Z. Phys. C – Particles and Fields 31, 245–248 (1986)

Zeitschrift für Physik C Particles and Fields

Intermediate 16 + 16 Multiplets in $N=1$ Supergravity

J.C. Romão and A.B. Henriques

CFMC, Instituto Nacional de Investigação Cientifica, Av. Gama Pinto 2, 1699 Lisboa, Portugal

Received 14 October 1985

Abstract. Using the formalism developed by Ogievetsky and Sokatchev for $N=1$ Supergravity, we analyse the intermediate $16+16$ sets (with one auxiliary spinor).

1. Introduction

At the present moment several off-shell versions of the simple $N=1$ supergravity are already known. Besides the two distinct minimal formulations, the "old-minimal" [1] and the "new-minimal" [2] ones, another irreducible formulation is known, Breitenlohner's non-minimal $\underline{20}+\underline{20}$ component description [3]. Recently, some effort has been devoted to the study of "intermediate" $\underline{16}+\underline{16}$ formulations with one auxiliary spinor. Two distinct such versions with chiral gauge transformations are known, one described in Sohnius and West paper [4], the other presented by Galperin et al. [5]. These two versions were introduced with the help of two different types of formalism, leaving somehow obscured the relations between them.

In this Note we use the prepotential formalism, developed by Ogievetsky and Sokatchev [6,7], to show the connections between those two $\underline{16}+\underline{16}$ component multiplets, namely, how they can be obtained from the same pair of prepotentials, the axial superfield $H^m(x,\theta,\bar\theta)$ and the spinor superfield $H^\mu(x,\theta,\bar\theta)$ (expressions (1.5) below). We shall also see that the two invariant constraints to be imposed on the initial $\underline{20}+\underline{20}$ multiplet defined with the help of H^m and H^μ (2.2) do correspond to known criteria of reducibility [8].

We use superspace techniques and make use, throughout, of the two-component formalism following Ogievetsky and Sokatchev [7]:

We shall describe Einstein supergravity by applying the restriction [5,9]

$$\left[\mathrm{Ber}\left(\frac{\partial Z'_L}{\partial Z_L}\right)\right]^{3n+1}=\left[\mathrm{Ber}\left(\frac{\partial z'_L}{\partial z_L}\right)\right]^{2n}; \tag{1.1}$$

selecting $n=0$, we get infinitesimally

$$\frac{\partial\bar\rho^{\dot\mu}}{\partial\bar\phi^{\dot\mu}_L}=\frac{\partial\lambda^m}{\partial x^m_L}-\frac{\partial\lambda^\mu}{\partial\theta^\mu_L}. \tag{1.2}$$

Here $\{Z_L\}$ refers to the left-handed parametrization of the complex superspace $\mathbb{C}^{4,4}=\{x^m_L,\theta^\mu_L,\bar\phi^{\dot\mu}_L\}$ and $\{z_L\}$ to $\mathbb{C}^{4,2}=\{x^m_L,\theta^\mu_L\}$ a chiral subspace of $\mathbb{C}^{4,4}$ [5]. Similarly, we have a right-handed parametrization $\{Z_R\}=\{x^m_R,\bar\theta^{\dot\mu}_R,\phi^\mu_R\}$, with $x^m_R=(x^m_L)^+$, $\bar\theta^{\dot\mu}_R=(\theta^\mu_L)^+$ and $\phi^\mu_R=(\bar\phi^{\dot\mu}_L)^+$.

"Real" superspace $R^{4,4}=\{Z\}=\{x^m,\theta^\mu,\bar\theta^{\dot\mu}\}$ is introduced as a hypersurface in $\mathbb{C}^{4,4}$ [5], through

$$\begin{aligned}&x^m=\mathrm{Re}\,x^m_L,\quad \theta^\mu=\theta^\mu_L,\quad \bar\theta^{\dot\mu}=\bar\theta^{\dot\mu}_R\\ &H^m(x,\theta,\bar\theta)=\mathrm{Im}\,x^m_L\\ &H^\mu(x,\theta,\bar\theta)=\phi^\mu_R-\theta^\mu_L,\quad \bar H^{\dot\mu}(x,\theta,\bar\theta)=\bar\phi^{\dot\mu}_L-\bar\theta^{\dot\mu}_R.\end{aligned} \tag{1.3}$$

We now define the group of analytic transformations of the coordinates leaving $\mathbb{C}^{4,2}$ invariant:

$$\begin{aligned}&\delta x^m_L=\lambda^m(x_L,\theta_L)\\ &\delta\theta^\mu_L=\lambda^\mu(x_L,\theta_L)\\ &\delta\bar\phi^{\dot\mu}_L=\bar\rho^{\dot\mu}(x_L,\theta_L,\bar\phi_L),\end{aligned} \tag{1.4}$$

where λ^m and λ^μ are chiral superfunctions and $\bar\rho^{\dot\mu}$ is a general superfunction.

We thus have the geometric framework for our work.

Equations (1.1) and (1.2) impose constraints on the superfunctions λ^m, λ^μ and $\bar\rho^{\dot\mu}$. Most of the remaining parameters can be used to gauge H^m and H^μ into the form

$$\begin{aligned}H^m(x,\theta,\bar\theta)=&\ \theta^\mu\bar\theta^{\dot\mu}\sigma^a_{\mu\dot\mu}e^m_a+\bar\theta^2\theta^\mu\psi^m_\mu+\theta^2\bar\theta_{\dot\mu}\bar\psi^{m\dot\mu}\\ &+\theta^2\bar\theta^2C^m,\end{aligned} \tag{1.5a}$$

and

$$\begin{aligned}H^\mu(x,\theta,\bar\theta)=&\ \theta^\nu iA\,\delta^\mu_\nu+\theta^\mu\bar\theta_{\dot\mu}\bar\xi^{\dot\mu}+\bar\theta^2\theta^\mu B\\ &+\theta^2\bar\theta_{\dot\mu}\bar\sigma^{a\dot\mu\mu}(V+iW)_a+\theta^2\bar\theta^2\beta^\mu,\end{aligned} \tag{1.5b}$$

$B\equiv B_R+iB_I$.

After this gauging, we are left with the following free parameters (from λ^m, λ^μ and $\bar\rho^{\dot\mu}$):

$a^m(x)$, for general coordinate transformations
$\varepsilon^\mu(x)$, for supersymmetric (SS) transformations
$\omega\begin{pmatrix}\mu\\ \nu\end{pmatrix}(x)$, for Lorentz (structure) transformations
$b(x)$, for local chiral transformations. (1.6)

This leaves us with a reducible $\underline{20} + \underline{20}$ component description. The corresponding fields and their mass dimensions are as follows:

e_a^m	the vierbein (dimension = 0)
ψ_ν^m	the gravitino ($d = 3/2$)
A	a pseudoscalar ($d = 1$)
ξ^μ	a spinor ($d = 3/2$)
B_R, B_I	a scalar-pseudoscalar pair ($d = 2$)
$V^m(\equiv e_a^m V^a)$	an axial gauge vector ($d = 2$)
$\tilde{W}^a$	a vector field ($d = 2$)
$\tilde{V}^a$	an axial vector ($d = 2$)
β^μ	a spinor ($d = 5/2$). (1.7)

In our formalism the following mass dimensions hold: $[K \equiv$ gravitational constant$] = -1$, $[H^m] = [x^m] = -1$, $[H^\mu] = -1/2$ and $[\theta^\mu] = -1/2$; the constant K is assumed everywhere to multiply all fields, except e_a^m.

Fields $\tilde{W}^a$ and $\tilde{V}^a$ are defined by

$$\tilde{W}_a = W_a + \tfrac{1}{2}\partial_m e_a^m \quad \text{and} \quad \tilde{V}_a = V_a - C_a + \tfrac{1}{2}\partial_a A,$$
$$C_a \equiv e_a^m C^m.$$

After subtraction of the gauge degrees of freedom we indeed obtain $\underline{20} + \underline{20}$ components. Transformations (1.4), together with restrictions (1.1) and (1.2), will induce variations in H^m and H^μ,

$$\delta^* H^m(x,\theta,\bar\theta) \equiv H'^m(x,\theta,\bar\theta) - H^m(x,\theta,\bar\theta)$$
$$\delta^* H^\mu(x,\theta,\bar\theta) \equiv H'^\mu(x,\theta,\bar\theta) - H^\mu(x,\theta,\bar\theta), \tag{1.8}$$

appropriate to Einstein supergravity. The proof that the corresponding transformation algebra of fields closes has been given by Ogievetsky and Sokatchev [7].

2. Invariant Constraints on the Prepotentials

From the preservation of $\mathbb{C}^{4,4}$ supervolume for the case $n = 0$, we can introduce the following invariant superfunction:

$$U(x,\theta,\bar\theta) = \mathrm{Ber}\left(\frac{\partial Z_L}{\partial Z}\right)\cdot \mathrm{Ber}^{-1}\left(\frac{\partial Z_R}{\partial Z}\right)$$
$$= \frac{\det(\delta_n^m + i\partial_n H^m)}{\det(\delta_{\dot\nu}^{\dot\mu} + \bar\Delta^{\dot\mu}\bar H_{\dot\nu})}\cdot\frac{\det(\delta_\nu^\mu + \Delta_\nu H^\mu)}{\det(\delta_n^m - i\partial_n H^m)}, \tag{2.1}$$

which will be used to impose constraints on the fields.

From $U^+ U = 1$, we have that $U = \exp[iu(x,\theta,\bar\theta)]$, $u(x,\theta,\bar\theta)$ being a real superfield. We shall use the constraints [5]

$$U = 1\,(u = 0) \tag{2.2a}$$

or

$$\varepsilon^{\mu\nu}\frac{\partial}{\partial\phi_R^\mu}\frac{\partial}{\partial\phi_R^\nu}U = 0. \tag{2.2b}$$

These constraints can be solved to yield the new-minimal set and a new intermediate $\underline{16} + \underline{16}$ multiplet (with one spinor). At the linear level (linear in K) the real superfield $u(x,\theta,\bar\theta)$ can be written as

$$\begin{aligned} u(x,\theta,\bar\theta) = {} & 4A + \theta^\nu(2i\xi_\nu) + \bar\theta_{\dot\nu}(-2i\bar\xi^{\dot\nu}) + \theta^2(2i\bar B) \\ & + \bar\theta^2(-2iB) + \theta^\nu\bar\theta^{\dot\nu}(4W_a + 2\partial_m e_a^m)\sigma^a_{\nu\dot\nu} \\ & + \theta^2\bar\theta^{\dot\nu}(2i\bar\beta_{\dot\nu} - \tfrac{1}{2}\sigma^a_{\nu\dot\nu}e_a^m\partial_m\xi^\nu + 2i\partial_m\bar\psi_{\dot\nu}^m) \\ & + \bar\theta^2\theta^\nu(\text{c.c.}) + \theta^2\bar\theta^2(2\partial_m C^m - e_a^m\partial_m V^a). \end{aligned} \tag{2.3}$$

It is immediately seen that $u(x,\theta,\bar\theta) = 0$, constraint (2.2a), gives the following $\underline{8} + \underline{8}$ conditions:

$$A = 0, \quad B_R = B_I = 0, \quad \xi_\nu = 0,$$
$$\tilde{W}^a = 0, \quad \bar\beta_{\dot\nu} = -2i\partial_m\bar\psi_{\dot\nu}^m, \quad \partial_a\tilde{V}^a = 0. \tag{2.4}$$

Applying (2.4), we get the new-minimal version of $N = 1$ supergravity, as is well known [5].

We now solve $\varepsilon^{\mu\nu}(\partial/\partial\phi_R^\mu)(\partial/\partial\phi_R^\nu)\,U = 0$. To do this, we first express $\partial/\partial\phi_R^\mu$ as derivatives in terms of x^m, $\bar\theta^{\dot\mu}$ and θ^μ, using

$$x_R^m = x^m - iH^m$$
$$\theta_R^{\dot\mu} = \bar\theta^{\dot\mu}$$
$$\phi_R^\mu = \theta^\mu + H^\mu$$

relating $\{Z_R\}$ with $\{Z\} \equiv R^{4,4}$. We then solve constraint (2.2b) at the linear level to obtain the $\underline{4} + \underline{4}$ conditions

$$\partial_a\tilde{W}^a = \partial_a\tilde{V}^a = 0,$$
$$B \equiv B_R + iB_I = 0,$$
$$\bar\beta^{\dot\mu} = -i\partial_m\bar\psi^{m\dot\mu} + \tfrac{3}{4}i\bar\sigma^{a\dot\mu\mu}e_a^m\partial_m\xi_\mu. \tag{2.5}$$

3. Field Transformations

The transformation rules applied to our fields will be obtained from variations (1.8), with $\delta^* H^m$ and $\delta^* H^\mu$ given by

$$\delta^* H^m = \frac{1}{2i}(\lambda^m - \bar\lambda^m) - \tfrac{1}{2}(\lambda^n + \bar\lambda^n)\partial_n H'^m - \left(\lambda^\nu\frac{\partial}{\partial\theta^\nu} + \bar\lambda^{\dot\nu}\frac{\partial}{\partial\bar\theta^{\dot\nu}}\right)H'^m \tag{3.1a}$$

and

$$\delta^* H^\mu = (\rho^\mu(x_R,\bar\theta_R,\phi_R) - \lambda^\mu) - \tfrac{1}{2}(\lambda^n + \bar\lambda^n)\partial_n H'^\mu - \left(\lambda^\nu\frac{\partial}{\partial\theta^\nu} + \bar\lambda^{\dot\nu}\frac{\partial}{\partial\bar\theta^{\dot\nu}}\right)H'^\mu. \tag{3.1b}$$

These transformations are those induced by the parameters (1.6), left free after gauge fixing H^m and H^μ. Remaining at the linear level in K, we focus our attention on the SS transformations defined by the parameter $\varepsilon^\mu(x)$ and the local chiral transformations given by $b(x)$.

First, the chiral gauge transformations of the $\underline{20}+\underline{20}$ fields (1.7):

$$\begin{aligned}
&\delta_b^* e_a^m = 0\\
&\delta_b^* \psi_\nu^m = ib\psi_\nu^m\\
&\delta_b^* V^m = \partial^m b\\
&\delta_b^* A = 0\\
&\delta_b^* \bar{\xi}^{\dot\mu} = ib\bar{\xi}^{\dot\mu}\\
&\delta_b^* B_R = -2bB_I \quad \text{and} \quad \delta_b^* B_I = +2bB_R\\
&\delta_b^* \tilde{V}_a = 0\\
&\delta_b^* \tilde{W}_a = 0\\
&\delta_b^* \beta'^\mu = ib\beta'\mu,
\end{aligned} \tag{3.2}$$

having introduced the following linear redefinition of β^μ: $\beta'^\mu = \beta^\mu - i\partial_m\psi^{m\mu} + \frac{3}{4}i\bar{\sigma}^{a\dot\mu\mu}e_a^m\partial_m\bar{\xi}_{\dot\mu}$.

The SS transformation rules require some non-linear redefinitions of the above fields to be recast into a standard form (namely, by eliminating terms proportional to $\partial_m\varepsilon^\nu$). Such redefinitions are assumed everywhere. There still is a further linear redefinition of the gravitino field,

$$\psi'^{m\mu} \equiv \psi^{m\mu} - \tfrac{1}{8}\bar{\xi}_{\dot\mu}\bar{\sigma}^{a\dot\mu\mu}e_a^m,$$

giving, as before, $\delta_b^*\psi_\nu'^m = ib\psi_\nu'^m$. Primes will be dropped from now on.

The important SS transformations for our purpose are those of the $\underline{8}+\underline{8}$ multiplet carried by $u(x,\theta,\bar\theta)$:

$$\begin{aligned}
\delta_\varepsilon^* A &= -\tfrac{1}{2}i(\varepsilon^\mu\xi_\mu - \bar\varepsilon_{\dot\mu}\bar\xi^{\dot\mu})\\
\delta_\varepsilon^* \bar\xi^{\dot\mu} &= 2(\varepsilon_\mu\bar\sigma^{a\dot\mu\mu}e_a^m\partial_m A + i\varepsilon_\mu\bar\sigma^{a\dot\mu\mu}\tilde W_a - \bar\varepsilon^{\dot\mu}B)\\
\delta_\varepsilon^* B_R &= \tfrac{1}{2}(\varepsilon^\nu\beta_\nu + \bar\varepsilon_{\dot\nu}\bar\beta^{\dot\nu})\\
\delta_\varepsilon^* B_I &= -\tfrac{1}{2}i(\varepsilon^\nu\beta_\nu - \bar\varepsilon_{\dot\nu}\bar\beta^{\dot\nu})\\
\delta_\varepsilon^* \tilde W^a &= -\tfrac{1}{2}\varepsilon^\mu\sigma^a_{\mu\dot\mu}\bar\beta^{\dot\mu} - \tfrac{1}{2}\bar\varepsilon^{\dot\mu}\sigma^a_{\mu\dot\mu}\beta^\mu\\
&\quad - \varepsilon_\mu(\sigma^{ba})_\nu^\mu\partial_b\xi^\nu + \bar\varepsilon_{\dot\mu}(\bar\sigma^{ba})_{\dot\nu}^{\dot\mu}\partial_b\bar\xi^{\dot\nu}\\
\delta_\varepsilon^* \beta^\mu &= -2(\varepsilon^\mu\partial_a\tilde W^a - i\varepsilon^\mu\partial_a\tilde V^a + i\bar\varepsilon_{\dot\mu}\bar\sigma^{a\dot\mu\mu}\partial_a B)\\
\delta_\varepsilon^*(\partial_a\tilde V^a) &= -\tfrac{1}{2}\varepsilon^\mu\sigma^a_{\mu\dot\mu}\partial_a\bar\beta^{\dot\mu} + \tfrac{1}{2}\bar\varepsilon^{\dot\mu}\sigma^a_{\mu\dot\mu}\partial_a\beta^\mu.
\end{aligned} \tag{3.3}$$

We see that A is a pseudoscalar, its transformation rule being proportional to $\varepsilon^\mu\xi_\mu - \bar\varepsilon_{\dot\mu}\bar\xi^{\dot\mu}$, instead of $\varepsilon^\mu\xi_\mu + \bar\varepsilon_{\dot\mu}\bar\xi^{\dot\mu}$ as appropriate to a scalar. This multiplet is the parity flipped, linearized version of what Sohnius and West [4] call the general multiplet.

If in (3.3) we take $\delta_\varepsilon^*(\partial_a\tilde W^a)$ the terms in ξ^μ drop out, due to the properties of σ^{ab}, and we can immediately read out the submultiplet

$$(B_R, B_I; \beta^\mu; \partial_a\tilde W^a, \partial_a\tilde V^a). \tag{3.4}$$

We can now consistently put equal to zero this submultiplet. This is equivalent to the solution of constraint (2.2b) and also equivalent to the application to the $\underline{8}+\underline{8}$ set (3.3) of what Taylor and Rivelles call the broader criterion for reducibility [8]. When we do that we obtain the $\underline{4}+\underline{4}$ multiplet

$$(A; \xi^\mu; \tilde W^a), \quad \partial_a\tilde W^a = 0. \tag{3.5}$$

If we apply this same constraint (2.2b) to the initial $\underline{20}+\underline{20}$ description, we get Ogievetsky–Sokatchev $\underline{16}+\underline{16}$ intermediate set, with auxiliary spinor ξ^μ of mass dimension 3/2:

$$(e_a^m; A; \psi_\mu^m, \xi^\mu; V^m, \tilde V^a, \tilde W^a), \tag{3.6}$$

with $\partial_a\tilde W^a = \partial_a\tilde V^a = 0$.

As for the constraint (2.2a), its application is equivalent to setting to zero the whole $\underline{8}+\underline{8}$ multiplet defined in (3.3). We obtain the new-minimal $\underline{12}+\underline{12}$ description. Again, this is the same as using Taylor's criterion.

It is important to notice that adding the $\underline{4}+\underline{4}$ submultiplet (3.4) to the new-minimal set

$$(e_a^m; \psi_\nu^m; V^m, \tilde V^a), \quad \partial_a\tilde V^a = 0,$$

we find Sohnius and West $\underline{16}+\underline{16}$ set, with auxiliary spinor β^μ of mass dimension 5/2:

$$(e_a^m; \psi_\nu^m; B_R, B_I, V^m, \tilde V^a; \beta^\mu; \partial_a\tilde W^a), \tag{3.7}$$

$\partial_a\tilde V^a = 0$ where the correspondence with the notation of [4] is given by

$$B_R, B_I \equiv M, N; V^m \equiv A^m, \tilde V^a \equiv b^a; \beta^\mu \equiv \lambda^\mu; \partial_a\tilde W^a \equiv S.$$

Adding, instead, to the new-minimal set, the $\underline{4}+\underline{4}$ multiplet defined in (3.5) ($\partial_a\tilde W^a = 0$), we regain Ogievetsky–Sokatchev $\underline{16}+\underline{16}$, which is thus also reducible.

4. Summary

Ogievetsky and Sokatchev (O–S) have shown that by the imposition of the geometrical constraints (2.2) on the $\underline{20}+\underline{20}$ set, corresponding to $n = 0$ in (1.1), we can reduce this set either to a $\underline{16}+\underline{16}$ multiplet (constraint 2.2b) or all the way down to the new-minimal set (constraint 2.2a). In this work we have shown that these constraints are equivalent to Taylor's criterion of reducibility, in the following sense: a) If we apply the criterion to the $\underline{8}+\underline{8}$ general multiplet contained in $u(x,\theta,\bar\theta)$, we get the $\underline{16}+\underline{16}$ set of O–S; b) if the criterion is applied to the whole $\underline{20}+\underline{20}$ set, we get down to the new-minimal set, therefore implying that the $\underline{16}+\underline{16}$ set of O–S is itself reducible.

On the other hand the $\underline{16}+\underline{16}$ set of Sohnius and West is not obtained in this reduction chain, notwithstanding the fact that the fields of both intermediate $\underline{16}+\underline{16}$ sets come from the same pair of superfields, H^m and H^μ in (1.5). The difficulty arises from the fact that the SS transformations of the fields A, ξ^μ and $\tilde W^a$ as members of the original $\underline{8}+\underline{8}$ multiplet only close through the remaining fields of this set, as can be seen from inspection of (3.3). But Sohnius and West $\underline{16}+\underline{16}$ set can be obtained by adding the chiral submultiplet (3.4), contained in $u(x,\theta,\bar\theta)$, to the new-minimal set. It is thus also a reducible set.

Combining these results with those of O–S we conclude that, although all the irreducible sets can be obtained in a geometrical way, the same is not true for reducible sets, an example being the $\underline{16}+\underline{16}$ set of Sohnius and West.

It will be interesting to explore these ideas in extended supergravity where non-minimal sets can be of importance.*

Acknowledgements. We thank P.C. Nascimento for calling our attention to the work of Ogievetsky and Sokatchev.

* After having completed this work we were called the attention to the work of G. Girardi, R. Grimm, M. Müller and J. Wess [10, 11]. Their 16 + 16 multiplet [11] cannot be obtained using the methods of our present paper

References

1. K.S. Stelle, P.C. West: Phys. Lett. **74B**, 330 (1978); S. Ferrara, P. Van Nieuwenhuizen: Phys. Lett. **74B**, 333 (1978)
2. M.F. Sohnius, P.C. West: Phys. Lett. **105B**, 353 (1981)
3. P. Breitenlohner: Phys. Lett. **67B**, 49 (1977); Nucl. Phys. **B124**, 500 (1977)
4. M.F. Sohnius, P.C. West: Nucl. Phys. **B216**, 100 (1983)
5. A.S. Galperin, V.I. Ogievetsky, E.S. Sokatchev: Geometries Inherent to $N = 1$ Supergravities, II International Seminar on Quantum Gravity, Moscow Oct. 1981, Dubna preprint E2-81-854
6. V.I. Ogievetsky, E.S. Sokatchev: Phys. Lett. **79B**, 222 (1978)
7. V.I. Ogievetsky, E.S. Sokatchev, Yad. Fiz. **31**, 264 (1980); Yad. Fiz. **31**, 821 (1980)
8. V.O. Rivelles, J.G. Taylor: J. Phys. A: Math. Gen. **16**, 3019 (1983); J. Phys. A: Math. Gen. **16**, 3023 (1983)
9. W. Siegel, S.J. Gates Jr.: Nucl. Phys. **B147**, 77 (1979)
10. G. Girardi, R. Grimm, M. Müller, J. Wess: Z. Phys. C—Particles and Fields **26**, 123 (1984)
11. G. Girardi, R. Grimm, M. Müller, J. Wess: Phys. Lett. **147B**, 81 (1984)