the encouraging branching ratios

$$B(\tau^- \to e^-e^-e^+) \lesssim 2 \times 10^{-6}, \qquad B(\tau^- \to e^-\mu^-\mu^+) \lesssim 1 \times 10^{-6}. \tag{5.4}$$

The present experimental upper limits on these decays are given by [56]

$$B(\tau^- \to e^-e^-e^+) < 1.3 \times 10^{-5},$$
$$B(\tau^- \to e^-\mu^-\mu^+) < 1.9 \times 10^{-5}, \quad \mathrm{CL} = 90\%. \tag{5.5}$$

Even if we assume smaller values for the mixing angles, $(s_L^{\nu_\tau})^2 = 0.035$ and $(s_L^{\nu_e})^2 = 0.01$ $((s_L^{\nu_\mu})^2 = 0)$, the lepton-flavour-violating decays of the τ lepton can still be significant. From Fig. 5 one has that

$$B(\tau^- \to e^-e^-e^+) \lesssim 5 \times 10^{-7}, \qquad B(\tau^- \to e^-\mu^-\mu^+) \lesssim 3 \times 10^{-7}, \tag{5.6}$$

and the possibility of observing such decays at future LEP experiments appears feasible. Note that the branching ratio increases with the heavy neutrino mass to the fourth power and hence allows to reach measurable values. To demonstrate this fact, we have just neglected contributions of seemingly suppressed terms $\mathcal{O}((s_L^{\nu_l})^4)$ in the transition elements and found a reduction of our numerical values up to $\sim 10^{-2}$. In the low-mass range of heavy neutrinos (i.e. for $m_{N_i} < 200$ GeV) the difference between the two

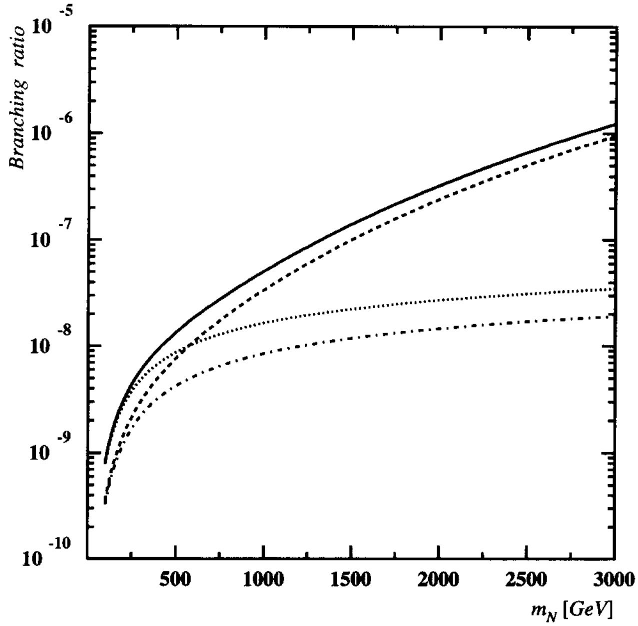

Fig. 4. $B(\tau^- \to e^-e^-e^+)$ (solid line) and $B(\tau^- \to e^-\mu^-\mu^+)$ (dashed line) as a function of the heavy neutrino mass $m_N (= m_{N_1} = m_{N_2})$ assuming $(s_L^{\nu_\tau})^2 = 0.07$, $(s_L^{\nu_e})^2 = 0.015$ and $(s_L^{\nu_\mu})^2 \simeq 0$. Numerical results obtained when seemingly suppressed terms of $\mathcal{O}((s_L^{\nu_l})^4)$ are neglected, are also presented for $B(\tau^- \to e^-e^-e^+)$ (dotted line) and $B(\tau^- \to e^-\mu^-\mu^+)$ (dash-dotted line).

distinct computations is quite small and consistent with results obtained in Ref. [34]. In the high-mass regime, however, the situation changes drastically (see also Figs. 4 and 5), since in the transition amplitude, terms proportional to $(s_L^{\nu_l})^2$ depend logarithmically on the heavy neutrino mass m_N, i.e. $\ln(m_N^2/M_W^2)$, while terms of $\mathcal{O}((s_L^{\nu_l})^4)$ show a strong quadratic dependence in the heavy neutrino mass, i.e. m_N^2/M_W^2.

Fig. 6 represents genuine Majorana-neutrino quantum effects, since we have computed the branching ratios as a function of the ratio m_{N_2}/m_{N_1} for the selective values of m_{N_1} = 200 GeV and 500 GeV. Although the most stringent constraints on the heavy Majorana neutrino masses result from Eq. (5.3), it is, however, important to notice that for lower neutrino masses the maximum of $B(\tau^- \to e^-e^-e^+)$ and $B(\tau^- \to e^-\mu^-\mu^+)$ is not given by the degenerate case where $\rho = 1$. In fact, if $m_{N_2}/m_{N_1} \simeq 3$ the branching ratios show up a maximum which can be up to two times greater than the case where both the heavy neutrinos, N_1 and N_2, are degenerate. We have thus found that for m_{N_1} = 500 GeV and $m_{N_2} \simeq 1.5$ TeV,

$$B(\tau^- \to e^-e^-e^+) \lesssim 2 \times 10^{-8}, \qquad B(\tau^- \to e^-\mu^-\mu^+) \lesssim 1.5 \times 10^{-8}. \qquad (5.7)$$

Such effects might be accessible at τ factories if one assumes an upgrade in the luminosity of the LEP collider by a factor of 10.

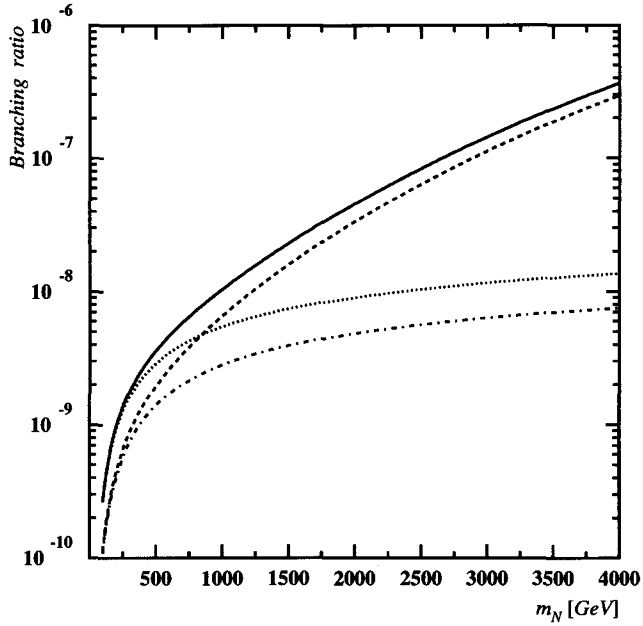

Fig. 5. $B(\tau^- \to e^- e^- e^+)$ (solid line) and $B(\tau^- \to e^- \mu^- \mu^+)$ (dashed line) as a function of the heavy neutrino mass $m_N(= m_{N_1} = m_{N_2})$ using $(s_L^{\nu_\tau})^2 = 0.035$, $(s_L^{\nu_e})^2 = 0.010$ and $(s_L^{\nu_\mu})^2 \simeq 0$. We also display numerical results obtained by neglecting seemingly suppressed terms of $\mathcal{O}((s_L^{\nu_l})^4)$ in the calculation of $B(\tau^- \to e^- e^- e^+)$ (dotted line) and $B(\tau^- \to e^- \mu^- \mu^+)$ (dash-dotted line).

In the following, we will discuss possible constraints that might arise from lepton-flavour-violating decays of the Z boson. Since we always assume that $(s_L^{\nu_\mu})^2 = 0$, we will focus our analysis on the decays $Z \to e^- \tau^+ + e^+ \tau^-$. Within the perturbatively allowed range of heavy neutrino masses as determined by Eq. (5.3), Fig. 7 gives

$$\begin{aligned} B(Z \to e^-\tau^+ + e^+\tau^-) &\lesssim 4.0\ 10^{-6}, \quad \text{for} \quad (s_L^{\nu_\tau})^2 = 0.070, \quad (s_L^{\nu_e})^2 = 0.015, \\ B(Z \to e^-\tau^+ + e^+\tau^-) &\lesssim 1.1\ 10^{-6}, \quad \text{for} \quad (s_L^{\nu_\tau})^2 = 0.035, \quad (s_L^{\nu_e})^2 = 0.010, \\ B(Z \to e^-\tau^+ + e^+\tau^-) &\lesssim 6.0\ 10^{-7}, \quad \text{for} \quad (s_L^{\nu_\tau})^2 = 0.020, \quad (s_L^{\nu_e})^2 = 0.010. \end{aligned} \tag{5.8}$$

In our numerical estimates, we have used values for $(s_L^{\nu_l})^2$ compatible with the updated upper bounds given in Eq. (3.3). Although all the branching ratios in Eq. (5.8) could be detected at future LEP data, they cannot impose any severe constraints on the τ decays into three charged leptons for the present analysis. The experimental sensitivity at LEP is currently given by [56]

$$B(Z \to e^-\tau^+ + e^+\tau^-) < 1.3 \times 10^{-5}, \quad \mathrm{CL} = 95\%. \tag{5.9}$$

Here, some comments are in order. In Fig. 7, the branching ratios for the three different mixing-angle sets in the order stated in Eq. (5.8) show a minimum at the positions

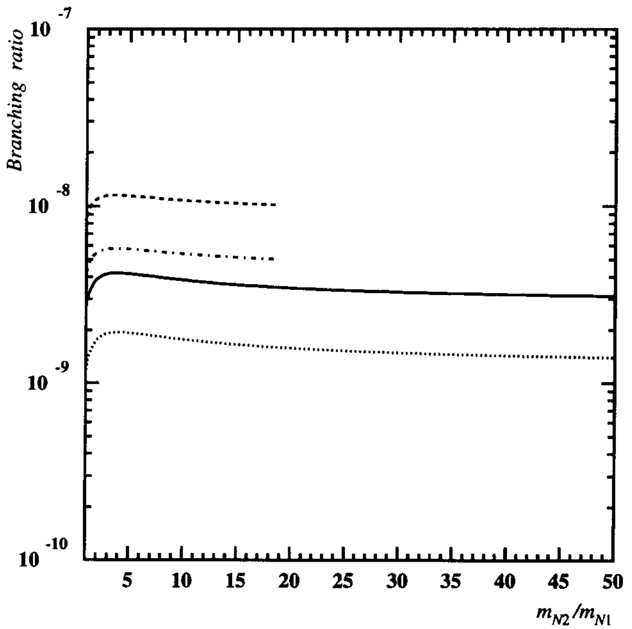

Fig. 6. $B(\tau^- \to e^-e^-e^+)$ as a function of the ratio m_{N_2}/m_{N_1} for m_{N_1} = 200 GeV (solid line) and 500 GeV (dashed line). We have assumed $(s_L^{\nu_\tau})^2 = 0.07$, $(s_L^{\nu_e})^2 = 0.015$ and $(s_L^{\nu_\mu})^2 \simeq 0$. The corresponding numerical results of $B(\tau^- \to e^-\mu^-\mu^+)$ are shown for m_{N_1} = 200 GeV (dotted line) and 500 GeV (dash-dotted line).

m_N = 700, 900 and 1200 GeV, respectively. The reason is that $\mathcal{O}((s_L^{\nu_l})^2)$ and $\mathcal{O}((s_L^{\nu_l})^4)$ terms of $\mathcal{F}_Z^{ll'}$ in Eq. (3.7) cancel each other and the whole transition amplitude becomes pure absorptive. In the range of very heavy neutrinos, terms proportional to $(s_L^{\nu_l})^4$ will dominate in the amplitude for the same reasons mentioned above. The effect of such a dynamical cancellation of the dispersive part of the amplitude could be shown up as a difference between the charge-conjugate decay modes of $Z \to e^-\tau^+$ and $Z \to e^+\tau^-$, leading to sizeable CP-violating effects [25].

In Fig. 8 we display genuine Majorana-neutrino virtual effects by examining the behaviour of the branching ratio as a function of the quantity m_{N_2}/m_{N_1} for rather modest values of m_{N_1}. Here, the situation is more involved and depends strongly on the value of m_{N_1} we choose. The fact that the amplitude could be dominated by $(s_L^{\nu_l})^2$ terms for relatively light heavy Majorana neutrinos (i.e. $m_{N_1} < 400$ GeV) or by $(s_L^{\nu_l})^4$ terms for larger values of m_{N_1} plays a crucial rôle for the shape of the different lines drawn in Fig. 8. The common feature is, however, that the case where both the heavy Majorana neutrinos, N_1 and N_2, have the same mass does not again correspond to the situation yielding the biggest branching-ratio value.

Finally, τ leptons can also decay hadronically via the channels: $\tau \to l_i\eta$, $\tau \to l_i\pi^0$, etc. [34]. However, the present experimental sensitivity to these decays seems to be rather weak [56] so as to set constraints on our analysis. For example, $B(\tau \to e\pi^0) < 1.4 \times 10^{-4}$, at CL = 90%.

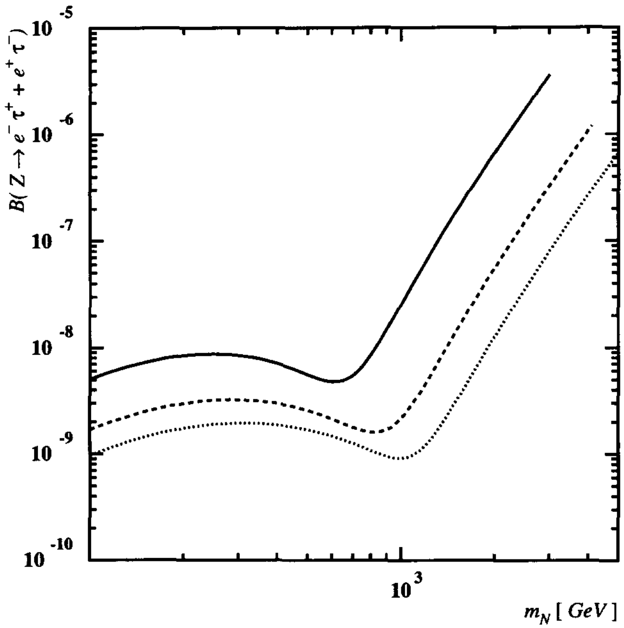

Fig. 7. Numerical estimates of $B(Z \to e^-\tau^+) + B(Z \to e^+\tau^-)$ as a function of the heavy neutrino mass $m_N (= m_{N_1} = m_{N_2})$ for three representative values of the mixing parameters $(s_L^{\nu_\tau})^2$ and $(s_L^{\nu_e})^2$ ($(s_L^{\nu_\mu})^2 = 0$): (i) $(s_L^{\nu_\tau})^2 = 0.070$ and $(s_L^{\nu_e})^2 = 0.015$ (solid line), (ii) $(s_L^{\nu_\tau})^2 = 0.035$ and $(s_L^{\nu_e})^2 = 0.010$ (dashed line), and (iii) $(s_L^{\nu_\tau})^2 = 0.020$ and $(s_L^{\nu_e})^2 = 0.010$ (dotted line).

6. Conclusions

We have explicitly shown that seesaw-type extensions of the minimal SM, which naturally contain left-handed and/or right-handed weak isosinglets, can favourably account for sizeable branching ratios of τ decays into three charged leptons that can be as large as $\sim 10^{-6}$. Using updated constraints for the mixings $(s_L^{\nu_l})^2$, we have found that our numerical estimates of $B(\tau \to eee)$ and $B(\tau \to e\mu\mu)$ are in qualitative agreement with those obtained in Ref. [34] for $m_N < 200$ GeV. However, our branching-ratio values can be up to 100 times larger than the results reported in Ref. [34] when the heavy neutrinos have TeV masses. The reason is that the flavour-violating decays of the τ lepton show a strong *quadric* mass dependence of the heavy neutrino mass in a complete calculation, which gives a unique chance for such decays to be seen at LEP or planned collider machines.

Apart from general constraints that our minimal models should satisfy and have been taken into account, we have found that $B(Z \to e^-\tau^+ + e^+\tau^-) \lesssim 4 \times 10^{-6}$ within the range allowed by perturbative unitarity. The latter do not yet impose any stringent constraints on the phenomenological parameters of the theory. Moreover, heavy Majorana neutrinos introduce a different behaviour in the transition amplitude via loop effects as compared to heavy Dirac ones. For example, Fig. 6 shows that an appreciably large mass difference between the two heavy Majorana neutrinos N_1 and N_2 (i.e. $m_{N_2}/m_{N_1} \simeq 3$)

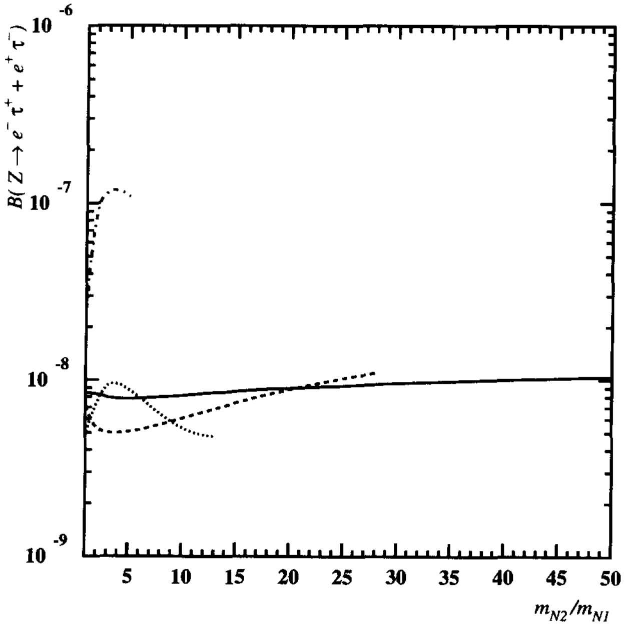

Fig. 8. Numerical estimates of $B(Z \to e^-\tau^+) + B(Z \to e^+\tau^-)$ versus m_{N_2}/m_{N_1} for selected values of $m_{N_1} = 200$ GeV (solid line), 400 GeV (dashed line), 600 GeV (dotted line), and 1 TeV (dash-dotted line). We have used $(s_L^{\nu_\tau})^2 = 0.07$ and $(s_L^{\nu_e})^2 = 0.015$.

will give rise to an enhancement of a factor of two to the corresponding branching-ratio value obtained for $m_{N_1} = m_{N_2}$.

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Appendix A. Loop integrals of leptonic Z-boson decays

After computing the Feynman graphs shown in Fig. 2, we find that the analytic expression of the form factor $\mathcal{F}_Z^{ll'}$ defined in Eq. (3.7) can be cast into the form [11]

$$\mathcal{F}_Z^{ll'} = \sum_{i,j=1}^{n_G+n_R} B_{l'i}B_{lj}^* \Big\{ \delta_{ij} \Big[-\tilde{I}(\lambda_i) - 3c_W^2 L_1(\lambda_i) - s_W^2 \lambda_i I(\lambda_i)$$

$$-\frac{1}{8}(1-2s_W^2)\lambda_i\left(2L_1(\lambda_i)+\frac{3}{2}-\frac{3}{1-\lambda_i}-\frac{(\lambda_i+2)\lambda_i\ln\lambda_i}{(1-\lambda_i)^2}\right)\Bigg]$$
$$+C_{ij}\left(\frac{1}{2}L_2(\lambda_i,\lambda_j)-\frac{1}{2}\lambda_Z[K_1(\lambda_i,\lambda_j)-K_2(\lambda_i,\lambda_j)+\tilde{K}(\lambda_i,\lambda_j)]-\frac{1}{4}\lambda_i\lambda_jK_1(\lambda_i,\lambda_j)\right)$$
$$+C^*_{ij}\sqrt{\lambda_i\lambda_j}\left(\frac{1}{2}K_1(\lambda_i,\lambda_j)+\frac{1}{4}\lambda_Z\tilde{K}(\lambda_i,\lambda_j)-\frac{1}{4}L_2(\lambda_i,\lambda_j)\right)\Bigg\}, \tag{A.1}$$

where $\lambda_i = m_i^2/M_W^2$, $\lambda_Z = M_Z^2/M_W^2$, and the definition of the loop integrals, I, $\tilde{I}$, L_1, K_1, K_2, $\tilde{K}$, and L_2, may be found in Ref. [13]. The analytic expressions of these loop integrals are listed below

$$I(\lambda_i)=\int_0^1\int_0^1\frac{\mathrm{d}x\,\mathrm{d}y\;y}{B_1(\lambda_i)}=-\frac{1}{\lambda_Z}\Bigg[\mathrm{Li}_2\left(\frac{1-\lambda_i}{1-\lambda_i-\lambda_Z\rho_+}\right)-\mathrm{Li}_2\left(\frac{1-\lambda_i-\lambda_Z}{1-\lambda_i-\lambda_Z\rho_+}\right)$$
$$+\mathrm{Li}_2\left(\frac{1-\lambda_i}{1-\lambda_i-\lambda_Z\rho_-}\right)-\mathrm{Li}_2\left(\frac{1-\lambda_i-\lambda_Z}{1-\lambda_Z-\lambda_Z\rho_-}\right)$$
$$-\mathrm{Li}_2\left(\frac{(1-\lambda_i)^2}{(1-\lambda_i)^2+\lambda_i\lambda_Z}\right)+\mathrm{Li}_2\left(\frac{(1-\lambda_i)(1-\lambda_i-\lambda_Z)}{(1-\lambda_i)^2+\lambda_i\lambda_Z}\right)\Bigg], \tag{A.2}$$

$$\tilde{I}(\lambda_i)=\int_0^1\int_0^1\frac{\mathrm{d}x\,\mathrm{d}y\;y^2[1-yx(1-x)]}{B_1(\lambda_i)}=\frac{1}{\lambda_Z}\Bigg[\frac{5}{2}-\frac{2(1-\lambda_i)}{\lambda_Z}$$
$$+2\frac{\lambda_i}{\lambda_Z}\ln\lambda_i-\frac{2\lambda_i}{1-\lambda_i}\ln\lambda_i+4\left(\frac{1-\lambda_i}{\lambda_Z}-1\right)\eta\tan^{-1}\left(\frac{1}{\eta}\right)$$
$$-\frac{2(1-\lambda_i-\lambda_Z)(1-\lambda_i)+\lambda_i\lambda_Z}{\lambda_Z}I(\lambda_i)\Bigg], \tag{A.3}$$

$$L_1(\lambda_i)=\int_0^1\int_0^1\mathrm{d}x\,\mathrm{d}y\;y\ln B_1(\lambda_i)=-\frac{3}{2}+\frac{1-\lambda_i}{\lambda_Z}+\left(1-\frac{2(1-\lambda_i)}{\lambda_Z}\right)\eta\tan^{-1}\left(\frac{1}{\eta}\right)$$
$$-\frac{\lambda_i}{\lambda_Z}\ln\lambda_i+\frac{(1-\lambda_i)^2+\lambda_i\lambda_Z}{\lambda_Z}I(\lambda_i), \tag{A.4}$$

$$K_1(\lambda_i,\lambda_j)=\int_0^1\int_0^1\frac{\mathrm{d}x\,\mathrm{d}y\;y}{B_2(\lambda_i,\lambda_j)}=-\frac{1}{\lambda_Z}\Bigg[\mathrm{Li}_2\left(\frac{1-\lambda_j}{1-\lambda_j+\lambda_Z\xi_+}\right)-\mathrm{Li}_2\left(\frac{1-\lambda_j+\lambda_Z}{1-\lambda_j+\lambda_Z\xi_+}\right)$$
$$+\mathrm{Li}_2\left(\frac{1-\lambda_j}{1-\lambda_j+\lambda_Z\xi_-}\right)-\mathrm{Li}_2\left(\frac{1-\lambda_j+\lambda_Z}{1-\lambda_j+\lambda_Z\xi_-}\right)$$
$$-\mathrm{Li}_2\left(\frac{(1-\lambda_i)(1-\lambda_j)}{(1-\lambda_i)(1-\lambda_j)+\lambda_Z}\right)+\mathrm{Li}_2\left(\frac{(1-\lambda_i)(1-\lambda_j+\lambda_Z)}{(1-\lambda_i)(1-\lambda_j)+\lambda_Z}\right)\Bigg], \tag{A.5}$$

$$K_2(\lambda_i,\lambda_j) = \int_0^1\int_0^1 \frac{\mathrm{d}x\,\mathrm{d}y\ y^2}{B_2(\lambda_i,\lambda_j)} = -\frac{1}{\lambda_Z}\Bigg[-1+\frac{1}{1-\lambda_i}\ln\lambda_i$$
$$-\left(\frac{1}{2}-\frac{\lambda_i-\lambda_j}{2\lambda_Z}\right)\ln\left(\frac{\lambda_i}{\lambda_j}\right)+\frac{\sqrt{w}}{\lambda_Z}\tan^{-1}\left(\frac{\sqrt{w}}{\lambda_i+\lambda_j-\lambda_Z}\right)$$
$$+(1-\lambda_j)K_1(\lambda_i,\lambda_j)\Bigg]+(\lambda_i\leftrightarrow\lambda_j), \tag{A.6}$$

$$\tilde{K}(\lambda_i,\lambda_j) = \int_0^1\int_0^1 \frac{\mathrm{d}x\,\mathrm{d}y\ y^3x(1-x)}{B_2(\lambda_i,\lambda_j)} = -\frac{1}{\lambda_Z}\Bigg[\frac{1}{2}+\frac{2-\lambda_i-\lambda_j}{\lambda_Z}$$
$$-\frac{1}{\lambda_Z}\ln(\lambda_i\lambda_j)+\frac{(2-\lambda_i-\lambda_j+\lambda_Z)(\lambda_j-\lambda_i)}{2\lambda_Z^2}\ln\left(\frac{\lambda_i}{\lambda_j}\right)$$
$$-\frac{2-\lambda_i-\lambda_j}{\lambda_Z}\,\frac{\sqrt{w}}{\lambda_Z}\tan^{-1}\left(\frac{\sqrt{w}}{\lambda_i+\lambda_j-\lambda_Z}\right)$$
$$-\left(1+\frac{2(1-\lambda_i)(1-\lambda_j)}{\lambda_Z}\right)K_1(\lambda_i,\lambda_j)\Bigg], \tag{A.7}$$

$$L_2(\lambda_i,\lambda_j) = \int_0^1\int_0^1 \mathrm{d}x\,\mathrm{d}y\ y\ln B_2(\lambda_i,\lambda_j) = -\frac{3}{2}-\frac{2-\lambda_i-\lambda_j}{2\lambda_Z}+\frac{1}{2}\left(\frac{1}{2}+\frac{1}{\lambda_Z}\right)\ln(\lambda_i\lambda_j)$$
$$+\frac{\lambda_i-\lambda_j}{4\lambda_Z^2}(2+2\lambda_Z-\lambda_i-\lambda_j)\ln\left(\frac{\lambda_i}{\lambda_j}\right)+\frac{2-\lambda_i-\lambda_j+\lambda_Z}{2\lambda_Z}$$
$$\times\frac{\sqrt{w}}{\lambda_Z}\tan^{-1}\left(\frac{\sqrt{w}}{\lambda_i+\lambda_j-\lambda_Z}\right)+\frac{(1-\lambda_i)(1-\lambda_j)+\lambda_Z}{\lambda_Z}K_1(\lambda_i,\lambda_j), \tag{A.8}$$

where $\rho_\pm = (1\pm i\eta)/2$ with $\eta = \sqrt{4\lambda_Z^{-1}-1}$, $\xi_\pm = (\lambda_Z-\lambda_i+\lambda_j\pm i\sqrt{w})/2\lambda_Z$ with $w = 4\lambda_i\lambda_j-(\lambda_Z-\lambda_i-\lambda_j)^2$, and

$$B_1(\lambda_i) = (1-y)\lambda_i + y[1-\lambda_Z yx(1-x)], \tag{A.9}$$
$$B_2(\lambda_i,\lambda_j) = 1-y+y[x\lambda_i+(1-x)\lambda_j-\lambda_Z yx(1-x)]. \tag{A.10}$$

Note that $w\geqslant 0$ for $|\sqrt{\lambda_i}-\sqrt{\lambda_j}|\leqslant\sqrt{\lambda_Z}\leqslant\sqrt{\lambda_i}+\sqrt{\lambda_j}$. If $\sqrt{\lambda_i}+\sqrt{\lambda_j}<\sqrt{\lambda_Z}$, then one has to analytically continue the function

$$\sqrt{w}\tan^{-1}\left(\frac{\sqrt{w}}{\lambda_i+\lambda_j-\lambda_Z}\right) = 2\sqrt{w}\tan^{-1}\left(\sqrt{\frac{\lambda_Z-(\sqrt{\lambda_i}-\sqrt{\lambda_j})^2}{(\sqrt{\lambda_i}+\sqrt{\lambda_j})^2-\lambda_Z}}\right)$$
$$\to\sqrt{-w}\ \ln\left(\frac{\sqrt{1-\frac{(\sqrt{\lambda_i}-\sqrt{\lambda_j})^2}{\lambda_Z}}+\sqrt{1-\frac{(\sqrt{\lambda_i}+\sqrt{\lambda_j})^2}{\lambda_Z}}}{\sqrt{1-\frac{(\sqrt{\lambda_i}-\sqrt{\lambda_j})^2}{\lambda_Z}}-\sqrt{1-\frac{(\sqrt{\lambda_i}+\sqrt{\lambda_j})^2}{\lambda_Z}}}\right)-i\pi\sqrt{-w}. \tag{A.11}$$

The dilogarithmic function $\mathrm{Li}_2(x)$ (with x being real) should also be continued analytically as follows:

$$\mathrm{Li}_2(x \pm i\varepsilon) = -\int_0^x dt \frac{\ln|1-t|}{t} \pm i\theta(x-1)\,\pi \ln x. \tag{A.12}$$

For $\sqrt{\lambda_Z} < |\sqrt{\lambda_i} - \sqrt{\lambda_j}|$, we have checked that Eq. (A.5) and the l.h.s of Eq. (A.11) do not contain any imaginary part. This implies that $\mathcal{F}_Z^{ll'}$ is pure dispersive in this specific kinematic range.

As we are interested in heavy neutrinos with masses larger than M_Z, the absorptive part of $\mathcal{F}_Z^{ll'}$ will solely originate from Fig. 2(i) in which only intermediate light neutrinos can come kinematically on-mass shell. Neglecting light neutrino masses in the calculation, we get

$$\mathrm{Abs}(\mathcal{F}_Z^{ll'}) = i\pi \sum_{i=1}^{n_R} B_{lN_i} B^*_{l'N_i} \left[-\frac{3}{2} - \frac{1}{\lambda_Z} + \left(1 + \frac{1}{\lambda_Z}\right)^2 \ln(1+\lambda_Z) \right]. \tag{A.13}$$

Appendix B. Loop functions of flavour-violating decays of charged leptons

In Section 4 the amplitudes of the flavour-violating decays of l, $l \to l' l_1 \bar{l}_2$ and $l \to l'\gamma$, are expressed in terms of all possible form factors that are derived by an explicit calculation of the Feynman graphs shown in Figs. 1–3. The photonic form factors, F_γ and G_γ in Eq. (4.1), vanish in the limit of zero external momenta and lepton masses due to the electromagnetic gauge invariance. One has consistently to expand the corresponding loop integrals up to the next order of q^2 [57] in order to obtain a nonvanishing result. After a straightforward computation, we find that

$$F_\gamma(x) = \frac{7x^3 - x^2 - 12x}{12(1-x)^3} - \frac{x^4 - 10x^3 + 12x^2}{6(1-x)^4} \ln x, \tag{B.1}$$

$$G_\gamma(x) = -\frac{2x^3 + 5x^2 - x}{4(1-x)^3} - \frac{3x^3}{2(1-x)^4} \ln x, \tag{B.2}$$

$$F_Z(x) = -\frac{5x}{2(1-x)} - \frac{5x^2}{2(1-x)^2} \ln x, \tag{B.3}$$

$$G_Z(x,y) = -\frac{1}{2(x-y)} \left[\frac{x^2(1-y)}{1-x} \ln x - \frac{y^2(1-x)}{1-y} \ln y \right], \tag{B.4}$$

$$H_Z(x,y) = \frac{\sqrt{xy}}{4(x-y)} \left[\frac{x^2 - 4x}{1-x} \ln x - \frac{y^2 - 4y}{1-y} \ln y \right], \tag{B.5}$$

$$\begin{aligned} F_{\mathrm{Box}}(x,y) = \frac{1}{x-y} \Bigg[& \left(1 + \frac{xy}{4}\right) \left(\frac{1}{1-x} + \frac{x^2 \ln x}{(1-x)^2} - \frac{1}{1-y} - \frac{y^2 \ln y}{(1-y)^2} \right) \\ & -2xy \left(\frac{1}{1-x} + \frac{x \ln x}{(1-x)^2} - \frac{1}{1-y} - \frac{y \ln y}{(1-y)^2} \right) \Bigg], \end{aligned} \tag{B.6}$$

$$G_{\text{Box}}(x,y) = -\frac{\sqrt{xy}}{x-y}\left[(4+xy)\left(\frac{1}{1-x}+\frac{x\ln x}{(1-x)^2}-\frac{1}{1-y}-\frac{y\ln y}{(1-y)^2}\right)\right.$$
$$\left.-2\left(\frac{1}{1-x}+\frac{x^2\ln x}{(1-x)^2}-\frac{1}{1-y}-\frac{y^2\ln y}{(1-y)^2}\right)\right]. \tag{B.7}$$

Although F_γ, G_γ, F_Z, and F_{Box} are already known in the literature [57-59], the form factors G_Z, H_Z and G_{Box} are newly obtained by Eqs. (B.4), (B.5) and (B.7), respectively.

For completeness, we list below expressions of the form factors computed at some special values of the arguments

$$F_\gamma(1) = -\frac{25}{72}, \qquad F_\gamma(0) = 0; \tag{B.8}$$

$$G_\gamma(1) = \frac{1}{8}, \qquad G_\gamma(0) = 0; \tag{B.9}$$

$$F_Z(1) = -\frac{5}{4}, \qquad F_Z(0) = 0; \tag{B.10}$$

$$G_Z(x,x) = -\frac{x}{2}-\frac{x\ln x}{1-x}, \qquad G_Z(0,x) = -\frac{x\ln x}{2(1-x)}, \qquad G_Z(1,x) = \frac{1}{2},$$
$$G_Z(0,0) = 0, \qquad G_Z(1,0) = \frac{1}{2}, \qquad G_Z(1,1) = \frac{1}{2}; \tag{B.11}$$

$$H_Z(x,x) = \frac{3}{4}-\frac{x}{4}-\frac{3}{4(1-x)}-\frac{x^3-2x^2+4x}{4(1-x)^2}\ln x,$$
$$H_Z(1,x) = \frac{\sqrt{x}}{4}\left[\frac{3}{1-x}-\frac{x^2-4x}{(1-x)^2}\ln x\right],$$
$$H_Z(0,x) = 0, \qquad H_Z(0,0) = 0, \qquad H_Z(1,0) = 0, \qquad H_Z(1,1) = \frac{1}{8}; \tag{B.12}$$

$$F_{\text{Box}}(x,x) = -\frac{x^4-16x^3+19x^2-4}{4(1-x)^3}-\frac{3x^3+4x^2-4x}{2(1-x)^3}\ln x,$$
$$F_{\text{Box}}(1,x) = -\frac{5x^3-8x^2+7x-4}{8(1-x)^3}-\frac{x^3-4x^2}{4(1-x)^3}\ln x,$$
$$F_{\text{Box}}(0,x) = \frac{1}{1-x}+\frac{x\ln x}{(1-x)^2},$$
$$F_{\text{Box}}(0,0) = 1, \qquad F_{\text{Box}}(1,0) = \frac{1}{2}, \qquad F_{\text{Box}}(1,1) = \frac{3}{4}; \tag{B.13}$$

$$G_{\rm Box}(x,x) = \frac{2x^4-4x^3+8x^2-6x}{(1-x)^3} - \frac{x^4+x^3+4x}{(1-x)^3}\ln x,$$

$$G_{\rm Box}(1,x) = -\sqrt{x}\left[\frac{x^3-2x^2+7x-6}{2(1-x)^3} + \frac{x^2-4x}{(1-x)^3}\ln x\right],$$

$$G_{\rm Box}(0,x)=0,\quad G_{\rm Box}(0,0)=0,\quad G_{\rm Box}(1,0)=0,\quad G_{\rm Box}(1,1)=\frac{3}{2}. \tag{B.14}$$

Since all the form factors given in Eqs. (B.1)-(B.7) are multiplied by certain combinations of B and C matrices in the decay amplitudes (4.1), (4.6), (4.7) and (4.8), it will be helpful to define the following composite form factors:

$$F_\gamma^{ll'} = \sum_i B^*_{li}B_{l'i}F_\gamma(\lambda_i) = \sum_{N_i} B^*_{lN_i}B_{l'N_i}F_\gamma(\lambda_{N_i}), \tag{B.15}$$

$$G_\gamma^{ll'} = \sum_i B^*_{li}B_{l'i}G_\gamma(\lambda_i) = \sum_{N_i} B^*_{lN_i}B_{l'N_i}G_\gamma(\lambda_{N_i}), \tag{B.16}$$

$$\begin{aligned}F_Z^{ll'} &= \sum_{ij} B^*_{li}B_{l'j}\Big[\delta_{ij}F_Z(\lambda_i) + C^*_{ij}G_Z(\lambda_i,\lambda_j) + C_{ij}H_Z(\lambda_i,\lambda_j)\Big]\\ &= \sum_{N_iN_j} B^*_{lN_i}B_{l'N_j}\Big[\delta_{N_iN_j}(F_Z(\lambda_{N_i}) + 2G_Z(0,\lambda_{N_i})) + C^*_{N_iN_j}(G_Z(\lambda_{N_i},\lambda_{N_j})\\ &\quad -G_Z(0,\lambda_{N_i}) - G_Z(0,\lambda_{N_j})) + C_{N_iN_j}H_Z(\lambda_{N_i},\lambda_{N_j})\Big],\end{aligned} \tag{B.17}$$

$$\begin{aligned}F_{\rm Box}^{ll'l_1l_2} &= \sum_{ij} B^*_{li}B^*_{l_2j}(B_{l'i}B_{l_1j} + B_{l_1i}B_{l'j})\,F_{\rm Box}(\lambda_i,\lambda_j)\\ &\quad + \sum_{ij} B^*_{li}B^*_{l_2i}B_{l'j}B_{l_1j}\,G_{\rm Box}(\lambda_i,\lambda_j)\\ &= \sum_{N_iN_j}\Big[(B^*_{lN_i}B_{l'N_i}\delta_{l_1l_2} + B^*_{lN_i}B_{l_1N_i}\delta_{l'l_2})\delta_{N_iN_j}\Big[F_{\rm Box}(0,\lambda_{N_i}) - F_{\rm Box}(0,0)\Big]\\ &\quad + B^*_{lN_i}B^*_{l_2N_j}(B_{l'N_i}B_{l_1N_j} + B_{l_1N_i}B_{l'N_j})\\ &\quad \times\Big[F_{\rm Box}(\lambda_{N_i},\lambda_{N_j}) - F_{\rm Box}(0,\lambda_{N_j}) - F_{\rm Box}(0,\lambda_{N_i}) + F_{\rm Box}(0,0)\Big]\\ &\quad + B^*_{lN_i}B^*_{l_2N_i}B_{l'N_j}B_{l_1N_j}G_{\rm Box}(\lambda_{N_i},\lambda_{N_j})\Big],\end{aligned} \tag{B.18}$$

where we have made use of the identities of Eq. (2.9) in the final step of the Eqs. (B.15)-(B.18) and re-expressed all the composite form factors as a sum over the heavy neutrino states. This simplification enables us to study the behaviour of these form factors in the heavy neutrino limit.

For the purpose of illustration, we will discuss the results of this asymptotic limit in a model with two heavy Majorana neutrinos. Employing Eqs. (5.1) and (5.2) for the mixing matrices B and C, we find that for $\lambda_{N_1} = m^2_{N_1}/M^2_W \gg 1$ and $\rho = m^2_{N_2}/m^2_{N_1} \gg 1$,

$$F_\gamma^{ll'} \to -\frac{1}{6}\, s_L^{\nu_l} s_L^{\nu_{l'}} \ln \lambda_{N_1}, \tag{B.19}$$

$$G_\gamma^{ll'} \to \frac{1}{2}\, s_L^{\nu_l} s_L^{\nu_{l'}}, \tag{B.20}$$

$$\begin{aligned} F_Z^{ll'} \to & -\frac{3}{2} s_L^{\nu_l} s_L^{\nu_{l'}} \ln \lambda_{N_1} \\ & + s_L^{\nu_l} s_L^{\nu_{l'}} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2 \frac{\lambda_{N_1}}{(1+\rho^{\frac{1}{2}})^2} \left(-\frac{3}{2}\rho + \frac{-\rho + 4\rho^{\frac{3}{2}} - \rho^2}{4(1-\rho)} \ln \rho \right), \end{aligned} \tag{B.21}$$

$$\begin{aligned} F_{\mathrm{Box}}^{ll'l_1l_2} \to & -\left(s_L^{\nu_l} s_L^{\nu_{l'}} \delta_{l_1l_2} + s_L^{\nu_l} s_L^{\nu_{l_1}} \delta_{l'l_2} \right) \\ & + s_L^{\nu_l} s_L^{\nu_{l'}} s_L^{\nu_{l_1}} s_L^{\nu_{l_2}} \frac{\lambda_{N_1}}{(1+\rho^{\frac{1}{2}})^2} \left(-\rho - \frac{\rho + \rho^{\frac{3}{2}} + \rho^2}{1-\rho} \ln \rho \right). \end{aligned} \tag{B.22}$$

In the limit $\rho \to 1$ and for $\lambda_{N_1} \gg 1$, Eqs. (B.21) and (B.22) take the form

$$F_Z^{ll'} \to -\frac{3}{2} s_L^{\nu_l} s_L^{\nu_{l'}} \ln \lambda_{N_1} - \frac{1}{2} s_L^{\nu_l} s_L^{\nu_{l'}} \sum_{i=1}^{n_G} (s_L^{\nu_i})^2 \lambda_{N_1}, \tag{B.23}$$

$$F_{\mathrm{Box}}^{ll'l_1l_2} \to -\left(s_L^{\nu_l} s_L^{\nu_{l'}} \delta_{l_1l_2} + s_L^{\nu_l} s_L^{\nu_{l_1}} \delta_{l'l_2} \right) + \frac{1}{2} s_L^{\nu_l} s_L^{\nu_{l'}} s_L^{\nu_{l_1}} s_L^{\nu_{l_2}} \lambda_{N_1}. \tag{B.24}$$

From Eqs. (B.19)–(B.24), it is obvious that all the composite form factors, $F_\gamma^{ll'}$, $G_\gamma^{ll'}$, $F_Z^{ll'}$, and $F_{\mathrm{Box}}^{ll'l_1l_2}$, violate the decoupling theorem [28]. Note that terms of $\mathcal{O}((s_L^{\nu_l})^2)$ in $F_Z^{ll'}$ depend logarithmically on the heavy neutrino mass, m_{N_1}, while terms proportional to $(s_L^{\nu_l})^4$ in Eqs. (B.23) and (B.24) show a strong quadratic, $m_{N_1}^2/M_W^2$, dependence and should not be neglected in the calculation when $m_{N_1} > 200$ GeV (see, e.g., Fig. 4).

Appendix C. Three-body phase-space integrals

As we have seen from Eq. (4.6), the γ-mediated amplitude of the decay $l(p) \to l'(p')l_1(p_1)\bar{l}_2(p_2)$ contains a non-local interaction which leads to a collinear singularity in the limit $q^2 \equiv (p_1+p_2)^2 \to 0$. This divergency can only be avoided if one assumes that the leptons, l_1 and l_2, coupled to the virtual photon are not strictly massless, i.e. $m_{l_1} = m_{l_2} = \varepsilon \neq 0$. Thus, after performing phase-space integration, we neglect all those terms that vanish as $\varepsilon \to 0$. On the other hand, the mass of l' can safely be set to zero. Then, the phase-space boundaries can be given by

$$4\varepsilon^2 \leqslant s_2 \leqslant m^2, \qquad s_1^{\pm} = \frac{1}{2}(m^2 - s_2)\left[1 \pm \sqrt{1 - \frac{4\varepsilon^2}{s_2}} \right] + \varepsilon^2, \tag{C.1}$$

where $s_1 = (p'+p_2)^2$, $s_2 = (p_1+p_2)^2$, m is the mass of the decaying lepton l, and $s_1^{+(-)}$ is the upper (lower) limit of the Mandelstam variable s_1.

The divergent phase-space integrals relevant for the decay $l \to l' l_1 \bar{l}_2$ (with $l' \neq l_2$) are given by the following expressions:

$$P_1 = \int\int ds_2 ds_1 \frac{1}{s_2} = m^2 \left(\ln \frac{m^2}{\varepsilon^2} - 3 \right) + \mathcal{O}(\varepsilon), \tag{C.2}$$

$$P_2 = \int\int ds_2 ds_1 \frac{s_1}{s_2} = m^4 \left(\frac{1}{2} \ln \frac{m^2}{\varepsilon^2} - \frac{7}{4} \right) + \mathcal{O}(\varepsilon), \tag{C.3}$$

$$P_3 = \int\int ds_2 ds_1 \frac{s_1^2}{s_2} = m^6 \left(\frac{1}{3} \ln \frac{m^2}{\varepsilon^2} - \frac{4}{3} \right) + \mathcal{O}(\varepsilon). \tag{C.4}$$

Note that a different result would have been obtained in Eqs. (C.2)-(C.4) if we had originally expanded the square root existing in $s_1^{\pm}$ in terms of ε and then performed the phase-space integration. Apparently, this technical problem seems to have caused some confusion in the literature, as far as the correct analytic expression of the non-local interaction in Eqs. (4.9) and (4.10) is concerned. In the three-body leptonic decays of l where $l' = l_1 = l_2$, one may have to take into account an additional divergent phase space integral when interfering the two possible, s_1-channel and s_2-channel, γ-exchange amplitudes, i.e.

$$P_4 = \int\int ds_2\, ds_1 \frac{1}{s_2 s_1} = -\frac{\pi^2}{12} - \ln^2 2 + \frac{1}{2} \ln^2 \frac{m^2}{\varepsilon^2} + \mathcal{O}(\varepsilon). \tag{C.5}$$

The integral P_4 in Eq. (C.5), however, is always multiplied by the small mass ε of the final leptons and therefore goes to zero as $\varepsilon \to 0$. As a result, the only type of divergency for $\varepsilon \to 0$ that appears in Eqs. (4.9) and (4.10) is the logarithmic one, $\ln(m^2/\varepsilon^2)$.

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