Large Muon- and Electron-Number Nonconservation in Supergravity Theories

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We discuss muon and electron lepton-number nonconservation due to renormalization effects of the scalar-lepton mass matrix in spontaneously broken $N=1$ supergravity theories. Differently from the case of massless or Dirac neutrinos, we find that if neutrinos are Majorana particles in a "seesaw" scheme the couplings photino–lepton–scalar-lepton lead to sizable rates for the processes $\mu \to e\gamma$ and $\mu+\text{nucleus} \to e+\text{nucleus}$. We discuss the constraints derived from the present experimental bounds.

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In the supersymmetric standard model (SSM) no leptonic flavor-changing processes ($\mu \to e\gamma$, $\mu \to ee\bar{e}$, . . .) can occur. However, as soon as we consider an extension of the standard model (SM), obtained by embedding of the SM in a unified scheme or simply by addition of new particles and couplings, and we supersymmetrize it, we expect, in general, violations of conservation of the electron and muon lepton numbers L_e and L_μ. Indeed, this occurs even when the extension of the SM still conserves L_e and L_μ[1]: For instance, the supersymmetrization of SU(5) leads to L_e and L_μ nonconservation. The point is that in general an extension of the SSM contains new trilinear couplings in the superpotential with interactions between the usual particles of the SSM and new (possibly superheavy) particles. These new couplings are responsible for the presence of additional contributions to the renormalization of the scalar-quark ($\tilde{q}$) and scalar-lepton ($\tilde{l}$) masses[1]; consequently, couplings such as photino ($\tilde{\gamma}$)-q-$\tilde{q}$ or $\tilde{\gamma}$-l-$\tilde{l}$ can induce flavor changes which were absent or smaller in the unsupersymmetrized theory. Already in the SSM the radiative contributions to the $\tilde{q}$ mass matrix provide the leading contribution to flavor-changing neutral currents in the hadronic sector.[2] However, their role can be drastically enhanced when we supersymmetrize any extension of the SM; they can lead to changes of otherwise conserved quantities or, in any case, to much larger rates for rare processes which were not strictly forbidden already in the unsupersymmetrized versions of the theory. We study here renormalization effects in supersymmetric models which entail flavor changes in the leptonic sector.

Our major result is that the renormalization of the charged–scalar-lepton mass matrix leads to sizable L_e, L_μ nonconservation in any extension of the SSM where light Majorana neutrinos are obtained involving the "seesaw" mechanism.[3] The rates for $\mu \to e\gamma$, $\mu \to ee\bar{e}$, and $\mu+\text{nucleus} \to e+\text{nucleus}$ are much larger than in the corresponding unsupersymmetrized version and even the present experimental upper bounds impose nontrivial constraints on these supersymmetric models.[4] Clearly the possibility of large nonconservation of L_e, L_μ which is characteristic of this class of supersymmetric theories makes the improvement of bounds on the above rare processes even more attractive.

On the other hand, in supersymmetric models where neutrinos are massless [for instance, SU(5)][1] or are Dirac particles[5] the rates for $\mu \to e\gamma$, $\mu \to ee\bar{e}$, . . . , are predicted to be some orders of magnitude below the present experimental bounds with dim hopes of observability in the future. Indeed, these models lead more or less to the same predictions as in extensions of the SM with massive (Majorana or Dirac)[6] neutrinos.

Our analysis is performed in the context of low-energy supersymmetric models coming from spontaneously broken minimal $N=1$ supergravity theories. In order to appreciate the crucial difference between supersymmetric frameworks with Dirac or Majorana neutrinos concerning the size of the L_e, L_μ nonconservation, we consider the charged–scalar-lepton mass matrix. At tree level it reads

$$(\tilde{l}_L^\dagger \;\; \tilde{l}_L^{c\dagger}) \begin{pmatrix} m^2 + m_l m_l^\dagger & A m m_l \\ A m m_l & m^2 + m_l m_l^\dagger \end{pmatrix} \begin{pmatrix} \tilde{l}_L \\ \tilde{l}_L^c \end{pmatrix}, \qquad (1)$$

where $\tilde{l}_L$ and $\tilde{l}_L^c$ denote the scalar partners of l_L and l_L^c, m_l is the charged-lepton mass matrix, m sets the scale of the breaking of the residual $N=1$ global supersymmetry, and A is a parameter ~ 1. If we denote by N the chiral superfield which contains the right-handed neutrino, i.e., a superfield neutrino under SU(3) $\otimes$ SU(2) $\otimes$ U(1), the superpotential contains the two couplings $hLHN$ and MNN. The first coupling is responsible for the "Dirac entry" in the neutrino mass matrix, $\bar{\nu}_L \nu_R h \langle H^0 \rangle$, which we denote by m_ν^D, whereas the second one gives rise to the Majorana entry $O(M)$ in the NN sector. This leads to a light, mainly left-handed Majorana neutrino with a mass $m_\nu \sim (m_\nu^D)^2/$

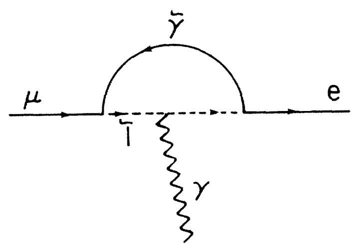

FIG. 1. Penguin diagram for $\mu \to e\gamma$ with exchange of $\tilde{\gamma}$.

M and a heavy, mainly right-handed Majorana neutrino of mass M. Clearly m_ν^D is much larger than m_ν; for instance, to get $m_\nu \sim 1$ eV, m_ν^D should be taken to be ~ 300 GeV in a grand unified scheme where $M \sim 10^{14}$ GeV or ~ 1–10 GeV in models where M corresponds to some intermediate scale $M \sim 10^{10}$ GeV. The presence in the superpotential of $hLHN$ induces a renormalization effect on the mass of $\tilde{l}$ proportional to $\tilde{l}_L^\dagger hh^\dagger l_L$; the entry $\tilde{l}_L^\dagger \tilde{l}_L$ in (1) gets modified to[1,2]

$$\tilde{l}_L^\dagger (m^2 + m_l m_l^\dagger + c m_\nu^D m_\nu^{D\dagger}) l_L, \tag{2}$$

where c is a parameter which can be calculated by solving the system of renormalization-group equations of the parameters in the superpotential. We take $c = 0.5$ throughout this paper. We choose to work in a basis where the $\tilde{\gamma}$ and $\tilde{Z}$ couplings are flavor-diagonal, but the left-handed charged–scalar-lepton mass matrix presents the flavor-changing radiative contribution

$$\Delta \tilde{m}_{l_L}^2 = m^2 \Delta, \quad \Delta = c U m_\nu^D m_\nu^{D\dagger} U^\dagger, \tag{3}$$

where U is the unitary matrix which diagonalizes $m_l m_l^\dagger$. For instance, the mixing mass term $\tilde{\mu}_L \tilde{e}_L^*$ is proportional to $c(U m_\nu^D m_\nu^{D\dagger} U^\dagger)_{12} = \Delta_{12}$, i.e., to the Dirac entries m_ν^D of the neutrino mass matrix and not to the effective neutrino mass m_ν. In a model with Dirac neutrino $m_\nu^D = m_\nu$ and so, clearly, L_e, L_μ-nonconserving effects due to $\tilde{\mu}_L$-$\tilde{e}_L$ mixings are smaller than in the Majorana case by powers of (m_ν / m_ν^D).

Before we give the results of our calculations, a few comments are in order. The $\tilde{\gamma}$ exchange turns out to be more important than the $\tilde{Z}$ or neutral–Higgs-fermion exchanges and so we shall consider only l-$\tilde{l}$-$\tilde{\gamma}$ vertices. For definiteness, we shall assume $m_{\tilde{\gamma}} < m$ and, hence, diagrams with $m_{\tilde{\gamma}}$ insertions shall be neglected in favor of diagrams where the $\tilde{\gamma}$ momentum is picked up. Finally, diagrams with $\tilde{l}_L$-$\tilde{l}_L^c$ mixings have extra powers of m_l/m and are therefore suppressed. It is remarkable that our entire ignorance of the leptonic sector should be parametrized in our results by the single parameter Δ defined in Eq. (3). If we take $(U m_\nu^D m_\nu^{D\dagger} U^\dagger)_{12} \simeq 0.2 m_\nu^{D2}$, where m_ν^D denotes some typical Dirac entry in the neutrino mass matrix, the mixing $\tilde{\mu}_L$-$\tilde{e}_L$ is given by $\Delta_{12}^2 \simeq 0.1 m_\nu^{D2}$, so that we can alternatively parametrize our results in terms of m_ν^D.

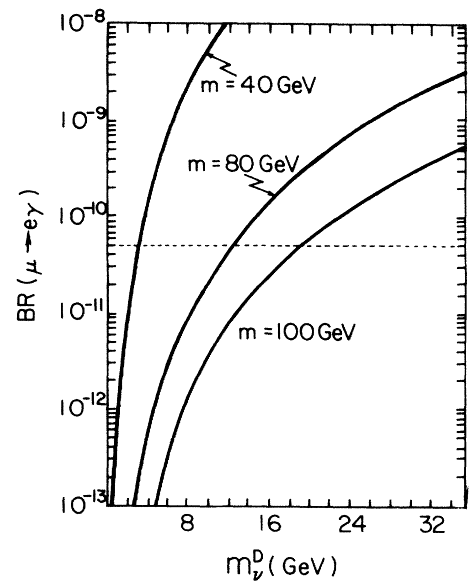

FIG. 2. Branching ratio for $\mu \to e\gamma$ as a function of m_ν^D, for different values of m. The dashed line indicates the present experimental upper bound.

The most relevant L_e, L_μ-nonconserving processes turn out to be $\mu \to e\gamma$, $\mu \to e e \bar{e}$, and μ + nucleus $\to e$ + nucleus.

(i) $\mu \to e\gamma$. The diagram in Fig. 1 yields

$$B(\mu \to e\gamma) = \frac{\alpha^3}{G_F^2} 12\pi \frac{\Delta_{12}^2 F(x)^2}{m^8}, \tag{4}$$

where

$$F(x) = \tfrac{1}{12}(1-x)^{-5}\{17x^3 - 9x^2 - 9x + 1 - 6x^2(x+3)\ln x\}$$

and $x = m_{\tilde{\gamma}}^2 / m^2$. For $0 < x < 1$, we obtain $\frac{1}{40} < F(x) < \frac{1}{12}$. If we choose $F(x) = \frac{1}{20}$, $B(\mu \to e\gamma)$ becomes a function solely of m and m_ν^D. The plot in Fig. 2 shows that the present experimental upper limit,[7] $B(\mu \to e\gamma) < 4.9 \times 10^{-11}$, already rules out large regions of m_ν^D values for $m < 100$ GeV. For large values of m, for instance $m \sim 150$ GeV, one still gets $B(\mu \to e\gamma) \sim 10^{-12}$–$10^{-13}$ for $m_\nu^D \sim 10$–20 GeV.

(ii) $\mu \to e e \bar{e}$. The leading contribution comes from the box diagram depicted in Fig. 3, yielding

$$B(\mu \to e e \bar{e}) = \tfrac{1}{8}(\alpha^4 / G_F^2) \Delta_{12}^2 G(x)^2 / m^8, \tag{5}$$

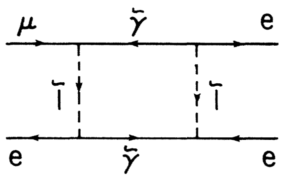

FIG. 3. Box diagram for $\mu \to \bar{e}ee$.

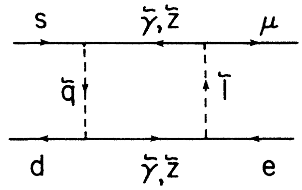

FIG. 4. Box contribution to $K_L \to \mu e$.

where

$$G(x) = \tfrac{1}{2}(1-x)^{-5}\{-x^3 - 9x^2 + 9x + 1 + 6x(1+x)\ln x\}.$$

Taking the same value for $x = m\tilde{\gamma}/m$ as in the $\mu \to e\gamma$ calculation, we obtain $B(\mu \to e\gamma)/B(\mu \to ee\bar{e}) \simeq 10^3$, so that, in view of the recent impressive improvement of the experimental bound,[8] $B(\mu \to ee\bar{e}) < 2.4 \times 10^{-12}$, it turns out that also $\mu \to ee\bar{e}$ constitutes an important test for the class of supersymmetric models that we are considering.

(iii) μ + nucleus $\to e$ + nucleus. Evaluating the leading diagram in Fig. 4 and using the procedure given in Ref. 9, we find

$$\frac{\omega(\mu^- + N \to e^- + N)}{\omega(\mu^- + N_z \to \nu_\mu + N_{z-1})} \equiv R_{eN} = \left(\frac{Z+A}{Z}\right)^2 \frac{1}{5184\pi^2} \frac{\alpha^2}{G_F^2} \frac{\Delta_{12}^2 G(x)^2}{m^8}, \tag{6}$$

where Z and A denote the number of protons and nucleons in N, respectively. The best experimental bound[10] on μ conversion from $\mu + \mathrm{Ti} \to e + \mathrm{Ti}$ is $R_{eN} < 1.6 \times 10^{-11}$. From (4), (5), and (6) we obtain

$$R_{eN}/B(\mu \to e\gamma) \simeq 0.026, \quad R_{eN}/B(\mu \to ee\bar{e}) \simeq 29.61. \tag{7}$$

These ratios are quite interesting since they are independent of Δ_{12}; furthermore, since the same function $G(x)$ appears in (5) and (6), the second ratio in (7) is not plagued by our ignorance of x.

The processes $\pi^0 \to \mu e$ and $K_L \to \mu e$ turn out to be very suppressed, typically between 8 and 10 orders of magnitude below the present experimental bounds. For $\pi^0 \to \mu e$ this is due to the rapidity of the decay $\pi^0 \to \gamma\gamma$:

$$\frac{\Gamma(\pi^0 \to \mu e)}{\Gamma(\pi^0 \to \gamma\gamma)} = \tfrac{1}{3} \times 10^{-8} \frac{\Gamma(\pi^0 \to \mu e)}{\Gamma(\pi^0 \to \mu\nu)},$$

whereas in the case of $K_L \to \mu e$ the suppression arises from the presence of the double Glashow-Iliopoulos-Maiani suppression in the hadronic and leptonic flavor changes. In this case, differently from what happens in K^0-$\bar{K}^0$, this double Glashow-Iliopoulos-Maiani suppression is effective, leading to a suppression factor $(1/m^4)\Delta^2\Delta_q^2/m^8$, where Δ_q is the analog of Δ in the hadronic sector [typically $\Delta_q^2/(m^4 \sin^2\theta_<) \sim 10^{-5}$–$10^{-6}$].

In conclusion, we have shown that large renormalization effects in the $\tilde{l}_L \tilde{l}_L^*$ sector of the charged–scalar-lepton mass matrix are present in supersymmetric versions of models with the heavy "seesaw" mechanism. In turn they give rise to sizeable violations of nonconservation of L_e, L_μ through l-$\tilde{l}$-$\tilde{\gamma}$ vertices leading to $\mu \to e\gamma$, $\mu \to ee\bar{e}$, and μ conversion in a range accessible to the present experiments. For values of the Dirac entry in the neutrino mass matrix in the gigaelectronvolt range and $m \simeq 150$ GeV they might be detectable in the very near future.

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