## SPONTANEOUS *CP* VIOLATION IN SUSY MODELS: A NO-GO THEOREM

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It is shown that in a large class of supersymmetric  $SU_c(3) \times SU_L(2) \times U(1)$  models there is no spontaneous CP violation.

The standard explanation of CP violation through the Kobayashi-Maskawa (KM) model [1] has recently been pushed to the limit due to the new measurements of  $\epsilon'/\epsilon$  [2]. There is therefore a renewed interest in looking for other sources of CP violation besides the KM mechanism.

One possibility is to have spontaneous CP violation. In the context of the  $SU_c(3) \times SU_L(2) \times U_Y(1)$  gauge theory it can be achieved if at least two Higgs doublets are introduced [3]. If in addition natural flavour conservation (NFC) is required then one must have three Higgs doublets [4,5] or two Higgs doublets and one singlet.

Supersymmetric (SUSY)  $SU_c(3) \times SU_L(2) \times U_Y(1)$  models (see ref. [6] for a review) need at least two Higgs doublets and incorporate NFC in a natural way. So it is interesting to ask if spontaneous *CP* violation can be accommodated in these models.

The purpose of this letter is to examine this question in a large class of "minimal" SUSY  $SU_c(3) \times SU_L(2) \times U_Y(1)$  models where the breaking of SUSY is accomplished through the coupling to N = 1 supergravity [6]. In these models the potential part of the lagrangian is (we follow the notation and conventions of ref. [7])

$$\mathcal{L}_{\text{pot}} = -\sum_{A} \left| \frac{\partial f}{\partial z_{A}} \right|^{2} - \sum_{j=1}^{3} \frac{1}{2} (\boldsymbol{D} \cdot \boldsymbol{D})_{j} + \mathcal{L}_{\text{SB}} , \qquad (1)$$

where  $z_A(\chi_A)$  denotes the scalar (spinor) component of a general chiral superfield and the first sum is over all the chiral superfields. In the D terms the index *j* refers to the gauge group and we have

$$D_j = g_j z^{*A} T_A{}^B z_B , \qquad (2)$$

where  $g_j$  are the coupling constants and T are the generators. If we do not introduce mass terms for the gauginos the supersymmetry breaking lagrangian is

$$\mathcal{L}_{SB} = -m_{3/2}^2 \sum_A z^{*A} z_A - m_{3/2} \left( (A-3)f(z) + \sum_A \frac{\partial f}{\partial z_A} z_A + h.c. \right),$$
(3)

where  $m_{3/2}$  is the gravitino mass and A is a parameter of order unity.

As we are interested in studying the possibility of having spontaneous CP violation in these models we want to consider the most general situation. Hence we include also the possibility that the scale at which supersymmetry is broken via eq. (3) is much higher then the low energy scale,  $O(M_{\omega})$ , at which we want to study the models. Then radiative corrections will change the parameters in eqs. (1)-(3).

Different models are defined by the f-function. As is well known [7,8], it is not a trivial task to assure that the true vacuum of the potential does not break colour and electric charges. Let us assume that a set of parameters can be found where that is true. Then we have only to consider the Higgs sector. If we would get spontaneous CP

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violation for some set of parameters, then we would have to check if the conditions for the non breaking of colour and electric charges were satisfied.

In the minimal SUSY model we have

$$f^{(0)} = \mu \epsilon^{\alpha\beta} h_{\alpha} h_{\beta}' , \qquad (4)$$

where h and h' are scalar components of the two Higgs doublets chiral superfields and  $\mu$  is a parameter with dimensions of mass. As these SUSY models incorporate natural flavour conservation (NFC), this gives a particular case of a potential for the standard model with NFC and two Higgs doublets, therefore this model has no spontaneous *CP* violation [4,5].

Hence, if we want to have spontaneous CP violation one must include more Higgs fields. In many of the "minimal" models considered in the literature [6] one Higgs singlet chiral superfield is also introduced. Thus the possibility of having spontaneous CP violation deserves a careful study in this situation. To be specific we have studied the following three types of models:

$$f^{(1)} = \lambda y \epsilon^{\alpha\beta} h_{\alpha} h'_{\beta} - \epsilon y , \quad f^{(2)} = \lambda y \epsilon^{\alpha\beta} h_{\alpha} h'_{\beta} + \sigma y^2 , \quad f^{(3)} = \lambda y \epsilon^{\alpha\beta} h_{\alpha} h'_{\beta} + \lambda' y^3 , \quad (5, 6, 7)$$

where y is the scalar component of the Higgs singlet chiral superfield and  $\epsilon$ ,  $\sigma$ ,  $\lambda$  and  $\lambda'$  are parameters with appropriate dimensions (remember that f has dimension 3 in units of mass). The models with  $f^{(1)}$  and  $f^{(3)}$  have been considered before [7,9]. The model with  $f^{(2)}$  is considered here for completeness (f is a polynomial in the fields of degree at most three).

For these types of models we prove the following:

Theorem. For the models specified by eqs. (1)-(3) with an *f*-function given by eqs. (5)-(7), and with radiative corrections, it is impossible to have spontaneous *CP* violation.

*Proof.* Although we have proved the theorem for the three cases, let us outline the proof only for the  $f^{(3)}$  case, since the other cases are very similar. Allowing for radiative corrections the Higgs potential is

$$V = \frac{1}{8} (g^2 - g'^2) (v_1^2 - v_2^2)^2 + \lambda^2 v_1^2 v_2^2 + 9\lambda'^2 v_3^4 + \lambda^2 v_3^2 (v_1^2 + v_2^2) + \frac{1}{2} m_1^2 v_1^2 + \frac{1}{2} m_2^2 v_2^2 + \frac{1}{2} m_2^2 v_3^2 + \frac{1$$

where

$$\langle h \rangle = \begin{pmatrix} v_1 \exp(i\Theta_1) \\ 0 \end{pmatrix}, \quad \langle h' \rangle = \begin{pmatrix} 0 \\ v_2 \exp(i\Theta_2) \end{pmatrix}, \quad \langle y \rangle = v_3 \exp(i\Theta_3).$$
(9)

At unification scale

$$m_1^2 = m_2^2 = m_3^2 = m_{3/2}^2$$
, and  $B = A$ , (10)

but due to radiative corrections they will be different at low energy. As we are going to demonstrate a no-go theorem we do not worry about the renormalization group (RG) evolution but allow the parameters to be arbitrary. Only if there was spontaneous CP violation for some set of parameters we would have then to check if they could be obtained via the RG equations.

It is convenient to make the following redefinitions:

$$V = (A^4 m_{3/2}^4 / \lambda^2) \hat{V}, \quad v_i = (m_{3/2} / \lambda) A \hat{v}_i, \quad 2\hat{\Theta}_1 = \Theta_1 + \Theta_2 + \Theta_3, \quad 2\hat{\Theta}_3 = 3\Theta_3, \quad (11a, b, c, d)$$

then the potential can be written

$$\hat{V} = a_1 (\hat{v}_1^2 - \hat{v}_2^2)^2 + \hat{v}_1^2 \hat{v}_2^2 + \hat{v}_3^2 (\hat{v}_1^2 + \hat{v}_2^2) + \frac{1}{4} c_1^2 \hat{v}_3^4 + \frac{1}{2} \hat{m}_1^2 \hat{v}_1^2 + \frac{1}{2} \hat{m}_2^2 \hat{v}_2^2 + \frac{1}{2} \hat{m}_3^2 \hat{v}_3^2 + 2 \hat{v}_1 \hat{v}_2 \hat{v}_3 \cos 2\hat{\Theta}_1 + c_1 \hat{v}_1 \hat{v}_2 \hat{v}_3^2 \cos (2\hat{\Theta}_1 - 2\hat{\Theta}_3) + c_2 \hat{v}_3^3 \cos 2\hat{\Theta}_3 \equiv U(\hat{v}_i) + W(\hat{v}_i, \hat{\Theta}_i) , \qquad (12)$$

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(15)

where all the quantities are dimensionless and can be related easily to the original parameters.

The part of the potential that depends on the phases is

$$W(\hat{v}_i, \hat{\Theta}_i) \equiv D_{12} \cos 2\hat{\Theta}_1 + D_{13} \cos (2\hat{\Theta}_1 - 2\hat{\Theta}_3) + D_{23} \cos (2\hat{\Theta}_3), \qquad (13)$$

which is of the type studied in ref. [5]. This has a nontrivial minimum in the angle variable if  $|D_{12}D_{13}|$ ,  $|D_{12}D_{23}|$  and  $|D_{13}D_{23}|$  can form a triangle [5] and if

$$D_{12}D_{13}/D_{23} > 0 . (14)$$

Condition (14) implies in our language

$$c_1/c_2 > 0$$
.

Let us assume that the parameters are such that a minimum with non trivial phases exists. Then

$$W(\hat{v}_i, \hat{\Theta}_i = \hat{\Theta}_i^{\min}) = -\frac{1}{2} [2(c_2/c_1)\hat{v}_3^2 + 2(c_1/c_2)\hat{v}_1^2\hat{v}_2^2 + \frac{1}{2}c_1c_2\hat{v}_3^4].$$
(16)

If we introduce the variables

$$\Sigma = \hat{v}_1^2 + \hat{v}_2^2, \qquad \Delta = \hat{v}_1^2 - \hat{v}_2^2, \tag{17}$$

the potential to be minimized in the remaining variables is

$$\hat{V} = a_1 \Delta^2 + \frac{1}{4} (\Sigma^2 - \Delta^2) (1 - c_1/c_2) + \hat{v}_3^2 \Sigma + \hat{v}_3^4 \frac{1}{4} c_1(c_1 - c_2) + \frac{1}{4} (m_1^2 + m_2^2) \Sigma + \frac{1}{4} (m_1^2 - m_2^2) \Delta + (\frac{1}{2} m_3^2 - c_2/c_1) \hat{v}_3^2.$$
(18)

The extremum conditions are

$$\partial \hat{V} / \partial \Delta = 2a_1 \Delta - \frac{1}{2} (1 - c_1 / c_2) \Delta + \frac{1}{4} (\hat{m}_1^2 - \hat{m}_2^2) = 0, \quad \partial \hat{V} / \partial \Sigma = \frac{1}{2} (1 - c_1 / c_2) \Sigma + \hat{v}_3^2 + \frac{1}{4} (\hat{m}_1^2 + \hat{m}_2^2) = 0,$$
  

$$\partial \hat{V} / \partial v_3^2 = \frac{1}{2} c_1 (c_1 - c_2) v_3^2 + (\Sigma + \frac{1}{2} m_3^2 - c_2 / c_1) = 0.$$
(19)

These conditions can be easily solved, but they never yield a minimum because the hessian matrix

$$\begin{bmatrix} 2a_1 - \frac{1}{2}(1 - c_1/c_2) & 0 & 0 \\ 0 & \frac{1}{2}(1 - c_1/c_2) & 1 \\ 0 & 1 & \frac{1}{2}c_1(c_1 - c_2) \end{bmatrix}$$
(20)

is not positive definite. In fact the conditions of positive definiteness are

(i) 
$$2a_1 - \frac{1}{2}(1 - c_1/c_2) > 0$$
, (ii)  $\frac{1}{2}(1 - c_1/c_2) > 0$ ,  
(iii)  $c_1^2 > c_1c_2 \Rightarrow (c_1/c_2)^2 > c_1/c_2$ , (iv)  $(1 - c_1/c_2)(c_1^2 - c_1c_2) > 4$ . (21)

Condition (i) can be verified with arbitrary  $a_1$ . However condition (iii) and eq. (15) imply

$$c_1/c_2 > 1$$
, (22)

which is in contradiction with condition (ii). This ends the proof of our theorem.

We have checked, both analytically and numerically that a potential of the form (12) but without SUSY can have spontaneous *CP* violation. The requirement of SUSY amounts to fixing the coefficients of the second, third and fourth terms in eq. (12). If we allow, for instance, the coefficient of  $\hat{v}_1^2 \hat{v}_2^2$  to be *d*, arbitrary (SUSY puts d = 1), then one can easily show that there is spontaneous *CP* violation if

$$d > c_1/c_2 + 4/(c_1^2 - c_1c_2) .$$
<sup>(23)</sup>

Using conditions (ii) and (iii) of eq. (21) one can verify that in order to have a minimum one must have d > 1, which is in agreement with the theorem we proved.

Although we have shown the absence of spontaneous CP violation in these minimal SUSY models, there are many other ways of obtaining additional CP violation. If we consider the model of ref. [7] without radiative corrections, then we can give a phase to the parameter A in eq. (3). This could still be thought of as a kind of spontaneous CP violation in the hidden sector that performed the breaking of supersymmetry. CP violating phases will then appear for the usual particles in the Yukawa interactions with the neutral Higgs, and for their supersymmetric partners in the wino-quark-squark coupling [10].

Another possibility is to consider models with radiative corrections. Then the different evolution under the RG of the up and down quark matrices gives origin to new phases. This possibility has been studied by many authors [11].

In conclusion, we have shown that in some classes of supersymmetric extensions of the standard model no spontaneous CP violation can take place. One should however note that our result was proved only for some classes of f functions. It remains an open possibility that more complicated f functions could produce spontaneous CP violation.

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