

## FLAVOUR VIOLATION IN SUPERSYMMETRIC THEORIES

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We investigate in detail the question of lepton-flavour violation in a  $SU(2) \times U(1)$  supersymmetric model, where the breaking of supersymmetry (SUSY) is achieved through the coupling to  $N=1$  supergravity. It is shown that in the limit of degenerate neutrino masses, lepton flavour is exactly conserved. Allowing for neutrino masses compatible with present experimental limits, we analyse SUSY contributions to several lepton-flavour violating processes, comparing the size of these contributions with those already present in the standard Glashow-Salam-Weinberg model. In the case of  $\mu \rightarrow e\gamma$ , SUSY leads to a branching ratio two or three orders of magnitude larger than the corresponding branching ratio in the standard model, for gravitino and photino masses compatible with the experimental limits on the muon anomalous magnetic moment. In contrast, SUSY contributions to  $K_L \rightarrow \bar{\mu}e$  are always small, of the order of  $10^{-2}$  of the corresponding amplitudes in the standard model, if the gravitino and photino masses are constrained by the  $K_L - K_S$  mass difference.

### 1. Introduction

There are various reasons for the growing belief that supersymmetry (SUSY) may play a fundamental role in elementary particle physics. On the one hand, it has been shown [1] that among all the graded Lie algebras, only supersymmetric algebras generate symmetries of the  $S$ -matrix consistent with relativistic quantum field theory. On the other hand, SUSY provides the hope of understanding the smallness of  $M_W$ , when compared with the scale of grand unification or the Planck mass [2]. Nevertheless, one has to face the fact that so far there is no experimental indication that nature is supersymmetric. Clearly, the best experimental evidence for SUSY would be through the discovery of the supersymmetric partners of the known particles. Several recent theoretical papers (for a recent review see [3]) have been devoted to the study of the production and signature of these superpartners, while present experiments [4] only give moderate constraints on the masses of some of these new particles. Another way in which SUSY could become manifest would be through the contribution of the superpartners to processes which are highly suppressed or forbidden in the standard  $SU(3) \times SU(2) \times U(1)$  model.

In this paper, we analyse in detail the supersymmetric contributions to some lepton-flavour violating processes, within the framework of a model [5] where SUSY is spontaneously broken through supergravity interactions. We organize our paper

in the following way. In sect. 2 the SUSY model is summarized and trivially extended to the case of massive neutrinos. Most of the material in this section is not new, but we have decided to include it for the sake of completeness. Our aim is to give sufficient detail so that a non-SUSY expert can easily understand the origin of the new vertices present in the theory. Furthermore, we adopt the standard metric conventions [6], which is not often done in the original papers but is certainly more convenient for those who want to do SUSY phenomenology. In sect. 3 we describe our main calculations regarding the lepton violating processes  $\mu \rightarrow e\gamma$  and  $K_L \rightarrow \bar{\mu}e$ . We also re-examine the SUSY contributions to the muon anomalous magnetic moment and to the  $K_L - K_S$  mass difference in order to see if a possible enhancement of the lepton violating transitions does not conflict with the experimental bounds on these quantities. Several authors [7–14] have previously calculated these two effects. However, most of the calculations were done in the framework of global SUSY and others are very phenomenological. In contrast, our attitude is to be as complete as possible within the framework of what is considered to be a minimal local SUSY model. Following Barbieri, Ferrara and Savoy [5] we want to explore the low-energy consequences of a SUSY  $SU(3) \times SU(2) \times U(1)$  without considering grand unification (GUT). Therefore, the  $SU(2) \times U(1)$  breaking will be induced by a light Higgs singlet\*.

## 2. The model

### 2.1. FIELD CONTENT

In this section we review the supersymmetric extension of the Glashow–Weinberg–Salam model with leptons and quarks [5].

**2.1.1. Gauge superfields.** We need four gauge vector supermultiplets  $V'$  and  $V_a$  ( $a = 1, 2, 3$ ) for the gauge groups  $U_Y(1)$  and  $SU_L(2)$ , respectively. In the following we often use a matrix notation for the  $SU_L(2)$  gauge superfields,  $V = V_a \frac{1}{2} \sigma^a$  where  $\sigma^a$  are the Pauli matrices. Together with the vector gauge bosons,  $B_\mu$  and  $A_\mu^a$  we have their spin- $\frac{1}{2}$  partners (Majorana spinors), the so-called gauginos,  $\lambda'$  and  $\lambda^a$  ( $a = 1, 2, 3$ ).

**2.1.2. Matter superfields.** Leptons and quarks are described by chiral (left-handed) superfields. In order to account for the 3 generations we need the following structure:

$$\begin{aligned} \mathbb{L}_i(2, -\tfrac{1}{2}), \quad \ell_i^c(1, 1), \\ \mathbb{Q}_i(2, \tfrac{1}{6}), \quad d_i^c(1, \tfrac{1}{3}), \quad u_i^c(1, -\tfrac{2}{3}), \end{aligned} \quad (2.1)$$

where the numbers in parentheses denote the transformation properties under  $SU_L(2) \times U_Y(1)$  and the index  $i$  is a generation index. Together with the leptons

\* We are aware that this procedure extended to GUTs might lead to some instability in the mass hierarchy [15].

and quarks we have their spin-zero partners. In order to simplify the notation, the scalar partners of quarks and leptons will be denoted by a tilde over the corresponding lepton and quark symbols:

$$\mathbb{L}_i \equiv (\tilde{L}_i, L_i), \quad \ell_i^c \equiv (\tilde{\ell}_i^c, \ell_{iL}^c),$$

with

$$\tilde{L}_i \equiv \begin{pmatrix} \tilde{\nu}_i \\ \tilde{\ell}_i \end{pmatrix}, \quad L_i \equiv \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix} \quad i = 1, 2, 3. \quad (2.2)$$

Similar conventions apply to the quark sector.

**2.1.3. Higgs superfields.** It is well known that, to give masses to both components of a doublet one needs two Higgs doublets of chiral superfields,  $\mathbb{H}(2, -\frac{1}{2})$  and  $\mathbb{H}^c(2, \frac{1}{2})$ , whereas the singlet  $Y(1, 0)$  is introduced to trigger the  $SU(2) \times U(1)$  breaking.

The spin-zero Higgs and their partners, the so-called higgsinos (spin- $\frac{1}{2}$  fermions) are

$$\begin{aligned} Y &= (y, \psi_L), \\ \mathbb{H}_\alpha &= (h_\alpha, \chi_{L\alpha}), \quad \alpha = 1, 2, \\ \mathbb{H}_\alpha^c &= (h_\alpha^c, \chi_{L\alpha}^c), \quad \alpha = 1, 2. \end{aligned}$$

## 2.2. THE LAGRANGIAN

Since in a realistic model SUSY has to be broken, it is convenient to write the total lagrangian as a sum of two pieces:

$$\mathcal{L} = \mathcal{L}(\text{SUSY}; f) + \mathcal{L}_{\text{SB}}, \quad (2.3)$$

where  $\mathcal{L}(\text{SUSY}; f)$  is a globally supersymmetric lagrangian and  $\mathcal{L}_{\text{SB}}$  provides the breaking of supersymmetry. The supersymmetric part can be written [16]

$$\mathcal{L}(\text{SUSY}; f) = \mathcal{L}_K + \mathcal{L}_{\text{pot}} + \mathcal{L}_Y, \quad (2.4)$$

where  $\mathcal{L}_K$  contains the kinetic terms for all the fields appearing in the model and also includes the supersymmetric extension of the minimal coupling. Its expression in terms of the superfields is given in appendix A, where the relevant notation is introduced.

Let  $z_A$  and  $\chi_{LA}$ ,  $A = 1, \dots, N$ , denote the scalar and spinor components of the  $N$  chiral multiplets. Then, the other two terms of the lagrangian (2.4) are

$$\mathcal{L}_Y = - \sum_{A,B} \left[ \frac{\partial^2 f}{\partial z_A \partial z_B} \chi_{LA}^T C \chi_{LB} + \text{h.c.} \right], \quad (2.5)$$

$$\mathcal{L}_{\text{pot}} = -\frac{1}{2} D^a D^a - \frac{1}{2} (D')^2 - \sum_A \left| \frac{\partial f(z)}{\partial z_A} \right|^2, \quad (2.6)$$

where

$$\begin{aligned} D^a &= g z^{*A} T_A^a z_B \quad (a = 1, 2, 3), \\ D' &= g' z^{*A} T'_A z_B. \end{aligned} \quad (2.7)$$

In the previous equations  $g$  and  $g'$  are the SU(2) and U(1) gauge couplings while  $T_A^{a\ B}$  and  $T'_A{}^B$  denote the SU(2) and U(1) generators, respectively.

In order to define the model, it is crucial to specify the function  $f(z)$  from which the Yukawa and potential terms can be derived. Here we choose for  $f(z)$  the form introduced by Fayet [17] in order to provide the correct  $SU(2) \times U(1) \rightarrow U(1)$  breaking. Without considering generation mixing we write [5, 16]

$$f = \beta(y\epsilon^{\alpha\beta}h_\alpha h_\beta^c - \epsilon y) + g_i\epsilon^{\alpha\beta}h_\alpha \tilde{L}_{\beta i} \tilde{e}_i^c + g_u\epsilon^{\alpha\beta}h_\beta^c \tilde{Q}_\alpha \tilde{u}_i^c + g_d\epsilon^{\alpha\beta}h_\alpha \tilde{Q}_\beta \tilde{d}_i^c. \quad (2.8)$$

Finally we need to specify the supersymmetry breaking part  $\mathcal{L}_{SB}$ . Here we adopt the viewpoint that the most promising scheme is the one where the breaking is achieved through the coupling to  $N=1$  supergravity [18]. In a wide class of models [5] the low-energy effective lagrangian can then be written

$$\mathcal{L}_{SB} = \frac{1}{2}m_\lambda(\bar{\lambda}^a\lambda^a + \bar{\lambda}'\lambda') - m_{3/2}^2 \sum_A z^{*A} z_A - m_{3/2}[h(z) + \text{h.c.}], \quad (2.9)$$

where  $m_{3/2}$  is the gravitino mass,  $m_\lambda$  a tree-level mass for the gauginos and the function  $h(z)$  is an analytical function of  $z$  given by [5]

$$h(z) = (A-3)f(z) + \sum_A \frac{\partial f}{\partial z_A} z_A, \quad (2.10)$$

where  $A$  is a model dependent numerical constant. Following Cremmer et al. [5] and Barbieri and Ferrara [5], we choose  $A=3$ .

### 2.3. HIGGS MECHANISM AND MASS EIGENSTATES

It has been shown [5] that the Higgs potential is minimized for

$$\langle h \rangle = \begin{pmatrix} v \\ 0 \end{pmatrix}, \quad \langle h^c \rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle y \rangle = v', \quad (2.11)$$

with all other fields having zero v.e.v. The parameters  $v$  and  $v'$  are

$$v = \frac{m_{3/2}}{\beta} \sqrt{1 + \frac{\epsilon\beta^2}{m_{3/2}^2}}, \quad v' = -\frac{m_{3/2}}{\beta}. \quad (2.12)$$

The Higgs mechanism leads to mass matrices which after diagonalization give the following results [19].

**2.3.1 Vector bosons.** As far as vector bosons are concerned, everything goes as in the standard model. The combination  $W_\mu^\pm = \sqrt{\frac{1}{2}}(A_\mu^\pm \mp iA_\mu^3)$  acquires a mass  $m_W = gv$ , while for  $Z_\mu = A_\mu^3 \cos \theta_w - B_\mu \sin \theta_w$  we have  $m_Z = m_W/\cos \theta_w$  with  $\tan \theta_w = g'/g$ .

The orthogonal combination  $A_\mu = A_\mu^3 \sin \theta_w + B_\mu \cos \theta_w$  describes the photon.

**2.3.2 Matter fields.** Leptons and quarks acquire a mass via the Yukawa couplings. One obtains the usual masses for leptons and quarks, i.e.  $m_i = g_i v$ ;  $m_u = g_u v$ ;

$m_{d_i} = g_{d_i} v$ . The mass matrices for the scalar partners of leptons and quarks have the same structure. For instance, for the sleptons we have the mass matrix

$$\mathcal{L}_{\text{slepton mass}} = -(\tilde{\ell}_i^*, \tilde{\ell}_i^c) \begin{pmatrix} m_{3/2}^2 + m_{\ell_i}^2 & 3m_{3/2}m_{\ell_i} \\ 3m_{3/2}m_{\ell_i} & m_{3/2}^2 + m_{\ell_i}^2 \end{pmatrix} \begin{pmatrix} \tilde{\ell}_i \\ \tilde{\ell}_i^{c*} \end{pmatrix}, \quad (2.13)$$

which has the following eigenstates:

$$\tilde{\ell}_{\alpha i} = \sqrt{\frac{1}{2}} (\tilde{\ell}_i + (-1)^\alpha \tilde{\ell}_i^{c*}), \quad \alpha = 1, 2, \quad (2.14)$$

and eigenvalues

$$m_{\ell_{\alpha i}}^2 = m_{3/2}^2 + m_{\ell_i}^2 + (-1)^\alpha 3m_{3/2}m_{\ell_i}. \quad (2.15)$$

**2.3.3. Gauginos and higgsinos.** We define the following combinations:

$$\begin{aligned} \lambda_- &= \sqrt{\frac{1}{2}} (\lambda' + i\lambda''), \\ \lambda_z &= \lambda' \cos \theta_w - \lambda'' \sin \theta_w, \\ \lambda_\gamma &= \lambda' \sin \theta_w + \lambda'' \cos \theta_w. \end{aligned} \quad (2.16)$$

The photino  $\lambda_\gamma$  does not acquire mass via the Higgs mechanism, therefore its mass has the tree-level value  $m_{\tilde{\gamma}} = m_\lambda$ . The charged gaugino  $\lambda_-$  and the combination  $\lambda_z$  acquire mass through the Higgs mechanism besides the contribution from explicit tree-level terms. In the diagonalization of the mass matrices  $\lambda_-$  and  $\lambda_z$  get mixed with the charged and neutral higgsinos, respectively. We consider separately the charged and neutral gauginos.

(i) Winos and charged higgsinos

In order to write the mass matrix in the standard form we have to make some redefinitions:

$$\begin{aligned} \lambda'_- &\equiv -\gamma_5 \lambda_-, \\ \xi_- &\equiv -i(\chi_{2L} + \chi_{1R}^c), \end{aligned} \quad (2.17)$$

where  $\lambda_-$  was defined in eq. (2.16) and  $\chi_2$  and  $\chi_1^c$  are the charged higgsinos. Then

$$\mathcal{L}_{\text{charged gauginos}}^{\text{charged}} = -(\bar{\lambda}'_-, \bar{\xi}_-) \begin{pmatrix} m_\lambda & m_w \\ m_w & m_{3/2} \end{pmatrix} \begin{pmatrix} \lambda'_- \\ \xi_- \end{pmatrix}. \quad (2.18)$$

The physical eigenstates are\*

$$\begin{aligned} \tilde{w}_1 &= (\gamma_5)[\lambda'_- \cos \omega_1 - \xi_- \sin \omega_1], \\ \tilde{w}_2 &= [\lambda'_- \sin \omega_1 + \xi_- \cos \omega_1], \end{aligned} \quad (2.19)$$

with

$$\begin{aligned} m_{\tilde{w}_1} &= \left[ \frac{1}{2}(m_{3/2} + m_\lambda) - \sqrt{m_w^2 + \frac{1}{4}(m_{3/2} - m_\lambda)^2} \right], \\ m_{\tilde{w}_2} &= \left[ \frac{1}{2}(m_{3/2} + m_\lambda) + \sqrt{m_w^2 + \frac{1}{4}(m_{3/2} - m_\lambda)^2} \right]. \end{aligned} \quad (2.20)$$

\* The  $(\gamma_5)$  factor must be included if  $m_{3/2}m_\lambda < m_w^2$ .

The mixing angle is

$$\tan 2\omega_1 = \frac{2m_w}{m_{3/2} - m_\lambda}. \quad (2.21)$$

(ii) Zinos and neutral higgsinos

We introduce the following Majorana spinors:

$$\begin{aligned} \xi_0 &= \sqrt{\frac{1}{2}} (\chi_1 - \chi_2^c), \\ \eta_0 &= \sqrt{\frac{1}{2}} i\gamma_5 (\chi_1 + \chi_2^c), \\ \lambda'_z &= i\gamma_5 \lambda_z, \\ \psi' &= -i\gamma_5 \psi, \end{aligned} \quad (2.22)$$

where  $\chi_1$ ,  $\chi_2^c$  and  $\psi$  are the spin- $\frac{1}{2}$  components of the Higgs chiral superfields  $H$ ,  $H_c$  and  $Y$  respectively. Then the mass terms in the lagrangian are

$$\begin{aligned} \mathcal{L}_{\text{neutral}} &= -\frac{1}{2} (\bar{\lambda}'_z, \bar{\xi}_0) \begin{pmatrix} m_\lambda & m_z \\ m_z & m_{3/2} \end{pmatrix} \begin{pmatrix} \lambda'_z \\ \xi_0 \end{pmatrix} \\ &\quad -\frac{1}{2} (\bar{\psi}', \bar{\eta}_0) \begin{pmatrix} 0 & \beta v \sqrt{2} \\ \beta v \sqrt{2} & m_{3/2} \end{pmatrix} \begin{pmatrix} \psi' \\ \eta_0 \end{pmatrix}, \end{aligned} \quad (2.23)$$

with eigenstates

$$\begin{aligned} \tilde{z}_1 &= (i\gamma_5)(\lambda'_z \cos \omega_2 - \xi_0 \sin \omega_2), \\ \tilde{z}_2 &= (\lambda'_z \sin \omega_2 + \xi_0 \cos \omega_2), \quad \tan 2\omega_2 = \frac{2m_z}{m_{3/2} - m_\lambda}, \end{aligned} \quad (2.24)$$

$$\begin{aligned} \tilde{h}_1 &= i\gamma_5(\psi' \cos \omega_3 - \eta_0 \sin \omega_3), \\ \tilde{h}_2 &= \eta_0 \cos \omega_3 + \psi' \sin \omega_3, \quad \tan 2\omega_3 = \frac{2\beta v \sqrt{2}}{m_{3/2}}, \end{aligned} \quad (2.25)$$

and eigenvalues

$$\begin{aligned} m_{\tilde{z}_1} &= \left| \frac{1}{2}(m_{3/2} + m_\lambda) - \sqrt{m_w^2 + \frac{1}{4}(m_{3/2} - m_\lambda)^2} \right|, \\ m_{\tilde{z}_2} &= \frac{1}{2}(m_{3/2} + m_\lambda) + \sqrt{m_w^2 + \frac{1}{4}(m_{3/2} - m_\lambda)^2}, \end{aligned} \quad (2.26)$$

$$\begin{aligned} m_{\tilde{h}_1} &= \sqrt{\frac{1}{4}m_{3/2}^2 + 2\beta^2 v^2} - \frac{1}{2}m_{3/2}, \\ m_{\tilde{h}_2} &= \frac{1}{2}m_{3/2} + \sqrt{\frac{1}{4}m_{3/2}^2 + 2\beta^2 v^2}. \end{aligned} \quad (2.27)$$

#### 2.4. MIXING IN THE LEPTONIC SECTOR

In the standard model, mixing in the leptonic sector can only occur if the neutrinos have non-degenerated masses. One can easily show that this remains true in this class of supersymmetric  $SU(2) \times U(1)$  models. Using matrix notation we change the

leptonic part of the  $f$  function to become

$$f = \frac{h_1}{v} \tilde{\ell}^T M \tilde{\ell}^c + \frac{h_2^c}{v} \tilde{\nu}^T M' \tilde{\nu}^c - \frac{h_2}{v} \tilde{\nu}^T M \tilde{\ell}^c \frac{h_1^c}{v} \tilde{\ell} M' \tilde{\nu}^c \quad (2.28)$$

where using the notation introduced before,  $\nu_i^c \equiv (\tilde{\nu}_i^c, \nu_{iL}^c)$ . The matrices  $M$  and  $M'$  can be diagonalized by

$$M = U^T D U, \quad M' = U'^T D' U', \quad (2.29)$$

where  $U$  and  $U'$  are unitary matrices and  $D$  and  $D'$  are diagonal in generation space. If we define the rotated states as

$$\begin{aligned} \tilde{\ell}'^c &= U \tilde{\ell}^c, & \tilde{\nu}'^c &= U' \tilde{\nu}^c, \\ \tilde{\ell}' &= U \tilde{\ell}, & \tilde{\nu}' &= U' \tilde{\nu}, \end{aligned} \quad (2.30)$$

we obtain

$$\begin{aligned} f &= \frac{h_1}{v} \tilde{\ell}'^T D \tilde{\ell}'^c + \frac{h_2^c}{v} \tilde{\nu}'^T D' \tilde{\nu}'^c \\ &\quad - \frac{h_2}{v} \tilde{\nu}'^T V^* D \tilde{\ell}'^c - \frac{h_1^c}{v} \tilde{\ell}'^T V^T D' \tilde{\nu}'^c, \end{aligned} \quad (2.31)$$

where  $V = U' U^+$  is analogous to the Kobayashi-Maskawa (KM) matrix for the leptons. One sees that the terms that give mass to the lepton and their partners are diagonal in generation space. It is straightforward to check that in terms of the mass eigenstates all couplings remain diagonal except those connecting the upper and lower components of the doublet, namely  $(W_\mu^-, \ell_i, \nu_i)$  and  $(\lambda_-, \ell_i, \tilde{\nu}_i)$ . Note that the same mixing matrix  $V$  appears. If the neutrinos do not have mass one can choose  $U' = U$  and then  $V = \mathbb{1}$ . Therefore for massless neutrinos we do not have mixing. Furthermore, if the neutrinos have equal mass one can still show that the mixing can be rotated away. In this case  $D' = m \mathbb{1}$ , and we can define

$$\begin{aligned} \tilde{\nu}''^c &= V^T \tilde{\nu}'^c, \\ \tilde{\nu}'' &= V^\dagger \tilde{\nu}'. \end{aligned} \quad (2.32)$$

Then  $\tilde{\nu}'^T D' \tilde{\nu}'^c = m \tilde{\nu}''^T \tilde{\nu}''^c$  and the function  $f$  becomes diagonal, i.e.,

$$f = \frac{h_1}{v} \tilde{\ell}'^T D \tilde{\ell}'^c + \frac{h_2^c}{v} m \tilde{\nu}''^T \tilde{\nu}''^c - \frac{h_2}{v} \tilde{\nu}''^T D \tilde{\ell}'^c - \frac{h_1^c}{v} m \tilde{\ell}'^T \tilde{\nu}''^c. \quad (2.33)$$

Also it is straightforward to check using the expressions of appendix A, that the same redefinition can be applied in the  $\lambda^-, \ell, \tilde{\nu}$  and  $\lambda^-, \tilde{\ell}, \nu$  couplings. Therefore we conclude that only for non-degenerate neutrinos do we have a mixing among generations.

### 3. SUSY processes

In the framework of the model summarized before we shall now calculate the SUSY contributions to several processes. Our aim is twofold. On the one hand we want to investigate to what extent these processes constrain the model. On the other hand we look for possible SUSY enhancements in transitions which are highly suppressed in the standard  $SU(3) \times SU(2) \times U(1)$  model,

#### 3.1. MUON ANOMALOUS MAGNETIC MOMENT

The weak contribution to the muon anomalous magnetic moment  $a = \frac{1}{2}(g-2)$  is very small (e.g. ref. [19]), namely

$$a_W \simeq 10^{-9}.$$

In fig. 1 we show the diagrams corresponding to the lowest-order contribution to  $a(\text{SUSY})$ . Diagrams 1a, b and c give the wino, photino and zino vertex correction, respectively. For a massless photino diagram (1b) was evaluated in the previous work of Fayet [5], who concluded that  $a(\text{SUSY})$  is small ( $10^{-9}$ ) for s-muon masses larger than 15 GeV. Following the observation [6] that a large Majorana mass for the neutral gauginos could invalidate Fayet's conclusion, all diagrams of fig. 1 were calculated by Ellis, Hagelin and Nanopoulos (EHN) [7] and by Barbieri and Maiani [8] in the framework of global SUSY. Their conclusions are essentially the same but EHN pointed out that for the potentially large contributions, proportional to  $m_\mu$  rather than  $m_\mu^2$ , there is a kind of GIM [21] suppression mechanism. This is transparent in our results which are

$$a_{\tilde{W}} = \frac{\alpha}{8\pi \sin^2 \theta_w} \sum_{l,m=1}^2 \left( \frac{m_\mu}{m_{\tilde{W}_l}} \right)^2 X_l^2 I_1(m_{\tilde{W}_l}, m_{\tilde{\nu}_m}), \quad (3.1)$$

$$a_{\tilde{\gamma}} = -\frac{\alpha}{2\pi} \sum_{m=1}^2 \left[ \left( \frac{m_\mu}{m_{\tilde{\mu}_m}} \right)^2 I_1(m_{\tilde{\mu}_m}, m_{\tilde{\gamma}}) + \frac{m_\mu m_{\tilde{\gamma}}}{m_{\tilde{\mu}_m}^2} (-1)^m I_2(m_{\tilde{\mu}_m}, m_{\tilde{\gamma}}) \right], \quad (3.2)$$

$$a_{\tilde{Z}} = -\frac{\alpha}{2\pi} \sum_{l,m=1}^2 X_l^2 \left[ \left( \frac{m_\mu}{m_{\tilde{\mu}_m}} \right)^2 \frac{1}{2} (A^2 + B^2) I_1(m_{\tilde{\mu}_m}, m_{\tilde{Z}_l}) + \frac{m_\mu m_{\tilde{Z}_l}}{m_{\tilde{\mu}_m}} (-1)^{m+l} A B I_2(m_{\tilde{\mu}_m}, m_{\tilde{Z}_l}) \right]. \quad (3.3)$$

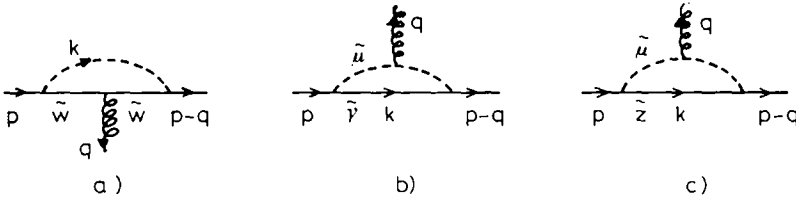


Fig. 1. Diagram (a) gives the  $\mu \rightarrow e\gamma$  amplitude in SUSY. Diagrams (a), (b) and (c) contribute to the  $\mu$  anomalous magnetic moment.



In the previous equations the index  $m$  denotes the scalar-lepton partners of a given lepton,  $l$  denotes the two partners of  $W^-$  and  $Z$ :

$$X_l = \begin{cases} \cos \omega_i & \text{if } l = 1 \\ \sin \omega_i & \text{if } l = 2, \end{cases} \quad (3.4)$$

where the angles  $\omega_i$ , with  $i = 1$  for the  $\tilde{W}$  diagram and  $i = 2$  for the  $\tilde{Z}$ , are given by eqs. (2.21) and (2.24).

$$A = \frac{1 - 2 \sin^2 \theta_w}{2 \sin \theta_w \cos \theta_w},$$

$$B = \tan \theta_w, \quad (3.5)$$

where  $\theta_w$  is the Weinberg angle, and  $I_i(m, m')$  are the following integrals:

$$I_1(m, m') = \int_0^1 dx \frac{x(1-x)^2}{1-x + (m'/m)^2 x - x(1-x)(m_\mu/m)^2}, \quad (3.6)$$

$$I_2(m, m') = \int_0^1 dx \frac{x(1-x)}{1-x + (m'/m)^2 x - x(1-x)(m_\mu/m)^2}. \quad (3.7)$$

One can see that for the neutral gauginos there are quadratic and linear terms in  $m_\mu$ . The linear term would be large without the presence of the phase  $(-1)^m$  which provides the EHN suppression mechanism. Comparing  $a_{\tilde{\gamma}}$  with  $a_{\tilde{Z}}$  one sees that  $a_{\tilde{Z}}$  is further suppressed by the angles contained in  $X_l$  and  $AB$ . So, one would expect that the photino gives the dominant SUSY contribution to the muon anomalous magnetic moment. This is indeed the case and we have found that  $a_{\tilde{\gamma}}$  is larger than  $a_{\tilde{W}}$  or  $a_{\tilde{Z}}$  by roughly an order of magnitude. In fig. 2 we show  $a(\text{SUSY})$  as a function of the photino mass  $m_{\tilde{\gamma}}$  for two values of the gravitino mass  $m_{3/2}$ ,  $m_{3/2} = 10 \text{ GeV}$

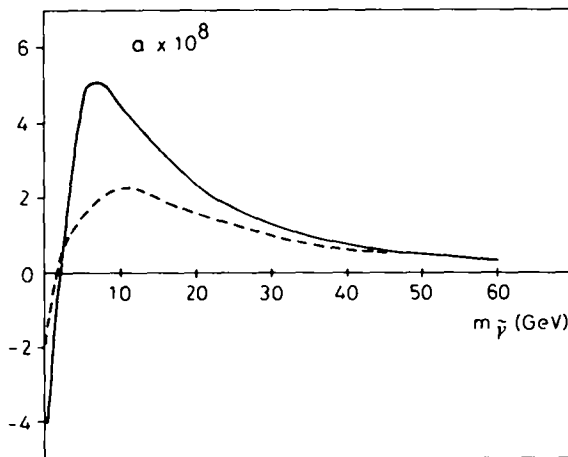


Fig. 2. Muon anomalous magnetic moment as a function of the photino mass  $m_{\tilde{\gamma}}$ , for  $m_{3/2} = 10 \text{ GeV}$  (full curve) and  $m_{3/2} = 15 \text{ GeV}$  (dashed curve).

(full curve) and  $m_{3/2} = 15$  GeV (dashed curve). For a given  $m_{\tilde{\gamma}}$ ,  $a(\text{SUSY})$  is a decreasing function of  $m_{3/2}$ . Fig. 2 illustrates our main conclusion, i.e., if  $m_{3/2} > 15$  GeV the SUSY contribution to  $a$  is too small to upset the present agreement between the theoretical and experimental values of the anomalous magnetic moment. For smaller values of  $m_{3/2}$  our results would imply a constraint on the photino mass. However, this constraint is, probably, not very interesting since it is known that scalar leptons do not exist up to an energy of 18 GeV [4] and this implies  $m_{3/2} \geq 18$  GeV.

Obviously, our numerical results reflect our choice  $A = 3$  (cf. eq. (2.10)). Recently, while this work was in progress, Kosower, Krauss and Sakai [11] published another calculation of  $a$  in models with local SUSY where the possibility of having different values of  $A$  was also considered. For values of  $A$  or order 1 their results [11] agree with ours and, as they have pointed out, only for  $A > 50$  can the values of  $a$  restrict the range of  $m_{\tilde{\gamma}}$ .

### 3.2. $\mu \rightarrow e\gamma$

In the Glashow-Weinberg-Salam model with massless neutrinos the decay  $\mu \rightarrow e\gamma$  is strictly forbidden since the degeneracy of the neutrino masses imply lepton flavour conservation. As we have shown before, this statement is also true in the present SUSY model.

It is worthwhile recalling that in the GWS model if one allows for neutrino masses compatible with the current experimental limits, namely

$$m_{\nu_e} \leq 30 \text{ eV} [22],$$

$$m_{\nu_\mu} \leq 520 \text{ keV} [23],$$

$$m_{\nu_\tau} \leq 250 \text{ MeV} [24],$$

and assumes the leptonic mixing matrix  $V_{ij}$  to be equal to the corresponding matrix in the quark sector, one obtains [25] a branching ratio  $B_W = \Gamma(\mu \rightarrow e\gamma) / \Gamma(\mu \rightarrow e\bar{\nu}_e\nu_\mu)$  of the order of  $10^{-16}$ , which is far below the experimental limit,  $B_{\text{exp}} = 1.9 \times 10^{-10}$  [26]. Evaluating diagram 1a we obtain

$$B_{\text{SUSY}} = \frac{3\alpha}{\pi} \left[ I_1^2 + \left( \frac{m_e}{m_\mu} \right)^2 I_2^2 \right], \quad (3.8)$$

with

$$I_i = \sum_{j=1}^3 V_{j\mu} V_{je}^* \sum_{m,l=1}^2 X_l \left( \frac{m_w}{m_{\tilde{w}l}} \right)^2 \int_0^1 dx_1 \int_0^{x_1} dx_2 \frac{x_2(x_2 - z_i)}{f(x_1, x_2)}, \quad (3.9)$$

where  $z_1 = x_1$ ,  $z_2 = 1$ ,

$$f(x_1, x_2) = 1 - x_2 + \left( \frac{m_{jm}}{m_{\tilde{w}l}} \right)^2 x_2 + \left( \frac{m_\mu}{m_{\tilde{w}l}} \right)^2 (x_2^2 - x_2) + \frac{m_\mu^2 - m_e^2}{m_{\tilde{w}l}^2} x_2(1 - x_1), \quad (3.10)$$

and the  $m_{jm}$  denote masses of the s-neutrinos of the  $j$ th generation.

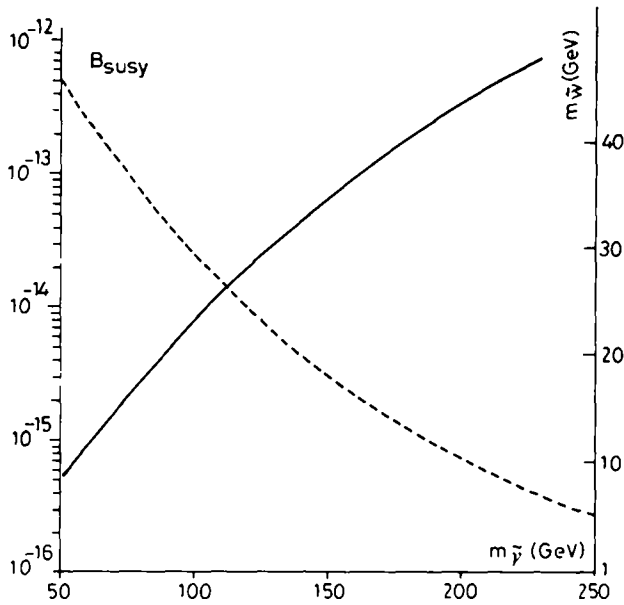


Fig. 3. The full curve (left scale) represents the SUSY  $\mu \rightarrow e\gamma$  branching ratio for  $m_{3/2} = 20$  GeV. The dashed curve (right scale) shows the mass of the lightest wino.

The relevant question regarding this process is whether, for the same values of the neutrino masses and mixing angles,  $B_{\text{SUSY}}$  can be larger than  $B_{\text{W}}$ . To answer this question we evaluate eq. (3.8) for gravitino and photino masses in the range 20–250 GeV. Our conclusions are the following:

- (i) If  $m_{\tilde{\gamma}} = 0$ ,  $B_{\text{SUSY}}$  can be larger than  $B_{\text{W}}$  by at most a factor of three;
- (ii) For a given  $m_{3/2}$ ,  $B_{\text{SUSY}}$  increases with  $m_{\tilde{\gamma}}$ . This is illustrated in fig. 3 where we plot (full curve) the SUSY branching ratio as a function of  $m_{\tilde{\gamma}}$  for  $m_{3/2} = 20$  GeV. For higher values of the gravitino mass this enhancement is also present but it is not as large.

The reason why  $B(\text{SUSY})$  increases with  $m_{\tilde{\gamma}}$  is due to the fact that, for a given  $m_{3/2}$ , as  $m_{\tilde{\gamma}}$  increases one of the winos becomes very light (cf. eq. (2.20)) and then the ratio  $(m_{\text{w}}/m_{\tilde{\text{w}}})^2$  present in eq. (3.9) becomes an enhancement factor. The dashed curve in fig. 3 shows this variation of  $m_{\tilde{\text{w}}}$  with  $m_{\tilde{\gamma}}$ . Obviously, a w-ino with a mass  $m_{\tilde{\text{w}}} \leq 15$  GeV would be stable since all s-leptons would then be at a mass scale of  $m_{3/2} = 20$  GeV. Hence, the non-existence of heavy leptons up to a mass of 18 GeV [4] provides an upper bound on  $B_{\text{SUSY}}$ , namely

$$B_{\text{SUSY}} \leq 10^{-13}.$$

### 3.3. $K_L - K_S$ MASS DIFFERENCE

Despite the fact that in this paper we are mainly concerned with lepton-violating processes, the study of our next example  $K_L \rightarrow \bar{\mu}e$  lead us to re-examine [12–14] the

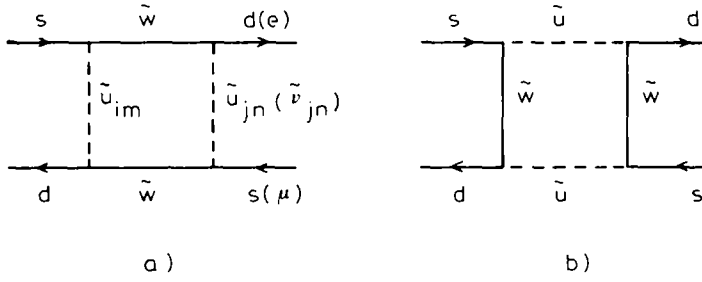


Fig. 4. Diagrams (a) and (b) represent the SUSY amplitude for the  $K_L - K_S$  mass difference. Diagram (a) gives also the amplitude for  $K_L \rightarrow \bar{\mu} e$ .

problem of SUSY contributions to the  $K_L - K_S$  mass difference. Evaluating the diagrams of fig. 4 one can see that, after a Fierz transformation, the total amplitude in the zero external momentum approximation becomes

$$\begin{aligned}
 iT = & 2 \left( \frac{g}{\sqrt{2}} \right)^4 \frac{1}{64\pi^2} \bar{d} \gamma^\mu \frac{1-\gamma_5}{2} s \bar{d} \gamma_\mu \frac{1-\gamma_5}{2} s \\
 & \times \sum_{i,j=1}^3 U_{is} U_{id}^* U_{js} U_{jd}^* \sum_{m,n=1}^2 [X_{im}^2 X_{jn}^2 I(\varepsilon_{im}, \varepsilon_{jn}, \tilde{w}_1) \\
 & + Y_{im}^2 Y_{jn}^2 I(\varepsilon_{im}, \varepsilon_{jn}, \tilde{w}_2) + 2 Y_{im}^2 X_{jn}^2 J(\varepsilon_{im}, \varepsilon_{jn}, \tilde{w}_1, \tilde{w}_2)], \quad (3.11)
 \end{aligned}$$

with

$$X_{im} = \cos \omega_1 + (-1)^m \frac{m_i}{m_w} \sin \omega_1, \quad (3.12)$$

$$Y_{in} = \sin \omega_1 + (-1)^{1+n} \frac{m_i}{m_w} \cos \omega_1, \quad (3.13)$$

$$\begin{aligned}
 I(\varepsilon, \delta, \tilde{w}_1) = & -\frac{i}{m_{\tilde{w}_1}^2} \{ [(1-\varepsilon)(1-\delta)]^{-1} - (\delta-\varepsilon)^{-1} \\
 & \times [\varepsilon^2(1-\varepsilon)^{-2} \ln \varepsilon - \delta^2(1-\delta)^{-2} \ln \delta] \}, \quad (3.14)
 \end{aligned}$$

$$\begin{aligned}
 J(\varepsilon, \delta, \tilde{w}_1, \tilde{w}_2) = & \frac{i}{m_{\tilde{w}_1}^2} \left\{ \frac{\beta^2 \ln \beta}{(1-\beta)(\varepsilon-\beta)(\delta-\beta)} + \frac{\varepsilon^2 \ln \varepsilon}{(1-\varepsilon)(\beta-\varepsilon)(\delta-\varepsilon)} \right. \\
 & \left. + \frac{\delta^2 \ln \delta}{(1-\delta)(\beta-\delta)(\varepsilon-\delta)} \right\}, \quad (3.15)
 \end{aligned}$$

$$\varepsilon_{im} = \left( \frac{m_{im}}{m_{\tilde{w}_1}} \right)^2, \quad \beta = \left( \frac{m_{\tilde{w}_2}}{m_{\tilde{w}_1}} \right)^2. \quad (3.16)$$

In the previous cases we have always neglected the component of the physical W-ino which is proportional to the Higgs coupling. However, since the mass of the

top quark can be large, this approximation is no longer reliable and therefore was not used. This is the reason why the amplitude  $T$  depends on the mixing angle  $\omega_1$  in a different way. For the sake of consistency, the standard model amplitude was also calculated in the  $R_\xi$  gauge including the box diagrams proportional to the unphysical scalars [27]. We have verified that for a light top quark ( $m_t < 20$  GeV) these extra contributions are negligible, and even for  $m_t = 40$  GeV they only amount to a 30% correction.

Neglecting the Higgs wino coupling, diagram 4b was calculated before [12] for two generations. In this limit we confirm their order of magnitude estimate, namely, the SUSY contribution is smaller than the standard one by a large factor. The interesting point is that this is no longer true for three families. In fact, if we define  $R$  as the ratio of the SUSY amplitude over the standard model amplitude, our results show that  $R$  can be larger than 1. Not surprisingly,  $R$  also depends on the mass of the top quark,  $m_t$ . Hence, without knowledge of  $m_t$  it is very difficult to use the  $K_L - K_S$  mass difference to obtain a meaningful constraint on the SUSY model. Nevertheless, to illustrate the sensitivity of  $R$  on the model parameters, we arbitrarily choose  $m_t = 40$  GeV and evaluate  $R$ . The left-hand side of fig. 5 shows  $R$  as a function of the photino mass for  $m_{3/2} = 113$  GeV. The full curve is for 3 families,

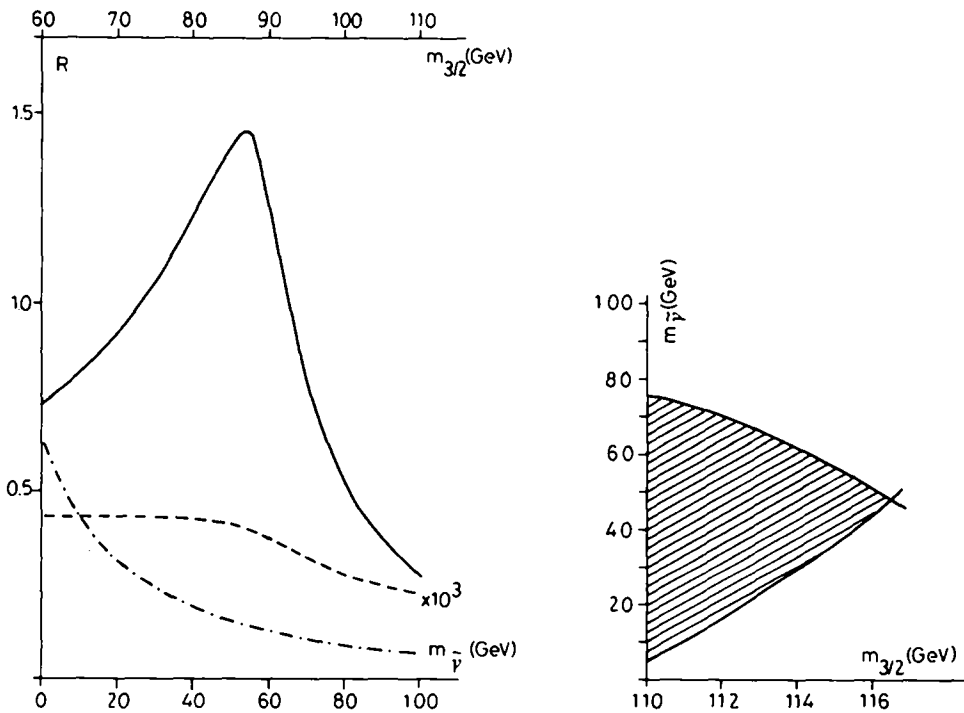


Fig. 5.  $K_L - K_S$  mass difference.  $R$  is the ratio of SUSY over the standard model amplitudes. The crossed area represents the excluded values of  $m_{\tilde{\nu}}$  and  $m_{3/2}$  for  $m_t = 40$  GeV.

while the dashed one is for 2 families. In this latter case  $R$  is practically constant and about  $4 \times 10^{-4}$ . We have checked that this is qualitatively true for a wide range of values of the top quark mass ( $20 \leq m_t \leq 100$  GeV). In the same figure the dashed-dotted curve gives  $R$  as a function of  $m_{3/2}$  for  $m_t = 20$  GeV and  $m_{\tilde{\gamma}} = 0$ , showing that even in this case SUSY can give a sizeable contribution.

If we insist that the  $K_L - K_S$  mass difference is fully explained in the framework of the standard model  $R$  must be smaller than one. This in turn excludes certain values of the SUSY model parameters. On the right-hand side of fig. 5 the dashed area corresponds to values of  $m_{3/2}$  and  $m_{\tilde{\gamma}}$  which would be excluded by this criterion. Notice that for  $m_t = 40$  GeV eq. (2.15) implies that  $0 \leq m_{3/2} \leq 15$  GeV or  $m_{3/2} \geq 105$  GeV. The first mass region for the gravitino is excluded on the basis of our previous analysis (see (3.2)) and for  $105 \leq m_{3/2} \leq 110$  GeV the mass of the lighter s-top would be  $m_t \leq 20$  GeV, which is perhaps in contradiction with present experimental evidence [4]. Let us stress that this figure only illustrates the fact that SUSY can give substantial contributions to the  $K_S - K_L$  mass difference. We did not try to constrain the model since there are further uncertainties associated with the mixing angles and with the evaluation of the matrix elements [14] which make such an attempt premature. Nevertheless, we should point out that in a recent paper, Lahanas and Nanopoulos [13] tried to use the  $K_L - K_S$  mass and the bound on  $K_L \rightarrow \mu^+ \mu^-$  to put limits on the gravitino and top quark masses. Their aim was to escape the Buras [28] upper limit for  $m_t$  in a grand unified model where the breaking of  $SU(2) \times U(1)$  occurs radiatively. Besides this model difference, they have kept the wino mass equal to the w-mass while in our case  $m_{\tilde{w}}$  varies as a function of  $m_{\tilde{\gamma}}$  and  $m_{3/2}$ . This probably explains the differences in our conclusions.

### 3.4. $K_L \rightarrow \bar{\mu} e$

For the process  $K_L \rightarrow \bar{\mu} e$  only the diagram 4a contributes since the s-fermions do not mix quarks with leptons. Using again the external momenta approximation we have

$$\begin{aligned}
 iT = & \left( \frac{g}{\sqrt{2}} \right)^4 \frac{1}{64 \pi^2} \bar{d} \gamma^\mu \frac{1 - \gamma_5}{2} s \bar{e} \gamma_\mu \frac{1 - \gamma_5}{2} \mu \\
 & \times \sum_{i,j=1}^3 U_{is} U_{id}^* V_{jc} V_{j\mu}^* \sum_{m,n=1}^2 [X_{im}^2 I(\varepsilon_{im}, \delta_{jm}, \tilde{w}_1) \cos^2 \omega_1 \\
 & + Y_{im}^2 I(\varepsilon_{im}, \delta_{jm}, \tilde{w}_2) \sin^2 \omega_1 \\
 & + 2Z_{im}^2 J(\varepsilon_{im}, \delta_{jm}, \tilde{w}_1, \tilde{w}_2) \sin^2 \omega_1 \cos^2 \omega_1], \quad (3.17)
 \end{aligned}$$

with

$$Z_{im} = 1 - \left( \frac{m_i}{m_w} \right)^2 + (-1)^{1+m} \tan^{-1} 2\omega_1. \quad (3.18)$$

The integrals  $I$  and  $J$  were defined before and  $\delta_{jn} = (m_{jn}/m_{\tilde{w}_l})^2$ , where  $m_{jn}$  denote the s-neutrino masses.

In a previous paper [29] the upper bound  $B \leq 10^{-17}$  was obtained in the standard model with massive neutrinos. Using the experimental upper limit for the neutrino masses and a leptonic mixing matrix equal to the corresponding one in the quark sector we evaluate  $R = B(\text{BUSY})/B(\text{STAND})$  for  $m_t = 20, 40$  and  $100$  GeV. Our results ( $R \leq 10^{-2}$ ) show that SUSY does not play any role in this process.

#### 4. Conclusions

We have done a detailed analysis of the question of leptonic flavour violation in a class of models where SUSY is spontaneously broken through supergravity interactions. We have shown that in the limit where neutrino masses become degenerate, lepton flavour is still exactly conserved, as is the case in the standard model. Assuming neutrinos to be massive and allowing their masses to vary within the present experimental limits, the SUSY contribution to specific lepton-flavour violating processes was calculated. It turns out that in the case of  $\mu \rightarrow e\gamma$  SUSY can give a branching ratio which is two or three orders of magnitude larger than the corresponding branching ratio in the standard model. We have verified that this enhancement is achieved for photino and gravitino masses which do not give a too large contribution to the muon anomalous magnetic moment. In the case of  $K_L \rightarrow \bar{\mu}e$ , we have used limits on gravitino and photino masses, together with constraints derived from the  $K_L - K_S$  mass difference, to show that SUSY contributions are always small, of order  $10^{-2}$  of the corresponding amplitudes in the standard model.

#### Appendix A

In this appendix we write the globally supersymmetric lagrangian  $\mathcal{L}(\text{SUSY}; f)$  in terms of the superfields introduced in the model. This is by now a well-known procedure [16]. However, our conventions for the metric and  $\gamma$ -matrices are those of Bjorken and Drell [6], and therefore the expressions found in the literature are in most cases slightly different from ours.

Our covariant derivative  $D_\alpha$  is

$$D_\alpha = \frac{\partial}{\partial \theta^\alpha} - i(\gamma_\mu \theta)_\alpha \partial^\mu. \quad (\text{A.1})$$

A left-handed chiral superfield  $S(D_R S = 0)$  is then given by

$$S = e^{i\theta_L^T C \gamma^\mu \theta_L \partial_\mu} (z + \sqrt{2} \theta_L^T C \psi_L + \theta_L^T C \theta_L y), \quad (\text{A.2})$$

where  $\theta$  and  $\psi$  are Majorana spinors ( $\theta_L = \frac{1}{2}(1 - \gamma_5)\theta$ ) and  $C$  is the charge conjugation matrix. The so-called  $F$ -component of a chiral superfield is the coefficient of the

term  $\theta_L^T C \theta_L$ . For the superfield  $S$  given above we have

$$[S]_F = y. \quad (\text{A.3})$$

The right-handed chiral superfield  $S^*(D_L S^* = 0)$  can then be written as

$$S^* = e^{-i\theta_R^T C \gamma^\mu \theta_L \partial_\mu (z^* + \sqrt{2} \theta_R^T C \psi_R + \theta_R^T C \theta_R y^*)}, \quad (\text{A.4})$$

the  $F$ -component being now the coefficient of  $\theta_R^T C \theta_R$ .

The vector gauge multiplet is written in the Wess and Zumino gauge as

$$V = -\theta_R^T C \gamma^\mu \theta_L A_\mu + i\theta_R^T C \theta_R \theta_L^T C \lambda_L - i\theta_L^T C \theta_L \theta_R^T C \lambda_R - \theta_R^T C \theta_R \theta_L^T C \theta_2^1 D, \quad (\text{A.5})$$

the gauge transformation being

$$\delta V = \frac{i}{2e} (\Lambda - \Lambda^*), \quad (\text{A.6})$$

where  $\Lambda$  is a chiral (left-handed) superfield. To write  $V$  in the form (A.5) all parameters in the chiral gauge function have been fixed with the exception of a real scalar  $a(x)$ . Then the remaining gauge transformations in  $V$  are

$$\begin{aligned} \delta A_\mu &= \frac{1}{e} \partial_\mu a(x), \\ \delta \lambda &= 0, \\ \delta D &= 0. \end{aligned} \quad (\text{A.7})$$

Our convention is that the so-called  $D$ -component of a vector superfield is the coefficient of  $-\theta_R^T C \theta_R \theta_L C \theta_L$ . For the example given above we have

$$[V]_D = \frac{1}{2} D. \quad (\text{A.8})$$

The generalization of these expressions for non-abelian gauge theories is straightforward. With these conventions we can then write the globally  $SU(2) \times U(1)$  supersymmetric lagrangian

$$\begin{aligned} \mathcal{L}(\text{SUSY}; f) &= \frac{1}{128g^2} \text{Tr} [W^T C W + \text{h.c.}]_F + \frac{1}{64} [W'^T C W' + \text{h.c.}]_F \\ &+ [\mathbb{L}_i^+ e^{(2gV - g'V')} \mathbb{L}_i + \ell_i^{c*} e^{2g'V'} \ell_i^c]_D \\ &+ [\mathbb{Q}_i^+ e^{(2gV + (1/3)g'V')} \mathbb{Q}_i + u_i^{c*} e^{-(4/3)g'V'} u_i^c + d_i^{c*} e^{(2/3)g'V'} d_i^c]_D \\ &+ [\mathbb{H}^+ e^{(2gV - g'V')} \mathbb{H} + \mathbb{H}^{c+} e^{(2gV + g'V')} \mathbb{H}^c + Y^* Y]_D \\ &+ [g_i \mathbb{H} \mathbb{L}_i \ell_i^c + g_u \mathbb{Q}_i \mathbb{H}^c u_i^c + g_d \mathbb{H} \mathbb{Q}_i d_i^c + \text{h.c.}]_F \\ &+ [\beta (Y \mathbb{H} \mathbb{H}^c - \varepsilon Y) + \text{h.c.}]_F, \end{aligned} \quad (\text{A.9})$$

where  $g$  and  $g'$  are the  $SU(2)$  and  $U(1)$  coupling constants and

$$\begin{aligned} W'_\alpha &= \bar{D}_R D_R D_{L\alpha} V', \\ W_\alpha &= \bar{D}_R D_R (e^{-2gV} D_{\alpha L} e^{2gV}). \end{aligned} \quad (\text{A.10})$$



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