Available online at www.sciencedirect.com
ScienceDirect
Physics Letters B 643 (2006) 115–123
PHYSICS LETTERS B
www.elsevier.com/locate/physletb

Complex CKM from spontaneous CP violation without flavor changing neutral current

G.C. Branco [a,b,*], R.N. Mohapatra [a,c,d]

[a] Theoretische Physik, Technische Universität München, Garching, Germany
[b] Departmento de Fisica and CFTP, Instituto Superior Técnico, Lisboa, Portugal
[c] Department of Physics, University of Maryland, College Park, MD 20742, USA
[d] Sektion Physik, Universität München, München, Germany

Received 4 August 2006; received in revised form 19 October 2006; accepted 20 October 2006

Available online 7 November 2006

Editor: G.F. Giudice

Abstract

We analyse the general constraints on unified gauge models with spontaneous CP breaking that satisfy the conditions that (i) CP violation in the quark sector is described by a realistic complex CKM matrix, and (ii) there is no significant flavor changing neutral current effects in the quark sector. We show that the crucial requirement in order to conform to the above conditions is that spontaneous CP breaking occurs at a very high scale by complex vevs of standard model singlet Higgs fields. Two classes of models are found, one consisting of pure Higgs extensions and the other one involving fermionic extensions of the standard model. We give examples of each class and discuss their possible embeddings into higher unified theories. One of the models has the interesting property that spontaneous CP violation is triggered by spontaneous P violation, thereby linking the scale of CP violation to the seesaw scale for neutrino masses.

1. Introduction

It is now becoming increasingly clear that the dominant contribution to low energy CP violation arises from the complex CKM matrix which parameterizes the weak quark current coupling to the W-boson. Indeed the recent measurement [1] of the angle $\gamma = -\operatorname{Arg}(V_{ud} V_{cb} V_{cd}^* V_{ub}^*)$ provides evidence [2] for a complex CKM matrix even if one allows for New Physics (NP) contributions to B_d–$\bar{B}_d$ mixing and B_s–$\bar{B}_s$ mixings.

However, this cannot be the full story of CP violation in elementary particle interaction [3] since it is believed that the explanation of the only cosmic manifestation of CP nonconservation i.e. the asymmetry between matter and anti-matter must come from sources other than the CKM CP violation; similarly the solution to the QCD θ problem may also imply new forms of CP violating interactions. Moreover, there is the fundamental question of the origin and nature of CP violation and its relation to other constituents and forces.

Even before the full story of CP violation is clear, one can ask the question as to whether the observed CKM CP violation is spontaneous in origin [4] or intrinsic to the Yukawa couplings in the theory. This question has nontrivial cosmological implications since spontaneous CP violation will lead to domain walls and in order to avoid conflict with observations such as WMAP data, one must have the scale of this breaking to be above that of the inflation reheating, thus imposing constraints on both cosmological as well as particle physics aspects of models.

* Corresponding author.
E-mail address: gbranco@mail.ist.utl.pt (G.C. Branco).

0370-2693/$ – see front matter © 2006 Published by Elsevier B.V.
doi:10.1016/j.physletb.2006.10.039

In practical construction of models with spontaneous CP breaking, one must have one or more Higgs fields to have complex vevs [4]. It is obvious that implementing this requires extending the standard model, by having either more Higgs/or fermion fields plus Higgs because gauge invariance allows no room for Higgs vevs to be complex in the standard model. Furthermore, since spontaneous CP violation (SCPV) requires nontrivial constraints on the realistic gauge models, it is not surprising that the process of implementing it can lead to unpleasant side effects. One such unpleasant effect is the plethora of flavor changing neutral current (FCNC) effects induced in the process of obtaining spontaneous CP breaking.

Therefore, the challenge in constructing realistic models with spontaneous CP violation is twofold:

(i) One should achieve genuine spontaneous CP violation and assure that the vacuum phase does lead to a non-trivially complex CKM matrix. This is not an easy task since CP invariance of the Lagrangian requires the Yukawa couplings to be real.
(ii) One should find a natural suppression mechanism for FCNC in the Higgs sector. Again, this is a challenging task, since there is in general a close connection [5] between the appearance of FCNC and the possibility of generating a complex CKM matrix through CP violating vacuum phases.

The above link between SCPV and FCNC can be seen by considering a two Higgs extension ($\phi_{1,2}$) of the standard model to implement SCPV. It is well known (and we repeat the derivation in Section 2 and in Appendix A) that general two Higgs models have FCNC mediated by neutral Higgs fields. In order to suppress these FCNC effects one may consider two possibilities. One consists of the introduction of extra symmetries which eliminate FCNC and guarantee natural flavour conservation (NFC) [6] in the Higgs sector. It is well known that the introduction of such symmetries in the two Higgs doublet framework eliminates the possibility of having spontaneous CP violation [5]. With three Higgs doublets one can have NFC and yet achieve spontaneous CP violation but the resulting CKM matrix is real, in contradiction with recent data. Above we have considered the case where FCNC are avoided through the introduction of extra symmetries, not by fine-tuning. It has been shown that even if one considers elimination of FCNC through fine-tuning, for three generations one cannot generate a realistic complex CKM matrix [7]. The other possibility for suppressing FCNC effects is by choosing a large mass for the neutral Higgs which violate flavour. Indeed the strength of FCNC effects is proportional to $1/M_H^2$ where H denotes the new neutral Higgs field (we will denote the standard model Higgs by h). So clearly, suppression of FCNC effects require that M_H become very large. On the other hand, as we show below, the magnitude of the CP phase (denoted by δ in the text) in this model is given by $\delta \sim \frac{M_W}{M_H}$ so that as $M_H \to$ very large, $\delta \to 0$ and the theory becomes almost CP conserving. Note that to obtain CKM CP violation, we need $\delta \sim 1$. We will thus show that in the context of models with SCPV at the electroweak scale, it is not possible to obtain a complex CKM matrix while suppressing FCNC effects. In this class of SCPV models, obtaining a large CP phase and having significant FCNC seem to go together.

In this Letter, we discuss the conditions under which this connection can be avoided. We point out that the crucial point is to have CP broken at a high energy scale. We present two classes of models: one where the extension involves only the Higgs sector of the standard model and another one which involves the fermion sector as well. In the latter case, there is a small departure from unitarity of the CKM matrix.

Several of the models we discuss have already been considered in the literature. We present a systematic classification of these models, adding some new ones and sharpening the connection between SCPV and FCNC. In particular, we present criteria for constructing realistic SCPV models free of FCNC constraints.

This Letter is organized as follows: in Section 2, we discuss the connection between SCPV and FCNC in doublet Higgs extension of the SM. In Section 3, we discuss spontaneous CP breaking at high scale in a pure Higgs extension and show how one can avoid the FCNC effects in this case. In Section 4, we present a fermionic extension of the SM with spontaneous CP breaking at high scale. In Section 5, we discuss these two classes models into a left–right model and discuss two models one of which has the interesting property that spontaneous CP violation is triggered by spontaneous P violation. In Section 6, we briefly comment on how our ideas can be extended to supersymmetric models and finally in Section 7, we present our conclusions. In the Appendices A and B we present a detailed demonstration of the results of sections in Sections 2 and 3.

2. Two Higgs doublet model for SCPV and FCNC

The simplest extension of the standard model that can accommodate spontaneous CP violation is the two Higgs doublet model. If we denote the two Higgs doublets as $\phi_{1,2}$, and define $V_0(x,y) = -\mu_1^2 x - \mu_2^2 y + \lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 xy$, we can write the potential as follows:

$$V(\phi_{1,2}) = V_0\big(\phi_1^\dagger\phi_1, \phi_2^\dagger\phi_2\big) + V_{12}, \tag{1}$$

where

$$V_{12}(\phi_1,\phi_2) = \mu_{12}^2\phi_1^\dagger\phi_2 + \lambda_4\big(\phi_1^\dagger\phi_2\big)^2 + \lambda_5\phi_1^\dagger\phi_2\phi_1^\dagger\phi_1 + \lambda_6\phi_1^\dagger\phi_2\phi_2^\dagger\phi_2 + \text{h.c.} + \lambda_3'\phi_1^\dagger\phi_2\phi_2^\dagger\phi_1. \tag{2}$$

We can now write down the potential in terms of the electrically neutral components of the doublets. It looks exactly the same as the above potential as long as we understand the various fields as the neutral components of the fields.

In order to discuss spontaneous CP violation [8], we look for a minimum of the form:

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v_2 e^{i\delta} \end{pmatrix}. \tag{3}$$

The potential at this minimum looks like

$$V\left(v_1^2, v_2^2, \delta\right) = V_0\left(v_1^2, v_2^1\right) + \frac{1}{4}\lambda_3' v_1^2 v_2^2 + \mu_{12}^2 v_1 v_2 \cos\delta + \frac{1}{2}\lambda_4 v_1^2 v_2^2 \cos 2\delta + \frac{1}{2}\left(\lambda_5 v_1^2 + \lambda_6 v_2^2\right) v_1 v_2 \cos\delta. \tag{4}$$

The three extremum equations are:

$$\left[-\mu_1^2 + \lambda_1 v_1^2 + (\lambda_3 + \lambda_3')v_2^2 + \lambda_4 v_2^2 \sin 2\delta\right] v_1 + v_2\left[\mu_{12}^2 \cos\delta + \frac{1}{2}\left(3\lambda_5 v_1^2 + \lambda_6 v_2^2\right)\cos\delta\right] = 0, \tag{5}$$

$$\left[-\mu_1^2 + \lambda_2 v_2^2 + \frac{1}{2}(\lambda_3 + \lambda_3')v_1^2 + \lambda_4 v_1^2 \sin 2\delta\right] v_2 + v_1\left[\mu_{12}^2 \cos\delta + \frac{1}{2}\left(\lambda_5 v_1^2 + 3\lambda_6 v_2^2\right)\cos\delta\right] = 0, \tag{6}$$

$$-\sin\delta\left[\mu_{12}^2 v_1 v_2 + 2\lambda_4 v_1^2 v_2^2 \cos\delta + v_1 v_2\left(\lambda_5 v_1^2 + \lambda_6 v_2^2\right)\right] = 0. \tag{7}$$

Now let us study the implications of the extremum equations for SCPV and FCNC. Writing the Yukawa couplings as $\mathcal{L}_Y = \sum_{a,b;i} h_{ab}^{u,i}(\bar{Q}_{La}\phi_i u_{R,b} + u \to d) + \text{h.c.}$, it is straightforward to see that in general there will be FCNC mediated by neutral Higgs. We will consider next two possibilities for suppressing these FCNC. One involves the introduction of extra symmetries in order to implement Natural Flavour Conservation (NFC) [6] in the Higgs sector; the other considers the possibility of making very heavy the neutral Higgs which mediate FCNC. We will see that both possibilities do not work as far as generating a viable complex CKM matrix, but the discussion is useful in order to motivate the breaking of CP at a high energy scale which will be considered in Sections 3 and 4.

2.1. Eliminating FCNC through extra symmetries

It is well known that it is possible to avoid FCNC by introducing for example a Z_2 symmetry which restricts the Yukawa couplings so that only one Higgs doublet gives mass terms to the down quarks while the other doublet gives mass to the up quarks. However, it has been shown [5] that the same symmetry which leads to these selective Yukawa couplings prevents the occurrence spontaneous CP breaking. A possible way out of this difficulty involves the introduction of a third Higgs doublet. In this case it is possible to obtain a CP violating vacuum [5] but the CKM matrix is real, in conflict with the recent experimental findings. The reason why CKM matrix is real in this case has to do with the fact that due to the selective Yukawa couplings, the vacuum phase which appears in the quark mass matrices can be eliminated by rephasing right handed quark fields.

2.2. Suppressing FCNC effects through large Higgs masses

It is straightforward to see that we could diagonalize one set of Yukawa couplings $h^{u,d,1}$ so that the neutral Higgs (h) coming from the doublet ϕ_1 has flavor conserving couplings whereas that from ϕ_2 (H) has flavor violating couplings. In general of course the two neutral Higgs fields mix and therefore the $h^{u,2}$ coupling which in the symmetry limit involves only the H Higgs field will have an admixture of the light Higgs h but mixing is always proportional to the mass ratio m_h^2/M_H^2 assuming $M_H \gg m_h$.

Thus FCNC processes will arise via the tree level exchange of H boson and will be proportional to M_H^{-2} and a contribution from the mixing term which due to the mixing will also have the same kind of power dependence on M_H. Therefore in order to suppress FCNC interactions, we must demand that M_H be very large. This can be achieved by making $-\mu_2^2 > 0$ and $|\mu_2^2| \gg v_{wk}$. Let us now study Eq. (6): This equation tells us the scale of the vev v_2 which depends on the scale of the mixing term μ_{12}. (Note that getting the correct weak scale fixes μ_1^2 to be of order v_{wk} and stopping FCNC tells us that $|\mu_2^2| \gg v_{wk}^2$ but so far μ_{12}^2 remains a free parameter.) We have two cases: (i) $\mu_{12}^2 \sim v_{wk}^2$ and (ii) $\mu_{12}^2 \sim M_H^2 \sim |\mu_2^2| \gg v_{wk}^2$. In case (i), it is easy to see using the middle equation above that:

$$v_2 \sim \lambda_5 \frac{v_1^3}{|\mu_2^2|} \ll v_1 \tag{8}$$

i.e. the vev of ϕ_2 is highly suppressed in the limit of no FCNC. Note that the mass of the second neutral Higgs is not of order v_2 since in this case the vev is induced by a tadpole like diagram. Substituting this small value of v_2 in Eq. (8), we then see that for natural values of the parameters (λ_i), the only solution for the CP violating phase is $\delta = 0, \pi, \ldots$.

On the other hand in case (ii), $v_2 \sim v_{wk}$ but Eq. (7) above tells us that in this case also the expression in the bracket cannot give a nonzero δ since $\mu_{12}^2 v_1 v_2 \gg 2\lambda_4 v_1^2 v_2^2$ and the term within the bracket cannot vanish meaning that $\sin\delta = 0$ and hence no SCPV.

We therefore conclude that in this simple model, the requirement of suppression of the neutral current effects implies no SCPV. The main point is that to get a large enough SCPV phase, Eq. (7) tells us that v_2 must be comparable in magnitude to v_1. For this to happen, we must have $|\mu_2^2| \sim v_{wk}^2$ which again means that there must be large FCNC effects at low energies.

The above result can also be seen as follows: In a two Higgs doublet theory, one can change the basis of Higgs bosons to pass to a basis where the new doublets are $\Phi_1 = (v_2 e^{i\delta}\phi_1 - v_1\phi_2)/\sqrt{v_1^2 + v_2^2}$ and Φ_2 is the orthogonal combination to Φ_1, where we have anticipated the vevs of the fields in the original basis, as discussed above. Now we see that $\langle\Phi_1\rangle = 0$ while $\langle\Phi_2\rangle \neq 0$ and it leads to the same mass matrices for quarks as before. Now we can choose parameters of the Higgs potential such that the mass of Φ_1 is very large to avoid FCNC effects. In this case, the effective theory below the mass of Φ_1 i.e. M_{Φ_1} is same as the standard model up to zeroth order in M_W/M_{Φ_1}. Therefore, to this order, the vev of Φ_2 (which is the equivalent of the standard model Higgs) will be real, and there will be no spontaneous CP violation in the theory (to order M_W/M_{Φ_1}). This again proves that in the limit of zero FCNC, there will be no SCPV. In Appendix A, we give explicit calculations in the mass basis that substantiates this conclusion.

This result can be generalized to the case of arbitrary number of Higgs doublets. For example for the case of three doublets, the argument is that as long as all the doublets couple to quark fields, at least two of the neutral Higgs bosons i.e. $H_{1,2}$ must be heavy in order to avoid large FCNC effects and this implies that $|\mu_{2,3}^2| \gg v_{wk}^2$; in that case their vev's must be suppressed and of order $\frac{v_{wk}^3}{|\mu_{2,3}^2|}$ and therefore small. The potential will then be forced to choose the minimum such that all SCPV phases are zero.

3. High scale spontaneous CP violation leading to complex CKM while avoiding FCNC: Model with extra Higgs only

In this section, we show how the FCNC problem is avoided if spontaneous violation of CP symmetry arises at a high scale. First we discuss this using a model with two $SU(2)_L \times U(1)_Y$ Higgs doublets $\phi_{1,2}$ as before and a complex singlet σ. The potential for this case can be written as follows:

$$V_{\phi_1,\phi_2,\sigma} = V(\phi_{1,2}) + V(\sigma) + V(\phi,\sigma), \tag{9}$$

where $V(\phi_{1,2})$ is defined in Eqs. (1), (3) and the other two terms are given by

$$V(\sigma) = -M_0^2\sigma^*\sigma + M_1^2\sigma^2 + \lambda_\sigma(\sigma^*\sigma)^2 + \lambda'_\sigma\sigma^4 + \lambda''_\sigma\sigma^3\sigma^* + \text{h.c.} \tag{10}$$

and

$$V(\phi,\sigma) = M_{2,ab}\phi_a^\dagger\phi_b\sigma + \kappa_{1,ab}\phi_a^\dagger\phi_b\sigma^2 + \kappa_{2,ab}\phi_a^\dagger\phi_b\sigma^*\sigma + \text{h.c.} \tag{11}$$

It is clear that the minimum of the potential $V(\sigma)$ corresponds to $\langle\sigma\rangle = \Lambda e^{i\alpha}$, where $\Lambda \sim M_{0,1,2} \gg v_{wk}$ and α can be large. Substituting this vev in the potential, we can write the effective tree level potential for the $\phi_{1,2}$ fields at low energies to be:

$$V_{\text{eff}}(\phi_1,\phi_2) = V(\phi_{1,2}) + V_{\text{new}}, \tag{12}$$

where $V_{\text{new}} = (M_{2,ab}\Lambda e^{i\alpha} + \kappa_{1,ab}\Lambda^2 e^{2i\alpha} + \kappa_{2,ab}\Lambda^2)\phi_a^\dagger\phi_b + \text{h.c.} \equiv \Lambda^2(\lambda_{11}\phi_1^\dagger\phi_1 + \lambda_{22}\phi_2^\dagger\phi_2 + \lambda_{12}e^{i\beta}\phi_1^\dagger\phi_2) + \text{h.c.}$ If we keep only the neutral components of the Higgs doublets, then the form of the potential is

$$V_{\text{eff}} = \Lambda^2\big(\lambda_{11}\phi_1^\dagger\phi_1 + \lambda_{22}\phi_2^\dagger\phi_2 + \lambda_{12}e^{i\beta}\phi_1^\dagger\phi_2 + \text{h.c.}\big) + \sum\lambda_{abcd}\phi_a^\dagger\phi_b\phi_c\phi_d + \text{h.c.}, \tag{13}$$

where $\Lambda \gg v_{wk}$. It is clear that although CP is spontaneously broken at a high scale Λ, at low energies, one has CP explicitly softly broken [9] by the bilinear terms in λ_{12}. Note that both the fields $\phi_{1,2}$ have Yukawa couplings and we can make a redefinition of the phase of one of the doublet fields (say ϕ_2) i.e. $\phi_2 \to e^{-i\beta}\phi_2$ so that all the bilinear and $O(\Lambda^2)$ terms in the potential become phase independent but the Yukawa couplings become complex. Thus the effective theory at low energies looks naively like hard CP violation, even though it is spontaneous CP violation at a very high scale. The Yukawa coupling Lagrangian looks like

$$\mathcal{L}_Y = \bar{Q}_{La}\big(h_{ab}^{u,1}\phi_1 + h_{ab}^{u,2}e^{-i\beta}\phi_2\big)u_{R,b} + \bar{Q}_{La}\big(h_{ab}^{d,1}\tilde{\phi}_1 + h_{ab}^{d,2}e^{i\beta}\tilde{\phi}_2\big)d_{R,b} + \text{h.c.} \tag{14}$$

This still does not imply a viable complex CKM matrix; to achieve that, we must show that the vev of ϕ_2 where the phase resides, does not become very tiny when we demand the suppression of FCNC. In order to show this, let us write down the extremization of the potential as in Section 2. For simplicity, we keep only the λ_{1111}, λ_{2222} and λ_{1122} terms in the potential but our results follow in general:

$$\big(-\mu_1^2 + \Lambda^2\lambda_{11} + \lambda_{1111}v_1^2 + \lambda_{1122}v_2^2\big)v_1 + v_2\big(\Lambda^2\lambda_{12}\big) = 0, \tag{15}$$

$$\big(-\mu_2^2 + \Lambda^2\lambda_{22} + \lambda_{2222}v_2^2 + \lambda_{1122}v_1^2\big)v_2 + v_1\big(\Lambda^2\lambda_{12}\big) = 0. \tag{16}$$

From these two equations, we find that both v_1 and v_2 are in general of the same order regardless of what the neutral Higgs masses are. This gives the CKM CP violation. As far as the masses of the neutral Higgs fields go, we can fine tune one set of parameters to keep one Higgs field light i.e. $\ll \Lambda$ and another will remain heavy thus suppressing the FCNC effects.

Of course we do not need to make the rephasing $\phi_2 \to e^{-i\beta}\phi_2$ and eliminate the phase from the bilinear terms. If we do not do rephasing, the extremum equation of the Higgs potential would look like:

$$-\Lambda^2\lambda_{12}v_1v_2\sin(\beta+\delta) - \sin\delta\bigl[2\lambda_4 v_1^2v_2^2\cos\delta + v_1v_2\bigl(\lambda_5^2v_1^2+\lambda_6^2v_2^2\bigr)\bigr] = 0. \tag{17}$$

Since $\Lambda^2 \gg v^2$, it is clear that to an excellent approximation one has:

$$\beta = -\delta. \tag{18}$$

The phase δ would then appear in the quark mass matrices which will be nontrivially complex, thus leading to a complex CKM matrix. In Appendix B, we discuss how the fine tuning needed to keep the standard model Higgs at the electroweak scale does not prevent the components of the extra Higgs become superheavy in order to suppress the FCNC effects.

4. SCPV without FCNC problem in fermionic extensions of standard model

In this section, we turn to SCPV models which extend the fermionic sector of the standard model to solve the flavor changing neutral current problem while giving complex CKM at the weak scale [10,11]. Typical features of these models are new SM singlet quarks and SM singlet Higgs fields, the latter used to generate SCPV. We briefly review the model in [10] which is a typical model of this type to illustrate the main points of our discussion i.e. spontaneous violation of CP at high scale without FCNC problem but with complex CKM.

In the model of Ref. [10], the new SM singlet vector like fermion are of down type: $(D_{L,R})$ with $U(1)_Y$ quantum number $-2/3$ and a complex singlet Higgs field σ as in Section 3. The potential for the σ field is the same as in Eq. (11). As a result, the σ field has a complex vev leading to high scale spontaneous CP violation (since $\langle\sigma\rangle = \Lambda \gg v_{wk}$).

The CP violation is transmitted to the weak scale via its couplings given below:

$$\mathcal{L}_\sigma = \sum_a \bar{D}_L d_{a,R}(g_a\sigma + g'_a\sigma^*) + (f\sigma + f'\sigma^*)\bar{D}_L D_R + \text{h.c.}, \tag{19}$$

where g_a, g'_a, f, f' are real due CP conservation. But after symmetry breaking, the mass matrix contains terms mixing the heavy D quarks with the light d quarks [10]. This can be seen by writing down the full down quark mass matrix (in the notation $\bar{\psi}_L M_{dD}\psi_R$):

$$M_{dD} = \begin{pmatrix} m_d & 0 \\ \Lambda(ge^{i\delta}+g'e^{-i\delta}) & \Lambda(fe^{i\delta}+f'e^{-i\delta}) \end{pmatrix}, \tag{20}$$

where g and g' denote the row vectors (g_1, g_2, g_3) and (g'_1, g'_2, g'_3). Diagonalizing $M_{dD}M_{dD}^\dagger$, we can get the generalized 4×4 CKM matrix which indeed has a complex phase in the 3×3 sector involving the standard model quarks even in the limit of heavy D quark masses. This is an example of a breakdown of the decoupling theorem [10]. Clearly since there is only one neutral Higgs boson coupling to the effective down quark mass matrix, there is no FCNC effects at the tree level as in the case of the standard model. Clearly, if the masses of the vectorlike quarks were at the weak scale, the mixing between the light d quarks and D would be significant and lead to large FCNC effects at low energies.

This provides a second way to introduce spontaneous CP violation without simultaneously having flavor changing neutral current effects. Note that the common thread between the examples in Sections 3 and 4 is the fact that CP is violated spontaneously at high scale, which highlights the main point of this Letter. In the remainder of this Letter, we show how these ideas can be embedded into extended models on the way towards a possible grand unified scheme where spontaneous CP violation occurs at the GUT scale.

5. Embedding high scale SCPV into left–right symmetric models

The left–right symmetric models are based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c$ with fermions assigned in a left–right symmetric manner [14] and Higgs belonging to bidoublet field $\Phi(2,2,0)$ and a pair of fields of either $(\chi_L(2,1,-1) \oplus \chi_R(1,2,-1))$ type (called χ-type below) or $(\Delta_L(3,1,+2) \oplus \Delta_R(1,3,+2))$ type (called Δ-type below). The left–right symmetric models are ideally suited to embed the first class of high scale SCPV models since the bidoublet Higgs field already contains the necessary two standard model doublet Higgs fields in it. All we have to do is to embed the high scale singlet field into a left–right Higgs field. We present two different ways to do this embedding in the two subsections below.

5.1. Left–right SCPV: Model I

The first way to implement high scale SCPV is by choosing two pairs of χ type or Δ-type fields. Two pairs are needed since with a single pair, constraint that W_R scale must be much higher than W_L scale suppresses the SCPV phase by a factor M_{W_L}/M_{W_R} [15]. The two Δ type model has been discussed in [12] where at the high scale, the Δ_R's have vevs as follows: $\langle\Delta^0_{1,R}\rangle = v_{1,R}$ and

$\langle\Delta_{2,R}\rangle = v_{2,R}e^{i\delta}$. The coupling of the form $\mathrm{Tr}(\Phi^\dagger\tau_2\Phi^*\tau_2)\,\mathrm{Tr}(\Delta^\dagger_{1,R}\Delta_{2,R})$ then induces the term $\lambda_{12}e^{i\delta}\Lambda^2\phi_1^*\phi_2$ at low energies and the rest of the discussion is as in Section 3 above.

Let us now turn our attention to embedding of the model of Ref. [10] into the left–right model. We consider the left–right model without the bidoublet but with the $(\chi_L(2,1,-1)\oplus\chi_R(1,2,-1))$ pair and three pairs of $SU(2)_L\times SU(2)_R$ singlet vector-like quarks ($P_{L,R}(1,1,4/3)$ and $N_{L,R}(1,1,-2/3)$). Such models were extensively studied in the early 90's but not from the point of view of spontaneous CP violation [13]. We take a complex singlet Higgs field σ as before and assume the theory to be CP conserving prior to symmetry breaking so that all couplings in the theory are real. Again, we assume the potential for the σ field to be as in Eq. (11) so that its minimum corresponds to a complex vev for $\langle\sigma\rangle = \Lambda e^{i\delta}$ as before. The vevs for the fields $\chi_{L,R}$ are real.

To study the implications of the theory for low energy quark mixings, let us write down the quark Yukawa couplings:

$$\begin{aligned}\mathcal{L}_Y = {}& h^u_{ab}[\bar Q_{L,a}\chi_L P_{R,b} + \bar Q_{R,a}\chi_R P_{L,b}] + h^d_{ab}[\bar Q_{L,a}\tilde\chi_L N_{R,b} + \bar Q_{R,a}\tilde\chi_R N_{L,b}] + \text{h.c.}\\ &+ \big[f^u_{ab}\sigma + f^{u,\prime}\sigma^*\big]\bar P_{L,a}P_{R,b} + \big[f^d_{ab}\sigma + f^{d,\prime}\sigma^*\big]\bar N_{L,a}N_{R,b} + \text{h.c.}\end{aligned} \tag{21}$$

After spontaneous symmetry breaking we have $\langle\sigma\rangle = \Lambda e^{i\delta}$, $\langle\chi^0_{L,R}\rangle = v_{L,R}$ with $v_R \sim \Lambda \gg v_L$. This leads to the mass matrix of the form:

$$\mathcal{M}_{uP} = \begin{pmatrix} 0 & h^u_{ab}v_L \\ h^u_{ba}v_R & M_P \end{pmatrix}, \tag{22}$$

$$\mathcal{M}_{dN} = \begin{pmatrix} 0 & h^d_{ab}v_L \\ h^d_{ba}v_R & M_N \end{pmatrix}. \tag{23}$$

Left–right symmetry requires that $M_{P,N} = M^\dagger_{P,N}$ whereas the matrices $h^{u,d}$ are real. After diagonalization, the effective up and down quark mass matrices become:

$$M_{u,d} \simeq v_L v_R h^{u,d,T} M^{-1}_{P,N} h_{u,d}. \tag{24}$$

These matrices are Hermitean and therefore lead to equal left and right handed CKM matrices as in the usual left–right models with bi-doublets and lead to complex CKM matrices. In fact one can write the rotation matrices for both the up and down sector as follows in a basis where the couplings $h^{u,d}$ are diagonal:

$$V^{u,d} = M^{-1/2}_{u,d} h^{u,d} U_{P,N} M^{\mathrm{diag}\,-1/2}_{P,N}\sqrt{v_L v_R}. \tag{25}$$

Clearly since $U_{P,N}$ is a unitary matrix with complex phases, $V^{u,d}$ will lead to complex CKM matrix i.e. $U_{\mathrm{CKM}} = V^u V^{d,\dagger}$.

As far as the FCNC effects are concerned, they arise only in order $m_{u,d}/M_{P,N}$ and therefore suppressed when $\Lambda \to$ large values. Note however that the quark mixing effects arise in zeroth order of this parameter.

5.2. *Left–right SCPV model II: Connecting the CP violation and seesaw scales*

In this subsection, we present a more economical left–right embedding of the high scale spontaneous CP violation with suppressed FCNC. The model consists of the usual left–right assignment of the fermions [14] and Higgs system consists of a single bidoublet $\phi(2,2,0)$ and the $\chi_L(2,1,-1)\oplus\chi_R(1,2,-1)$. Here spontaneous CP violation is implemented via the vev of a CP and P odd real singlet scalar field η [16]. The CP invariant Higgs potential for the theory can be written as:

$$V(\chi_{L,R},\eta,\phi) = V_0(\phi) + i\mu\eta\,\mathrm{Tr}\big(\phi_1^\dagger\phi_2\big) + M'\chi_L^\dagger\phi\chi_R + V_2(\eta,\chi_{L,R}), \tag{26}$$

where

$$V_0(\phi) = -\mu^2_{ab}\,\mathrm{Tr}\big(\phi_a^\dagger\phi_b\big) + \sum\kappa_{abcd}\,\mathrm{Tr}\big(\phi_a^\dagger\phi_b\phi_c^\dagger\phi_d\big) + \kappa'_{abcd}\,\mathrm{Tr}\big(\phi_a^\dagger\phi_b\big)\,\mathrm{Tr}\big(\phi_c^\dagger\phi_d\big) + \text{h.c.} \tag{27}$$

with (a,b) going over $(1,2)$ with $\phi_1 = \phi$ and $\phi_2 = \tau_2\phi^*\tau_2$.

$$\begin{aligned}V_2(\eta,\chi_{L,R}) = {}& M^2_\eta\eta^2 + \lambda_\eta\eta^4 - M^2_\chi\big(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R\big) + \lambda_\chi\big(\chi_L^\dagger\chi_L + \chi_R^\dagger\chi_R\big)^2\\ &+ \lambda'_\chi\big(\chi_L^\dagger\chi_L - \chi_R^\dagger\chi_R\big)^2 + M'_\eta\eta\big(\chi_L^\dagger\chi_L - \chi_R^\dagger\chi_R\big).\end{aligned} \tag{28}$$

We have assumed that under CP transformation $\eta \to -\eta$ and $\chi_L \to \chi_R^\dagger$ and $\phi \to \phi^\dagger$. Invariance under this transformation requires that all parameters in the potential be real (except for one imaginary coupling shown explicitly in the above equation).

Note now that if the term in the potential connecting η and χ fields was absent, we would have $\langle\eta\rangle = 0$ since $M^2_\eta > 0$. However as soon as $SU(2)_R$ symmetry is broken by $\langle\chi^0_R\rangle \neq 0$, the M'_η term in the potential introduces a tadpole term for η thereby generating

$$\langle\eta\rangle \simeq \frac{+M'_\eta v_R^2}{2M^2_\eta}. \tag{29}$$

Since η is CP odd, this breaks CP spontaneously. The way it manifests is that the $i\mu\langle\eta\rangle\,\mathrm{Tr}(\phi_1^\dagger\phi_2)$ term now combines with the $\mu_{12}^2\,\mathrm{Tr}\,\phi_1^\dagger\phi_2$ to generate at low energies an effective soft CP breaking term as in Eq. (13) where $\phi_{1,2}$ are the two doublets contained in the bidoublet ϕ of the left–right model. The same arguments as in the Appendix B then guarantee that in this model the FCNC can be suppressed by making one of the left–right Higgs doublets superheavy.

This can also be seen in an alternative manner by minimizing the potential, noting that there is a range of values of the parameters in the potential for which we have $\langle\chi_R^0\rangle = v_R \neq 0$; $\langle\eta\rangle \neq 0$; $\langle\chi_L\rangle = 0$ provided $M'_\eta\langle\eta\rangle > 2\lambda'_\chi u^2$. The vevs of χ_R and η fields are much larger than the weak scale.

An interesting point worth stressing is that in this model, the scale of CP violation and the seesaw [17] scale for neutrino masses are connected. To see this, note that the right handed neutrino masses come from the higher dimensional term $(L_R\bar{\chi}_R)^2/M_{\mathrm{Pl}}$ leading to seesaw right handed neutrino masses given by $M_{\mathrm{seesaw}} \simeq \frac{v_R^2}{M_{\mathrm{Pl}}}$ and from Eq. (29), the CP violating scale $\langle\eta\rangle$ and M_{seesaw} owe their origin to the same scale v_R i.e. violation of parity. Since in grand unified theories, v_R can be identified with the GUT scale, one would therefore relate several scales of the theory i.e. M_{SCPV}, M_{seesaw} and M_{GUT}.

6. Possible extensions to supersymmetry and SUSY CP problem

As is well known, generic minimal supersymmetric extensions of the standard model (MSSM) are plagued with the SUSY CP problem. There have been many solutions suggested to solve this problem [18]. A simple solution to this problem would of course be to have CP spontaneously broken. However, in MSSM, CP cannot be spontaneously broken. Furthermore, it has also been pointed out that [19] it is particularly hard to have spontaneous CP breaking by considering multi-Higgs generalizations of the MSSM. A possibility for achieving spontaneous CP breaking within SUSY involves the introduction of singlet chiral fields [20]. As far as the FCNC effects are concerned, in these models one may fine tune the μ terms to make of the extra Higgs doublets heavy thereby eliminating large FCNC effects. However, the early versions of these models are no longer viable since they had a real CKM matrix, in contradiction with recent experimental data.

Therefore, the ideas described in this Letter may be particularly useful if one wants to solve the SUSY CP problem by spontaneous CP violation in a viable scenario, where vacuum phases do lead to a complex CKM matrix, while at the same time suppressing FCNC effects. In fact, recently it has been suggested one such model which includes two singlet Higgs superfields and adds an extra vector like singlet fermion to MSSM [21] to break CP spontaneously and generate a complex CKM matrix. One can embed this scheme into the SUSY left–right model. Detailed analysis of SUSY models that exploit the ideas of this Letter is under way and will be taken up in a forthcoming publication.

7. Conclusion

We have emphasized the close connection between spontaneous CP violation and FCNC effects in theories where CP breaking vev is at the weak scale. We have also shown that in order to avoid FCNC effects while at the same time generating a complex CKM matrix through vacuum phases, one is naturally led to have spontaneous CP breaking at a high energy scale, well above the electroweak scale.We then describe two classes of models one without and one with extra heavy fermions where having a high vev break CP spontaneously leads to complex CKM matrix as given by experiment without simultaneously having large FCNC effects. We then show how these models can be embedded into the high scale left–right models where parity violation and neutrino mass are connected via the seesaw mechanism. We find one particular model where spontaneous parity violation triggers the spontaneous CP violation thus connecting the three scales: Seesaw scale for neutrino masses, scale of spontaneous parity and CP violation.

In conclusion, if our view on the origin of CP violation is correct, then small neutrino masses and CP violation at low energies would have in common the fact that they are both manifestations of physics occuring at very high energy scale.

Acknowledgements

The work of R.N.M. is supported by the National Science Foundation grant No. Phy-0354401 and the work of G.C.B. is supported by Fundacao para a Ciencia e a Tecnologia (FCT, Portugal), through the projects POCTI/FNU/44409/2002, PDCT/FP/FNU/50250/2003, POCI/FP/63415/2005, POCTI/FP/FNU/50167/2003, which are partially funded through POCTI (FEDER). Both the authors are very grateful for the Alexander von Humboldt Senior Research Award which made this collaboration possible. They are also grateful to A. Buras and M. Lindner at TUM and R.N.M. to H. Fritzsch at LMU for kind hospitality when the work was done.

Appendix A

In this appendix, we elaborate on the connection between SCPV and FCNC and complex CKM in a two Higgs doublet model. For this purpose, we write the Yukawa Lagrangian as:

$$\mathcal{L}_Y = \sum_{a,b}\left(h^{u,i}_{ab}\bar{Q}^0_{La}\phi_i u^0_{R,b} + u \to d\right) + \text{h.c.} \tag{30}$$

It can be readily seen [5,22] that in the quark mass eigenstate basis, the scalar coupling can be written as:

$$\begin{aligned}\mathcal{L}_{\text{scalar}} = {}& [\bar{u}D_u u + \bar{d}D_d d]\frac{H}{v} - \left[\bar{u}\left(N_u P_R + N_u^\dagger P_L\right)u + \bar{d}\left(N_d P_R + N_d^\dagger P_L\right)d\right]\frac{R}{v}\\ &+ i\left[\bar{u}\left(N_u P_R - N_u^\dagger P_L\right)u - \bar{d}\left(N_d P_R - N_d^\dagger P_L\right)d\right]\frac{I}{v},\end{aligned} \tag{31}$$

where

$$H = \frac{1}{v}[v_1 R_1 + v_2 R_2], \qquad R = \frac{1}{v}[v_2 R_1 - v_1 R_2], \qquad I = \frac{1}{v}[v_2 I_1 - v_1 I_2] \tag{32}$$

with $\phi^0_1 = \frac{1}{\sqrt{2}}[v_1 + R_1 + iI_1]$, $\phi^0_2 = \frac{1}{\sqrt{2}}e^{i\delta}[v_2 + R_2 + iI_2]$, where

$$N_d = U^\dagger_{dL}\left[\frac{v_2}{\sqrt{2}}Y^d_1 - \frac{v_1}{\sqrt{2}}e^{i\delta}Y^d_2\right]U_{dR}, \tag{33}$$

where $U_{d_{L,R}}$ are the unitary matrices which diagonalize the down quark mass matrix M_d. Analogous expressions are there for N_u. It is clear that $N_{d,u}$ are in general not diagonal and therefore R and I mediate FCNC.

The quark mass matrices are in the form

$$M_d M_d^\dagger = H_{\text{real}} + 2iv_1 v_2 \sin\delta\left(Y^{d2}Y^{d1^T} - Y^{d1}Y^{d2^T}\right), \tag{34}$$

where H_{real} is a symmetric real matrix. It is clear that $M_d M_d^\dagger$ (and similarly $M_u M_u^\dagger$) is an arbitray complex matrix and therefore CKM is a complex matrix.

If one fine tunes such that $Y^d_1 \propto Y^2_d$, N_d would be diagonal and FCNC would be eliminated. But in that case, Eq. (34) implies that $M_d M_d^\dagger$ becomes real. This illustrates the connection between FCNC and the possibility of generating a complex CKM by a vacuum phase.

Appendix B

In this appendix, we discuss how the extra neutral Higgs fields in the model of Section 3 that are potential mediators of FCNC effects can be made heavy while at the same time the SM Higgs can be kept light by one fine tuning. We will work with the potential in Eqs. (13), (14).Clearly, the minimum of this potential corresponds to:

$$\langle\phi_1\rangle = \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \qquad \langle\phi_2\rangle = \begin{pmatrix} 0 \\ v_2 e^{i\delta} \end{pmatrix}. \tag{35}$$

Let us work in a basis in which

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \frac{1}{v}\begin{pmatrix} v_1 & v_2 \\ v_2 & -v_1 \end{pmatrix}\begin{pmatrix} \phi_1 \\ e^{-i\beta}\phi_2 \end{pmatrix}. \tag{36}$$

The potential in Eq. (13) looks as follows:

$$\begin{aligned}V(H_{1,2}) = {}& \Lambda^2\left(\lambda_{11}H_1^\dagger H_1 + \lambda_{22}H_2^\dagger H_2 + \left(\lambda_{12}H_1^\dagger H_2 + \text{h.c.}\right)\right) + \lambda_1\left(H_1^\dagger H_1\right)^2 + \lambda_2\left(H_2^\dagger H_2\right)^2\\ &+ \lambda_3\left(H_1^\dagger H_1\right)\left(H_2^\dagger H_2\right) + \lambda_4\left(H_2^\dagger H_1\right)\left(H_1^\dagger H_2\right) + \left[\lambda_5 H_1^\dagger H_2 + \lambda_6 H_1^\dagger H_1 + \lambda_7 H_2^\dagger H_2\right]H_1^\dagger H_2 + \text{h.c.}\end{aligned} \tag{37}$$

Even though we use the same λ's in both Eq. (13) and here, they are different and in fact now λ_{12}, $\lambda_{5,6,7}$ are in general complex while the other λ's are real.

Now we can write the $H_{1,2}$ in terms of their components:

$$H_1 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + H + iG) \end{pmatrix}, \qquad H_2 = \begin{pmatrix} C^+ \\ \frac{S+iP}{\sqrt{2}} \end{pmatrix}. \tag{38}$$

As already discussed in [3], the stability of vacuum demands that, the coefficients of the linear terms in (H, S, P) vanish and gives

$$\Lambda^2\lambda_{11} + 2\lambda_1 v^2 = 0, \qquad \Lambda^2\lambda_{12} + \lambda_6 v^2 = 0. \tag{39}$$

These are the fine tuning conditions in the $(H_{1,2})$ basis to have SM Higgs field light and have the correct electroweak symmetry breaking. We can now write down the mass matrix for the other neutral Higgs fields (H, S, P) as follows [3]:

$$\mathcal{M}_{H,S,P} = \begin{pmatrix} 4v^2\lambda_1 & 2v^2 \operatorname{Re}\lambda_6 & -2v^2 \operatorname{Im}\lambda_6 \\ 2v^2 \operatorname{Re}\lambda_6 & \lambda_2\Lambda^2 + (\lambda_3 + \lambda_4 + 2\operatorname{Re}\lambda_5)v^2 & -2v^2 \operatorname{Im}\lambda_5 \\ -2v^2 \operatorname{Im}\lambda_6 & -2v^2 \operatorname{Im}\lambda_5 & \lambda_2\Lambda^2 + \lambda_3 v^2 + (\lambda_4 - 2\operatorname{Re}\lambda_5)v^2 \end{pmatrix}. \tag{40}$$

From this expression, we can explicitly see that the beyond the standard model neutral Higgs particles (S, P) have masses of order Λ whereas the SM Higgs field has mass of order of the electroweak scale. Also the mixings of the SM Higgs which can generate FCNC effects are of order v^2/Λ^2 and hence very small as (S, P) are made heavy. Also $\lambda_2\Lambda^2 + \lambda_3 v^2$ gives the mass of the charged Higgs field from the second Higgs field H_2. Thus we have complex CKM from SCPV while at the same time suppressing the FCNC effects.

References

[1] A. Poluektov, et al., Belle Collaboration, Phys. Rev. D 73 (2006) 112009, hep-ex/0604054;
B. Aubert, et al., BaBar Collaboration, hep-ex/0507101.
[2] F.J. Botella, G.C. Branco, M. Nebot, M.N. Rebelo, Nucl. Phys. B 725 (2005) 155, hep-ph/0502133.
[3] G.C. Branco, L. Lavoura, J.P. Silva, CP Violation, Oxford Univ. Press, London, 1998.
[4] T.D. Lee, Phys. Rev. D 8 (1973) 1226.
[5] G.C. Branco, Phys. Rev. Lett. 44 (1980) 504;
G.C. Branco, Phys. Rev. D 22 (1980) 2901;
G.C. Branco, A.J. Buras, J.M. Gerard, Nucl. Phys. B 259 (1985) 306.
[6] S.L. Glashow, S. Weinberg, Phys. Rev. D 15 (1977) 1958.
[7] G. Ecker, W. Grimus, H. Neufeld, Phys. Lett. B 228 (1989) 401;
G. Ecker, W. Grimus, H. Neufeld, Phys. Lett. B 194 (1987) 251;
M. Gronau, A. Kfir, G. Ecker, W. Grimus, H. Neufeld, Phys. Rev. D 37 (1988) 250.
[8] For a recent discussion of CP violation in two Higgs models, see J.F. Gunion, H.E. Haber, Phys. Rev. D 72 (2005) 095002, hep-ph/0506227;
G.C. Branco, M.N. Rebelo, J.I. Silva-Marcos, Phys. Lett. B 614 (2005) 187, hep-ph/0502118;
S. Davidson, H.E. Haber, Phys. Rev. D 72 (2005) 035004, hep-ph/0504050;
S. Davidson, H.E. Haber, Phys. Rev. D 72 (2005) 099902, Erratum.
[9] G.C. Branco, M.N. Rebelo, Phys. Lett. B 160 (1985) 117.
[10] L. Bento, G.C. Branco, P.A. Parada, Phys. Lett. B 267 (1991) 95;
L. Bento, G.C. Branco, Phys. Lett. B 245 (1990) 599.
[11] S.M. Barr, Phys. Rev. Lett. 53 (1984) 329;
A.E. Nelson, Phys. Lett. B 143 (1984) 165.
[12] J. Basecq, J. Liu, J. Milutinovic, L. Wolfenstein, Nucl. Phys. B 272 (1986) 145.
[13] K.S. Babu, R.N. Mohapatra, Phys. Rev. Lett. 62 (1989) 1079;
K.S. Babu, R.N. Mohapatra, Phys. Rev. D 41 (1990) 1286.
[14] J.C. Pati, A. Salam, Phys. Rev. D 10 (1974) 275;
R.N. Mohapatra, J.C. Pati, Phys. Rev. D 11 (1975) 566;
G. Senjanović, R.N. Mohapatra, Phys. Rev. D 12 (1975) 1502.
[15] A. Masiero, R.N. Mohapatra, R.D. Peccei, Nucl. Phys. B 192 (1981) 66.
[16] D. Chang, R.N. Mohapatra, M.K. Parida, Phys. Rev. Lett. 52 (1984) 1072.
[17] P. Minkowski, Phys. Lett. B 67 (1977) 421;
M. Gell-Mann, P. Ramond, R. Slansky, in: P. van Nieuwenhuizen, et al. (Eds.), Supergravity, North-Holland, Amsterdam, 1980, p. 315;
T. Yanagida, in: O. Sawada, A. Sugamoto (Eds.), Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe, KEK, Tsukuba, Japan, 1979, p. 95;
S.L. Glashow, The future of elementary particle physics, in: M. Lévy, et al. (Eds.), Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons, Plenum Press, New York, 1980, pp. 671–687;
R.N. Mohapatra, G. Senjanović, Phys. Rev. Lett. 44 (1980) 912.
[18] T. Ibrahim, P. Nath, Phys. Rev. D 58 (1998) 111301;
T. Ibrahim, P. Nath, Phys. Rev. D 60 (1999) 099902, Erratum;
K.S. Babu, B. Dutta, R.N. Mohapatra, Phys. Rev. D 65 (2002) 016005.
[19] M. Masip, A. Rasin, Nucl. Phys. B 460 (1996) 449.
[20] G.C. Branco, F. Kruger, J.C. Romao, A.M. Teixeira, JHEP 0107 (2001) 027;
C. Hugonie, J.C. Romao, A.M. Teixeira, JHEP 0306 (2003) 020, hep-ph/0304116.
[21] G.C. Branco, D. Emmanuel-Costa, J.C. Romao, hep-ph/0604110.
[22] L. Lavoura, Int. J. Mod. Phys. A 9 (1994) 1873.