$K_L \rightarrow \bar{\mu}e$: CAN IT BE OBSERVED?

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The decay rate for $K_L \rightarrow \bar{\mu}e$ is evaluated in minimal extensions of the Glashow–Salam–Weinberg model. It is shown that in the left–right symmetric theories this branching ratio can be larger than in the corresponding left-handed theories. The prospects for detecting this decay in proposed experiments are analyzed.

There are good prospects of obtaining very stringent limits on some lepton-flavour non-conserving process (or discovering them!) in the next round of proposed experiments at KEK and BNL as well as in the planned kaon factories at LAMPF II and TRIUMF [1]. On the other hand, the recent experimental indications [2] of a non-zero mass for the electron neutrino together with the fact that neutrino masses arise naturally in some grand unified theories, provide further motivation to analyze lepton-flavour violating processes.

In this letter, we calculate in detail the branching ratio $B(K_L \rightarrow \bar{\mu}e)$, which at KEK and BNL will be measured at the level of 10^{-10}, while the kaon factories may even reach sensitivities of the order of 10^{-12} to 10^{-13}. We address ourselves to the question of whether the observation of a decay rate in this range would be an indication of some "new physics". We shall use minimal extensions of the standard Glashow–Salam–Weinberg (GSW) model Our aim is to give a yardstick against which the forthcoming experimental results should be compared.

Since in the GSW model neutrinos are degenerate in mass, lepton-flavour is conserved by gauge interactions. Furthermore, with only one Higgs doublet, leptonic flavour is also conserved by Higgs interactions. However, in a theory with non-degenerate neutrinos, leptonic flavour is not conserved by gauge interactions and the dominant contribution to $K_L \rightarrow \bar{\mu}e$ comes [3] from the box diagrams of fig. 1a. In the limit where the external momenta are neglected when compared with the loop momentum, the amplitude for this diagram is

$$iT_{LL} = \left(\frac{g}{\sqrt{2}}\right)^4 \bar{d}\gamma^\mu(1-\gamma_5)s\,\bar{e}\gamma_\mu(1-\gamma_5)\mu \sum_{i,j=1}^n U_{i2}U_{i1}^* V_{j1} V_{j2}^* I(\epsilon_i, \delta_j), \tag{1}$$

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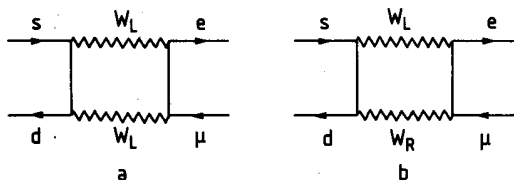
Fig. 1. Leading diagram contributing to $K_L \rightarrow \bar{\mu}e$ in the LL model. Diagram (b) and a similar one with W_L, W_R interchanged are also present in the LRS model.

with

$$I(\epsilon_i, \delta_j) = -[i/4(4\pi)^2 M_W^2]\,\{[(1-\epsilon_i)(1-\delta_j)]^{-1} + (\delta_j - \epsilon_i)^{-1}\,[(1-\epsilon_i)^{-2}\epsilon_i \ln \epsilon_i - (1-\delta_j)^{-2}\delta_j^2 \ln \delta_j]\}\,, \qquad (2)$$

$$\epsilon_i = (M_i/M_W)^2\,, \quad \delta_j = (m_j/M_W)^2\,, \qquad (3)$$

where M_W is the W boson mass, M_i and m_j $(i, j = 1, ..., n)$ stand for the quark and neutrino masses, respectively, n is the number of families and $U(V)$ is the mixing matrix for the quark (lepton) sector. It is clear from eqs. (1) and (2) that the rate for $K_L \to \bar{\mu}e$ is suppressed by the Glashow–Iliopoulos–Maiani (GIM) mechanism [4] both in the quark and leptonic sectors. Thus, in order to have an appreciable rate, neutrinos should not be almost degenerate in comparison with the relevant mass scale (M_W). In fact, one obtains the upper bound $B(K_L \to \bar{\mu}e) \lesssim 10^{-17}$ for a mass of the τ neutrino of 200 MeV. Hence, in the following, we shall assume the existence of a fourth family with a heavy neutral lepton. Since the dominant contribution comes from this heavy neutral lepton, we set $\delta_1 = \delta_2 = \delta_3 = 0$, $\delta_4 \equiv \delta$. Then, in the free quark approximation, we obtain

$$\Gamma(K_L \to e\bar{\mu})/\Gamma(K^+ \to \nu_\mu \bar{\mu}) = (\alpha/4\sqrt{2}\,\pi \sin^2\theta_W \sin\theta_c)^2 \,\frac{m_K^2(m_e^2 + m_\mu^2) - (m_\mu^2 - m_e^2)^2}{m_K^2(m_\mu^2 + m_{\nu_\mu}^2) - (m_\mu^2 - m_{\nu_\mu}^2)^2}\,|I|^2\,, \qquad (4)$$

where the relation

$$\langle 0|\bar{s}\gamma^\mu\gamma_5 d + \bar{d}\gamma^\mu\gamma_5 s|K_L\rangle \cong \sqrt{2}\langle 0|\bar{s}\gamma^\mu\gamma_5 u|K^+\rangle = \sqrt{2}f_K q^\mu \qquad (5)$$

has been used. One can see from eqs. (2) and (4) that $B(K_L \to \bar{\mu}e)$ crucially depends on $|V_{41}V_{42}^*|$ and δ. Note that a limit on these two quantities can be extracted from the present upper bound on $\mu \to e\gamma$. The heavy lepton contribution to $B(\mu \to e\gamma)$ comes from the one-loop diagram which gives [5]

$$B(\mu \to e\gamma) = (3\alpha/32\pi)\delta^2|V_{41}V_{42}^*|^2\,. \qquad (6)$$

On the other hand, from the experimental bound [6] on the non-orthogonality between ν_e and ν_μ it follows that

$$|V_{31}V_{32}^* + V_{41}V_{42}^*| \leqslant 5 \times 10^{-2}\,. \qquad (7)$$

If we take for $|V_{41}V_{42}^*|^2$ the maximum value allowed by the previous equations, then $B(\mu \to e\gamma) < 10^{-10}$ [7] implies $m_4 \leqslant 10$ GeV. Our numerical results for $B(K_L \to \bar{\mu}e)$ are shown in fig. 2. The two dashed curves correspond to the upper and lower values of $B(K_L \to \bar{\mu}e)$ as a function of the heavy lepton mass, when the quark mixing angles are allowed to vary in the following range:

$$U_{11}U_{12} = 0.213\,, \quad -0.21 \leqslant U_{22}U_{21} \leqslant -0.17\,. \qquad (8)$$

We have taken [*1] $m_T = 40$ GeV and $|V_{41}V_{42}^*|^2 \approx 2.5 \times 10^{-3}$.

Next, we shall consider a left–right symmetric model [8] based on $SU(2)_L \times SU(2)_R \times U(1)$. Allowing again for neutrino mixing and the existence of a fourth family, the new contribution to $K_L \to \bar{\mu}e$, from diagram (b) of of fig. 1 is

$$iT_{LR} = -\tfrac{1}{2}g^4(\bar{d}\gamma_5 s)\bar{e}\,\gamma_5\mu\sum_{i,j}M_i m_i U_{i2}^{*R} U_{i1}^L V_{j2}^{*L} V_{j1}^R J(\epsilon_i, \delta_j)\,, \qquad (9)$$

where

[*1] When summing over quark families, we have considered only the contributions of the u, c and t quarks. We thus consider that the "effective" contribution of flavours beyond charm, can be parametrized by the top quark contribution, with M_t and mixing angles varying in an allowed range.

$$J(\epsilon_i, \delta_j) = -[i/(4\pi)^2 M_W^4]\,[\epsilon_i \ln(\epsilon_i\beta)/(\beta\epsilon_i - 1)(\delta_j - \epsilon_i)(1 - \epsilon_i)$$

$$+ \beta\delta_j \ln(\delta_j\beta)/(\beta\delta_j - 1)(\epsilon_i - \delta_j)(1 - \delta_j) + \beta \ln(\beta)/(\beta - 1)(\epsilon_i - 1)(\delta_j - 1)] \tag{10}$$

and $\beta = (M_W/M_R)^2$, with M_R denoting the mass of the right-handed gauge boson. We write the branching ratio as

$$B_{LR}(K_L \to \bar{\mu}e) = B_{LL}(K_L \to \bar{\mu}e)(1 + \beta R), \tag{11}$$

where R can be obtained [+2] from eqs. (1) and (9). We have assumed $U^R = U^L$, $V^R = V^L$ and used the relation

$$\langle 0|(\bar{d}\gamma_5 s + \bar{s}\gamma_5 d)|K_L\rangle = \sqrt{2}m_K^2 f_K/(m_s + m_d). \tag{12}$$

The numerical results for B_{LR} are shown in fig. 2. The full curves correspond to the previously chosen values of the top quark mass and mixing angles. We have set $M_R = 1$ TeV, which is the order of magnitude of the bound coming from $K_L - K_S$ mass difference [9]. It turns out that R is, somewhat surprisingly, a strong enhancement factor ranging from 5×10^4 to 2×10^2, as m_4 varies from 0.2 to 10 GeV. This large value of R has its origin in the PCAC equations as well as in the ratio J/I. Since no such enhancement is present in the decay $\mu \to e\gamma$, $B_{LR}(K_L \to \bar{\mu}e)$ can be larger than $B_{LL}(K_L \to \bar{\mu}e)$, without upsetting the experimental upper limit on $\mu \to e\gamma$. Obviously, if there are some cancellations between gauge and Higgs exchange contributions to the K_L-K_S mass difference [10], thus allowing for smaller values of M_R, the rate for $B_{LR}(K_L \to \bar{\mu}e)$ can be even larger. Notice that due to a different sum in mixing angles, the corresponding βR for the decay $K_L \to \mu\bar{\mu}$ is negligible and so the LL contribution dominates. We have explicitly checked that this is indeed the case. Finally, we observe that in both models the results for $B(K_L \to \bar{\mu}e)$ are almost insensitive to a variation of M_t within the range 20 GeV $< M_t < 40$ GeV.

[+2] In the evaluation of R, we have assumed no W_L-W_R mixing, which should be a good approximation, since there is an experimental limit on the W_L-W_R mixing angle, ($|\xi| \le 0.06$). We have also neglected the contribution of box graphs with unphysical scalars exchange. These graphs, although present in the 't Hooft–Feynman gauge are expected to give a small contribution, since they are suppressed by powers of m_f^2/M_W^2, m_f denoting quark, lepton masses.
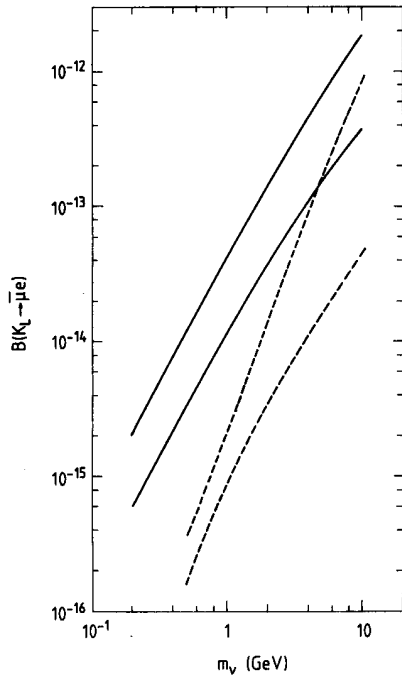

Fig. 2. $B(K_L \to \bar{\mu}e)$ as a function of the heavy neutral lepton mass.

Another simple extension of the standard model which can lead to leptonic flavour violation consists of introducing two Higgs doublets. In this case there are three neutral Higgs which will in general couple to flavour changing neutral currents [*3]. The effective four-fermion interaction contributing to $K_L \to \bar{\mu}e$ can then be written

$$\mathcal{L}_{eff} = \sum_{\alpha=1}^{3} \frac{G_F}{\sqrt{2}M_\alpha^2} (\xi_1^\alpha \bar{d}_L s_R + \xi_2^\alpha \bar{d}_R s_L)(\eta_1^\alpha \bar{\mu}_L e_R + \eta_2^\alpha \bar{\mu}_R e_L). \tag{13}$$

In order to estimate the rate for $K_L \to \bar{\mu}e$, we use constraints on ξ_i^α coming from the K_L-K_S mass difference. A simple calculation using the vacuum insertion approximation leads to

$$\langle K^0 | \mathcal{L}_{eff}^{\Delta s=2} | \bar{K}^0 \rangle = \frac{G_F}{\sqrt{2}} \frac{f_K^2 m_K^4}{4(m_s + m_d)^2} \sum_\alpha \frac{(\xi_1^\alpha - \xi_2^\alpha)^2}{M_\alpha^2}. \tag{14}$$

If, for simplicity, we assume that the dominant contribution comes from the lightest Higgs, with mass M_H, then the K_L-K_S mass difference implies the bound

$$(\xi_1 - \xi_2)^2/M_H^2 < 1.5 \times 10^{-8}. \tag{15}$$

Using (13), (15), one finally obtains

$$B(K_L \to \bar{\mu}e) \leqslant 10^{-5}(\eta_1^2 + \eta_2^2)/M_H^2. \tag{16}$$

There is a great arbitrariness in the values of $(\eta_1^2 + \eta_2^2)M_H^{-2}$. However, if one uses the present limit on $B(\mu \to 3e)$ and further assumes that the $\bar{e}e$ and $\bar{\mu}e$ couplings to Higgs are of the same order of magnitude, one obtains

$$B(K_L \to \bar{\mu}e) \leqslant 10^{-10}. \tag{17}$$

On the other hand, the more pessimistic assumption

$$(\eta_1^2 + \eta_2^2)M_H^{-2} \approx (\xi_1 - \xi_2)^2 M_H^{-2} \tag{18}$$

leads to

$$B(K_L \to \bar{\mu}e) \leqslant 10^{-13}. \tag{19}$$

We summarize our conclusions as follows:

(i) in the left-handed model, the present limit on $B(\mu \to e\gamma)$ imposes a strict constraint on $B(K_L \to \bar{\mu}e)$. Actually, a branching ratio at the level of 10^{-13} is possible only for a very small range of m_4 and mixing angles;

(ii) the predictions of the LRS model are somewhat different due to a large contribution of the W_L-W_R exchange box diagrams. As a consequence, $B(K_L \to \bar{\mu}e)$ can reach values larger than 10^{-13} for a wider range of the parameters;

(iii) theories without natural flavour conservation in the Higgs sector, have in general a great arbitrariness in the choice of parameters. Nevertheless, for "reasonable" values of these parameters, we estimate $B(K_L \to \bar{\mu}e)$ to be in the range 10^{-10} to 10^{-13}. These larger branching ratios reflect both the fact that the decay is allowed at tree level and that the experimental constraints on some of the couplings are not very restrictive. This is a feature common to other models with flavour changing neutral currents, such as technicolor and gauge theories of horizontal interactions, where relatively large values of $B(K_L \to \bar{\mu}e)$ can also arise [12,13].

[*3] For a general analysis of various virtual effects of Higgs particles, see. e.g., ref. [11].

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