Spectrum of Higgsonium in the SM and beyond

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Abstract. Using the formalism of the Bethe-Salpeter equation (BSE), the Higgsonium bound state is studied. The conditions for the formation of Higgsonium bound states are discussed in the SM and in a simple extension thereof.

Keywords: Higgs boson, Bethe-Salpeter equation, relativistic bound states
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INTRODUCTION

The Standard Model (SM) of particle physics involves a very minimalistic idea of the electroweak symmetry breaking scenario. Consequently it results with only one real scalar field and in fact its last directly unobserved particle — the Higgs boson. However, being inspired by the family pattern of the SM fermionic sector, it is quite natural to consider extensions of the SM with a richer structure in the scalar sector. By doubling the doublets or/and adding gauge singlets such next-to-minimal extensions of the SM have been considered and studied from various perspectives. Clear motivations for such extensions are to reduce some SM shortcomings, like a better agreement with precision electroweak fits, the theoretical problem of mass hierarchy or the dark matter problem.

Having more interacting scalar bosons one can expect qualitative changes in the scalar boson sector. In some circumstances the binding forces between scalars can appear strong enough to produce bound states. What the spectrum of appearing bound states is and how they exhibit their existence in collider experiments are important questions. Most notably, with the running of LHC, what do we observe if Higgsonium is realized in Nature?

The simplest model to be chosen for an actual calculation is the extension of the SM that involves the addition of a real scalar singlet S to the SM Lagrangian. The phenomenological implications for singlet extension SM (xSM) have been studied from the collider and cosmological perspectives [1]. The latter typically require a small mixing with the SM Higgs, and from the perspective of bound states it reduces to the SM. In such circumstances it was shown in [2] that a super-heavy Higgs of $m_H \simeq 1$ TeV would be needed to form a bound state. In our model we will consider a large mixing, which could produce two scalars H_1 and H_2 both having masses of a few hundred GeV.

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MODEL

In what follows I will consider the xSM, where the SM is obtained by putting the new couplings to zero. The Lagrangian density for the xSM model is

$$\mathscr{L} = (D_\mu H)^\dagger D^\mu H + \frac{1}{2}\partial_\mu S \partial^\mu S - V(H,S), \tag{1}$$

where H denotes the complex Higgs doublet and S the real scalar. The term linear in S is chosen to vanish after the spontaneous breaking. The potential is given by

$$\begin{aligned} V(H,S) = \lambda (H^\dagger H - \frac{v^2}{2})^2 + \frac{\delta_1}{2} H^\dagger H\, S \\ + \frac{\delta_2}{2} H^\dagger H\, S^2 + \delta_1 v^2 S + \frac{\kappa_2}{2} S^2 + \frac{\kappa_3}{3} S^3 + \frac{\kappa_4}{4} S^4. \end{aligned} \tag{2}$$

In the unitary gauge the charged component of the Higgs doublet H becomes the longitudinal components of the charged W-bosons, and the imaginary part of the neutral component becomes the longitudinal component of the Z-boson. In the unitary gauge the Higgs field doublet reads

$$\left(\begin{array}{c} 0 \\ \frac{1}{\sqrt{2}}(h+v) \end{array} \right) . \tag{3}$$

With our notation the mass terms in the scalar potential become

$$V_{\rm mass} = \frac{1}{2}\left(\mu_h^2 h^2 + \mu_S^2 S^2 + \mu_{hS}^2 hS\right), \tag{4}$$

where

$$\mu_h^2 = 2\lambda v^2 ,\ \ \mu_S^2 = \kappa_2 + \delta_2 v^2 ,\ \ \mu_{hS}^2 = 2\delta_1 v . \tag{5}$$

The mass eigenstate fields $H_{1,2}$ are linear combinations of the Higgs scalar field h and the singlet scalar field S. Explicitly, the inverse transformation reads

$$\begin{aligned} h &= c\, H_1 - s\, H_2 , \\ S &= s\, H_1 + c\, H_2 , \end{aligned} \tag{6}$$

where $c = \cos\theta$, $s = \sin\theta$, and the mixing is determined as [3]

$$\tan\theta = \frac{x}{1+\sqrt{1+x^2}} , \quad x = \frac{\mu_{hS}^2}{\mu_S^2 - \mu_h^2} . \tag{7}$$

For positive x and for the heavier singlet $(x<0)$, we have for the mixing angle

$$\tan\theta = \frac{1+\sqrt{1+x^2}}{|x|} . \tag{8}$$

The terms in the scalar potential that break the discrete $S \to -S$ symmetry are proportional to the couplings δ_1 and κ_3, and we do not consider these terms to be very small. We assume they are large enough to form a bound state and a sufficiently strong communication with the rest of the Standard Model. The small mixing scenario with the light singlet-like scalar has been considered in [4].

In this paper we will consider a relatively large mixing angle θ with Higgs masses less than 200 GeV, which could be promising for experimental observation in the LHC era. For such a case the constraints from electroweak precision observables and their implications for LHC Higgs phenomenology have already been analyzed in [1].

The mass eigenstates satisfy in any case

$$\begin{aligned} M_1^2 &= \mu_h^2 c^2 + \mu_S^2 s^2 + \mu_{hS}^2 cs \\ M_2^2 &= \mu_h^2 s^2 + \mu_S^2 c^2 - \mu_{hS}^2 cs \,. \end{aligned} \tag{9}$$

The rest of the scalar potential contains four new parameters which are added to the SM:

$$V_{\rm int} = \frac{\lambda}{4} h^4 + \frac{\kappa_4}{4} S^4 + \lambda v h^3 + \frac{\kappa_3}{3} S^3 + \frac{\delta_2}{2} h^2 S^2 + \frac{\delta_1}{2} h^2 S + \frac{\delta_2}{v} h S^2 \,. \tag{10}$$

Based on nonrelativistic considerations, the cubic interaction should be sufficiently enhanced against the quartic one, otherwise no bound state can be formed.

The purely cubic interaction between mass eigenstates can be written as

$$V_{\rm cub} = g_{111} H_1^3 + g_{112} H_1^2 H_2 + g_{122} H_1 H_2^2 + g_{222} H_2^3 \,, \tag{11}$$

where the new g couplings are known cubic polynomials of $\sin\theta$ and $\cos\theta$.

It is advantageous to evaluate new quartic couplings between physical states, i.e.,

$$V_4 = \frac{g_{111}}{4} H_1^4 + \frac{g_{1112}}{4} H_1^3 H_2 + \frac{g_{1122}}{4} H_1^2 H_2^2 + \frac{g_{1222}}{4} H_1 H_2^3 + \frac{g_{2222}}{4} H_2^4 \tag{12}$$

where the λ's are known quartic polynomials of sine and cosine of the mixing angle.

Higgsonium, just like any other two-body state in quantum field theory, is described by the two-body BSE

$$\Gamma = \int_k V G^{[2]} \Gamma \tag{13}$$

where we use the shorthand notation $\int_k = i \int \frac{d^4 q}{(2\pi)^4}$, and where $G^{[2]}$ is the two-particle propagator of the constituent Higgses H_1. In momentum space it can be conventionally written as

$$G^{[2]}(k,P) = D(k+P/2, M_1^2) D(-k+P/2, M_1^2) \,, \tag{14}$$

$$D(k, M^2) = \frac{1}{k^2 - M^2 - i\varepsilon} \,. \tag{15}$$

Let us assume that the attractive interaction between heavy Higgses H_2 is strong enough to form a bound state. Within the xSM, the irreducible BSE kernel in lowest order reads

$$V = 6\lambda_{1111} + \sum_{x=s,t,u} \left[\frac{4 g_{112}^2}{x - M_2^2} + \frac{36 g_{111}^2}{x - M_1^2} \right] \,, \tag{16}$$

where the first term represents the purely constant interaction, and s,t,u are the usual Mandelstam variables. Lower indices show which field — H_1 or/and H_2 — belong to a given interaction vertex.

It is advantageous to explicitly divide out the solution which is independent of the relative momenta of the constituents. Following the notation in [2], the original BSE can be rewritten in the form

$$\Gamma_p(p,P) = \Gamma_I(P)\int_k V_p(k,p,P)G^{[2]}(k,P) + \int_k V_p(k,p,P)G^{[2]}(k,P)\Gamma_p(k,P)\,, \tag{17}$$

where

$$V_I = V_c + V_s\ ,\ \ V_p = V_t + V_u\ . \tag{18}$$

The first term is supposed to collect all constant terms, i.e., the ones that do not depend on the relative momenta. In our tree approximation, the kernel reads

$$V_I = 6\lambda_{1111} + \frac{4g_{221}^2}{P^2 - M_1^2} + \frac{36g_{222}^2}{P^2 - M_2^2}\ , \tag{19}$$

where the full solution of the BSE is given by the sum

$$\Gamma(p,P) = \Gamma_I(P) + \Gamma_p(p,P)\ . \tag{20}$$

The equation for the function $\Gamma_I(P)$ is purely algebraic, viz.

$$\Gamma_I(P) = \frac{V_I \int_k \Gamma_p(k,P)G^{[2]}(k,P)}{1 - V_I\int_k G^{[2]}(k,P)}\ . \tag{21}$$

The BSE represents a singular equation which can be solved by some known method. One possibility is to perform a Wick rotation in the relative momenta of the constituents, while keeping the four-momentum-squared P^2 timelike.

Another well-known possibility is the Minkowski solution, performed by using the unique integral representation of the kernels and amplitudes that appear in the BSE [6, 7].

The bound-state vertex function can be expressed as

$$\Gamma(P,p) = \int_{-1}^{1} d\eta \int_{-\infty}^{\infty} d\alpha\, \frac{\rho^{[n]}(\alpha,\eta)}{[F(\alpha,\eta;P,p)]^n}\ , \tag{22}$$

where n is an integer and all the singularities are trapped by the zeros of the denominator in Eq. (22), which reads

$$F(\alpha,\eta;P,p) = \alpha - (p^2 + P.pz + \frac{P^2}{4}) - i\varepsilon\ . \tag{23}$$

Recall the known property of super-renormalizable models studied in previous work [5]: the function $\rho^{[n]}$ becomes smoother and smoother according as n is increased. In practice, the BSE was solved for generalized Wick-Cutkosky models, but only for the lowest n values, i.e., $n = 1, 2$. The studied models were very simple, with at most a cubic scalar interaction.

Here, having a quartic interaction as well, the generated inhomogeneous term is represented by $\Gamma_I(P)$, which is just a real constant for a given discrete value of bound-state mass $\sqrt{P^2}$. To avoid more complicated distributions, we naturally assume

$$\Gamma(P,p) = \Gamma_I(P) + \int_{-1}^{1} d\eta \int_{-\infty}^{\infty} d\alpha \, \frac{\rho_p(\alpha,\eta)}{[F(\alpha,\eta;P,p)]} \, , \tag{24}$$

where $\rho_p(\alpha,\eta)$ is supposed to be a real function, and not a delta distribution. It fully corresponds to the function Γ_p, and one should note that its structure is fully driven by a pure triplet Higgs interaction. Furthermore, we explicitly choose $n = 1$ in the integral representation (24), following the easiest integral representation of the inhomogeneous term in the expression, i.e., $\int_k V_p(k,p,P)G^{[2]}(k,P)$. This integral corresponds to the Feynman scalar triangle diagram.

One can show that the BSE can be converted to a regular integral equation for ρ_p. It reads

$$\rho_p(\alpha,\eta) = \frac{1}{\alpha - M_1^2} \left[\Gamma_I(P)\rho_I(\alpha,\eta) + \int_{-1}^{1} dz \int_{-\infty}^{\infty} da \, \rho_p(a,z) \mathscr{V}(\alpha,\eta,a,z) \right] \, , \tag{25}$$

where $\rho_I, \mathscr{V}$ are known regular functions, and the constant term arises due to the quartic interaction

$$\Gamma_I(P) = \frac{V_I^R \int_{-1}^{1} dz \int_{-\infty}^{\infty} da \rho_p(a,z) I_F(P^2;a,z)}{1 - V_I^R I_B^{[R]}(P^2)} \, , \tag{26}$$

with V_I^R the renormalized constant interaction, and I_F, I_B the known one-loop triangle and one-loop bubble integrals, respectively.

In order to renormalize, the momentum-subtraction renormalization scheme with zero momentum scale is used, i.e.,

$$V_I^R = 6\lambda_{1111}^R + \frac{4g_{112}^2}{P^2 - M_2^2} + \frac{36g_{111}^2}{P^2 - M_1^2} \, , \tag{27}$$

where λ_{1111}^R is the renormalized quartic coupling of heavier Higgs-mass eigenstates H_1.

RESULTS

After a suitable normalization, the BSE for the weight function (25) is solved by the method of iteration. The coupling constants are varied so as to obtain a real discrete spectrum of Higgsonia.

First I present the results for a SM Higgs. The only known input is the Higgs vacuum expectation value (VEV) $v = 275$ GeV, while the Higgs mass and the cubic coupling depend on the experimentally unknown λ, satisfying $m_h = \sqrt{2\lambda}v$ and $\lambda_3 = 2\lambda v$. There are no bound states below a certain critical coupling λ_3. The first Higgsonium state of mass $M = 2m_h$ is formed when $m_h = 1.3$ TeV. Such a result does not provide a reliable answer, and even raises more questions.

Such a heavy Higgs boson is ruled out by electroweak precision tests. Furthermore, for such a fat Higgs, the Higgs sector of the SM represents a strongly-coupled field theory, and our BSE becomes a merely rough estimate of reality. In addition, switching on the top-quark Yukawa coupling, the fat Higgs becomes a broad resonance and its fast decay should prevent the formation of bound states.

To have a reasonable model which is not completely ruled out by electroweak oblique correction constraints, we assume relatively light scalars in the xSM. There are no bound state unless the new cubic coupling is large enough. To compare various quantities, all dimensionful quantities are scaled in units of Higgs VEV v. Then, roughly speaking, the new cubic coupling must be several times larger than the rest of the Lagrangian parameters. This is the main conclusion from the numerical inspection of the large region of xSM parameter space.

Here, I present the first preliminary numerical results. The parameters used as input are the following: $v = 275$ GeV, $\lambda = 0.20$, $\delta_1 = 1.20v$, $\delta_2 = 0.40$, $\kappa_2 = 0.10v$, $\kappa_3 = 5.0v$, $\kappa_4 = 0.20$.

This gives rise to two massive eigenstates $M_1 = 179.5$ GeV, $M_2 = 177.7$ GeV, couplings $g_{111} \simeq 280$ GeV, $g_{222} \simeq 400$ GeV, $\lambda_{1111} \simeq 0.33$, and the appropriate mixing $\cos\theta = 0.696$. Solution of the BSE gives bound states 20% lighter than the Higgs production threshold:

$$M_B = 0.8 \times 2.0 \times M_1 = 286\,\mathrm{GeV}\,. \tag{28}$$

REFERENCES

1. V. Barger, P. Langacker, M. McCaskey, M. J. Ramsey-Musolf and G. Shaughnessy, arXiv:0706.4311v1 [hep-ph].
2. G. Rupp, Phys. Lett. **B** 288, 99 (1992).
3. N. V. Krasnikov, Phys. lett. **B** 291, 89 (1992); Mod. Phys. Lett. **A** 13, 893 (1998).
4. D. O'Connel, M. J. Ramsey-Musolf and Mark B. Wise, Phys. Rev. **D** 75, 037701 (2007).
5. V. Sauli and J. Adam, Phys. Rev. **D** 67, 085007 (2003).
6. N. Nakanishi, *Graph Theory and Feynman Integrals*, eds. Gordon and Breach, New York, 1971.
7. V. Sauli, PhD thesis.