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Natural gauge and gravitational coupling unification and the superpartner masses

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Abstract

The possibility to achieve unification at the string scale in the context of the simplest supersymmetric grand unified theory is investigated. We find conservative upper bounds on the superpartner masses consistent with the unification of gauge and gravitational couplings, $M_{\tilde{G}} \lesssim 5$ TeV and $M_{\tilde{f}} \lesssim 3 \times 10^7$ GeV, for the superparticles with spin one-half and zero, respectively. These bounds hint towards the possibility that this supersymmetric scenario could be tested at future colliders, and in particular, at the forthcoming LHC.

1. Introduction

The unification of all fundamental forces in nature is one of the main motivations for the physics beyond the Standard Model. More than two decades have passed since the remarkable observation that in the minimal supersymmetric extension of the Standard Model (MSSM) the gauge couplings unify at a very high-energy scale [1], $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV. Supersymmetric grand unified theories (GUTs) are considered as the most natural candidates to describe the physics at the M_{GUT} scale. Nevertheless, this scale is somehow below the generically predicted perturbative string unification scale $M_{\text{str}} \simeq 5 \times 10^{17}$ GeV [2,3].

Different paths to resolve the discrepancy between the GUT and string scales have been proposed in the literature [4]. In particular, the introduction of additional states, with masses below the unification scale, is one of the well-motivated possibilities. A simple example is provided by the addition of adjoint representations such as a color-$SU(3)$ octet (Σ_8) and a weak-$SU(2)$ color-neutral triplet (Σ_3) [5]. In this framework, the role of the adjoint scalars is to push the GUT scale up to M_{str}. These adjoint scalars are present in the $\widehat{\mathbf{24}}_{\mathbf{H}}$ representation of the minimal supersymmetric $SU(5)$ [6], where the MSSM matter superfields are unified in $\hat{\bar{\mathbf{5}}}$ and $\widehat{\mathbf{10}}$, and the Higgs sector is composed of $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$, $\hat{\mathbf{5}}_{\mathbf{H}}$ and $\widehat{\mathbf{24}}_{\mathbf{H}}$ representations.

As is well known, proton decay is the most dramatic prediction coming from grand unified theories [7]. However, it is interesting to look for alternative ways to test the idea of the unification of all fundamental forces in nature. In this Letter we investigate if the unification of the gauge and gravitational couplings at the string scale can give us some new insight in our quest for unification. We study the possibility to achieve unification of gauge couplings and gravity in the context of the simplest supersymmetric grand unified theory. We show that such a unification leads at one-loop level to a unique relation between the superpartner masses. Using the electroweak precision data and the current limits on the SUSY partner masses, we find upper bounds on the sfermion and fermionic superpartner masses. We conclude that in this minimal framework the fermionic superpartner masses are naturally at or below the TeV scale.

2. Upper bound on the superpartner masses

In this section we will explain the possibility to find upper bounds on the superpartner masses once the unification of all forces is assumed in the context of heterotic string scenarios.

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Table 1
The additional contributions to the one-loop beta coefficients in the context of the minimal SUSY $SU(5)$. Here $\tilde{G}$ stands for gauginos and higgsinos and $\tilde{f}$ for sfermions and the extra Higgs doublet

R	Δ_1^R	Δ_2^R	Δ_3^R
$\widetilde{G}$	2/3	2	2
$\widetilde{f}$	7/2	13/6	2
$\hat{\Sigma}_8 \subset \widehat{\mathbf{24}}_{\mathbf{H}}$	0	0	3
$\hat{\Sigma}_3 \subset \widehat{\mathbf{24}}_{\mathbf{H}}$	0	2	0

In a weakly-coupled heterotic string theory, gauge and gravitational couplings unify at tree level [2],

$$\alpha_{\text{str}} = \frac{2G_N}{\alpha'} = k_i \alpha_i, \tag{1}$$

where $\alpha_{\text{str}} = g_{\text{str}}^2/4\pi$ is the string-scale unification coupling constant, G_N is the Newton constant, α' is the Regge slope, $\alpha_i = g_i^2/4\pi$ $(i = 1, 2, 3)$ are the gauge couplings and k_i are the so-called affine or Kač–Moody levels at which the group factors $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ are realized in the four-dimensional string [4]. Including one-loop string effects, the unification scale M_{str} is predicted as [3]

$$M_{\text{str}} = \sqrt{4\pi\alpha_{\text{str}}}\,\Lambda_s, \tag{2}$$

where $\Lambda_s \approx 5.27 \times 10^{17}$ GeV.

Our main goal is to investigate the possibility to achieve unification of all interactions in the context of the minimal supersymmetric $SU(5)$ theory. The relevant one-loop renormalization group equations are given by

$$\alpha_{iZ}^{-1} = k_i \alpha_{\text{str}}^{-1} + \frac{b_i^{\text{SM}}}{2\pi} \log\frac{M_{\text{str}}}{M_Z} + \sum_R \frac{\Delta_i^R}{2\pi} \log\frac{M_{\text{str}}}{M_R}, \tag{3}$$

where $\alpha_{iZ} \equiv \alpha_i(M_Z)_{\overline{DR}}$ are the couplings defined in the $\overline{DR}$ renormalization scheme. The masses M_R are the different thresholds included in the running. We recall that in the Standard Model $b_i^{\text{SM}} = (41/6, -19/6, -7)$. The coefficients Δ_i^R are the additional contributions associated to each mass threshold M_R. In Table 1 we list their values for the minimal SUSY $SU(5)$ theory considered here. In the above equations we have used M_{str} as the most natural value for the superheavy gauge boson masses as well as for the mass of the colored triplets in $\hat{\mathbf{5}}_{\mathbf{H}}$ and $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$, relevant for proton decay [7]. Notice that since the contribution of the colored triplets to $b_1 - b_2$ $(b_2 - b_3)$ is positive (negative) the upper bounds presented below are the most conservative bounds. In other words, the lower the colored triplet mass scale is, the lighter the superpartner masses have to be to achieve unification.

The affine levels k_i are those corresponding to the standard $SU(5)$ theory, i.e. the canonical values $k_1 = 5/3$, $k_2 = 1$ and $k_3 = 1$. We remark that considering a higher Kač–Moody level k (as required, for instance, in string models having a $G \times G$ structure [8]) simply corresponds to the redefinition $\Lambda_s \to \sqrt{k}\,\Lambda_s$. This pushes the string scale M_{str} up and would require slightly lower values of the adjoint scalar masses and somewhat heavier sfermions to achieve unification.

Assuming a common mass $M_{\tilde{G}}$ for gauginos and higgsinos, as well as a common mass $M_{\tilde{f}}$ for sfermions and the extra Higgs doublet, and using $M_{\Sigma_3} = M_{\Sigma_8} \equiv M_\Sigma$ as predicted by the minimal supersymmetric $SU(5)$ model, the system of Eqs. (3) has the solution

$$M_{\tilde{f}} = \frac{M_{\text{str}}^6}{M_Z M_{\tilde{G}}^4} e^{\pi(3\alpha_{1Z}^{-1} - 15\alpha_{2Z}^{-1} + 10\alpha_{3Z}^{-1})}, \tag{4}$$

$$M_\Sigma = \frac{M_Z^{11/3}}{M_{\text{str}}^2 M_{\tilde{G}}^{2/3}} e^{\pi(\frac{1}{2}\alpha_{1Z}^{-1} - \frac{1}{2}\alpha_{2Z}^{-1} - \frac{1}{3}\alpha_{3Z}^{-1})}, \tag{5}$$

with the unification scale M_{str} given by

$$\frac{\Lambda_s^2}{M_{\text{str}}^2} = \frac{3}{8\pi^2} W_0\left[\frac{8\pi^2}{3} \frac{\Lambda_s^2 M_Z^{2/3}}{M_{\tilde{G}}^{8/3}} e^{\pi(\frac{5}{2}\alpha_{1Z}^{-1} - \frac{21}{2}\alpha_{2Z}^{-1} + 7\alpha_{3Z}^{-1})}\right], \tag{6}$$

where $W_0(x)$ is the principal branch of the Lambert function [9,10].

Notice that from Eqs. (4) and (6) we can find a unique relation between the gaugino and sfermion masses in this minimal framework. Our main result reads then as

$$M_{\tilde{f}} = \frac{(8\pi^2)^3 \Lambda_s^6 e^{\pi(3\alpha_{1Z}^{-1} - 15\alpha_{2Z}^{-1} + 10\alpha_{3Z}^{-1})}}{27 M_Z M_{\tilde{G}}^4 W_0^3\left[\frac{8\pi^2}{3} \frac{\Lambda_s^2 M_Z^{2/3}}{M_{\tilde{G}}^{8/3}} e^{\pi(\frac{5}{2}\alpha_{1Z}^{-1} - \frac{21}{2}\alpha_{2Z}^{-1} + 7\alpha_{3Z}^{-1})}\right]}. \tag{7}$$

In the case when $M_{\tilde{G}} = M_{\tilde{f}} \equiv M_{\text{SUSY}}$, from Eq. (4) we find that the common superpartner mass is given by

$$M_{\text{SUSY}} = \frac{M_{\text{str}}^{6/5}}{M_Z^{1/5}} e^{\pi(\frac{3}{5}\alpha_{1Z}^{-1} - 3\alpha_{2Z}^{-1} + 2\alpha_{3Z}^{-1})}, \tag{8}$$

which corresponds precisely to a degenerate SUSY threshold at a low-energy (TeV) scale. In this scenario, the mass scales M_{SUSY}, M_Σ and M_{str} are uniquely determined. We find $M_{\text{SUSY}} = 2.3$ TeV, $M_\Sigma = 7.2 \times 10^{12}$ GeV and $M_{\text{str}} = 3.9 \times 10^{17}$ GeV, taking at $M_Z = 91.187$ GeV the input values $\alpha_s(M_Z)_{\overline{\text{MS}}} = 0.1176$, $\sin^2\theta_W(M_Z)_{\overline{\text{MS}}} = 0.2312$ and $\alpha^{-1}(M_Z)_{\overline{\text{MS}}} = 127.906$ [11]. In Fig. 1 we show the evolution of the gauge couplings with the energy scale μ. The role of the adjoint scalars $\Sigma_{3,8}$ in lifting the unification scale to the string scale becomes evident from the figure. For comparison, a similar plot is presented in Fig. 2 for the case of a split-SUSY scenario [12] with a common gaugino mass $M_{\tilde{G}} = 200$ GeV, which corresponds to the presently available experimental lower bound [11]. In the latter case, we obtain from Eqs. (4)–(6) the following mass scales: $M_{\tilde{f}} = 3 \times 10^7$ GeV, $M_\Sigma = 4.2 \times 10^{13}$ GeV and $M_{\text{str}} = 3.7 \times 10^{17}$ GeV. Similar results can be obtained for a higher $k > 1$ affine level. For $k = 2$ we find $M_{\text{SUSY}} = 3.6$ TeV, $M_\Sigma = 2.7 \times 10^{12}$ GeV and $M_{\text{str}} = 5.6 \times 10^{17}$ GeV for the low-energy supersymmetric case, while $M_{\tilde{f}} = 2.3 \times 10^8$ GeV, $M_\Sigma = 2.2 \times 10^{13}$ GeV and $M_{\text{str}} = 5.2 \times 10^{17}$ GeV for the split-SUSY scenario with $M_{\tilde{G}} = 200$ GeV.

In our analysis we have assumed a common mass for all superpartners with the same spin. This is an approximation to a realistic spectrum that is produced in several scenarios of

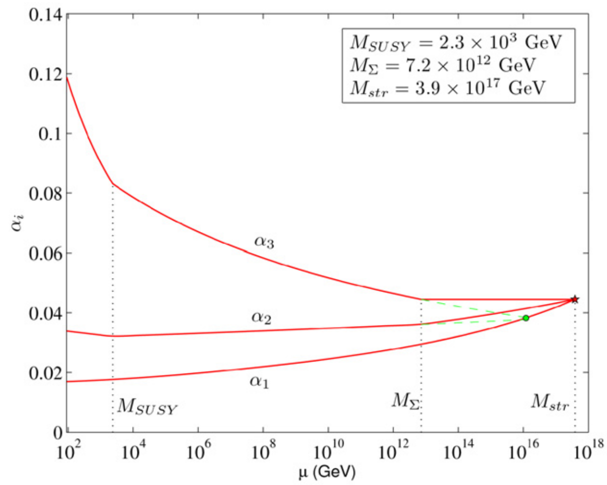

Fig. 1. (Color online.) The running of the gauge couplings in the minimal SUSY $SU(5)$ theory for the case of a degenerate SUSY threshold $M_{\tilde{G}} = M_{\tilde{f}} \equiv M_{\text{SUSY}}$ consistent with the unification with gravity. The dashed curves correspond to the standard running in the MSSM without imposing unification with gravity.

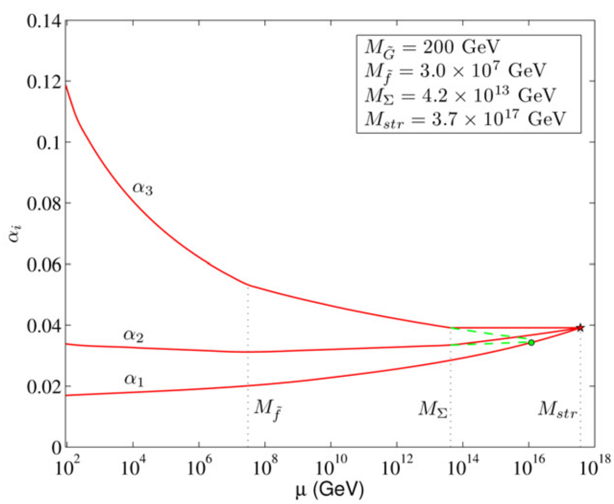

Fig. 2. (Color online.) Gauge coupling unification in the minimal SUSY $SU(5)$ theory for a common gaugino mass $M_{\tilde{G}} = 200$ GeV consistent with the unification with gravity. The dashed curves correspond to the MSSM case.

supersymmetry breaking as, for instance, in models based on minimal supergravity (mSUGRA) [13]. Our approximation represents averages of the mass spectra in these models. A more realistic analysis of the sparticle masses will not change the main conclusions of our work. We may ask ourselves how a mass splitting between the superpartners could modify the unification picture. In particular, one could expect different masses for the gluino ($\tilde{g}$), the weak-gauginos ($\tilde{W}$) and higgsinos ($\tilde{h}$). To illustrate the dependence of our results on the gaugino spectrum, and without committing ourselves to any specific SUSY breaking scenario, we present in Fig. 3 the gauge unification curves (solid lines) in the $(M_{\tilde{g}}, M_{\tilde{f}})$-plane for different mass ratios $M_{\tilde{g}}/M_{\tilde{W}}$. For simplicity we have assumed $M_{\tilde{h}} = M_{\tilde{W}}$. From Fig. 3 we conclude that the present experimental lower bound coming from sfermion searches, $M_{\tilde{f}} \gtrsim 100$ GeV [11],

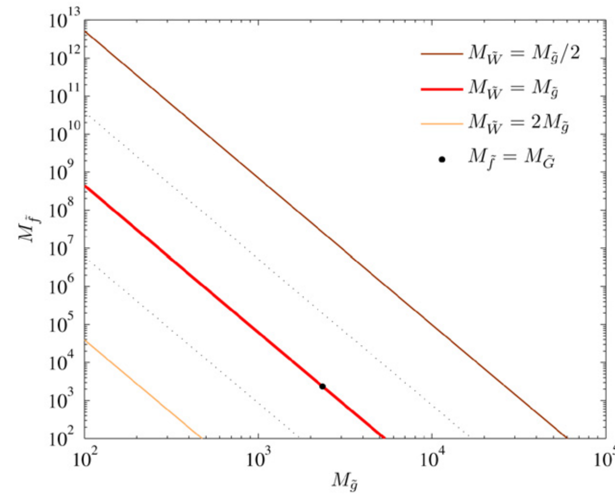

Fig. 3. (Color online.) Curves (solid lines) in the $(M_{\tilde{g}}, M_{\tilde{f}})$-plane consistent with the unification of gauge and gravitational couplings for different mass ratios $M_{\tilde{g}}/M_{\tilde{W}}$. The dotted lines reflect the $\alpha_s(M_Z)$ experimental uncertainty for the case of degenerate gaugino masses. The black dot corresponds to a fully degenerate SUSY partner spectrum at $M_{\text{SUSY}} = 2.3$ TeV.

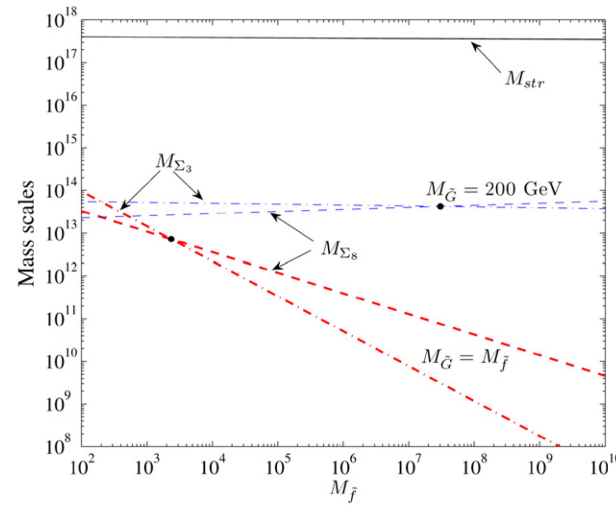

Fig. 4. (Color online.) The dependence of the adjoint scalar masses M_{Σ_8} (dashed lines) and M_{Σ_3} (dot-dashed lines) on the sfermion mass scale $M_{\tilde{f}}$ for two different scenarios: a low gaugino mass $M_{\tilde{G}} = 200$ GeV (blue thin lines) and a degenerate superpartner mass scale $M_{\tilde{G}} = M_{\tilde{f}}$ (red thick lines). The black dots correspond to the solutions presented in Figs. 1 and 2 for a degenerate mass $M_\Sigma = M_{\Sigma_3} = M_{\Sigma_8}$.

implies an upper bound on the gaugino masses. Using the central value of $\alpha_s(M_Z)$ we obtain for $M_{\tilde{G}} = M_{\tilde{g}} = M_{\tilde{W}}$ (solid red line) the upper limit

$$M_{\tilde{G}} \lesssim 5 \text{ TeV}. \tag{9}$$

Similarly, the experimental lower bound $M_{\tilde{g}} \gtrsim 200$ GeV yields an upper bound on the sfermion scale,

$$M_{\tilde{f}} \lesssim 3 \times 10^7 \text{ GeV}. \tag{10}$$

If we take into account the presently allowed α_s uncertainty, then there is a sizable shift of the curves (see dotted lines). Clearly, these bounds could also be subject to modifications if gaugino masses are non-degenerate, as can be seen from the figure. The result of Eq. (10) is consistent with the upper bound on the scalar masses of $\sqrt{m_{3/2}M_{\mathrm{Pl}}} \sim 10^{10}$ GeV [14] in SUGRA and string models coming from the cancellation of vacuum energy. We recall that $m_{3/2}$ is the gravitino mass. We also notice that the upper bound on the superpartner masses is in agreement with the cosmological constraints on the gluino lifetime [15].

In a similar way, one can consider the case when the adjoint scalars Σ_3 and Σ_8 have different masses [16]. In Fig. 4 we present the solutions for $M_{\tilde{G}}$ and $M_{\tilde{f}}$ consistent with unification. We notice that when the mass splitting is small the fermionic superpartner masses in agreement with unification are in the interesting region for LHC. However, if we restrict ourselves to the minimal supersymmetric $SU(5)$, where these adjoint fields have to be degenerate, the upper bounds given in Eqs. (9) and (10) hold.

Let us also comment on some other relevant effects. As explained before, when the colored triplets in $\hat{\mathbf{5}}_{\mathbf{H}}$ and $\hat{\bar{\mathbf{5}}}_{\mathbf{H}}$ are below the unification scale the masses of the superpartners have to be smaller. Therefore, the upper bounds on the superpartner masses are indeed those coming from the case when the colored triplets are at the unification scale. String threshold effects as well as two loop effects have been neglected in our analysis. These effects could be important and will be studied elsewhere. However, as we have pointed out, there are other relevant effects at one-loop level, such as the mass splitting between the fermionic superpartners, which already indicate that only in the simplest scenario conservative upper bounds on the superpartner masses can be found.

3. Summary

We have investigated the possibility to achieve unification of the gauge and gravitational couplings at the perturbative string scale in the context of the simplest supersymmetric grand unified theory. We have pointed out a unique one-loop relation between the superpartner masses consistent with the unification of all interactions. Conservative upper bounds on the superpartner masses were found, namely, $M_{\tilde{G}} \lesssim 5$ TeV and $M_{\tilde{f}} \lesssim 3 \times 10^7$ GeV, for the spin-1/2 and spin-0 superpartners, respectively. These bounds hint towards the possibility that this supersymmetric scenario could be tested at future colliders, and in particular, at the forthcoming LHC.

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