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Model for fermion masses and lepton mixing in $SO(10) \times A_4$

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The discrete flavor symmetry A_4 explains very well neutrino data at low energy, but it seems difficult to extend it to grand unified models since, in general, left-handed and right-handed fields belong to different A_4 representations. Recently a model has been proposed where all the fermions equally transform under A_4. We study here a concrete $SO(10)$ realization of such a model providing small neutrino masses through the see-saw mechanism. We fit the charged fermion masses run up to the unification scale. Some fermion masses properties come from the $SO(10)$ symmetry while lepton mixing angles are a consequence of the A_4 properties. Moreover, our model predicts the absolute value of the neutrino masses; these are in the range $m_\nu \simeq 0.005$–0.052 eV.

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I. INTRODUCTION

The existence of a grand unified theory (GUT) [1,2] has continued to be an attractive idea for physics beyond the standard model (SM) since the 70's. Among indications toward GUTs is the phenomenological tendency to unify of the gauge couplings, and the theoretical implicit possibility to explain charge quantization and anomaly cancellation. One of the main features of GUTs is their potential to unify the particle representations and the fundamental parameters in a hopefully predictive framework. There are many gauge groups that can accommodate the SM ($SU(5)$, $SU(6)$, $SO(10)$, E_6, etc.). Among them $SO(10)$ is the smallest simple Lie group for which a single anomaly-free irreducible representation (namely the spinor 16 representation) can accommodate the entire SM fermion content of each generation.

Once we fix the unification group, we deal with the flavor physics. The introduction of an extra horizontal symmetry acting on the fermion families may further constrain the neutrino mixing parameters and hopefully explain large mixing angles. After the recent neutrino evidence [3–13] we know very well almost all the parameters both in the quark [14] and lepton [15–33] sectors. We know all the quark and charged lepton masses and the value of the difference between the square of the neutrino masses: $\delta m^2_{12} = m^2_1 - m^2_2$ and $\delta m^2_{23} = |m^2_3 - m^2_2|$. We also know the value of the quark mixing angles and phases, and the two mixing angles θ_{12} and θ_{23} in the lepton sector. Moreover we have an upper bound for the θ_{13} mixing angle in the lepton sector. All these experimental informations seem to indicate a discrete flavor symmetry such as 2–3 [34–36], S_3 [37–40], S_4 [41,42], D_3, D_4 [43], A_4 [44–51], T' [52], etc., in the lepton sector. In particular, models with A_4 flavor symmetry, the case studied here, very easily give the tri-bi-maximal mixing matrix [53] that fits well the neutrino data. Non-Abelian discrete symmetries could arise from superstring theory, in particular, from the compactification of heterotic orbifolds [54], the case for A_4 is reported in [55]. Models with $SU(5) \times A_4$ [50] and $SU_L(2) \times SU_R(2) \times SU(4) \times A_4$ [51] symmetries have already been studied in literature. In these previous studies, fermion singlets and $SU_L(2)$ doublets do not equally transform under A_4. Thus this family symmetry seems not to be compatible with $SO(10)$ models where all the matter fields belong to the same representation. Only recently it has been proposed a generic phenomenological model with A_4 [56] which is suitable, as we will see in this work, for a $SO(10)$ GUT generalization.

The purpose of the paper is to construct an explicit $SO(10) \times A_4$ GUT model and to fit, at tree level, fermion masses and mixing. We propose here a non-SUSY GUT model with a Lagrangian invariant under $SO(10) \times A_4$. The matter fields are in a **16**, triplet of A_4. In the Higgs sector, we introduce a **10**, a $\mathbf{126}_s$ and three **45**s singlets of A_4, a **45** and a $\mathbf{126}_t$ triplets of A_4. The A_4 symmetry is dynamically broken by the vacuum expectation value (vev) of the Higgs A_4-triplets. The study of the problem of the vacuum alignment in A_4 just studied in the context of extra dimensions [48] and the MSSM [57] is beyond the scope of this work. The direction of the four vevs of the **45**s in the $SO(10)$ are simply assumed to be T_{3R}, Y and two other

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combinations of them. The **10** gives contributions to the Dirac mass matrices proportional to the identity. Because of the chosen vev directions and the fact that the **45**s appear only in a given combination, we get contributions to M^u, M^d, M^l, but not to $M^\nu_{\rm Dirac}$ from higher dimension operators. The **126** gives contributions only to the Majorana neutrino mass matrix. The low energy neutrino mass matrix is obtained with the see-saw mechanism (for a phenomenological realization in A_4 see [49]).

The paper is organized as follows. In Sec. II we define the matter and Higgs fields transformations under the $SO(10)$ and A_4 groups. In Sec. III we write the Lagrangian of our model. In Sec. IV we show the relations between the Dirac mass matrices and the Higgs vevs. We show similar relations for the Majorana mass matrix of the neutrinos. In Sec. V we write the mixing matrices and masses as function of the Higgs vevs. In Sec. VI we show how the experimental data constrain our model. In Sec. VI A, we perform a numerical analysis of the experimental data by using a Monte Carlo minimization fit. In Sec. VI B we investigate some predictions of our model. Section VII is devoted to conclusions. We list some relevant, well known, A_4 group and representation properties in Appendix A.

II. MATTER AND HIGGS FIELDS

The smaller spinorial representation of $SO(10)$ is the **16** dimensional one. All the fermionic matter fields of one family can be accommodated within the **16** by including the right-handed neutrino. The Higgs electroweak doublet can be taken in the **10** as well as one of the **126** representations. For simplicity we assume that the electroweak doublet Higgs belongs to the **10** representation. Since leptons and quarks mass matrices cannot be symmetric, we need to break the $SO(10)$ left-right symmetry at the unification scale. We perform this job by introducing sets of fields in the **45** representation. The scalar **45** representations can get vev in any combination of the extra Abelian factors Y and T_{3R} directions. The matter fields and scalar fields transform under A_4 as in Table I, where the index of the **45**s refers to the vev's direction. C and D are linear combinations of Y and T_{3R}. We will determine these combinations latter, by using the experimental constraints.

III. THE LAGRANGIAN

Let us write our Lagrangian as,

$$\begin{aligned} L_{\rm Y} = {}& h_0^{ij}\mathbf{16}^i\,\mathbf{10}\,\mathbf{16}^j + h_0'^{ij}\mathbf{16}^i\,\mathbf{10}\,\mathbf{45}_{T_{3R}}\,\mathbf{45}_Y\,\mathbf{16}^j \\ & + h^{ijk}\mathbf{16}^i\,\mathbf{10}\,\mathbf{45}_{T_{3R}}\,\mathbf{45}_Y\,\mathbf{45}_C^j\,\mathbf{45}_D\,\mathbf{16}^k \\ & + \sigma^{il}\mathbf{16}^i\,\mathbf{45}_{T_{3R}}\,\mathbf{126}_s\,\mathbf{45}_{T_{3R}}\,\mathbf{16}^j \\ & + \lambda^{ijk}\mathbf{16}^i\,\mathbf{45}_{T_{3R}}\,\mathbf{126}_t^j\,\mathbf{45}_{T_{3R}}\,\mathbf{16}^k \end{aligned} \tag{1}$$

$$\equiv L_{\rm Dirac} + L_{\rm Majo} \tag{2}$$

TABLE I. Matter and Higgs field representations.

$SO(10)$	**16**	**10**	$\mathbf{45}_{T_{3R}}$	$\mathbf{45}_Y$	$\mathbf{45}_C$	$\mathbf{45}_D$	$\mathbf{126}_s$	$\mathbf{126}_t$
A_4	3	1	1	1	3	1	1	3

where the indices $\{i, j, k, l\}$ are A_4 indices and the sum over the gauge indices is understood. As shown in [58] any Lagrangian of the form in Eq. (1) can be easily obtained from a renormalizable Lagrangian, by including a set of heavy spinor fields, with the inclusion of an $U(1)$ charge and/or supersymmetry. We reserve to a further investigation the question of how general our Lagrangian is, and how it can be obtained in a renormalizable theory.

As we will better clarify in the Appendix A, in the second and in the last terms of Eq. (1) there are two ways of contracting the three A_4 indices in an invariant way. We have to choose to which representation of A_4 the **10** scalar field belongs. Because we want only one Higgs, we excluded the triplet possibility but we still have three possibilities that correspond to how the **10** transforms with respect to A_4: as **1**, **1**$'$, **1**$''$. The fermion mass matrices M_f (with $f = u, d, l, \nu$) coming from the first term in L_Y will be, respectively

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}. \tag{3}$$

In any case we have three degenerate eigenvalues, namely $m_u = m_c = m_t$, that are corrected by the additional terms in Eq. (1). Let us assume that the A_4 triplets $\mathbf{45}_C$ and $\mathbf{126}_t$ get vevs, respectively, in the following directions of A_4

$$\langle \mathbf{45}_C \rangle = v_{\mathbf{45}_C}(1, 1, 1), \qquad \langle \mathbf{126}_t \rangle = v_{\mathbf{126}_t}(1, 0, 0), \tag{4}$$

where the $SO(10)$ indices are understood on both left and right sides. After symmetry breaking, once the Higgs acquire vevs, the quadratic part for the fermions of the Lagrangian in Eq. (1) can be rewritten as

$$L_{\rm Dirac} = h_0(\mathbf{16}_1\mathbf{16}_1 + \mathbf{16}_2\mathbf{16}_2 + \mathbf{16}_3\mathbf{16}_3)v_{\mathbf{10}} + \tag{5a}$$

$$+ h_0'(\mathbf{16}_1\mathbf{16}_1' + \mathbf{16}_2\mathbf{16}_2' + \mathbf{16}_3\mathbf{16}_3')v_{\mathbf{10}} + \tag{5b}$$

$$+ h_1(\mathbf{16}_1\mathbf{16}_2'' + \mathbf{16}_2\mathbf{16}_3'' + \mathbf{16}_3\mathbf{16}_1'')v_{\mathbf{10}} + \tag{5c}$$

$$+ h_2(\mathbf{16}_1\mathbf{16}_3'' + \mathbf{16}_2\mathbf{16}_1'' + \mathbf{16}_3\mathbf{16}_2'')v_{\mathbf{10}} + \tag{5d}$$

$$\begin{aligned} L_{\rm Majo} = {}& \sigma(\mathbf{16}_1'''\mathbf{16}_1''' + \mathbf{16}_2'''\mathbf{16}_2''' + \mathbf{16}_3'''\mathbf{16}_3''')v_{\mathbf{126}_s} \\ & + \lambda\mathbf{16}_2'''\mathbf{16}_3'''v_{\mathbf{126}_t} \end{aligned} \tag{5e}$$

where

$$\begin{aligned} \mathbf{16}_i'' &\equiv v_{\mathbf{45}_{T_{3R}}}v_{\mathbf{45}_Y}v_{\mathbf{45}_C}v_{\mathbf{45}_D}\mathbf{16}_i \qquad \mathbf{16}_i''' \equiv v_{\mathbf{45}_{T_{3R}}}\mathbf{16}_i \\ \mathbf{16}_i' &\equiv v_{\mathbf{45}_{T_{3R}}}v_{\mathbf{45}_Y}\mathbf{16}_i \qquad \text{with } i = 1, 2, 3 \end{aligned} \tag{6}$$

We obtain the following expression by absorbing the vevs of the **45**s into the coupling constants

TABLE II. Quantum numbers for the low energy matter fields.

	X	Y	$B-L$	T_{3R}
q	1	1/3	1	0
u^c	1	$-4/3$	-1	1/2
d^c	-3	2/3	-1	$-1/2$
l	-3	-1	-3	0
e^c	1	2	3	$-1/2$
ν^c	5	0	3	1/2

$$\mathbf{16}' = (x_{qL}q, x_{uR}u^c, x_{dR}d^c, x_{lL}l, x_{eR}e^c, x_{\nu R}\nu_R)^T, \tag{7a}$$
$$\mathbf{16}'' = (x'_{qL}q, x'_{uR}u^c, x'_{dR}d^c, x'_{lL}l, x'_{eR}e^c, x'_{\nu R}\nu_R)^T, \tag{7b}$$
$$\mathbf{16}''' = (x''_{qL}q, x''_{uR}u^c, x''_{dR}d^c, x''_{lL}l, x''_{eR}e^c, x''_{\nu R}\nu_R)^T, \tag{7c}$$

where $x_{fL,R}$, $x'_{fL,R}$, and $x''_{fL,R}$ are the quantum numbers, respectively, of the product of the charges T_{3R} with Y, of the product of the charges T_{3R}, Y, C, and D, and of the charge T_{3R} reported in Table II [58].

IV. FROM VEVS TO MASS MATRICES

From Table II we observe that $x'_{\nu R} = 0$ (because Y of the right-handed neutrino is zero) and $x'_{lL} = 0$ (because T_{3R} of the lepton doublet is zero). These two conditions imply that the terms $\mathbf{16_i}\,\mathbf{16''_j}v_{\mathbf{10}}$ in the Lagrangian L_{Dirac} do not contribute to the Dirac neutrino mass term. Therefore, once the **45**s get a vev, from Eq. (5b) we have that the Dirac neutrino mass matrix M^ν_{Dirac} is proportional to the identity.

$$M^\nu_{\text{Dirac}} = h_0 v^u \mathbf{I}, \tag{8}$$

where $\mathbf{I}$ is the identity matrix and v^u is the vev of the up component of the **10** [59]. The fact that the M^ν_{Dirac} is proportional to the identity will be important in order to realize the see-saw mechanism and not spoiling the main consequence of the A_4 symmetry; the explanation of the appearence of a tri-bi-maximal mixing matrix in the lepton sector. With the conventions $x_{uL} = x_{dL} \equiv x_{qL}$, $x_{eL} = x_{\nu L} \equiv x_{lL}$, and $v^e = v^d$, the interactions $h_1\mathbf{16_1}\,\mathbf{16''_2}$ and $h_2\mathbf{16_2}\,\mathbf{16''_1}$ in Eqs. (5c) and (5d) give the following mass terms

$$h_1 v^f(x'_{fL}\bar\psi_{L1}\psi_{R2} + x'_{fR}\bar\psi_{L2}\psi_{R1}) + h_2 v^f(x'_{fL}\bar\psi_{L2}\psi_{R1} + x'_{fR}\bar\psi_{L1}\psi_{R2}) + \text{H.c.} \tag{9}$$

namely,

$$v^f\begin{pmatrix} 0 & h_1x'_{fL} + h_2x'_{fR} \\ h_1x'_{fR} + h_2x'_{fL} & 0 \end{pmatrix}_{12}$$

and so on for the other interactions (in the flavor planes 31 and 23). If we introduce

$$A^f = (h_1x'_{fL} + h_2x'_{fR}) \quad \text{and} \quad B^f = (h_1x'_{fR} + h_2x'_{fL}) \tag{10}$$

the full contribution to the Dirac mass matrices, coming from the operators proportional to the **45** representations, is

$$v^f\begin{pmatrix} 0 & A^f & B^f \\ B^f & 0 & A^f \\ A^f & B^f & 0 \end{pmatrix}. \tag{11}$$

The charged fermion mass matrices are then

$$M^u = v^u\begin{pmatrix} h_0^u & A^u & B^u \\ B^u & h_0^u & A^u \\ A^u & B^u & h_0^u \end{pmatrix}; \quad M^d = v^d\begin{pmatrix} h_0^d & A^{d,l} & B^{d,l} \\ B^{d,l} & h_0^d & A^{d,l} \\ A^{d,l} & B^{d,l} & h_0^d \end{pmatrix}; \quad M^l = v^d\begin{pmatrix} h_0^l & A^{d,l} & B^{d,l} \\ B^{d,l} & h_0^l & A^{d,l} \\ A^{d,l} & B^{d,l} & h_0^l \end{pmatrix} \tag{12}$$

where v^u and v^d are the vevs of the up and down components of the **10**, while the A and B coefficients are defined in Eq. (10). The h_0^f are defined by the combinations of h_0 and h'_0 with the weight corresponding to the charge x_{fR}

$$h_0^u = h_0 + x_{uR}h'_0, \tag{13a}$$
$$h_0^d = h_0 + x_{dR}h'_0, \tag{13b}$$
$$h_0^l = h_0 + x_{eR}h'_0. \tag{13c}$$

We observe that the general form of the mass matrices $M^{u,d,l}$, are of the same type of the one reported in Ref. [60] (see Eq. (20)). Moreover the Majorana mass matrix for the right-handed neutrino is given by

$$M_R = \begin{pmatrix} a & 0 & 0 \\ 0 & a & b \\ 0 & b & a \end{pmatrix} \tag{14}$$

where $a = \sigma v_{\mathbf{126}_s}$ and $b = \lambda v_{\mathbf{126}_t}$. The Dirac neutrino mass matrix has been previously given in Eq. (8).

V. MASSES AND MIXING

It has been recently shown in [60] that, if the Dirac mass matrices are given by Eq. (12), the charged fermion mass matrices are diagonalized by

$$U = \frac{1}{\sqrt{3}}\begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \tag{15}$$

and then we have

$$M^f = U\begin{pmatrix} (h_0^f + A^f + B^f)v^f & 0 & 0 \\ 0 & (h_0^f + \omega A^f + \omega^2 B^f)v^f & 0 \\ 0 & 0 & (h_0^f + \omega B^f + \omega^2 A^f)v^f \end{pmatrix}U^\dagger \tag{16}$$

where $f = u, d, l$, $v^l = v^d$, and h_0^f, A^f and B^f are complex parameters.

From the Lagrangian in Eq. (1), the light neutrino mass matrix comes from a type-I see-saw mechanism as below

$$M^\nu = M^\nu_{\rm Dirac}\frac{1}{M_R}(M^\nu_{\rm Dirac})^T \tag{17}$$

where the Dirac neutrino mass matrix $M^\nu_{\rm Dirac}$ is proportional to the identity (see Eq. (8)), while M_R is the right-handed Majorana neutrino matrix. We observe that our Lagrangian does not give the left-handed M_L Majorana neutrino mass matrix since we have introduced the T_{3R} fields. In the basis where the charged leptons are diagonal, the mass matrix of the low energy neutrino $\bar{M}_\nu$ is given by

$$\bar{M}_\nu = U^T M_\nu U = M^\nu_{\rm Dirac}\frac{1}{U^\dagger M_R U^\star}(M^\nu_{\rm Dirac})^T \tag{18}$$

where we used the fact that $M^\nu_{\rm Dirac}$ is proportional to the identity. We have

$$U^\dagger M_R U^\star = \begin{pmatrix} a + 2b/3 & -b/3 & -b/3 \\ -b/3 & 2b/3 & a - b/3 \\ -b/3 & a - b/3 & 2b/3 \end{pmatrix} \tag{19}$$

and it is diagonalized by a tri-bi-maximal mixing matrix. Consequently $\bar{M}_\nu$ is diagonalized by the same tri-bi-maximal mixing matrix too. The eigenvalues of $\bar{M}^\nu$ are

$$m_1 = \frac{(h_0 v^u)^2}{a + b}, \tag{20a}$$

$$m_2 = \frac{(h_0 v^u)^2}{a}, \tag{20b}$$

$$m_3 = \frac{(h_0 v^u)^2}{b - a}. \tag{20c}$$

VI. NUMERICAL FITTING AND MODEL PREDICTIONS

In the following Subsec. VI A, we analyze how to translate all the information from the experimental data into constraints for the parameters of our theory. Then, in Subsec. VI B we will show how well the charged fermion mass matrices in Eq. (16) can be fitted. We also include some theoretical predictions of our model about the absolute neutrino masses.

A. Experimental constraints

From Eq. (16) we have that the tree mass eigenvalues for the charged fermions are of the form

$$(h_0^f + A^f + B^f)v^f = m_1^f, \tag{21a}$$

$$(h_0^f + \omega A^f + \omega^2 B^f)v^f = m_2^f, \tag{21b}$$

$$(h_0^f + \omega^2 A^f + \omega B^f)v^f = m_3^f, \tag{21c}$$

where the masses m_i^f are in general complex and their phases are unphysical. The parameters h_0^f, A^f, and B^f are complex. The v^f are the vevs of the scalar Higgs doublets in the **10** and $v^l = v^d$. The most general solution of the system in Eq. (21) is

$$h_0^f = \frac{1}{v^f}\frac{m_1^f + m_2^f + m_3^f}{3} \tag{22a}$$

$$A^f = \frac{1}{v^f}\frac{m_2^f\omega^2 + m_1^f + m_3^f\omega}{3} \tag{22b}$$

$$B^f = \frac{1}{v^f}\frac{m_3^f\omega^2 + m_1^f + m_2^f\omega}{3}. \tag{22c}$$

The numerical values of h_0^f, A^f and B^f in Eq. (22) are then fixed, up to phases, by the fermion masses. The absolute value of h_0^f can be written as

$$\begin{aligned} |h_0^f|^2 = \left(\frac{1}{3v^f}\right)^2 [(m_1^f + m_2^f + m_3^f)^2 \\ - 2(m_1^f m_3^f(1 - \cos\phi_1) \\ + m_1^f m_2^f(1 - \cos(\phi_1 - \phi_2)) \\ + m_2^f m_3^f(1 - \cos\phi_2))] \end{aligned} \tag{23}$$

where ϕ_1 and ϕ_2 are the relative phases between m_1 and m_3 and between m_2 and m_3 respectively. From Eq. (23) and by assuming that $m_3 > m_1 + m_2$, we obtain

$$\frac{1}{3v^f}(m_1^f + m_2^f + m_3^f) \geq |h_0^f| \geq \frac{1}{3v^f}(m_3^f - m_1^f - m_2^f). \tag{24a}$$

In the same manner we get

$$\frac{1}{3v^f}(m_1^f + m_2^f + m_3^f) \geq |A^f| \geq \frac{1}{3v^f}(m_3^f - m_1^f - m_2^f) \tag{24b}$$

$$\frac{1}{3v^f}(m_1^f + m_2^f + m_3^f) \geq |B^f| \geq \frac{1}{3v^f}(m_3^f - m_1^f - m_2^f). \tag{24c}$$

Under the condition that $m_3 \gg m_1, m_2$, the phases among h_0^f, A^f and B^f are strongly constrained by the last equation in Eq. (21). From the solutions in Eq. (22) we get

$$\frac{A^f}{h_0} \simeq \omega \quad \text{and} \quad \frac{B^f}{h_0} \simeq \omega^2. \tag{25}$$

From the solution in Eq. (22) and by using the definitions of A^f, B^f in Eq. (10) we obtain

$$x'_+ = \frac{1}{3v^f}\frac{m_3^f + m_2^f - 2m_1^f}{h_1 + h_2} \quad \text{and} \quad x'_- = \frac{i}{\sqrt{3}v^f}\frac{m_3^f - m_2^f}{h_1 - h_2} \tag{26}$$

where we have introduced the notation $x'_\pm \equiv x'_L \pm x'_R$. In Eq. (26) we must remember that each mass includes an undetermined phase. We notice that the ratios x'_+/x'_+ and x'_-/x'_- do not depend on h_i, then they are experimentally determined (up to the undetermined phases). In fact we have

$$\frac{x'_+}{x'_+} = \frac{v^d}{v^u}\frac{m_t + m_c - 2m_u}{m_b + m_s - 2m_d}, \tag{27a}$$
$$\frac{x'_-}{x'_-} = \frac{v^d}{v^u}\frac{m_t - m_c}{m_b - m_s}, \tag{27b}$$
$$\frac{x'_+}{x'_+} = \frac{v^d}{v^u}\frac{m_t + m_c - 2m_u}{m_\mu + m_\tau - 2m_e}, \tag{27c}$$
$$\frac{x'_-}{x'_-} = \frac{v^d}{v^u}\frac{m_t - m_c}{m_\tau - m_\mu}. \tag{27d}$$

By using the masses run up to the $2 \cdot 10^{16}$ GeV scale in the (non-SUSY) standard model given in Table III, we performed a Monte Carlo analysis of Eq. (27). For the masses we took two sided Gaussian distributions with central values and standard deviations taken from Table III. For the unknown phases we took flat random distributions in the interval $[0, 2\pi]$. Our results can be summarized as

TABLE III. Quark masses run at the $2 \cdot 10^{16}$ GeV scale in non-SUSY standard model (see Ref. [61]).

m_u (MeV)	$0.8351^{+0.1636}_{-0.1700}$
m_c (MeV)	$242.6476^{+23.5536}_{-24.7026}$
m_t (GeV)	$75.4348^{+9.9647}_{-8.5401}$
m_d (MeV)	$1.7372^{+0.4846}_{-0.2636}$
m_s (MeV)	$34.5971^{+4.8857}_{-5.1971}$
m_b (GeV)	$0.9574^{+0.0037}_{-0.0169}$
m_e (MeV)	$0.4414^{+0.0001}_{-0.0001}$
m_μ (MeV)	$93.1431^{+0.0136}_{-0.0101}$
m_τ (GeV)	$1.5834^{+10.4664}_{-13.6336}$

$$\frac{x'_+}{x'_+} = 0.972^{+0.073}_{-0.013} \qquad \frac{x'_-}{x'_-} = 1.034^{+0.007}_{-0.072} \tag{28a}$$
$$\frac{x'_+}{x'_+} = 0.573^{+0.079}_{-0.011} \qquad \frac{x'_-}{x'_-} = 0.640^{+0.011}_{-0.077} \tag{28b}$$
$$\frac{x'_+}{x'_+} = 0.590^{+0.085}_{-0.048} \qquad \frac{x'_-}{x'_-} = 0.619^{+0.054}_{-0.075}. \tag{28c}$$

Notice that, if we neglect the undetermined phases in the masses, we get similar central values but wrong errors in the constraints. For example we would obtain in such a case

$$\frac{x'_+}{x'_+} = 0.972 \pm 0.005, \qquad \frac{x'_-}{x'_-} = 1.034 \pm 0.006. \tag{29}$$

B. The theoretical prediction

In our model we are able to fit all the masses of quarks and leptons. Moreover we obtain, thanks to the A_4 structure of the model, a tri-bi-maximal lepton mixing matrix. Let us investigate the fermion masses. As obtained in the previous section, the quantities to be fitted are the ratios in Eq. (28). The theoretical result for the ratios $x_+^f/x_+^{f'}$ and $x_-^f/x_-^{f'}$ are determined from Table I and the definitions of $x_\pm^f$. By using, for example, the direction $C = (28X - 249Y)$ and $D = (238X - 9Y)$ we get

$$\frac{x'_+}{x'_+} = 1 \quad \text{and} \quad \frac{x'_-}{x'_-} = 1; \tag{30a}$$
$$\frac{x'_+}{x'_+} = \frac{300}{517} \quad \text{and} \quad \frac{x'_-}{x'_-} = \frac{300}{517} \tag{30b}$$

in good agreement with the experimental values in Eq. (28). The absolute neutrino mass scale is fixed, because the presence of, essentially, only two free parameters, a and b, in the neutrino sector. If we impose the experimental constraints on $\delta m^2_{12} = 7.92(1 \pm 0.09) \times 10^{-5}$ eV^2 and $|\delta m^2_{13}| = 2.4(1^{+0.21}_{-0.26}) \times 10^{-3}$ eV^2 we get the following neutrino masses:

$$m_1 = 0.052 \pm 0.005 \text{ eV}, \quad m_2 = 0.052 \pm 0.005 \text{ eV}, \quad m_3 = 0.017 \pm 0.002 \text{ eV} \tag{31}$$

$$m_1 = 0.0051 \pm 0.0005 \text{ eV}, \quad m_2 = 0.0102 \pm 0.0005 \text{ eV}, \quad m_3 = 0.049 \pm 0.004 \text{ eV} \tag{32}$$

where the first results correspond to an Inverted Hierarchy case, while the second ones would correspond to the Normal Hierarchy.

VII. CONCLUSIONS

Neutrino data at low energy are well explained by a A_4 symmetry, nevertheless it is difficult to include this symmetry in grand unified theories. In this paper we investigate the possibility to construct an explicit model with a Lagrangian invariant under $SO(10) \times A_4$. We assumed that the matter fields are in a **16** dimensional $SO(10)$ representation, triplet of A_4. In the Higgs sector, we introduced a **10**, a **126** and three **45**s singlets of A_4, a **45** and a **126** triplets of A_4. The A_4 symmetry is dynamically broken by the vevs of the Higgs A_4-triplets. The direction of the vevs of the **45**s in the $SO(10)$ are assumed to be T_{3R}, Y and two other combinations of them, C and D. The Lagrangian contains three terms with the **10** that give contributions to the Dirac mass matrices, and two terms with the **126**s that determine the Majorana neutrino mass matrix. The first two terms containing the **10** give a contribution to the Dirac mass matrices which is proportional to the identity (the second term is used to avoid the τ bottom unification). The third term, because of the fact that the **45**s appear only in the given combination, provides contributions to M^u, M^d, M^l, but not to $M^\nu_{\rm Dirac}$. For these reasons $M^\nu_{\rm Dirac}$ results to be proportional to the identity. Finally the **126** terms give contribution only to the right-handed neutrino Majorana mass matrix M_R. The low energy neutrino mass matrix is then obtained with the see-saw mechanism.

The mixing angle structure of the charged fermion mass matrices are fixed by the A_4 structure of the model. They are diagonalized by the mixing matrix in Eq. (15). The A_4 direction of the vev of the triplet **126** implies a particular form for M_R. This particular form of M_R and the fact that $M^\nu_{\rm Dirac}$ is proportional to the identity, imply that the low energy neutrino mass matrix, in the base with diagonal charged lepton, is diagonalized by the tri-bi-maximal mixing matrix.

We show that at tree level our model fits with great precision (within 1 standard deviation) the values of the fermion masses, run at $2 \cdot 10^{16}$ GeV scale in the (non-SUSY) standard model, if particular directions of the vevs of the $\mathbf{45}_C$ and $\mathbf{45}_D$ are assumed ($C = (28X - 249Y)$ and $D = (238X - 9Y)$).

One important consequence of the structure of this model is the prediction of an absolute scale for low mass neutrinos. We predict the absolute scale of the neutrino mass to be close to $\sim$0.05 eV. Normal or inverted hierarchies are allowed by the model.

In the model presented here, both up and down sector are diagonalized by the same mixing matrix. For this reason the resulting quark mixing matrix, the CKM matrix is proportional to the identity, in agreement with evidence only at first order. The explanation of the correct CKM matrix is beyond the scope of this work. However a deeper study of radiative corrections to the potential could posibly shed light on the right CKM structure.

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APPENDIX A: THE A_4 PROPERTIES

The group A_4 is the finite group of the even permutations of four object and contains 12 elements. Every finite group can be generated by a subset of elements, called generators. A set of elements is independent if none of them can be expressed in terms of the other. The group A_4 has two independent generators denoted as S and T, which can be chosen to verify the following defining relations:

$$S^2 = T^3 = (ST)^3 = I.$$

There are four irreducible representations for the A_4 group: denoted as 1, $1'$, $1''$ and the 3. In each of these representations the generators are explicitly written as follows:

$$\begin{aligned} 1&: S = 1, T = 1,\\ 1'&: S = 1, T = \omega,\\ 1''&: S = 1, T = \omega^2,\\ 3&: S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{A1}$$

where $\omega = e^{2\pi i/3}$ and then $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$. If $a = (a_1, a_2, a_3)$ is a triplet, then the action of the S and T operators is $Sa = (a_1, -a_2, -a_3)$ and $Ta = (a_2, a_3, a_1)$. If b is another analogous A_4 triplet, their tensorial product decomposes in irreducible representations as

$$3 \times 3 = 1 + 1' + 1'' + 3 + 3.$$

In order to explicitly construct a singlet from these quantities we first impose the invariance under S, the most generic term will be

$$xa_1b_1 + ya_2b_2 + za_3b_3 + ta_2b_3 + ra_3b_2,$$

where x, y, z, r and t are parameters. If we impose also the invariance under T, we have that the above term transforms like a 1 single, if and only if $x = y = z$ and $r = t = 0$. Then we have

$$1 = (ab) = (a_1b_1 + a_2b_2 + a_3b_3).$$

Similarly one can check that

$$1' = (ab)' = (a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3),$$
$$1'' = (ab)'' = (a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3).$$

Let us go now to construct the triplet. By imposing S invariance, the most generic triplet in the product of a and b is

$$(x a_1 b_1 + y a_2 b_2 + z a_3 b_3 + t a_2 b_3 + r a_3 b_2, \tilde{x} a_1 b_2 + \tilde{y} a_1 b_3 + \tilde{z} a_2 b_1 + \tilde{t} a_3 b_1, \ldots)$$

applying T we have

$$(x a_2 b_2 + y a_3 b_3 + z a_1 b_1 + t a_3 b_1 + r a_1 b_3, \ldots, \ldots)$$

from which we have the relation

$$x a_2 b_2 + y a_3 b_3 + z a_1 b_1 + t a_3 b_1 + r a_1 b_3 = \tilde{x} a_1 b_2 + \tilde{y} a_1 b_3 + \tilde{z} a_2 b_1 + \tilde{t} a_3 b_1$$

from which we get

$$x = y = \tilde{x} = \tilde{z} = z = 0, \qquad t = \tilde{t}, \qquad r = \tilde{y}.$$

The final result is

$$3 = (a_2 b_3, a_3 b_1, a_1 b_2) \quad \text{and} \quad 3 = (a_3 b_2, a_1 b_3, a_2 b_1)$$

where the first line comes from terms proportional to t while the second line is proportional to r. In summary, with this notation, if $v = (v_1, v_2, v_3)$ is an additional triplet, the product of the three triplet a, b and v that transform as a singlet 1 in A_4 is given by

$$h_1(a_2 b_3 v_1 + a_3 b_1 v_2 + a_1 b_2 v_3) + h_2(a_3 b_2 v_1 + a_1 b_3 v_2 + a_2 b_1 v_3) \qquad \text{(A2)}$$

where h_1 and h_2 are arbitrary parameters. The term in Eq. (A2) is invariant under A_4.

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