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Electromagnetic Properties of the Z Boson. I

A. Barroso [1], F. Boudjema, J. Cole and N. Dombey

Physics Division, University of Sussex, Brighton, BN1 9QH, England

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Abstract. The contribution of the fermion triangle diagram responsible for the ABJ anomaly to the $ZZ\gamma$ vertex is calculated paying particular attention to the symmetries which must be satisfied. Contrary to previous calculations no static electric dipole moment of the Z is found. Two other P-violating but CP-conserving couplings are demonstrated as is a new anomaly condition.

1. Introduction

The recent discovery at CERN [1] of the W and Z particles with the masses predicted by the standard electroweak model $SU(2)\times U(1)$ including radiative corrections [2] provides the most powerful confirmation of the model so far. Yet the question of whether the model is fundamental or simply phenomenological remains open. If the former is true then the W and the Z can be considered as elementary in the same way that the photon is elementary; if the latter, they are simply composite particles of spin-1. At low energies it is difficult to distinguish between these two possibilities: we can expect, however, that any composite structure of the W and Z will show at higher energies in deviations from the standard model. In particular radiative corrections should provide a sensitive test of the standard model compared with composite models.

Our aim in this paper is to examine one of these higher order processes which will be of interest when LEP and other electron positron machines have sufficient energy to produce a pair of Z's. In particular we want to consider the Z coupling to photons via the fermion triangle diagram. This diagram has already been studied by Renard [3] who reached the conclusion that the Z boson would have an electric dipole moment (EDM). Motivated by this surprising conclusion, which would violate the time reversal invariance of the theory, we have re-examined carefully the $ZZ\gamma$ triangle coupling. We shall prove that for real on-shell Z's and virtual photons the *only* coupling is a parity-violating anapole moment. This anapole moment, first examined by Zeldovich [4] for a spin-$\frac{1}{2}$ particle, is a coupling of the Z spin with the external electromagnetic current which violates parity but conserves CP-invariance.

We have organised our paper in the following way. In Sect. 2 we review the calculation (5) of the triangle graph with one axial current coupled to two photons and the Adler-Bell-Jackiw (ABJ) anomaly [6]. In Sect. 3 we generalise these results to obtain the fermion triangle contribution to the $ZZ\gamma$ three-point function and point out the existence of yet another anomaly. In Sect. 4 we consider the particular case of one Z and the photon on-shell and the other Z off-shell. We show that, in this case, there is an electric dipole transition, but time reversal invariance is preserved since it does not lead to a static EDM for the Z. Finally in Sect. 5 we consider the Z anapole moment.

2. The $Z\gamma\gamma$ Three Point Function

In the standard $SU(2)_L\times U(1)$ electroweak model the Z boson fermion coupling is

$$L_{\text{int}}=\frac{g}{\cos\theta_W}Z_\mu J^\mu \tag{1}$$

where the neutral current J^μ of fermion f is

$$J^\mu=\bar{\psi}_f(g_V^f\gamma^\mu-g_A^f\gamma^\mu\gamma_5)\psi_f \tag{2}$$

with

[1] Permanent address: CFN, Universidade de Lisboa, Gama Pinto 2, 1699 Lisboa, Portugal

$$g_V^f = \tfrac{1}{2} T_3^f - Q_f \sin^2\theta_W, \tag{3a}$$

$$g_A^f = \tfrac{1}{2} T_3^f. \tag{3b}$$

T_3^f and Q_f are the third component of the weak isospin and the fermion charge respectively. In the interaction Lagrangian, L_{int}, there is no direct Z-photon coupling. Hence, in lowest order, the $Z\gamma\gamma$ three point function, $G^{\rho\nu\mu}$, is represented by the diagrams of Fig. 1, with fermions in the loop. Evaluating these diagrams we obtain:

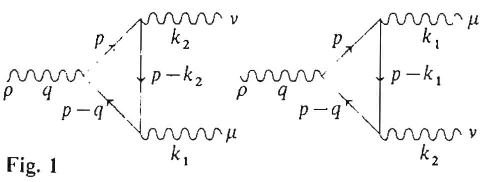

Fig. 1

$$G^{\rho\nu\mu}(q, k_2, k_1) = -\frac{e^2 g}{\cos\theta_W} \sum_f g_A^f Q_f^2 \int \frac{d^4 p}{(2\pi)^4}$$
$$\cdot \operatorname{Tr}\left(\frac{1}{\not p - m_f}\gamma^\rho \gamma_5 \frac{1}{\not p - \not q - m_f}\gamma^\nu \frac{1}{\not p - \not k_1 - m_f}\gamma^\mu\right)$$
$$+(k_1 \to k_2, \mu \to \nu). \tag{4}$$

Notice that the only contribution to $G^{\rho\nu\mu}$ is proportional to g_A^f since, by Furry's theorem, the triangle diagram with three vector vertices is zero.

The evaluation of the integral in (4) is by now well known [5]. After evaluating the trace one imposes the Ward identities, resulting in the conservation of the electromagnetic current

$$k_{1\mu} G^{\rho\nu\mu} = 0, \tag{5a}$$

$$k_{2\nu} G^{\rho\nu\mu} = 0 \tag{5b}$$

and obtains:

$$G^{\rho\nu\mu}(q, k_1, k_2) = -\frac{e^2 g}{2\pi^2 \cos\theta_W} \sum_f g_A^f Q_f^2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2$$
$$\cdot \frac{1}{\Delta}\left[\varepsilon^{\rho\nu\mu\alpha}(1-x_1-x_2)(x_2 k_1 - x_1 k_2)_\beta (k_2^\beta k_{1\alpha} + k_1^\beta k_{2\alpha})\right.$$
$$+(1-x_1-x_2)(\varepsilon^{\alpha\rho\beta\nu} k_{1\alpha} k_{2\beta}(x_2 k_1^\mu - x_1 k_2^\mu) + (\mu \to \nu))$$
$$\left.+\varepsilon^{\alpha\nu\beta\mu} k_{1\alpha} k_{2\beta}(x_2(x_2 - x_1 - 1) k_1^\rho - x_1(x_2 - x_1 + 1) k_2^\rho)\right] \tag{6a}$$

with

$$\Delta = m_f^2 + x_2(x_2-1) k_1^2 + x_1(x_1-1) k_2^2 - 2 x_1 x_2 k_1 \cdot k_2. \tag{6b}$$

We note that if we had only applied one of the conditions (5) the other would automatically follow from the necessary symmetry arising from the identical nature of the two photons. It is now easy to see that $G^{\rho\nu\mu}$ also satisfies the relation:

$$q_\rho G^{\rho\nu\mu} = (k_1 + k_2)_\rho G^{\rho\nu\mu}$$
$$= \frac{e^2 g}{\pi^2 \cos\theta_W} \sum_f g_A^f Q_f^2 \varepsilon^{\nu\mu\alpha\beta} k_{1\alpha} k_{2\beta}$$
$$\cdot \left[\frac{1}{2} - m_f^2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{\Delta}\right]. \tag{7}$$

The presence of a mass independent term on the right hand side of (7) constitutes a break-down of axial current conservation for massless fermions. This is the well-known ABJ anomaly [6]. Clearly, with the standard fermion multiplet assignment, the anomaly vanishes since

$$\sum_f g_A^f Q_f^2 = 0.$$

It is worthwhile remembering (6) that the 4-momentum integral defining the triangle is linearly divergent. So the value of the triangle graph is ambiguous and depends on the labelling convention of momenta and the method of evaluation of the integral. However this ambiguity is removed by imposing the two constraints given by (5). On the other hand the different terms in (6a) are not all independent. In fact, using the identity

$$(a \cdot b)[cdef] - (a \cdot c)[bdef] + (a \cdot d)[bcef]$$
$$-(a \cdot e)[bcdf] + (a \cdot f)[bcde] = 0 \tag{8}$$

where $[abcd]$ is a shorthand notation for $\varepsilon^{\alpha\beta\gamma\delta} a_\alpha b_\beta c_\gamma d_\delta$, it is easy to re-write (7) as

$$G^{\rho\nu\mu}(q, k_1, k_2) = -\frac{e^2 g}{\pi^2 \cos\theta_W} \sum_f g_A^f Q_f^2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{\Delta}$$
$$\cdot \left[\varepsilon^{\rho\nu\mu\alpha}(k_{1\alpha}(x_1(x_1-1) k_2^2 - x_1 x_2 k_1 \cdot k_2)\right.$$
$$- k_{2\alpha}(x_2(x_2-1) k_1^2 - x_1 x_2 k_1 \cdot k_2))$$
$$+\varepsilon^{\alpha\rho\beta\nu} k_{1\alpha} k_{2\beta}(x_2(1-x_2) k_1^\mu + x_1 x_2 k_2^\mu)$$
$$\left.-\varepsilon^{\alpha\rho\beta\mu} k_{1\alpha} k_{2\rho}(x_1(1-x_1) k_2^\nu + x_1 x_2 k_1^\nu)\right]. \tag{9}$$

This expression is precisely the one derived by Rosenberg [5] and later used by Adler (6) in his seminal paper on triangle anomalies.

Using (9) one can immediately write down the amplitude F for Z decay into two photons, i.e.

$$F(Z \to \gamma\gamma)$$
$$= e_\rho \varepsilon_{1\mu} \varepsilon_{2\nu} G^{\rho\nu\mu}(q, k_1^2 = k_2^2 = \varepsilon_1 \cdot k_1 = \varepsilon_2 \cdot k_2 = 0)$$
$$= -\frac{e^2 g}{\pi^2 \cos\theta_W} \sum_f g_A^f Q_f^2 \int_0^1 dx_1$$
$$\cdot \int_0^{1-x_1} dx_2 \frac{x_1 x_2}{m_f^2 - 2 x_1 x_2 k_1 \cdot k_2} e_\rho \varepsilon_{1\mu} \varepsilon_{2\nu}$$
$$\cdot \left[\varepsilon^{\rho\nu\mu\alpha}(k_2 - k_1)_\alpha + k_{1\alpha} k_{2\beta}(\varepsilon^{\alpha\rho\beta\nu} k_2^\mu - \varepsilon^{\alpha\rho\beta\mu} k_1^\nu)\right] = 0. \tag{10}$$

$k_1 \cdot k_2$

Again we obtain a well-known result: Bose statistics forbids a spin-1 particle to decay into two photons. We note that if one photon is off-shell the amplitude $F(Z \to \gamma\gamma_V)$ does not vanish.

3. The $ZZ\gamma$ Three Point Function

We shall now use the results of the previous section to calculate the $ZZ\gamma$ three point function $T^{\rho\nu\mu}$. For this it is convenient to have an expression for $G^{\rho\nu\mu}$ using q and k_2 as the independent variables. After some manipulations, which involve the use of the identity (8), we obtain:

$$G^{\rho\nu\mu}(q, k_1, k_2) = -\frac{e^2 g}{\pi^2 \cos\theta_W} \sum_f g_A^f Q_f^2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{\Delta'} \cdot [\varepsilon^{\rho\nu\mu\alpha}[q_\alpha(x_1+x_2-1)(x_2 k_2 \cdot q + x_1 k_2^2) + k_{2\alpha}(x_1(1-x_1)k_2^2 + x_2(1-x_2)q^2 - 2x_1 x_2 q \cdot k_2] + \varepsilon^{\alpha\rho\beta\nu} q_\alpha k_{2\beta}(x_2(1-x_2)q^\mu - x_1 x_2 k_2^\mu) + \varepsilon^{\alpha\rho\beta\mu} q_\alpha k_{2\beta}(x_1+x_2-1)(x_2 q^\nu + x_1 k_2^\nu)] \tag{11}$$

with

$$\Delta' = m_f^2 + k_2^2 x_1(x_1-1) + q^2 x_2(x_2-1) + 2x_1 x_2 q \cdot k_2. \tag{12}$$

To obtain the expression $\tilde{G}^{\rho\nu\mu}(q_1, q_2, k)$ with one photon replaced by a Z, where q_1, q_2 and $k(=q_1 - q_2)$ are the incoming Z, the outgoing Z and photon 4-momentum respectively, we must make the relevant momentum substitutions and the replacement

$$e^2 Q_f^2 \to e Q_f \frac{g}{\cos\theta_W} g_V^f \tag{13}$$

so that

$$\Delta' \to m_f^2 + q_2^2 x_1(x_1-1) + q_1^2 x_2(x_2-1) + 2x_1 x_2 q_1 \cdot q_2. \tag{14}$$

In this case, however, the two electromagnetic current conservation conditions, (5a, b) no longer apply. Instead we only have the one condition

$$k_\mu T^{\rho\nu\mu} = 0.$$

This is not sufficient to determine $T^{\rho\nu\mu}$ uniquely. In addition we must apply the condition that the two Z's are identical. This implies that $T^{\rho\nu\mu}$ is symmetric under $\rho \to \nu$, $q_1 \to -q_2$. We therefore write

$$T^{\rho\nu\mu}(q_1, q_2, k) = \tilde{G}^{\rho\nu\mu}(q_1, q_2, k) + \tilde{G}^{\nu\rho\mu}(-q_2, -q_1, k). \tag{15}$$

We postpone the evaluation of this quantity till later and consider now the triangle diagrams of Fig. 1, but with two axial currents, i.e. assume that the currents with 4-momentum k_1 and k_2 are axial currents and the third is a vector current. Then

$$G_{AAV}^{\rho\nu\mu} \propto \int \frac{d^4 p}{(2\pi)^4} \mathrm{Tr}\left(\frac{1}{\not p - m}\gamma^\rho \frac{1}{\not p - \not q - m}\gamma^\nu \gamma_5 \cdot \frac{1}{\not p - \not k_1 - m}\gamma^\mu \gamma_5\right) + (k_1 \to k_2, \mu \to \nu) \tag{16}$$

and if the loop fermions are massless, we have

$$G_{AAV}^{\rho\nu\mu}(m=0) = G_{VVV}^{\rho\nu\mu} = 0.$$

On the other hand, evaluating the trace in (16) and separating the terms proportional to m^2 it is easy to obtain

$$G_{AAV}^{\rho\nu\mu} = G_{AAV}^{\rho\nu\mu} - G_{VVV}^{\rho\nu\mu} \propto 8m^2 \cdot \left[\int \frac{d^4 p}{(2\pi)^4} \frac{q^\nu g^{\rho\mu} - q^\mu g^{\rho\nu} - (2p-q)^\rho g^{\mu\nu}}{(p^2-m^2)((p-q)^2-m^2)((p-k_1)^2-m^2)} - \int \frac{d^4 p}{(2\pi)^4} \frac{q^\nu g^{\rho\mu} - q^\mu g^{\rho\nu} + (2p-q)^\rho g^{\mu\nu}}{(p^2-m^2)((p-q)^2-m^2)((p-k_2)^2-m^2)}\right]. \tag{17}$$

Since now all the integrals are convergent, a change of integration variable, $p \to q - p$, in the second term proves that $G_{AAV}^{\rho\nu\mu} = 0$.

Having proved that the triangle with two axial currents is zero, let us now proceed with the calculation of $T^{\rho\nu\mu}$. From (15) we obtain:

$$T^{\rho\nu\mu}(q_1, q_2, k) = -\frac{2e}{\pi^2}\left(\frac{g}{\cos\theta_W}\right)^2 \sum_f g_A^f g_V^f Q_f \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{\Delta'} \cdot (\varepsilon^{\rho\nu\mu\alpha}[\tfrac{1}{2}k_\alpha(\Delta' - m_f^2) + q_{1\alpha}(x_2(1-x_2)q_1^2 + x_1 x_2 q_2^2 - x_2(1+x_1-x_2)q_1 \cdot q_2)] + \varepsilon^{\alpha\rho\beta\nu} q_{1\alpha} q_{2\beta}[x_2(1-x_2)q_1^\mu - x_1 x_2 q_2^\mu] + \varepsilon^{\alpha\rho\beta\mu} q_{1\alpha} q_{2\beta}(x_1+x_2-1)(x_2 q_1^\nu + x_1 q_2^\nu)). \tag{18}$$

It is easy to check that $T^{\rho\nu\mu}$ satisfies the following relations:

$$k_\mu T^{\rho\nu\mu} = 0, \tag{19a}$$

$$q_{1\rho} T^{\rho\nu\mu} = -\frac{e}{\pi^2}\left(\frac{g}{\cos\theta_W}\right)^2 \sum_f g_A^f g_V^f Q_f \varepsilon^{\rho\nu\mu\alpha} q_{1\rho} k_\alpha \cdot \left(\frac{1}{2} - m_f^2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{\Delta'}\right) \tag{19b}$$

and

$$q_{2\nu} T^{\rho\nu\mu} = \frac{e}{\pi^2}\left(\frac{g}{\cos\theta_W}\right)^2 \sum_f g_A^f g_V^f Q_f \varepsilon^{\rho\nu\mu\alpha} q_{2\nu} k_\alpha \cdot \left(\frac{1}{2} - m_f^2 \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{1}{\Delta'}\right). \tag{19c}$$

We should emphasize here that (19) show that neither of the quantities $q_{1\rho} T^{\rho\nu\mu}$ or $q_{2\nu} T^{\rho\nu\mu}$ vanish even in the limit of massless fermions when it would seem that both the vector and axial currents coupled to the Z are conserved. This is because both the quantities $q_{1\rho} T^{\rho\nu\mu}$ and $q_{2\nu} T^{\rho\nu\mu}$ have anomalous contributions: in the $ZZ\gamma$ coupling one of the Zs must couple to an axial current and by symmetry, it is not possible to say which Z couples to the axial current and which to the vector current. We stress that it is the presence of these anomalies which show that it is not correct to calculate $T^{\rho\nu\mu}$ by imposing naive current conservation for the current coupled to the Z. Renard [3] does not use (19) in his calculation: he seems simply to have evaluated the amplitude $F(Z\to\gamma\gamma_V)$ of (10) above with $k_2^2=M_Z^2$. Our results therefore are different and in particular we will show later that, contrary to Renard's claims, we do not obtain a Z EDM.

At this point we would like to point out a new anomaly. Clearly, for a massless fermion loop, one would expect that the difference

$$\chi^{\rho\nu\mu}=\tilde{G}^{\rho\nu\mu}(q_1,q_2,k)-\tilde{G}^{\nu\rho\mu}(-q_2,-q_1,k)$$

would be zero. However, using (11) to evaluate $\chi^{\rho\nu\mu}$, we obtain

$$\chi^{\rho\nu\mu}=-\frac{e}{\pi^2}\left(\frac{g}{\cos\theta_W}\right)^2\sum_f g_A^f g_V^f Q_f\,\varepsilon^{\rho\nu\mu\alpha}k_\alpha \cdot\left(\frac{1}{2}-m_f^2\int_0^1 dx_1\int_0^{1-x_1}dx_2\,\frac{1}{\Delta'}\right) \tag{20}$$

with Δ' given by (15). Like before, there is a mass independent anomalous term which again vanishes in the standard model since

$$\sum_f g_A^f g_V^f Q_f=0. \tag{21}$$

4. The Electric Dipole Transition of the Z

Let us consider the particular case where the initial Z and the photon are on-shell and the final Z is off-shell. Using the on-shell conditions

$$q_1^2=M_Z^2,$$
$$e_{1\rho}q_1^\rho=0,$$
$$k^2=0$$

and defining $q_2^2=s$ we obtain

$$\begin{aligned}T^{\rho\nu\mu}&(q_1^2=M_Z^2,\,q_2^2=s,\,k^2=0)\\&=-\frac{2e}{\pi^2}\left(\frac{g}{\cos\theta_W}\right)^2\sum_f g_A^f g_V^f Q_f(\varepsilon^{\alpha\rho\beta\mu}q_{1\alpha}q_{2\beta}q_2^\nu I(s,M_Z^2)\\&\quad+\tfrac{1}{2}\varepsilon^{\rho\nu\mu\alpha}k_\alpha[sI(s,M_Z^2)-M_Z^2 I(M_Z^2,s)])\end{aligned} \tag{22}$$

where $I(s,M_Z^2)$ is an integral defined in the appendix (A9). In the appendix we prove (A11) that the combination

$$\begin{aligned}A&=sI(s,M_Z^2)-M_Z^2 I(M_Z^2,s)\\&=\frac{s-M_Z^2}{6M_Z^2}\left[1-\tfrac{1}{2}\mu^2 f(\mu^2)+\tfrac{1}{2}\mu^4 f'(\mu^2)\right]\\&\quad+O\left(\left[\frac{s-M_Z^2}{M_Z^2}\right]^2\right)\end{aligned} \tag{23}$$

where $f(\mu^2)$ is given in the appendix and $\mu=m_f/M_Z$. We note that the second term in (22) is an apparent EDM term since it gives rise to a non-static parity-violating electromagnetic coupling of the form

$$e\beta(s)(s-M_Z^2)\,\varepsilon^{\rho\nu\mu\alpha}e_{1\rho}e_{2\nu}k_\alpha\varepsilon_\mu. \tag{24}$$

In the static limit this is proportional to $\boldsymbol{E}\cdot(\boldsymbol{e}_1\times\boldsymbol{e}_2)$, where $\boldsymbol{E}$ is the electric field, but vanishes as $s\to M_Z^2$, (since $\beta(s)$ is finite in this limit). Thus this term is an electric dipole transition but *not* an EDM. Thus we find that the Z does not have an EDM in contradiction to Renard [3].

The electric dipole transition, however, contributes to the amplitude for the process $Z\to e^+e^-\gamma$ as in Fig. 2, this contribution being:

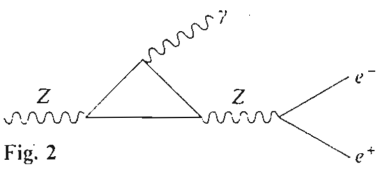

Fig. 2

$$\begin{aligned}iF=&\frac{e}{\pi^2}\left(\frac{g}{\cos\theta_W}\right)^3\varepsilon^{\rho\nu\mu\alpha}e_{1\rho}\varepsilon_\mu k_\alpha\bar{u}(p)\gamma_\nu\\&\cdot(\tfrac{1}{4}-\sin^2\theta_W-\tfrac{1}{4}\gamma_5)v(p')\\&\cdot\frac{1}{s-M_Z^2}\sum_f g_V^f g_A^f Q_f[sI(s,M_Z^2)-M_Z^2 I(M_Z^2,s)].\end{aligned} \tag{25}$$

Recalling that $s-M_Z^2=-2M_Z\omega$, in the rest frame of the Z, where ω is the photon energy, one sees that *apparently* F has the ω^{-1} behaviour characteristic of a bremsstrahlung process. However, this is not the case since the square bracket on the right hand side of (25) is also proportional to $s-M_Z^2$ on account of (23). Equation (25) therefore shows that there should be hard photon radiative decays of the Z. The radiative decays of the Z seen recently at CERN [7] may therefore be connected with an electric dipole transition of the form given in (24) and we have taken up this matter in a recent preprint [8]. Gounaris et al. [9] have also suggested a similar coupling to explain these unexpected decays.

We also note that the first term in (22) does not contribute when both Z's become real since $e_2 \cdot q_2 = 0$ in this case.

5. The Anapole Moment of the Z^0

In the previous section we have considered the coupling of Z's to real photons when one Z is real and the other virtual. In this section we consider the coupling of a real Z to virtual photons. We show that this leads to an anapole moment for the Z. To examine this it is convenient to write down the matrix elements of the current J^μ between an initial and final Z. Starting from the general expression given by (18) and introducing the relations

$$q_1^2 = q_2^2 = M_Z^2,$$
$$e_1 \cdot q_1 = e_2 \cdot q_2 = 0$$

we obtain

$$\begin{aligned} J_Z^\mu &= \langle e_2 q_2 | J^\mu | e_1 q_1 \rangle \\ &= -\frac{2eg^2}{\pi^2 \cos^2\theta_W} \sum_f g_A^f g_V^f Q_f I'(k^2) \\ &\quad \cdot (\varepsilon^{\alpha\rho\beta\nu} q_{1\alpha} e_{1\rho} q_{2\beta} e_{2\nu} k^\mu \\ &\quad + \tfrac{1}{2} k^2 \varepsilon^{\mu\rho\nu\alpha} e_{1\rho} e_{2\nu} (q_1 + q_2^2)_\alpha) \end{aligned} \tag{26}$$

where the integral I' is

$$I'(k^2) = \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \cdot \frac{x_1 x_2}{m_f^2 + (x_1 + x_2)(x_1 + x_2 - 1) M_Z^2 - x_1 x_2 k^2}. \tag{27}$$

In agreement with our previous analysis, (26) shows that the coupling vanishes for a real photon. Nevertheless for a virtual electromagnetic field, A_μ created by an external conserved current, J_μ^{ext}, i.e.

$$k^2 A_\mu = J_\mu^{\text{ext}}$$

one obtains an effective current-current coupling of the form

$$L_{\text{eff}} = -eF(k^2)\, \varepsilon^{\mu\rho\nu\alpha} J_\mu^{\text{ext}} e_{1\rho} e_{2\nu} (q_1 + q_2)_\alpha \tag{28}$$

with

$$F(k^2) = \frac{g^2}{\pi^2 \cos^2\theta_W} \sum_f g_A^f g_V^f Q_f I'(k^2). \tag{29}$$

Hence, in the static limit, $q_1 + q_2 \to (2M_Z, 0)$ we have

$$L_{\text{eff}} \to -2eaM_Z (e_1 \times e_2) \cdot J_{\text{eff}}$$

where $a = F(0)$ is the Z anapole moment. From our results for the integral I', given in the appendix, it can be seen that for $M_Z > 2m_f$ the anapole moment is a complex number. This is because Z is unstable and can decay into an $f\bar{f}$ pair when $M_Z > 2m_f$. We thus have

$$a = \frac{-g^2}{\pi^2 \cos^2\theta_W} \sum_f g_A^f g_V^f Q_f \frac{M_Z^2 - 3m_f^2}{6M_Z^2 m_f^2} S'\left(\frac{M_Z^2}{m_f^2}\right) \tag{30}$$

using the anomaly condition (21), where the function S is given in the appendix.

The current J_Z^μ is not a directly measurable quantity. If, for instance, one looks at the simplest process of electron-Z elastic scattering one realises that there are other diagrams of the same order that can contribute. What is observable are the $e^- Z \to e^- Z$ and the $e^+ e^- \to ZZ$ cross-sections and not the Z anapole moment by itself. A complete discussion of the latter process will be given in the next paper of this series.

Appendix

In this appendix we collect a few expressions useful to calculate the parametric integrals and give the results for those integrals that appear in (22) and (27).

Our definition of the Spence function, $\mathrm{Sp}(x)$, follows that of Lewin [8], namely

$$\mathrm{Sp}(x) = -\int_0^x \frac{\ln(1-z)}{z}\, dz = -\int_0^1 \frac{\ln(1-xz)}{z}\, dz. \tag{A1}$$

Consider the integral

$$S(\beta) = \int_0^1 \frac{dx}{x} \ln(1 - \beta x(1-x) + i\varepsilon). \tag{A2}$$

Writing $1 - \beta x(1-x) = (1 - \alpha_+ x)(1 - \alpha_- x)$ and splitting the logarithm in (A2) we obtain (10)

$$\begin{aligned} S(\beta) &= \int_0^1 \frac{dx}{x} (\ln(1 - \alpha_+ x) + \ln(1 - \alpha_- x)) \\ &\quad - 2i\pi\theta(\beta - 4) \ln(\min(\alpha_+, \alpha_-)) \end{aligned} \tag{A3}$$

where $\theta(x)$ is the step function and $\min(\alpha_+, \alpha_-)$ is the minimum of α_+ and α_- with

$$\alpha_\pm = \tfrac{1}{2}(\beta \pm \sqrt{(\beta^2 - 4\beta)}).$$

We can now write $S(\beta)$ in terms of Spence functions, and the result is:

i) $\beta < 0$, $\alpha_+ < 1$

$$S(\beta) = -\mathrm{Sp}(\alpha_+) - \mathrm{Sp}\left(\frac{\alpha_+}{\alpha_+ - 1}\right) = \frac{1}{2}\left[\cosh^{-1}\frac{2-\beta}{2}\right]^2. \tag{A4a}$$

ii) $0 < \beta < 4$, $\alpha_\pm = \mathrm{Re}^{\pm i\theta}$

$$S(\beta) = -\mathrm{Sp}(\mathrm{Re}^{i\theta}) - \mathrm{Sp}(\mathrm{Re}^{-i\theta}) = -2\left[\sin^{-1}\frac{\sqrt{\beta}}{2}\right]^2. \tag{A4b}$$

iii) $\beta>4,\ \alpha_+>1$

$$S(\beta)=-\mathrm{Sp}(\alpha_+)-\mathrm{Sp}\left(\frac{\alpha_+}{\alpha_+-1}\right)-2i\pi\ln(\alpha_-)$$
$$=\tfrac{1}{2}\ln^2\left(\tfrac{1}{2}(\beta-2+\sqrt{(\beta^2-4\beta)})\right)$$
$$+2i\pi\cosh^{-1}\left(\frac{\sqrt{\beta}}{2}\right). \qquad \text{(A4c)}$$

Now consider

$$L(\beta)=\int_0^1 dx\ln(1-\beta x(1-x)+i\varepsilon). \qquad \text{(A5)}$$

Integrating by parts we obtain

$$L(\beta)=\int_0^1 dx\,\frac{\beta x(1-2x)}{1-\beta x(1-x)+i\varepsilon}. \qquad \text{(A6)}$$

On the other hand, from (A2) we have

$$S'(\beta)=\frac{\partial S}{\partial\beta}=-\int_0^1 dx\,\frac{1-x}{1-\beta x(1-x)+i\varepsilon}$$
$$=-\frac{1}{2}\int_0^1 dx\,\frac{1}{1-\beta x(1-x)+i\varepsilon} \qquad \text{(A7)}$$

and, writing the numerator of (A6) as

$$-2(1-\beta x(1-x))+2-\beta x$$

we obtain

$$L(\beta)=-2+(\beta-4)\,S'(\beta). \qquad \text{(A8)}$$

The integral defined in (22) is

$$I(s,M_Z^2)=\int_0^1 dx_1\int_0^{1-x_1} dx_2$$
$$\cdot\frac{x_1(x_1+x_2-1)}{m_f^2-sx_1(1-x_1)+x_1x_2(s-M_Z^2)}$$

which can be written in terms of the functions L and S:

$$I(s,M_Z^2)=\frac{1}{s-M_Z^2}\left(\frac{1}{2}-\frac{m_f^2}{s-M_Z^2}\left[S\left(\frac{M_Z^2}{m_f^2}\right)-S\left(\frac{s}{m_f^2}\right)\right]\right.$$
$$\left.+\frac{M_Z^2}{2(s-M_Z^2)}\left[L\left(\frac{M_Z^2}{m_f^2}\right)-L\left(\frac{s}{m_f^2}\right)\right]\right). \qquad \text{(A9)}$$

Hence

$$A=sI(s,M_Z^2)-M_Z^2I(M_Z^2,s)$$
$$=\frac{s+M_Z^2}{2(s-M_Z^2)}+\frac{sM_Z^2}{(M_Z^2-s)^2}\left[L\left(\frac{M_Z^2}{m_f^2}\right)-L\left(\frac{s}{m_f^2}\right)\right]$$
$$-\frac{m_f^2(s+M_Z^2)}{(M_Z^2-s)^2}\left[S\left(\frac{M_Z^2}{m_f^2}\right)-S\left(\frac{s}{m_f^2}\right)\right]. \qquad \text{(A10)}$$

From this equation it looks as if A has a pole at $s=M_Z^2$. However it is easy to check that this is not true. In fact, writing $s-M_Z^2=\varepsilon m_f^2$, one can check that the terms in ε^{-1} and ε^0 vanish and that the first non-zero term of the expansion is of order ε, i.e.

$$A=\frac{s-M_Z^2}{6M_Z^2}\left[1-\tfrac{1}{2}\mu^2 f(\mu^2)+\tfrac{1}{2}\mu^4 f'(\mu^2)\right]$$
$$+O\left(\left[\frac{s-M_Z^2}{M_Z^2}\right]^2\right) \qquad \text{(A11)}$$

with

$$f(\mu^2)=\begin{cases}\dfrac{2}{\sqrt{(4\mu^2-1)}}\tan^{-1}\left(\dfrac{\sqrt{(4\mu^2-1)}}{2\mu^2-1}\right) & \text{if } \mu>\tfrac{1}{2}\\ -2 & \text{if } \mu=\tfrac{1}{2}\\ \dfrac{2}{\sqrt{(1-4\mu^2)}}\ln\left(\dfrac{2\mu^2-1+\sqrt{(1-4\mu^2)}}{2\mu^2}\right) & \\ -\dfrac{2i\pi}{\sqrt{(1-4\mu^2)}} & \text{if } \mu<\tfrac{1}{2}\end{cases} \qquad \text{(A12)}$$

and $\mu=m_f/M_Z$. The imaginary part of $f(\mu^2)$ goes to infinity as μ approaches $\frac{1}{2}$ from below. However this behaviour was artificially introduced by our expansion and it signals the breakdown of the Taylor series. Directly from (A10) one can check that for $\mu=\frac{1}{2}$ the value of A is finite.

Consider now the integral $I'(k^2)$ introduced in (27). In the limit $k^2\to 0$ and after the change of variable $x_1\to 1-x_1-x_2$ we obtain

$$I'(0)=\frac{1}{6}\int_0^1 dx\,\frac{x^3}{m_f^2-M_Z^2x(1-x)}$$
$$=\frac{1}{12M_Z^2}\left[3-2\,\frac{M_Z^2-3m_f^2}{m_f^2}\,S'\left(\frac{M_Z^2}{m_f^2}\right)\right]. \qquad \text{(A13)}$$

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