# $e^{+} e^{-} \rightarrow W^{-} e^{+} v$ and non-standard gauge couplings: another look 

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#### Abstract

The cross section for single $W^{-}$production in $e^{+} e^{-}$collisions is exactly computed for an arbitrary $W$ magnetic moment. It is found that this process may give a first look at the $W W \gamma$ vertex just below the two $W$ threshold. A detailed comparison is made with previous calculations that used the equivalent photon approximation.


## 1 Introduction

At present, the experimental success of the standard model of electroweak interactions is an unpleasant feature for the theorists who feel that there is something missing. Preferably, one would like to show that this model is just a good effective theory at energies $\lesssim G_{F}^{-1 / 2}$.
In this paper we address ourselves to the problem of finding deviations of the $W$ electromagnetic coupling from its standard model form, looking at single $W^{-}$production in $e^{+} e^{-}$collisions. This has been done before [1-2], but in a way that we do not find completely satisfying: only a subset of the diagrams has been taken into account and also use has been made of the equivalent photon (WeizsäckerWilliams) approximation. We have exactly computed the contribution to the cross section from all the twelve diagrams that contribute to this process at tree level. This was done using the spinor product technique of Kleiss [3]. We have found that in general it is necessary to consider the full contribution specially for high energies and for realistic cuts that are necessary to account for the impossibility of detection along the beam axis.
At LEP-I this process has a negligible cross section $\left(\sigma \simeq 6 \times 10^{-3} \mathrm{pb}\right.$ at $\left.\sqrt{s}=100 \mathrm{GeV}\right)$, which makes useless any attempt to find deviations from the standard model predictions. But this cross section rises fast with energy, being about 0.2 pb just below the $W^{+} W^{-}$production threshold. If the schedule [4] for LEP-II is to be followed, then at least one year will
be spent at energies $\sqrt{s} \simeq 150 \mathrm{GeV}$. This possibility will rend more relevant a precise study, because it is precisely at high energy that the approximations considered before are more inaccurate. If the process $e^{+} e^{-} \rightarrow W^{+} W^{-}$is expected to provide the most accurate measurement of the $W$ magnetic moment [5], single $W$ production just below the two $W$ production threshold could provide a first measurement (although more rough) before the completion of LEP-II.

The paper is organized as follows. In Sect. 2 we give the expression for the exact cross section. In Sect. 3 the equivalent photon approximation is discussed. The results and discussion are presented in Sect. 4. In the Appendix we collect the expressions for the helicity amplitudes and some other useful formulae.

## 2 The helicity amplitudes for $\boldsymbol{e}^{+} \boldsymbol{e}^{-} \rightarrow \boldsymbol{W}^{-} \boldsymbol{e}^{+} \boldsymbol{v}$

Let us denote by $p_{-}\left(p_{+}\right)$the momentum of the incoming electron (positron), by $p_{1}\left(p_{2}\right)$ the momentum of the outgoing neutrino (positron) and by $p_{W}$ the $W^{-}$ momentum. Then if we choose as independent phase space variables the solid angles of the $W^{-}$and of the positron in the C.M.-frame, and the invariant mass squared of the neutrino-positron pair,
$m_{12}^{2}=\left(p_{1}+p_{2}\right)^{2}$,
the differential cross section for the process can be written in the form

$$
\begin{align*}
d \sigma= & \frac{1}{2 s(2 \pi)^{5}} \sum_{\text {pol }}|T|^{2} \frac{\left|\mathbf{p}_{W}\right|}{16 \sqrt{s}} \frac{\left|\mathbf{p}_{2}\right|^{3}}{\left|\mathbf{p}_{2}\right|^{2}\left(\sqrt{s}-E_{W}\right)+E_{2} \mathbf{p}_{2} \cdot \mathbf{p}_{W}} \\
& \cdot d m_{12}^{2} d \Omega_{W} d \Omega_{2} \tag{2}
\end{align*}
$$

In the last expression $\sum_{\text {pol }}$ means, as usual, average over initial state and sum over final state polarizations, $\sqrt{s}$ is the C.M. energy and $T$ is the invariant $T$-matrix, i.e.
$T \equiv \mathscr{M}_{\mu} \varepsilon^{\mu}\left(p_{W}\right)$.

1

7

9


Fig. 1. Feynman diagrams for $e^{+} e^{-} \rightarrow W^{-} e^{+} v$

In Fig. 1 we show the twelve diagrams that contribute to $T$ at tree level. Diagrams 2 and 7 have the $W^{+} W^{-} \gamma$ coupling. As we want to test the possibility of having a non standard magnetic moment for the $W$, we write this coupling, with the conventions of Fig. 2, in the form

$$
\begin{align*}
\Gamma^{\mu \alpha \beta}= & -i e\left\{g^{\alpha \beta}(p-k)^{\mu}+g^{\beta \mu}[k-(1+\Delta k) q]^{\alpha}\right. \\
& \left.+g^{\mu \alpha}[(1+\Delta k) q-p]^{\beta}\right\} . \tag{4}
\end{align*}
$$

For the standard model we have $\Delta k=0$, while for a charged vector particle "minimally" coupled to the photon, $\Delta k=-1$.

We evaluate the helicity amplitudes using the spinor product formalism of [3]. In this technique the $W$ polarization vector is defined to be
$\varepsilon^{\mu}\left(p_{W}\right) \Leftrightarrow\left(\frac{3}{8 \pi M_{W}^{2}}\right)^{1 / 2} a^{u} ; \quad a^{\mu}=\bar{u}_{-}\left(r_{1}\right) \gamma^{\mu} u_{-}\left(r_{2}\right)$,
where $r_{1}$ and $r_{2}$ are light-like vectors such that $p_{W}=$ $r_{1}+r_{2}$. The sum over the $W$ polarizations is then


Fig. 2. The $W^{+} W^{-} \gamma$ vertex
replaced by an angular integration over $d \Omega_{r_{1}}^{*}$ in the frame where the $W$ is at rest. In fact one can show [3] that
$\int d \Omega_{\mathrm{r}_{1}}^{*}\left(\frac{3}{8 \pi M_{W}^{2}}\right) a^{* \mu} a^{v}=-g^{\mu v}+\frac{p_{W}^{\mu} p_{W}^{v}}{M_{W}^{2}}$.
It's convenient to define a dimensionless scattering amplitude $\widetilde{T}$ by
$\tilde{T} \equiv \tilde{\mathscr{M}}_{\mu} a^{\mu} ; \quad \mathscr{M}_{\mu}=\frac{1}{\sqrt{2}}\left(\frac{e}{\sin \theta_{W}}\right)^{3} \tilde{\mathscr{M}}_{\mu}$.
Then we can write the differential cross section as

$$
\begin{align*}
d \sigma= & \frac{3}{32}\left(\frac{\alpha}{2 \pi \sin ^{2} \theta_{W}}\right)^{3} \sum_{\mathrm{Pol}}|\tilde{T}|^{2} \cdot \frac{\left|\mathbf{p}_{W}\right|}{s \sqrt{s} M_{W}^{2}} \\
& \cdot \frac{\left|\mathbf{p}_{2}\right|^{3}}{\left(\mathbf{p}_{2}\right)^{2}\left(\sqrt{s}-E_{W}\right)+E_{2} \mathbf{p}_{2} \cdot \mathbf{p}_{W}} \cdot d m_{12}^{2} d \Omega_{W} d \Omega_{2} d \Omega_{\mathrm{r}_{1}}^{*} \tag{8}
\end{align*}
$$

For each diagram we have an helicity amplitude $\widetilde{T}_{i}\left(\sigma_{-}, \sigma_{+}\right)$where $\sigma_{-}\left(\sigma_{+}\right)$is the electron (positron) helicity. At high energy it is a very good approximation to neglect the electron mass everywhere except in the photon propagators of diagrams 5 and 7 . Then the amplitudes $\widetilde{T}_{i}\left(\sigma_{-}, \sigma_{+}\right)$are easily derived using the spinor product formalism [3]. If $u_{ \pm}(p)$ are chiral spinors satisfying the equation

$$
\begin{equation*}
u_{ \pm}(p) \bar{u}_{ \pm}(p)=\gamma_{ \pm} \not p, \tag{9}
\end{equation*}
$$

with $\gamma_{ \pm}=\left(1 \pm \gamma_{5}\right) / 2$, the only non-zero spinor products are [3],
$s\left(p_{1}, p_{2}\right)=\bar{u}_{+}\left(p_{1}\right) u_{-}\left(p_{2}\right)=-s\left(p_{2}, p_{1}\right)$
and
$t\left(p_{1}, p_{2}\right)=\bar{u}_{-}\left(p_{1}\right) u_{+}\left(p_{2}\right)=s^{*}\left(p_{2}, p_{1}\right)$.
The amplitudes $\widetilde{T}_{i}$ are given in the Appendix in terms of these spinor products. We have then

$$
\begin{align*}
\sum_{\text {pol }}|\widetilde{T}|^{2}= & \frac{1}{4}\left[|\widetilde{T}(+,-)|^{2}+|\widetilde{T}(-,+)|^{2}\right. \\
& \left.+|\widetilde{T}(-,-)|^{2}\right] . \tag{11}
\end{align*}
$$

From these expressions we see the simplicity of the spinor product formalism. The amplitude corresponding to each diagram is a complex number easily calculated from the expressions given in the Appendix. To sum all diagrams corresponding to a particular
helicity amplitude one has only to add the complex numbers corresponding to each diagram that contributes. This is a fantastic simplification compared with the usual trace techniques, even with the use of symbolic programs. Another advantage concerns the case of having polarization, because we directly have the necessary helicity amplitudes.

Of course the phase space integration has to be done numerically. This is not a serious drawback, because for phase spaces with three or more final particles most of it has to be done numerically anyway. We used RIWIAD [6], a Monte-Carlo integration routine from the CERN program Library.

Before we close this section let us discuss the inclusion of the finite width of the $W$. Clearly this will be important only near the threshold for the process $e^{+} e^{-} \rightarrow W^{-} e^{+} v$, that is, $\sqrt{s} \gtrsim M_{W}$ and close to the two $W$ production threshold, which is $\sqrt{s}=2 M_{W}$. These effects can be easily incorporated. If we neglect the masses of the $W^{-}$decay products, we get for the cross section corrected for finite $W$ width [7]

$$
\begin{align*}
\sigma_{\mathrm{corf}}= & \left(\frac{\Gamma_{W}}{\pi M_{W}}\right)_{\left(M_{W}^{-}\right.}^{\left.M_{W}+\Delta\right)^{2}} \int_{-}^{2} d m_{-}^{2} \frac{m_{-}^{2}}{\left(m_{-}^{2}-M_{W}^{2}\right)^{2}+\Gamma_{W}^{2} M_{W}^{2}} \\
& \cdot \sigma\left(e^{+} e^{-} \rightarrow W^{-}\left(m_{-}\right) e^{+} v\right), \tag{12}
\end{align*}
$$

where $\Gamma_{W}$ is the $W$ width and $\sigma\left(e^{+} e^{-} \rightarrow W^{-}\left(m_{-}\right) e^{+} v\right)$ is the cross section to produce a $W^{-}$with mass $m_{-}$ in the interval $M_{W}-\Delta \leqq m_{-} \leqq M_{W}+\Delta$.

In Sect. 4 we will present and discuss the results obtained with $(11,12)$.

## 3 The equivalent photon approximation (EPA)

The previous calculations of this process [1,2] used the equivalent photon approximation (EPA) of Weizsäcker and Williams [8]. In this approximation only diagrams 5 and 7 of Fig. 1 are considered and the cross section is given by
$\sigma\left(e^{+} e^{-} \rightarrow W^{-} e^{+} \nu\right)=\int_{\omega_{\text {min }}}^{\omega_{\text {max }}} \frac{d \omega}{\omega} N(\omega) \tilde{\sigma}\left(\gamma e^{-} \rightarrow W^{-} \nu\right)$
where $\tilde{\sigma}$ is the cross section for the process $\gamma e^{-} \rightarrow W^{-} v$ with real photons, $\omega$ is the photon energy and $N(\omega)$ is the equivalent photon spectrum.
In the following section the exact cross section is compared with the EPA result. Here we want to see how good the approximation is compared with the exact result for diagrams 5 and 7. In doing this we realized that the expressions for $N(\omega)$ used in $[1,2]$ were only approximate. In particular, when the positron scattering angle, $\theta$, was very small one should expect the EPA to be quite good but that was not the case, it was worse than if $\theta_{\min }$ was small but not extremelly small (for instance a few degrees).
To understand this difference we went back to the definition of $N(\omega)$ given, for instance, in [9]. We have

$$
\begin{align*}
N(\omega)= & \frac{\alpha}{\pi} \int_{\cos \theta_{\max }}^{\cos \theta_{\min }} d \cos \theta \frac{\omega^{2} E^{\prime}}{E}\left(\frac{1}{-q^{2}}\right)^{2} \\
& \cdot\left[-q^{2}+\frac{2 E^{2} E^{2}}{|\mathbf{q}|^{2}} \sin ^{2} \theta\right] \tag{14}
\end{align*}
$$

where $q \equiv(\omega, \mathbf{q})$ is the photon 4 -momentum, $E\left(E^{\prime}\right)$ is the incident (outgoing) positron energy.

The positron mass has to be retained in the photon denominator. As we want to compare the result of EPA with the exact result of diagrams 5 and 7 using the formalism of the previous section, and there we neglected the electron (positron) mass in every place except in the photon denominator, we should adopt the same procedure here. This means that in the square bracket in (14) we should put $m_{e}=0$. Then a trivial calculation gives

$$
\begin{align*}
N(\omega)= & \frac{\alpha}{2 \pi} \frac{\omega^{2}}{E^{2}}\left\{C_{1} \ln \left(\frac{\beta-\cos \theta_{\max }}{\beta-\cos \theta_{\min }}\right)\right. \\
& +C_{2} \ln \left(\frac{\gamma-\cos \theta_{\max }}{\gamma-\cos \theta_{\min }}\right) \\
& \left.+C_{3} \frac{\cos \theta_{\min }-\cos \theta_{\max }}{\left(\beta-\cos \theta_{\min }\right)\left(\beta-\cos \theta_{\max }\right)}\right\} \tag{15}
\end{align*}
$$

where

$$
\begin{align*}
& \beta=1+\frac{m_{e}^{2} \omega^{2}}{2 E^{2} E^{\prime 2}} \\
& \gamma=1+\frac{\omega^{2}}{2 E E^{\prime}} \tag{16}
\end{align*}
$$

and

$$
C_{1}=1 / 2-C_{2} / 2
$$

$$
\begin{equation*}
C_{2}=-\frac{1+\frac{4 E E^{\prime}}{\omega^{2}}}{\left(1-\frac{m_{e}^{2}}{E E^{\prime}}\right)^{2}} \tag{17}
\end{equation*}
$$

$$
C_{3}=-2 \frac{m_{e}^{2}}{E E^{\prime}} \frac{1+\frac{2 E E^{\prime}}{\omega^{2}}-\frac{m_{e}^{2} \omega^{2}}{4 E^{2} E^{\prime 2}}}{\left(1-\frac{m_{e}^{2}}{E E^{\prime}}\right)}
$$

The expression used in [1, 2],

$$
\begin{align*}
N(\omega)= & \frac{\alpha}{2 \pi}\left[1+\left(1-\frac{\omega}{E}\right)^{2}\right] \\
& \ln \left(\frac{1+\frac{m_{e}^{2}}{2 E^{2}} \frac{(\omega / E)^{2}}{1-\omega / E)^{2}}-\cos \theta_{\max }}{1+\frac{m_{e}^{2}}{2 E^{2}} \frac{(\omega / E)^{2}}{(1-\omega / E)^{2}}-\cos \theta_{\min }}\right), \tag{18}
\end{align*}
$$

considered only the term proportional to $C_{1}$ and neglected the other two. Of course, this term is the largest if $\theta_{\min }=0$, but even for this case, the last term


Fig. 3. Relative error of the EPA for various cuts. The solid lines correspond to $N(\omega)$ given in (15) while the dashed curves use $N(\omega)$ as in (18)
proportional to $C_{3} \simeq \mathcal{O}\left(m_{e}^{2} / s\right)$ gives a non negligible contribution because $\beta-\cos \theta_{\min } \simeq \mathcal{O}\left(m_{e}^{2} / s\right)$. For more realistic cuts for the positron, like $\theta_{\min }=5^{\circ}$ or $10^{\circ}$, there is no reason to neglect the term proportional to $C_{2}$.

In Fig. 3 we show the result of the comparison between the EPA and the exact result for diagrams $5+7$. We plot the relative error
$\varepsilon=\frac{\sigma_{\text {exact }}(5+7)-\sigma_{\text {ePA }}}{\sigma_{\text {exact }}(5+7)}$,
as a function of $\sqrt{s}$ for various cuts in the positron scattering angle. In those curves $\theta_{\text {min }}=5^{\circ}$, for instance, means that only angles in the interval $5^{\circ} \leqq \theta \leqq 175^{\circ}$ were allowed, and similarly for the other values of $\theta_{\text {min }}$. For numerical results we took $M_{Z}=92.0 \mathrm{GeV}, M_{W}=$ $80.7 \mathrm{GeV}, \sin ^{2} \theta_{W}=1-M_{W}^{2} / M_{Z}^{2}=0.23$ and $\Gamma_{W}=$ 2.8 GeV .

We can conclude that the EPA is a good approximation to diagrams $5+7$ only if very small scattering angles are included. Also the approximation gets worse at higher energies. This is due to the fact that the longitudinal part of the off-shell photon, that is neglected in EPA gets more important at those energies. We can also see that at this level of precision (the error in the Monte Carlo program was less than $0.1 \%$ ) we get very different results depending on which equivalent photon spectrum we take. If we use the approximate expression for $N(\omega)$ given in (18) we see (dashed curves) that the EPA does not get better as we go to very small angles as we discussed before. This is not the case for the more exact $N(\omega)$ given in (15)
(solid lines). Therefore, in the next section when comparing the exact result for the total cross section with the EPA we will take (15) for $N(\omega)$.

## 4 Results and discussion

For the numerical results presented in this section we used the same values for $M_{Z}, M_{W}, \sin ^{2} \theta_{W}$ and $\Gamma_{W}$ as given before.

In Fig. 4 we present the results for the cross section for two different cuts, with $\Delta k=0$ (standard model). The exact result (solid line) is compared with the equivalent photon approximation (dash-dotted line) and with the exact cross section corrected for the $W$ finite width (dashed line). We can conclude that just below the two $W$ production threshold this process will be seen at LEP-II. We can see that at these energies the EPA is not good specially if realistic cuts are taken into account. The fact that for $\sqrt{s}<140 \mathrm{GeV}$ the EPA is reasonably good for $\Delta k \gtrsim 0$ can be misleading. It does not mean that diagrams $5+7$ are much more important than all the others. In fact, we have verified, that even for $100 \mathrm{GeV} \leqq \sqrt{s} \leqq 140 \mathrm{GeV}$ diagrams $1+2+3$ are not negligible (for $5^{\circ} \leqq \theta \leqq 175^{\circ}$, diagrams $1+2+3$ are $20 \%$ of diagrams $5+7$ at $\sqrt{s}=$ 100 GeV and $35 \%$ at $\sqrt{s}=140 \mathrm{GeV}$ ). But when we include all diagrams the interferences are negative and the final result is, by accident, close to the EPA. The introduction of a finite width for the $W$ only affects the results close to the thresholds as it could be expected. The effect is bigger close to $\sqrt{s} \simeq 2 M_{W}$ and


Fig. 4. Total cross section for two different cuts. The solid line is the exact result, the dashed line the exact result with the $W$ width included and the dot-dashed line the EPA


Fig. 5. Relative error of EPA as a function of $\Delta k$ for several energies (in GeV )
it should be considered for a careful study at these energies.
In Fig. 5 we show the relative error of the EPA
$\varepsilon=\frac{\sigma_{\text {exact }}-\sigma_{\text {EPA }}}{\sigma_{\text {exact }}}$
as a function of $\Delta k$ for several energies, for two different cuts. As it is to be expected the error is much bigger if we consider realistic cuts. In fact for some values of $\Delta k$ this error can be even larger than the deviation of the exact result from the standard model $(\Delta k=0)$. Notice also that the error is not a flat function of $\Delta k$, being larger for $\Delta k<0$. This has to do with a change of sign in the interferences we mentioned above.

This can be seen in Fig. 6 where we show the variation of the cross section with $\Delta k$ for several energies and for the same cuts considered before. We see that


Fig. 6. Variation of the cross section with $\Delta k$ for two different cuts and for several energies.
if LEP operates, at it is scheduled [4], for a sufficient amount of time below the two $W$ threshold, it will be possible to put limits on $\Delta k$ much better than exist now. Of course after LEP is raised to its maximum energy the process $e^{+} e^{-} \rightarrow W^{+} W^{-}$will provide a better way to look at $\Delta k$ [5].
We did not attempt to turn our results into an upper limit for $|\Delta k|$. This will depend on the luminosity of the machine, the time of the running at those energies and the details of the experimental apparatus (cuts are very important). Instead, we give the detailed expressions that the experimentalists can use in their simulations. To the interested reader we can provide a FORTRAN program that evaluates the helicity amplitudes and the cross section.
After we had completed these calculations, we learned about the model of Kuroda, Maalampi, Schildknecht and Schwarzer [10] (KMSS for short). In this model there is a global $\operatorname{SU}(2)_{W_{I}}$ (weak isospin) symmetry broken by electromagnetism. As a consequence the trilinear couplings $\gamma W^{+} W^{-}$and $Z^{0} W^{+} W^{-}$are given by [10] (see Fig 2 for conventions),

$$
\begin{align*}
\Gamma^{\mu \alpha \beta}= & i g_{X W W}\left[g^{\alpha \beta}(p-k)^{\mu}+g^{\beta \mu}\left(k-\kappa_{X} q\right)^{\alpha}\right. \\
& \left.+g^{\mu \alpha}\left(\kappa_{X} q-p\right)^{\beta}\right] \tag{21}
\end{align*}
$$

where $X$ stands for $\gamma$ or $Z^{0}$. For $\gamma W^{+} W^{-}$they have $g_{\gamma W W}=-e$ and $\kappa_{y}=\kappa \equiv 1+\Delta k$, which is just the vertex we considered above (4). For the $Z^{0} W^{+} W^{-}$ coupling, corrections to the $\rho$-parameter [11] imply that $g_{Z W W}$ and $\kappa_{Z}$ are uniquely determined in terms

Table 1. Total cross section (in pb) for the cut $0^{\circ} \leqq \theta \leqq 180^{\circ}$ as a function of the C.M. Energy and of the anomalous magnetic moment of the W. The upper value corresponds to the KMSS model. (22-24) and the lower value to the model described in Sect. 2

|  |  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $(\mathrm{GeV})$ | 0 | 0.6 | 0.8 | 1 | 1.2 | 1.4 | 2.0 |
| 100 | $2.1 \times 10^{-3}$ | $2.5 \times 10^{-3}$ | $3.3 \times 10^{-3}$ | $4.5 \times 10^{-3}$ | $5.8 \times 10^{-3}$ | $7.6 \times 10^{-3}$ | $1.5 \times 10^{-2}$ |
|  | $2.0 \times 10^{-3}$ | $2.7 \times 10^{-3}$ | $3.4 \times 10^{-3}$ | $4.5 \times 10^{-3}$ | $5.7 \times 10^{-3}$ | $7.2 \times 10^{-3}$ | $1.3 \times 10^{-2}$ |
| 130 | $2.1 \times 10^{-2}$ | $2.9 \times 10^{-2}$ | $3.7 \times 10^{-2}$ | $4.9 \times 10^{-2}$ | $6.4 \times 10^{-2}$ | $8.2 \times 10^{-2}$ | $1.5 \times 10^{-1}$ |
|  | $1.8 \times 10^{-2}$ | $2.9 \times 10^{-2}$ | $3.8 \times 10^{-2}$ | $4.9 \times 10^{-2}$ | $6.3 \times 10^{-2}$ | $8.0 \times 10^{-2}$ | $1.5 \times 10^{-1}$ |
| 160 | $1.9 \times 10^{-1}$ | $1.9 \times 10^{-1}$ | $2.1 \times 10^{-1}$ | $2.4 \times 10^{-1}$ | $2.8 \times 10^{-1}$ | $3.4 \times 10^{-1}$ | $5.6 \times 10^{-1}$ |
| 200 | $1.6 \times 10^{-1}$ | $1.9 \times 10^{-1}$ | $2.1 \times 10^{-1}$ | $2.4 \times 10^{-1}$ | $2.8 \times 10^{-1}$ | $3.3 \times 10^{-1}$ | $5.2 \times 10^{-1}$ |
|  | 2.6 | 1.7 | 1.6 | 1.6 | 1.7 | 2.0 | 3.3 |

of $\kappa=1+\Delta k$, the magnetic moment of the $W$-boson, by the relations
$g_{Z W W}(\kappa)=\frac{e\left(\kappa-\sin ^{2} \theta_{W}\right)}{\sin \theta_{W} \cos \theta_{W}}=g_{Z W W}^{S M}\left(1+\frac{\Delta k}{\cos ^{2} \theta_{W}}\right)$
and
$\kappa_{Z}(\kappa)=\frac{\kappa \cos ^{2} \theta_{W}}{\kappa-\sin ^{2} \theta_{W}}=1-\frac{\sin ^{2} \theta_{W}}{\cos ^{2} \theta_{W}+\Delta k} \Delta k$.
The standard model values are obtained for $\kappa=1$ $(\Delta k=0)$, that is $\kappa=\kappa_{Z}=1, g_{Z W W}^{S M}=g \cos \theta_{W}$. As everything depends only on one parameter, $\kappa$, the KMSS model is very predictive.
It is very easy to incorporate the KMSS model in our calculations. We only have to change diagrams 3 and 8 . The necessary modifications are, in an obvious notation,

$$
\begin{align*}
& C_{3,8}^{\mathrm{KMSS}}=C_{3,8}\left(1+\frac{\Delta k}{\cos ^{2} \theta_{W}}\right)  \tag{24a}\\
& \tilde{T}_{3,8}^{\mathrm{KMSS}}(-,+)=C_{3,8}^{\mathrm{KMSS}} g_{+}\left[A_{2,12}+\left(1+\frac{\Delta k_{Z}}{2}\right) A_{3,13}\right], \tag{24b}
\end{align*}
$$

$\tilde{T}_{3}^{\mathrm{KMSS}}(+,-)=C_{3}^{\mathrm{KMSS}} g_{-}\left[A_{4}+\left(1+\frac{\Delta k_{Z}}{2}\right) A_{5}\right]$,
$\tilde{T}_{8}^{\text {KMSS }}(-,-)=C_{8}^{\text {KMSS }} g_{-}\left[A_{10}+\left(1+\frac{\Delta k_{Z}}{2}\right) A_{11}\right]$,
where $g_{ \pm}, C_{i}$ and $A_{i}$ are given in the Appendix.
We have evaluated the cross section for $e^{+} e^{-} \rightarrow$ $W^{-} e^{+} v$ in the framework of this model. The results are presented in Table 1 where they are compared with the situation that we have studied above, in which the $\gamma W^{+} W^{-}$vertex is allowed to be non-standard but the $Z^{0} W^{+} W^{-}$couplings are chosen as in the standard
model. We can see that below the two $W$ 's threshold, for which most of the above discussion applies, the differences between the two models are, at most, 15$20 \%$. As for $\kappa=1(\Delta k=0)$ the two models coincide with the standard model, the differences grow with $\Delta k$. If we go above the two $W$ 's threshold, then we will be able to distinguish among the various possibilities, at least for not too small values of $\Delta k$. We should also note that the KMSS model gives in most cases higher values for the cross section than the modification of the standard model we have considered in Sect. 2.

In conclusion we have shown that just below the threshold for two $W$ production, the process $e^{+} e^{-} \rightarrow$ $W^{-} e^{+} v$ can provide the first opportunity to test the $W W \gamma$ coupling. This will be specially important if, as planned, the LEP machine will stay at those energies for a reasonable amount of time, before it goes to its maximum energy.

In this energy region, and for reasonable experimental cuts, the equivalent photon approximation used before $[1,2]$ for this process is very bad. If one wants to calculate are upper limit to the value $|\Delta k|$ one must use the exact results presented here. Also the effects of the finite width of the $W$ must be included.

Finally one should mention, that if $\sqrt{s}>2 M_{W}$ so that the process $e^{+} e^{-} \rightarrow W^{+} W^{-}$is allowed, the calculations presented here should be useful to determine one of the backgrounds to that process. Also, in this energy region this process could serve as a test of the KMSS model.

## Appendix

The helicity amplitudes can be written in the form

$$
\begin{aligned}
& \widetilde{T}_{1}(-,+)=C_{1} A_{1} \\
& \widetilde{T}_{2}(-,+)=C_{2}\left[A_{2}+\left(1+\frac{\Delta k}{2}\right) A_{3}\right]
\end{aligned}
$$

$$
\begin{align*}
& \tilde{T}_{2}(+,-)=C_{2}\left[A_{4}+\left(1+\frac{\Delta k}{2}\right) A_{5}\right] \\
& \widetilde{T}_{3}(-,+)=C_{3} g_{+}\left[A_{2}+A_{3}\right] \\
& \widetilde{T}_{3}(+,-)=C_{3} g_{-}\left[A_{4}+A_{5}\right] \\
& \widetilde{T}_{4}(-,+)=C_{4} g_{+} A_{6} \\
& \widetilde{T}_{4}(+,-)=C_{4} g_{-} A_{7} \\
& \widetilde{T}_{5}(-,-)=C_{5} A_{8} \\
& \widetilde{T}_{5}(-,+)=C_{5} A_{9} \\
& \widetilde{T}_{6}(-,-)=C_{6} g_{-} A_{8} \\
& \widetilde{T}_{6}(-,+)=C_{6} g_{+} A_{9} \\
& \widetilde{T}_{7}(-,-)=C_{7}\left[A_{10}+\left(1+\frac{\Delta k}{2}\right) A_{11}\right] \\
& \tilde{T}_{7}(-,+)=C_{7}\left[A_{12}+\left(1+\frac{\Delta k}{2}\right) A_{13}\right] \\
& \widetilde{T}_{8}(-,-)=C_{8} g_{-}\left[A_{10}+A_{11}\right] \\
& \widetilde{T}_{8}(-,+)=C_{8} g_{+}\left[A_{12}+A_{13}\right] \\
& \widetilde{T}_{9}(-,-)=C_{9} g_{-} A_{14} \\
& \widetilde{T}_{9}(-,+)=-C_{9} g_{+} A_{1} \\
& \tilde{T}_{10}(-,+)=-C_{10} A_{6} \\
& \widetilde{T}_{11}(-,+)=-C_{11} A_{9} \\
& \widetilde{T}_{11}(+,-)=C_{11} A_{15} \\
& \widetilde{T}_{12}(-,+)=-C_{12} g_{+} A_{9} \\
& \widetilde{T}_{12}(+,-)=C_{12} g_{-} A_{15}, \tag{A1}
\end{align*}
$$

where

$$
\begin{align*}
& g_{-}=2 \sin ^{2} \theta_{W} \\
& g_{+}=2 \sin ^{2} \theta_{W}-1, \tag{A2}
\end{align*}
$$

and

$$
\begin{align*}
& C_{1}=-2\left[\left(\left(p_{1}+p_{2}\right)^{2}\right.\right. \\
&\left.\left.-M_{W}^{2}+i M_{W} \Gamma_{W}\right)\left(p_{-}-p_{W}\right)^{2}\right]^{-1} \\
& C_{2}=-4 \sin ^{2} \theta_{W}\left[\left(p_{+}+p_{-}\right)^{2}\right. \\
&\left.\cdot\left(\left(p_{1}+p_{2}\right)^{2}-M_{W}^{2}+i M_{W} \Gamma_{W}\right)\right]^{-1} \\
& C_{3}= 2\left[\left(\left(p_{1}+p_{2}\right)^{2}-M_{W}^{2}+i M_{W} \Gamma_{W}\right)\right. \\
&\left.\cdot\left(\left(p_{+}+p_{-}\right)^{2}-M_{Z}^{2}+i M_{Z} \Gamma_{Z}\right)\right]^{-1} \\
& C_{4}=-\sec ^{2} \theta_{W}\left[\left(\left(p_{+}+p_{-}\right)^{2}-M_{Z}^{2}+i M_{Z} \Gamma_{Z}\right)\right. \\
&\left.\cdot\left(\left(p_{2}+p_{W}\right)^{2}-m_{e}^{2}\right)\right]^{-1} \\
& C_{5}= 4 \sin ^{2} \theta_{W}\left[\left(p_{2}-p_{+}\right)^{2}\left(\left(p_{1}+p_{W}\right)^{2}-m_{e}^{2}\right)\right]^{-1} \\
& C_{6}= g_{+} \sec ^{2} \theta_{W}\left[\left(\left(p_{+}-p_{2}\right)^{2}-M_{Z}^{2}\right)\right. \\
&\left.\cdot\left(\left(p_{1}+p_{W}\right)^{2}-m_{e}^{2}\right)\right]^{-1} \\
& C_{7}= 4 \sin ^{2} \theta_{W}\left[\left(p_{2}-p_{+}\right)^{2}\left(\left(p_{1}-p_{-}\right)^{2}-M_{W}^{2}\right)\right]^{-1} \\
& C_{8}=-2\left[\left(\left(p_{1}-p_{-}\right)^{2}-M_{W}^{2}\right)\left(\left(p_{2}-p_{+}\right)^{2}-M_{Z}^{2}\right)\right]^{-1} \\
& C_{9}= \sec ^{2} \theta_{W}\left[\left(p_{-}-p_{W}\right)^{2}\left(\left(p_{2}-p_{+}\right)^{2}-M_{Z}^{2}\right)\right]^{-1} \\
& C_{10}= 2\left[\left(p_{2}+p_{W}\right)^{2}\left(\left(p_{1}-p_{-}\right)^{2}-M_{W}^{2}\right)\right]^{-1} \\
& C_{11}=-4 \sin ^{2} \theta_{W}\left[\left(p_{+}+p_{-}\right)^{2}\left(\left(p_{1}+p_{W}\right)^{2}-m_{e}^{2}\right)\right]^{-1} \\
& C_{12}=-g_{+} \sec ^{2} \theta_{W}\left[\left(\left(p_{1}+p_{W}\right)^{2}-m_{e}^{2}\right)\right. \\
&\left.\cdot\left(\left(p_{+}+p_{-}\right)^{2}-M_{Z}^{2}+i M_{Z} \Gamma_{Z}\right)\right]^{-1} . \tag{A3}
\end{align*}
$$

The expressions for $A_{i},(i=1, \ldots, 15)$ are written in terms of the spinor products:

$$
\begin{align*}
& A_{1}=s\left(p_{-}, r_{2}\right) s^{*}\left(p_{+}, p_{1}\right) \\
& \cdot\left[s\left(p_{-}, p_{2}\right) s^{*}\left(p_{-}, r_{1}\right)-s\left(p_{2}, r_{2}\right) s^{*}\left(r_{1}, r_{2}\right)\right] \\
& A_{2}=s\left(p_{2}, r_{2}\right) s^{*}\left(p_{1}, r_{1}\right) \\
& \cdot\left[s\left(p_{-}, r_{1}\right) s^{*}\left(p_{+}, r_{1}\right)+s\left(p_{-}, r_{2}\right) s^{*}\left(p_{+}, r_{2}\right)\right] \\
& A_{3}=s\left(p_{-}, p_{2}\right) s^{*}\left(p_{+}, p_{1}\right) \\
& \cdot\left[s\left(p_{-}, r_{2}\right) s^{*}\left(p_{-}, r_{1}\right)+s\left(p_{+}, r_{2}\right) s^{*}\left(p_{+}, r_{1}\right)\right] \\
& -\left\{\begin{array}{l}
r_{1} \leftrightarrow p_{1} \\
r_{2} \leftrightarrow p_{2}
\end{array}\right\} \\
& A_{4}=s\left(p_{2}, r_{2}\right) s^{*}\left(p_{1}, r_{1}\right) \\
& \cdot\left[s\left(p_{+}, r_{1}\right) s^{*}\left(p_{-}, r_{1}\right)+s\left(p_{+}, r_{2}\right) s^{*}\left(p_{-}, r_{2}\right)\right] \\
& A_{5}=s\left(p_{+}, p_{2}\right) s^{*}\left(p_{-}, p_{1}\right) \\
& \cdot\left[s\left(p_{-}, r_{2}\right) s^{*}\left(p_{-}, r_{1}\right)+s\left(p_{+}, r_{2}\right) s^{*}\left(p_{+}, r_{1}\right)\right] \\
& -\left\{\begin{array}{l}
r_{1} \leftrightarrow p_{1} \\
r_{2} \leftrightarrow p_{2}
\end{array}\right\} \\
& A_{6}=s\left(p_{2}, r_{2}\right) s^{*}\left(p_{+}, p_{1}\right) \\
& \cdot\left[s\left(p_{-}, r_{2}\right) s^{*}\left(r_{1}, r_{2}\right)-s\left(p_{-}, p_{2}\right) s^{*}\left(p_{2}, r_{1}\right)\right] \\
& A_{7}=s\left(p_{2}, r_{2}\right) s^{*}\left(p_{-}, p_{1}\right) \\
& \cdot\left[s\left(p_{+}, r_{2}\right) s^{*}\left(r_{1}, r_{2}\right)-s\left(p_{+}, p_{2}\right) s^{*}\left(p_{2}, r_{1}\right)\right] \\
& A_{8}=s\left(p_{-}, p_{+}\right) s^{*}\left(p_{1}, r_{1}\right) \\
& \cdot\left[s\left(p_{+}, r_{2}\right) s^{*}\left(p_{+}, p_{2}\right)+s\left(p_{-}, r_{2}\right) s^{*}\left(p_{-}, p_{2}\right)\right] \\
& A_{9}=s\left(p_{-}, p_{2}\right) s^{*}\left(p_{1}, r_{1}\right) \\
& \cdot\left[s\left(p_{2}, r_{2}\right) s^{*}\left(p_{+}, p_{2}\right)+s\left(p_{-}, r_{2}\right) s^{*}\left(p_{-}, p_{+}\right)\right] \\
& A_{10}=s\left(p_{-}, r_{2}\right) s^{*}\left(p_{1}, r_{1}\right) \\
& \cdot\left[s\left(p_{-}, p_{+}\right) s^{*}\left(p_{-}, p_{2}\right)+s\left(p_{+}, p_{1}\right) s^{*}\left(p_{1}, p_{2}\right)\right] \\
& A_{11}=s\left(p_{+}, r_{2}\right) s^{*}\left(p_{2}, r_{1}\right) \\
& \cdot\left[s\left(p_{-}, p_{+}\right) s^{*}\left(p_{+}, p_{1}\right)+s\left(p_{-}, p_{2}\right) s^{*}\left(p_{1}, p_{2}\right)\right] \\
& -\left\{\begin{array}{c}
p_{-} \leftrightarrow r_{2} \\
p_{1} \leftrightarrow r_{1}
\end{array}\right\} \\
& A_{12}=s\left(p_{-}, r_{2}\right) s^{*}\left(p_{1}, r_{1}\right) \\
& \cdot\left[s\left(p_{-}, p_{2}\right) s^{*}\left(p_{-}, p_{+}\right)+s\left(p_{1}, p_{2}\right) s^{*}\left(p_{+}, p_{1}\right)\right] \\
& A_{13}=s\left(p_{2}, r_{2}\right) s^{*}\left(p_{+}, r_{1}\right) \\
& \cdot\left[s\left(p_{-}, p_{+}\right) s^{*}\left(p_{+}, p_{1}\right)+s\left(p_{-}, p_{2}\right) s^{*}\left(p_{1}, p_{2}\right)\right] \\
& -\left\{\begin{array}{c}
p_{-} \leftrightarrow r_{2} \\
p_{1} \leftrightarrow r_{1}
\end{array}\right\} \\
& A_{14}=-s\left(p_{-}, r_{2}\right) s^{*}\left(p_{1}, p_{2}\right) \\
& \cdot\left[s\left(p_{+}, p_{2}\right) s^{*}\left(p_{2}, r_{1}\right)+s\left(p_{+}, p_{1}\right) s^{*}\left(p_{1}, r_{1}\right)\right] \\
& A_{15}=s\left(p_{+}, p_{2}\right) s^{*}\left(p_{1}, r_{1}\right) \\
& \cdot\left[s\left(r_{1}, r_{2}\right) s^{*}\left(p_{-}, r_{1}\right)+s\left(p_{1}, r_{2}\right) s^{*}\left(p_{-}, p_{1}\right)\right] . \tag{A4}
\end{align*}
$$

The spinor product $s(p, q)$ as a function of the components of the four-vectors $p$ and $q$ is [4]

$$
\begin{equation*}
s(p, q)=\left(p^{2}+i p^{3}\right)\left[\left(q^{0}-q^{1}\right) /\left(p^{0}-p^{1}\right)\right]^{1 / 2}-(p \leftrightarrow q) \tag{A5}
\end{equation*}
$$

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