

Measurements of new physics in $B \rightarrow \pi\pi$ decays

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If new physics (NP) is present in $B \rightarrow \pi\pi$ decays, it can affect the isospin $I = 2$ or $I = 0$ channels. In this paper, we discuss various methods for detecting and measuring this NP. The techniques have increasing amounts of theoretical hadronic input. If NP is eventually detected in $B \rightarrow \pi\pi$ —there is no evidence for it at present—one will be able to distinguish $I = 2$ and $I = 0$, and measure its parameters, using these methods.

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I. INTRODUCTION

The decay $B_d^0 \rightarrow \pi^+ \pi^-$ was originally thought to be dominated by a single weak decay amplitude, the tree amplitude. As such, it could be used to extract the Cabibbo-Kobayashi-Maskawa (CKM) angle α [1]. However, it was shown that this decay also receives significant contributions from the penguin amplitude, which has a different weak phase [2]. This is referred to as “penguin pollution.” Because of this, α cannot be obtained cleanly from measurements of $B_d^0(t) \rightarrow \pi^+ \pi^-$.

However, in 1990 it was shown that it is possible to use isospin to remove the penguin pollution and extract α [3]. Briefly, the argument goes as follows. In $B \rightarrow \pi\pi$ decays, the final $\pi\pi$ state can have only isospin $I = 0$ or $I = 2$. Thus, given that the standard model (SM) weak Hamiltonian has $\Delta I = 1/2$ or $3/2$, there are only two decay paths, implying that the amplitudes for the three decays $B^+ \rightarrow \pi^+ \pi^0$, $B_d^0 \rightarrow \pi^+ \pi^-$ and $B_d^0 \rightarrow \pi^0 \pi^0$ obey a triangle relation. In terms of the isospin amplitudes A_0 ($I = 0$) and A_2 ($I = 2$), the amplitudes can be decomposed as [3]

$$\begin{aligned} -\sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0) &= 3A_2, \\ -A(B_d^0 \rightarrow \pi^+ \pi^-) &= A_2 + A_0, \\ -\sqrt{2}A(B_d^0 \rightarrow \pi^0 \pi^0) &= 2A_2 - A_0, \end{aligned} \quad (1)$$

where A_0 and A_2 include both weak and strong phases. The amplitudes for the CP -conjugate processes are obtained from the above by changing the sign of the weak phases. From these expressions, we see that the triangle relation is $\sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0) = A(B_d^0 \rightarrow \pi^+ \pi^-) + \sqrt{2}A(B_d^0 \rightarrow \pi^0 \pi^0)$, and similarly for the CP -conjugate decays.

In the SM, the electroweak-penguin (EWP) contributions to $B \rightarrow \pi\pi$ decays are expected to be very small, so that the amplitude A_2 has a well-defined weak phase: $A_2 = |A_2| \exp(i\gamma) \exp(i\delta_2)$, where γ and δ_2 are the weak and strong phases, respectively. In the CP -conjugate process, the $I = 2$ contribution is thus simply $|A_2| \exp(-i\gamma) \times$

$\exp(i\delta_2)$. On the other hand, due to the presence of the penguin amplitude, A_0 does not have a well-defined weak phase. It can be written $A_0 = |A_0| \exp(i\theta)$, where $|A_0|$ and θ are complicated functions of both weak and strong phases. The $I = 0$ contribution to the CP -conjugate processes is therefore $|\bar{A}_0| \exp(i\bar{\theta})$, where $|\bar{A}_0|$ and $\bar{\theta}$ differ from $|A_0|$ and θ . We can now count the total number of theoretical parameters. Setting $\delta_2 = 0$ (an overall phase is not physical), we find there are six: $|A_2|$, $|A_0|$, $|\bar{A}_0|$, θ , $\bar{\theta}$ and γ . However, there are also six independent experimental measurements [4]: three average branching ratios (B_{+0} , B_{+-} and B_{00}), two direct CP asymmetries (C_{+-} and C_{00} ; the direct CP asymmetry C_{+0} is expected to vanish in the SM), and one interference CP asymmetry (S_{+-}). (The subindices refer to the charges of the physical pions in the final state.) We can therefore obtain all theoretical parameters, up to discrete ambiguities. In particular, combining the phase of B_d^0 - \bar{B}_d^0 mixing (β), we can extract $\alpha = \pi - \beta - \gamma$.

Obviously, it is also possible to extract the other five theoretical parameters. This was shown explicitly by Charles [5]. In his analysis, a diagrammatic decomposition of the $B \rightarrow \pi\pi$ amplitudes was used:

$$\begin{aligned} -\sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0) &= T + C, \\ -A(B_d^0 \rightarrow \pi^+ \pi^-) &= T + P, \\ -\sqrt{2}A(B_d^0 \rightarrow \pi^0 \pi^0) &= C - P. \end{aligned} \quad (2)$$

Here, T and C are (nominally) the color-allowed and color-suppressed tree amplitudes, respectively, while P is the penguin contribution. All three contain both weak and strong phases. The one complication here is that, while T and C are both proportional to $V_{ub}^* V_{ud} \sim \exp(i\gamma)$, the penguin amplitude contains three contributions, corresponding to the flavor of the internal quark. These can be written

$$\begin{aligned} &P_u V_{ub}^* V_{ud} + P_c V_{cb}^* V_{cd} + P_t V_{tb}^* V_{td} \\ &= (P_u - P_c) V_{ub}^* V_{ud} + (P_t - P_c) V_{tb}^* V_{td} \end{aligned} \quad (3)$$

where the P_i ($i = u, c, t$) contain only strong phases. Above, the unitarity of the CKM matrix has been used to eliminate the P_c term. The $(P_u - P_c)$ term can be absorbed in the definitions of T and C , so that

$$\begin{aligned} T &= [T_{tr} + (P_u - P_c)]V_{ub}^*V_{ud}, \\ C &= [C_{tr} - (P_u - P_c)]V_{ub}^*V_{ud}, \end{aligned} \quad (4)$$

where T_{tr} and C_{tr} denote the pure tree-level color-allowed and color-suppressed tree contributions, respectively. These contain only strong phases; the weak phases have been factored out. Thus, the diagrams T and C of Eq. (2) actually contain both tree and $(P_u - P_c)$ penguin contributions, while the penguin diagram P is actually $(P_t - P_c)V_{tb}^*V_{td}$.

With these (re)definitions, all three diagrams of Eq. (2) have well-defined weak phases—the phase of T and C is γ , while that of P is $-\beta$. Thus, in this parametrization, taking into account the phase of $B_d^0-\overline{B}_d^0$ mixing (β), there are again six parameters: the magnitudes $|T|$, $|C|$ and $|P|$, two relative strong phases, and the weak phase $\alpha = \pi - \beta - \gamma$. As above, it is possible to extract all theoretical parameters.

Having shown that it is possible to extract α and the remaining theoretical parameters from $B \rightarrow \pi\pi$ decays within the SM using an isospin analysis, it is natural to ask how things are affected should physics beyond the SM be present. In particular, we want to ascertain whether it is possible to detect the new physics (NP) and measure its parameters. If so, we want to perform the relevant fits using present data.

We have noted above that there are two isospin amplitudes in $B \rightarrow \pi\pi$, $I = 0$ and $I = 2$. Thus, if NP is present, it can contribute to either of these isospin channels. However, in a recent paper [6], we showed that if no assumptions are made concerning hadronic parameters, then the $I = 0$ NP amplitude can never be detected, and only one piece of the $I = 2$ NP amplitude can be seen. The main aim of this paper is to investigate what happens if one adds hadronic input. In particular, we consider the addition of theoretical assumptions about the size of $|C/T|$ and $|P/T|$, as well as the strong phases of T and the NP contributions. (The former assumptions are on stronger footing than the latter.)

The decompositions in terms of isospin amplitudes (A_0, A_2) and diagrams (T, C, P) are equivalent. Throughout this paper we will use a mixed notation, in which the SM is described by diagrams, but new physics is described in terms of isospin amplitudes.

Note that in this paper we assume that the isospin triangles close, even in the presence of NP. However, this can be obviated by electromagnetic interactions. For example, the authors of Ref. [7] consider the decays $B \rightarrow \pi\pi\gamma$ and note that the triangles will not close if the final-state photon is undetected. The electromagnetic effects are found to be about 2.5% at the amplitude level. An alter-

native, more general, analysis is presented in Ref. [8], in which an additional weak-interaction $\Delta I = 5/2$ piece is considered. This can be generated by electromagnetic interactions, among other things. Here it is noted that the triangles will not close if such an additional piece is present. Throughout the present paper, we therefore assume that the NP effects are large enough that the $\Delta I = 5/2$ amplitude can be ignored, and the isospin triangles close.

In Sec. II, we examine the SM explanation of current $B \rightarrow \pi\pi$ data, with no theoretical hadronic input. We find a good fit in this case. In Sec. III, we add theoretical hadronic estimates of $|C/T|$ and $|P/T|$. In this case, there is a discrepancy in the SM fit, but only at the level of about 1.5σ . We then consider the addition of new physics. If no theoretical input is added (Sec. IV), we again obtain a good fit. We consider the addition of such input in Sec. V. We parametrize the NP using reparametrization invariance. Here we fit for $I = 2$ and $I = 0$ NP separately. We find that the addition of $I = 2$ NP gives a fit which is not great, but still acceptable. There are not enough observables to measure the $I = 0$ NP parameters. In order to make more progress, we change parametrizations in the following sections. We assume that the NP strong phases are all small in Sec. VI. We find a good fit for $I = 2$ NP, but still cannot fit for $I = 0$ NP. Finally, in Sec. VII, we add a third piece of theoretical hadronic input, that the phases of T and the NP are equal. In this case, we are able to measure both $I = 2$ and $I = 0$ NP parameters, obtaining good fits in both cases. We conclude in Sec. VIII.

II. STANDARD MODEL: NO HADRONIC INPUT

We begin with a review of the standard-model fit to current measurements. In order to easily compare with the case of new physics in subsequent sections, our analysis is slightly different from what is usually presented.

As discussed in the introduction, the $B \rightarrow \pi\pi$ measurements alone allow one to extract the CP phase α . However, it is possible to measure the weak phases independently. The phase β has already been measured very precisely in $B_d^0(t) \rightarrow J/\psi K_S$: $\sin 2\beta = 0.726 \pm 0.037$ [9]. Note that this value is consistent with SM expectations. If new physics is present in $B \rightarrow \pi\pi$ decays, it could affect $B_d^0-\overline{B}_d^0$ mixing. However, the data show that, even if NP is present in $B_d^0-\overline{B}_d^0$ mixing, its effect on β is minimal. The measurement of $\sin 2\beta$ determines 2β up to a twofold ambiguity. Here we assume that $2\beta \approx 47^\circ$, or $\beta = 23.5^\circ$, in agreement with the SM.

The phase γ can in principle be measured without penguin pollution through CP violation in decays such as $B \rightarrow DK$ [10]. Alternatively, γ can be obtained from a fit to a variety of other measurements, some non- CP violating. The latest analysis gives $\gamma = 58.2_{-5.4}^{+6.7}$ [11]. Note that this error includes uncertainties in theoretical quantities.

TABLE I. Branching ratios, direct CP asymmetries C_f , and interference CP asymmetries S_f (if applicable) for the three $B \rightarrow \pi\pi$ decay modes. Data come from Refs. [12–14]; averages (shown) are taken from Ref. [15].

| | $BR[10^{-6}]$ | C_f | S_f |
|-------------------------------|-----------------|------------------|------------------|
| $B^+ \rightarrow \pi^+ \pi^0$ | 5.5 ± 0.6 | 0.02 ± 0.07 | |
| $B^0 \rightarrow \pi^+ \pi^-$ | 4.6 ± 0.4 | -0.37 ± 0.10 | -0.50 ± 0.12 |
| $B^0 \rightarrow \pi^0 \pi^0$ | 1.51 ± 0.28 | -0.28 ± 0.39 | |

For the purposes of the fits, we assume symmetric errors, and take $\gamma = (58.2 \pm 6.0)^\circ$.

It is therefore possible to obtain β and γ (or $\alpha = \pi - \beta - \gamma$) independently. Here and below, we assume that the SM weak phases are already known, having been obtained from other decays. The SM analysis therefore consists of a fit to all $B \rightarrow \pi\pi$ data, along with these independent determinations of the CP phases. The latest $B \rightarrow \pi\pi$ measurements are shown in Table I.

Performed in this way, the fit yields a single solution for the five unknown theoretical parameters (Table II). We find $\chi_{\min}^2/\text{d.o.f.} = 0.27/2$. We therefore see that the SM can acceptably account for the data (leaving aside the somewhat small value of $\chi_{\min}^2/\text{d.o.f.}$).

III. STANDARD MODEL: HADRONIC INPUT

There is one potentially worrisome aspect about the fit described in the previous section: naive theoretical estimates of the relative sizes of the magnitudes of the amplitudes yield $|C_{ir}/T_{ir}| \sim |(P_t V_{ib}^* V_{td})/(T_{ir} V_{ub}^* V_{ud})| \sim 0.2$ [16]. (T_{ir} and C_{ir} are the tree-level color-allowed and color-suppressed amplitudes, respectively.) While the $|P/T|$ ratio in Table II is in line with these expectations, $|C/T|$ is quite a bit larger than expected. We therefore see that, although the SM can account for the $B \rightarrow \pi\pi$ data, it appears to give hadronic parameters which are at odds with theoretical expectations. It is therefore worthwhile to re-examine the fit, including the theoretical hadronic inputs of $|C/T|$ and $|P/T|$.

Consider first the ratio $|C/T|$:

$$\left| \frac{C}{T} \right| = \left| \frac{C_{ir} - (P_u - P_c)}{T_{ir} + (P_u - P_c)} \right|. \quad (5)$$

A numerical value of this ratio depends on estimates of the magnitudes and strong phases of C_{ir}/T_{ir} and $(P_u - P_c)/T_{ir}$. The theoretical estimate of $|C_{ir}/T_{ir}|$ is ~ 0.2 [16]. We add a theoretical uncertainty and take $|C_{ir}/T_{ir}| =$

TABLE II. Results of a fit within the SM. Inputs are the $B \rightarrow \pi\pi$ data (Table I) and independent determinations of the CP phases. All angles are in degrees.

| $ T $ | $ C $ | $ P $ | $\delta_C - \delta_T$ | $\delta_P - \delta_T$ |
|----------------|----------------|---------------|-----------------------|-----------------------|
| 21.8 ± 1.1 | 18.5 ± 1.9 | 5.6 ± 1.9 | -66.6 ± 13.6 | -52.9 ± 20.7 |

0.2 ± 0.1 . We also have a theoretical estimate of $|(P_t V_{ib}^* V_{td})/(T_{ir} V_{ub}^* V_{ud})|$ [16]. Including an error, it is taken to be 0.2 ± 0.1 . However, it is expected that P_u and P_c are somewhat smaller than P_t . We assume no particular cancellations, and take $|(P_u - P_c)/T_{ir}| = 0.1 \pm 0.1$.

For the strong phases, we proceed as follows. All strong phases are due to rescattering from intermediate states [17]. Consider first the SM diagram P_c [Eq. (3)]. Its strong phase arises principally from the rescattering of the $\bar{b} \rightarrow \bar{c}c\bar{d}$ tree diagram, T_c . That is, the $(V - A) \times (V - A) T_c$ ($\bar{b} \rightarrow \bar{c}c\bar{d}$) rescatters into the $(V - A) \times V P_c$ ($\bar{b} \rightarrow \bar{u}u\bar{d}$). The Wilson coefficients in the effective Hamiltonian indicate that the size of the rescattered P_c amplitude is only about 5%–10% of that of the T_c amplitude. Thus, rescattering costs a factor of about 10–20. However, the Wilson coefficients also show that the T_c diagram is about 10–20 times as big as the P_c penguin diagram. Thus, the rescattered T_c contributes a strong phase of $O(1)$ to P_c .

There is also a contribution to the strong phase of P_c from rescattering from states created by P_c itself (“self-rescattering”). However, since rescattering costs a factor of 10–20, the strong phase generated from this self-rescattering is much smaller than that generated by rescattering from T_c . This effect is therefore subdominant, and the strong phase from this self-rescattering can be neglected.

The diagram P_u [Eq. (3)] also receives a large $O(1)$ strong phase due to rescattering from the $\bar{b} \rightarrow \bar{u}u\bar{d}$ tree diagram, T_u . Self-rescattering of P_u is negligible.

We therefore see that the principal source of strong phases is from rescattering from the tree diagrams T_c and T_u . Thus, the strong phases of P_u and P_c are large and can take essentially any value. On the other hand, P_t is essentially real and the strong phase of T_{ir} is very small (it is pure self-rescattering). Finally, the strong phase of the diagram C_{ir} comes from rescattering from T_c and is smaller than that of P_u/P_c , but not negligible. We take it to be 0 ± 0.2 .

The ratio $|P/T|$ is given by

$$\left| \frac{P}{T} \right| = \left| \frac{(P_t - P_c)V_{ib}^* V_{td}}{[T_{ir} + (P_u - P_c)]V_{ub}^* V_{ud}} \right|. \quad (6)$$

The same inputs as above are necessary for the estimate of this ratio.

In all cases, we assume that the magnitude ratios and strong phases have Gaussian and flat distributions, respectively. Allowing all quantities to vary in their allowed ranges according to their (assumed) distributions, we find the following estimates:

$$r_C = \left| \frac{C}{T} \right| = 0.3 \pm 0.2, \quad r_P = \left| \frac{P}{T} \right| = 0.2 \pm 0.1. \quad (7)$$

These are the only two theoretical inputs of ratios of magnitudes of amplitudes, and they are thought to be reasonably reliable. In performing the fit in this section

TABLE III. Results of a fit within the SM. Inputs are the $B \rightarrow \pi\pi$ data (Table I), independent determinations of the CP phases, and the theoretical expectations of Eq. (7). All angles are in degrees.

| $ T $ | $ C $ | $ P $ | $\delta_C - \delta_T$ | $\delta_P - \delta_T$ |
|----------------|----------------|---------------|-----------------------|-----------------------|
| 22.3 ± 1.0 | 16.7 ± 2.0 | 5.3 ± 1.5 | -61.6 ± 15.1 | -51.7 ± 20.1 |

we exclude other inputs, which have to do with estimates of the sizes of individual diagrams and/or strong phases, and which are considered to be less solid. [Note that the above procedure gives $\delta_C - \delta_T = (0 \pm 70)^\circ$. This is consistent with the result found in Table II.]

We now redo the SM fit, including the theoretical input of Eq. (7) above. We again find a single solution for the five unknown theoretical parameters (Table III). The results are similar to those found for the fit without theoretical expectations (Table II). However, in this case, we have a much worse fit: $\chi^2_{\min}/\text{d.o.f.} = 6.7/4$.

One explanation is statistical fluctuation. The solution is driven by the unexpectedly large branching ratio of $B^0 \rightarrow \pi^0\pi^0$. As more data is taken, this branching ratio might decrease, giving a fit which is more in line with theoretical expectations. (Note that $\chi^2_{\min}/\text{d.o.f.} = 6.7/4$ only corresponds to a deviation of about 1.5σ . Thus, we must stress that the statistical discrepancy in the SM fit is not very strong at present.) A second explanation is that the theoretical expectation of $|C/T|$ is wrong. For example, this can happen if P_u is larger than expected [18] (if P_c is larger, the size of P will in general be affected). The point is that the naive estimates of $|C_{tr}/T_{tr}| \sim |P_t/T_{tr}| \sim 0.2$ are for the amplitudes uncontaminated by penguin contributions, whereas the T , C and P amplitudes which appear in the fit all contain P_u and/or P_c contributions [Eqs. (3) and (4)]. If P_u is sizeable, and if the strong phases are such that there is destructive interference between the color-allowed tree amplitude T_{tr} and $P_u - P_c$ (for T) and/or constructive interference between the color-suppressed tree amplitude C_{tr} and $P_u - P_c$ (for C), one can account for the large $|C/T|$ ratio. However, this solution is somewhat fine-tuned, and we have no idea why P_u should be larger than expected.

A third explanation, which is the one investigated in this paper, is that new physics is manifesting itself in the $B \rightarrow \pi\pi$ data.

IV. NEW PHYSICS: NO HADRONIC INPUT

As discussed in the introduction, the $B \rightarrow \pi\pi$ amplitudes can be written in terms of isospin amplitudes or diagrams. In the SM these are related as

$$\begin{aligned} A_2 &= \frac{1}{3}(T + C)e^{i\gamma}, & \bar{A}_2 &= \frac{1}{3}(T + C)e^{-i\gamma}, \\ A_0 &= \frac{1}{3}(2T - C)e^{i\gamma} + Pe^{-i\beta}, \\ \bar{A}_0 &= \frac{1}{3}(2T - C)e^{-i\gamma} + Pe^{i\beta}, \end{aligned} \quad (8)$$

where the diagrams T , C , P include strong phases, but their weak phases have now been written explicitly. We therefore see that there are only two areas in which new physics can enter: the $I = 0$ or $I = 2$ amplitudes. In the most general case, these NP contributions have arbitrary phases, so that the isospin amplitudes can now be written

$$\begin{aligned} A_2 &= \frac{1}{3}(T + C)e^{i\gamma} + N_2, \\ \bar{A}_2 &= \frac{1}{3}(T + C)e^{-i\gamma} + \bar{N}_2, \\ A_0 &= \frac{1}{3}(2T - C)e^{i\gamma} + Pe^{-i\beta} + N_0, \\ \bar{A}_0 &= \frac{1}{3}(2T - C)e^{-i\gamma} + Pe^{i\beta} + \bar{N}_0, \end{aligned} \quad (9)$$

where N_0 , \bar{N}_0 , N_2 , and \bar{N}_2 are complex numbers. Although the following discussion is general, it is assumed that the NP effects, if present, are roughly the same size as the $B \rightarrow \pi\pi$ penguin amplitudes. There are several reasons for this. First, from the theoretical point of view, it is most likely that the NP will affect $\bar{b} \rightarrow \bar{d}q\bar{q}$ loop-level processes. Second, if the NP were larger, of tree-level size, it is probable that we would already have seen evidence for it, through branching ratios, direct CP asymmetries, etc. Finally, if the NP amplitudes are smaller, effects due to the SM EWP amplitudes become important, and the NP will be more difficult (if not impossible) to detect.

Now, it was shown in Ref. [19] that any complex amplitude can be written in terms of two weak phases as follows:

$$\begin{aligned} N &= N_{\phi_{A1}} e^{i\phi_{A1}} + N_{\phi_{A2}} e^{i\phi_{A2}}, \\ \bar{N} &= N_{\phi_{A1}} e^{-i\phi_{A1}} + N_{\phi_{A2}} e^{-i\phi_{A2}}, \end{aligned} \quad (10)$$

where

$$\begin{aligned} N_{\phi_{A1}} &= \frac{Ne^{-i\phi_{A2}} - \bar{N}e^{i\phi_{A2}}}{2i \sin(\phi_{A1} - \phi_{A2})}, \\ N_{\phi_{A2}} &= \frac{Ne^{-i\phi_{A1}} - \bar{N}e^{i\phi_{A1}}}{2i \sin(\phi_{A2} - \phi_{A1})}. \end{aligned} \quad (11)$$

Note that the same complex numbers $N_{\phi_{A1}}$ and $N_{\phi_{A2}}$ appear in the expressions for N and \bar{N} . This means that each includes a magnitude and a strong (CP -even) phase, but that the weak phases (ϕ_{A1} and ϕ_{A2} , respectively) have been taken out explicitly.

These relations, which are known as reparametrization invariance, can be applied to N_2 and N_0 . The key point is that any two weak phases can be used. We therefore choose to express N_0 in terms of γ and $-\beta$; N_2 is written in terms of γ and another weak phase. The choice of this second weak phase is arbitrary; the only condition is that it be known. For convenience, we take it to be zero. Writing N_2 and N_0 in this way, one obtains [6]

$$\begin{aligned}
A_2 &= \frac{1}{3}(t + c)e^{i\gamma} + N_{2,0}, \\
\bar{A}_2 &= \frac{1}{3}(t + c)e^{-i\gamma} + N_{2,0}, \\
A_0 &= \frac{1}{3}(2t - c)e^{i\gamma} + pe^{-i\beta}, \\
\bar{A}_0 &= \frac{1}{3}(2t - c)e^{-i\gamma} + pe^{i\beta},
\end{aligned} \tag{12}$$

where

$$\begin{aligned}
t + c &= T + C + 3N_{2,\gamma}, & 2t - c &= 2T - C + 3N_{0,\gamma}, \\
p &= P + N_{0,-\beta},
\end{aligned} \tag{13}$$

with

$$\begin{aligned}
N_{2,\gamma} &= i \frac{\bar{N}_2 - N_2}{2 \sin \gamma}, & N_{2,0} &= \frac{\bar{N}_2 + N_2}{2} - i \frac{\bar{N}_2 - N_2}{2 \tan \gamma}, \\
N_{0,\gamma} &= \frac{\bar{N}_0 + N_0}{2} \frac{\sin \beta}{\sin(\beta + \gamma)} + i \frac{\bar{N}_0 - N_0}{2} \frac{\cos \beta}{\sin(\beta + \gamma)}, \\
N_{0,-\beta} &= \frac{\bar{N}_0 + N_0}{2} \frac{\sin \gamma}{\sin(\beta + \gamma)} - i \frac{\bar{N}_0 - N_0}{2} \frac{\cos \gamma}{\sin(\beta + \gamma)}.
\end{aligned} \tag{14}$$

Comparing the expressions for the isospin amplitudes in Eqs. (8) and (12), one notices several things [6]. First, the expressions for A_0 and \bar{A}_0 in these two equations have the same form. This implies that one cannot detect $I = 0$ NP without hadronic input. Second, the expressions for A_2 and \bar{A}_2 do not have the same form. Thus, even without hadronic input, one can detect NP, but only the $N_{2,0}$ piece. Because $N_{2,0}$ is a complex number, there are two tests for NP, related to its magnitude and phase (assuming that $N_{2,0}$ is nonzero). These are: a nonzero direct CP asymmetry C_{+0} (if the strong phase of $N_{2,0}$ is different from that of $t + c$) and a difference between the value of γ extracted from $B \rightarrow \pi\pi$ decays and that obtained from other, independent measurements.

Absorbing the NP parameters as above, we can now count the number of parameters. There are nine: the four magnitudes of t , c , p and $N_{2,0}$, three relative strong phases, and the two weak phases β and γ . Thus, if the values of the weak phases are taken from independent determinations, the seven $B \rightarrow \pi\pi$ observables are sufficient to perform a fit (this fit is taken from Ref. [6].)

As in the SM fit, we assume that the values of weak CP phases are already known. The present $B \rightarrow \pi\pi$ measure-

ments are detailed in Table I. In fitting to the data, we find four solutions, shown in Table IV. Although the second solution gives a nonzero value of $|N_{2,0}|$, the other three solutions give values which are consistent with zero. We therefore conclude that, if one pays no attention to hadronic quantities, as done in this fit, the present $B \rightarrow \pi\pi$ data exhibit no compelling signs of new physics. This is to be expected since, as shown in Table II, the SM can account for the $B \rightarrow \pi\pi$ data if hadronic theoretical expectations are ignored. That is, there is no need for NP.

Note that we obtain $\chi_{\min}^2 \text{d.o.f.} = 0.0049/0$, instead of zero as expected. This occurs because the current data are slightly inconsistent with the isospin $\{A_0, A_2\}$ description. That is, for the central values

$$\cos(\delta_2 - \delta_0) = \frac{\frac{2}{3}|A_{+0}|^2 + |A_{+-}|^2 - 2|A_{00}|^2}{2\sqrt{2}|A_{+0}||A_0|} = 1.07, \tag{15}$$

where $|A_0|$ is given by

$$|A_0|^2 = \frac{2}{3} \left(-\frac{2}{3}|A_{+0}|^2 + |A_{+-}|^2 + |A_{00}|^2 \right). \tag{16}$$

Thus, our fit gives a nonzero value of $\chi_{\min}^2 \text{d.o.f.}$

V. NEW PHYSICS: HADRONIC INPUT

In the previous section, we performed a fit without hadronic input. However, as discussed earlier, we do have some theoretical hadronic information which can be added to the fit. This is done in this section.

A. Preliminary remarks

The equations in Eqs. (12)–(14) can be inverted to give

$$\begin{aligned}
p &= P + N_{0,-\beta} = \frac{\bar{A}_0 e^{i\gamma} - A_0 e^{-i\gamma}}{2i \sin(\beta + \gamma)}, \\
t &= T + (N_{2,\gamma} + N_{0,\gamma}) = -\frac{\bar{A}_2 - A_2}{2i \sin \gamma} - \frac{\bar{A}_0 e^{-i\beta} - A_0 e^{i\beta}}{2i \sin(\beta + \gamma)}, \\
c &= C + (2N_{2,\gamma} - N_{0,\gamma}) = -2\frac{\bar{A}_2 - A_2}{2i \sin \gamma} + \frac{\bar{A}_0 e^{-i\beta} - A_0 e^{i\beta}}{2i \sin(\beta + \gamma)}, \\
N_{2,0} &= \frac{\bar{A}_2 e^{i\gamma} - A_2 e^{-i\gamma}}{2i \sin \gamma}.
\end{aligned} \tag{17}$$

TABLE IV. Results of a fit including NP, using the parametrization of Eq. (12). Inputs are the $B \rightarrow \pi\pi$ data (Table I) and independent determinations of the CP phases. We have factored out the (unphysical) overall phase $\delta_{N_{2,0}} = \arg N_{2,0}$. The magnitudes are measured in eV and the phases in degrees.

| $ t $ | $ c $ | $ p $ | $ N_{2,0} $ | $\delta_t - \delta_{N_{2,0}}$ | $\delta_c - \delta_{N_{2,0}}$ | $\delta_p - \delta_{N_{2,0}}$ |
|----------------|-----------------|----------------|----------------|-------------------------------|-------------------------------|-------------------------------|
| 6.1 ± 2.7 | 9.9 ± 13.7 | 12.9 ± 3.2 | 9.6 ± 6.5 | 81.5 ± 70.5 | -40.5 ± 90.0 | 22.3 ± 74.1 |
| 2.8 ± 2.6 | 19.8 ± 23.8 | 11.4 ± 6.6 | 13.2 ± 1.3 | 41 ± 108 | -174 ± 9 | -48.6 ± 64.2 |
| 22.8 ± 4.0 | 18.2 ± 6.7 | 7.3 ± 6.5 | 2.7 ± 9.3 | -156 ± 52 | 155 ± 32 | 157 ± 20 |
| 19.6 ± 3.9 | 6.1 ± 22.4 | 6.4 ± 1.7 | 6.0 ± 8.4 | -19.1 ± 43.9 | 68.9 ± 174 | -127 ± 35 |

Now, in the preceding sections we introduced three equivalent sets of variables. The set closest to experiment is $B_{+0}, B_{+-}, B_{00}, C_{+0}, C_{+-}, C_{00}, S_{+-}$. From this we can extract the isospin set $|A_0|, |\bar{A}_0|, |A_2|, |\bar{A}_2|, \delta_2 - \bar{\delta}_0, \delta_0 - \bar{\delta}_0, \bar{\delta}_2 - \bar{\delta}_0$, or, equivalently—c.f. Eqs. (17)—the diagrammatic set $|t|, |p|, |c|, |N_{2,0}|, \delta_t - \delta_{N_{2,0}}, \delta_c - \delta_{N_{2,0}}$, and $\delta_p - \delta_{N_{2,0}}$. These three sets are completely determined from experiment.

Equations (17) show us that if theoretical hadronic input is added involving any combination of T, C , and/or P , we will be able to probe new physics in $N_{2,\gamma}$ and/or the $I = 0$ new-physics amplitudes. In particular, one sees that assumptions about P, T , and C will allow us to probe $N_{0,-\beta}, N_{2,\gamma} + N_{0,\gamma}$, and $2N_{2,\gamma} - N_{0,\gamma}$, respectively. For example, given the experimental fit of $|p|$ and a theoretical assumption about $|P|$, $N_{0,-\beta}$ is constrained by

$$|p|^2 = |P|^2 + |N_{0,-\beta}|^2 + 2|P||N_{0,-\beta}|\cos\theta_P, \quad (18)$$

where $\theta_P = \arg(N_{0,-\beta}P^*)$. Thus, $|p| \neq |P|$ is a sign of new physics in $N_{0,-\beta}$. Indeed,

$$\begin{aligned} ||p| - |P|| &\leq |N_{0,-\beta}| \leq |p| + |P|, \\ |t| - |T| &\leq |N_{2,\gamma} + N_{0,\gamma}| \leq |t| + |T|, \\ |c| - |C| &\leq |2N_{2,\gamma} - N_{0,\gamma}| \leq |c| + |C|. \end{aligned} \quad (19)$$

We can also identify the combinations that probe $N_{2,\gamma}$ and $N_{0,\gamma}$ independently

$$\begin{aligned} ||t + c| - |T + C|| &\leq 3|N_{2,\gamma}| \leq |t + c| + |T + C|, \\ ||2t - c| - |2T - C|| &\leq 3|N_{0,\gamma}| \leq |2t - c| + |2T - C|. \end{aligned} \quad (20)$$

Recall that one cannot identify new physics in $I = 0$ without hadronic assumptions. But, as we have just shown, it takes only *one single* theoretical assumption (about $|P|$) in order to be able to probe new physics in $I = 0$.

Unfortunately, estimates of the sizes of individual diagrams are thought to be less reliable than the two theoretical inputs of ratios of magnitudes of amplitudes in Eq. (7). Using Eqs. (17) we find

$$\begin{aligned} \frac{P}{T} - \frac{p}{t} &= -\frac{N_{0,-\beta}}{T} + \frac{N_{2,\gamma} + N_{0,\gamma}}{T} \frac{p}{t}, \\ \frac{C}{T} - \frac{c}{t} &= -\frac{2N_{2,\gamma} - N_{0,\gamma}}{T} + \frac{N_{2,\gamma} + N_{0,\gamma}}{T} \frac{c}{t}, \end{aligned} \quad (21)$$

leading to

$$\begin{aligned} \left| -\frac{N_{0,-\beta}}{T} + \frac{N_{2,\gamma} + N_{0,\gamma}}{T} \frac{p}{t} \right| &\geq \left| \left| \frac{P}{T} \right| - \left| \frac{p}{t} \right| \right|, \\ \left| -\frac{2N_{2,\gamma} - N_{0,\gamma}}{T} + \frac{N_{2,\gamma} + N_{0,\gamma}}{T} \frac{c}{t} \right| &\geq \left| \left| \frac{C}{T} \right| - \left| \frac{c}{t} \right| \right|. \end{aligned} \quad (22)$$

These equations show which combinations of new-physics

parameters are probed when the experimental observable $|p/t|$ ($|c/t|$) differs from the SM theoretical prediction for $|P/T|$ ($|C/T|$, respectively). Sadly, these combinations are rather complicated and, in particular, we cannot disentangle $N_{2,\gamma}$ from the $I = 0$ new-physics contributions using exclusively theoretical predictions for the SM $|P/T|$ and $|C/T|$.

B. Fitting for the new-physics parameters

We will now consider several scenarios for the NP, attempting to fit for all free parameters available within each scenario. If we assume that both $I = 0$ and $I = 2$ NP are present, then there are 13 unknown parameters: the seven magnitudes of $T, C, P, N_{0,\gamma}, N_{0,-\beta}, N_{2,\gamma}, N_{2,0}$ and the six relative phases (recall that we assume that the SM CP phases are known). Given the seven observables, a complete fit would require six theoretical hadronic inputs, which is unmanageable. For this reason, we assume that one type of NP dominates over the other, and consider $I = 2$ and $I = 0$ NP individually.

1. New physics exclusively in $I=2$

Consider only $I = 2$ NP. There are nine unknown parameters: five magnitudes of $T, C, P, N_{2,\gamma}$ and $N_{2,0}$, and the four relative phases. With seven $B \rightarrow \pi\pi$ observables, two additional theoretical hadronic constraints are necessary in order to measure all the $I = 2$ NP parameters, which we take to be those in Eq. (7).

Before performing this fit, we can make some general observations. Recall that $N_{2,0}$ is determined from experiment. In the absence of $I = 0$ new physics, Eqs. (13) become

$$t + c = T + C + 3N_{2,\gamma}, \quad 2t - c = 2T - C, \quad p = P, \quad (23)$$

which, combined with our estimates for r_C and r_P , allow us to find explicit analytical formulas for T, C, P , and $N_{2,\gamma}$. We do not include them, since they are not very instructive, but they do provide a few interesting bounds,

$$\left| \frac{N_{2,\gamma}}{T} \right| \geq \left| 1 - \frac{r_P}{|p/t|} \right|, \quad \left| \frac{N_{2,\gamma}}{T} \right| \geq \left| \frac{r_C - |c/t|}{2 - c/t} \right|. \quad (24)$$

Moreover, dividing the last two of Eqs. (23), we find

$$r_C e^{i(\delta_C - \delta_T)} = 2 - \left(\frac{2t - c}{p} \right) r_P e^{i(\delta_P - \delta_T)}, \quad (25)$$

from which

$$\left| 2 - \left| \frac{2t - c}{p} \right| r_P \right| \leq r_C \leq 2 + \left| \frac{2t - c}{p} \right| r_P. \quad (26)$$

Given the experimental determination of t, c , and p , this constrains the choices of (r_C, r_P) which are consistent with the hypothesis that all new physics appears exclusively in $I = 2$.

TABLE V. Results of a fit assuming only $I = 2$ NP, using the parametrization of Eq. (12). Inputs are the $B \rightarrow \pi\pi$ data (Table D), independent determinations of the CP phases, and the hadronic estimates of Eq. (7). All angles are in degrees.

| $ T $ | $ C $ | $ P $ | $ N_{2,\gamma} $ | $ N_{2,0} $ |
|-------------------------------|-------------------------------|-------------------------------|--|-----------------|
| $\delta_T - \delta_{N_{2,0}}$ | $\delta_C - \delta_{N_{2,0}}$ | $\delta_P - \delta_{N_{2,0}}$ | $\delta_{N_{2,\gamma}} - \delta_{N_{2,0}}$ | |
| 24.0 ± 3.8 | 7.6 ± 5.6 | 5.5 ± 1.7 | 10.3 ± 11.4 | 0.55 ± 2.81 |
| -116 ± 72 | -116 ± 195 | 163 ± 69 | 131 ± 65 | |

We now redo the NP fit of Table IV with the theoretical hadronic input. Table V shows the result of this fit. We obtain $\chi^2_{\min} \text{d.o.f.} = 0.27/0$. Note that we expect $\chi^2_{\min} = 0$ for zero degrees of freedom (i.e. a solution to the equations). The fact that we do not obtain this indicates that the equations to be solved are inconsistent, and suggests a poor fit. However, at this point we are not certain of how poor this fit is since we cannot test the goodness of fit with zero degrees of freedom.

Fortunately, there is an alternative: consider again the four solutions of Table IV, which correspond to the inclusion of NP without theoretical hadronic input. With hadronic input, Eq. (26) should be satisfied. However, the four solutions in Table IV lead to

$$\left| \frac{2t - c}{p} \right| = 1.50 \pm 0.95, 2.17 \pm 3.30, 5.00 \pm 6.55, 6.20 \pm 1.41, \quad (27)$$

respectively. Using Eq. (26), we see that none of the central values is consistent with $(r_C, r_P) = (0.3, 0.2)$. If we relied exclusively on the central values, this would show explicitly that none of the solutions in Table IV actually provides a good fit, assuming our theoretical input is correct. However, the errors are still quite large, so that no such conclusion can be drawn.

There is another way to test the goodness of fit: we calculate

$$\cos(\delta_T - \delta_C) = \frac{4 + r_C^2}{4r_C} - \frac{r_P^2}{4r_C} \left| \frac{2t - c}{p} \right|^2. \quad (28)$$

Using the values of t , c and p given in Table IV, we find

$$\cos(\delta_T - \delta_C) = \{3.33 \pm 2.13, 3.25 \pm 2.13, 2.57 \pm 2.84, 2.10 \pm 1.93\}. \quad (29)$$

All central values are much larger than 1, suggesting a poor fit. However, when we take into account the error, the third value is (just) consistent with $\cos(\delta_T - \delta_C) \leq 1$, so that this value is acceptable. (Note that the third solution also has the smallest χ^2 .)

We therefore conclude that, with this parametrization of the new physics, in which the $I = 2$ NP has zero degrees of freedom, this type of NP is still permitted, although the fit is not perfect. There may be a small deviation of about 1σ .

This shows that it is still possible to keep $(r_C, r_P) = (0.3, 0.2)$ by adding $I = 2$ NP.

Note that it is possible to improve the fit by choosing a particular value of the NP weak phase. For instance, we could take only $N_{2,\gamma}$ or $N_{2,0}$ to be nonzero. In both of these cases, the number of degrees of freedom is increased to 2, and we obtain acceptable fits. However, we prefer not to make any assumptions about the nature of the NP (i.e. its weak phase), so we do not consider this possibility here.

2. New physics exclusively in $I=0$

The case of $I = 0$ NP is more complicated. Here there are the same number of parameters and constraints—nine—but the effect of the $B \rightarrow \pi\pi$ observables is different. First, if there is no $I = 2$ NP, the direct CP asymmetry in charged B decays is automatically zero, i.e. $C_{+0} = 0$. This means that the measurements of $BR(B^+ \rightarrow \pi^+ \pi^0)$ and $BR(B^- \rightarrow \pi^- \pi^0)$ do not provide constraints on the NP. Instead, they give two different measurements of the same quantity, $|T + C|$. In addition, in the presence of only $I = 0$ NP, the situation is analogous to that of the SM: the $I = 2$ amplitude has a well-defined weak phase, while the $I = 0$ amplitude does not. In the SM, the 6 $B \rightarrow \pi\pi$ measurements are sufficient to obtain the various parameters, including the CP phase α . That is, some combination of $B \rightarrow \pi\pi$ measurements is equal to α . However, in our analysis, α is assumed to be known independently. Therefore the combination of $B \rightarrow \pi\pi$ measurements which is equal to α does not provide additional information.

Thus, for the case in which NP appears exclusively in $I = 0$, we have only five independent $B \rightarrow \pi\pi$ constraints, so that we need *four* additional theoretical inputs to measure *all* the $I = 0$ NP parameters. Since we have only the two theoretical inputs of Eq. (7), we conclude that it is not possible to measure the $I = 0$ NP parameters.

We note that one can fit for the $I = 0$ NP parameters if one chooses only one of the two terms $N_{0,\gamma}$ or $N_{0,-\beta}$, in which case there are an equal number of parameters and constraints. However, as was the case with $I = 2$ NP, we prefer not to fix the form of the NP at this time.

Notice that the quantities t , c and p can still be determined in this case. The key point is that these are *observables*, derivable from the $B \rightarrow \pi\pi$ measurements [Eqs. (12)–(14)]. In fact, since we are setting $N_{2,0} = 0$ here, the values of t , c and p can be determined from the SM fit in Table II, with the replacements $T \rightarrow t$, $C \rightarrow c$, and $P \rightarrow p$. What we cannot do is to disentangle all the parameters T , C , P , $N_{0,\gamma}$, and $N_{0,-\beta}$ from our knowledge of t , c and p .

Fortunately, although in this case it is impossible to fit all parameters, we may still gain some information about the new physics. In the absence of $I = 2$ new physics, Eqs. (13) become

$$t + c = T + C, \quad 2t - c = 2T - C + 3N_{0,\gamma}, \quad (30)$$

$$p = P + N_{0,-\beta},$$

while Eqs. (21) become

$$\frac{P}{T} - \frac{p}{t} = -\frac{N_{0,-\beta}}{T} + \frac{N_{0,\gamma}}{T} \frac{p}{t}, \quad (31)$$

$$\frac{C}{T} - \frac{c}{t} = \frac{N_{0,\gamma}}{T} \left(1 + \frac{c}{t}\right).$$

The last equation leads to

$$\left| \frac{r_C - |c/t|}{1 + c/t} \right| \leq \left| \frac{N_{0,\gamma}}{T} \right| \leq \frac{r_C + |c/t|}{|1 + c/t|}, \quad (32)$$

meaning that, if we are lucky, a hadronic input on r_C might be enough to detect the presence of $N_{0,\gamma}$ new physics. Using the central values for r_C and for t , c , and $\delta_c - \delta_t$ from Table II, we find

$$0.35 \leq \left| \frac{N_{0,\gamma}}{T} \right| \leq 0.74. \quad (33)$$

This means that, assuming the scenario in which new physics appears exclusively in the $I = 0$ channel and that $r_C \sim 0.3$, the conclusion is that there might be evidence for new physics in $N_{0,\gamma}$.

Given the bound on $|N_{0,\gamma}/T|$ in Eq. (32), one may check whether the magnitudes r_P , $|p/t|$, and $|(N_{0,\gamma}/T)(p/t)|$ can close a triangle. If they cannot, then the first of Eq. (31) can be used to place a lower bound on $|N_{0,-\beta}/T|$, thus detecting new physics in $N_{0,-\beta}$. We conclude that, given appropriate experimental numbers and the two hadronic inputs r_C and r_P , one might be able to get information on both $|N_{0,\gamma}/T|$ and $|N_{0,-\beta}/T|$. However, the current experimental data, which implies that $|N_{0,\gamma}| \neq 0$ in this scenario—c.f. Eq. (33)—is consistent with $N_{0,-\beta} = 0$.

Summarizing, we see that the theory input $(r_C, r_P) \sim (0.3, 0.2)$, along with the current experimental results, can be accommodated by NP in the $I = 0$ channel. (Note that the errors are still very large, so that new physics is not *required*.)

VI. NEW PHYSICS: STRONG PHASES

In the previous section, we saw that, with theoretical hadronic input, $I = 2$ new physics gave a poorish fit, and one could not even fit for all the $I = 0$ NP parameters. In both cases, the difficulties could be traced to the large number of theoretical parameters. This then begs the question: is it possible to reduce the number of parameters? As we explore in this section, the answer is yes, but it requires using a different parametrization to describe the new physics.

In the discussion surrounding the SM fit, we pointed out that strong phases are due to rescattering. This has implications for the strong phases of the NP amplitudes. The

strong phase for a particular NP operator can arise only due to rescattering from itself or another NP operator. In either case, we have self-rescattering. However, it is reasonable to expect that the strong phases generated by such rescattering are not $O(1)$, but only about 5%–10%. That is, the NP strong phases are expected to be small [20]. Since we can multiply any amplitude by an arbitrary phase, a more accurate statement is that all NP amplitudes (both $I = 2$ and $I = 0$) are expected to have the same strong phase. The theoretical error incurred by this assumption is expected to be at the level of 5%–10%.

Let us illustrate these features by considering $I = 0$, for example. The $I = 0$ NP amplitude is

$$N_0 = |N_{0,a}| e^{i\Phi_{0,a}^{NP}} e^{i\delta_{0,a}^{NP}} + |N_{0,b}| e^{i\Phi_{0,b}^{NP}} e^{i\delta_{0,b}^{NP}} + \dots, \quad (34)$$

where “ a, b, \dots ” labels the various possible contributions. Within the above model, the strong phases for all contributions to a particular NP amplitude are approximately equal. In this case, one can combine all contributions into a single effective NP amplitude, with well-defined weak and strong phases:

$$N_0 \rightarrow |N_{0,\text{eff}}| e^{i\Phi_{0,\text{eff}}^{NP}} e^{i\delta^{NP}}. \quad (35)$$

Here, $\Phi_{0,\text{eff}}^{NP}$ is a weak phase which changes sign in the CP -conjugate amplitudes; δ^{NP} is the strong phase. The $I = 2$ NP amplitude has a similar expression with the same NP strong phase.

It is also possible to rewrite the $I = 0$ and $I = 2$ amplitudes using reparametrization invariance with the specific choices in Eqs. (14), but this is not terribly illuminating, so we will stick to Eq. (35).

With this form for the NP amplitude, we write

$$-\sqrt{2}A(B^+ \rightarrow \pi^+ \pi^0) = (T + C)e^{i\gamma} + 3|N_{2,\text{eff}}| e^{i\Phi_{2,\text{eff}}^{NP}} e^{i\delta^{NP}},$$

$$-A(B_d^0 \rightarrow \pi^+ \pi^-) = Te^{i\gamma} + Pe^{-i\beta} + |N_{2,\text{eff}}| e^{i\Phi_{2,\text{eff}}^{NP}} e^{i\delta^{NP}} + |N_{0,\text{eff}}| e^{i\Phi_{0,\text{eff}}^{NP}} e^{i\delta^{NP}},$$

$$-\sqrt{2}A(B_d^0 \rightarrow \pi^0 \pi^0) = Ce^{i\gamma} - Pe^{-i\beta} + 2|N_{2,\text{eff}}| e^{i\Phi_{2,\text{eff}}^{NP}} e^{i\delta^{NP}} - |N_{0,\text{eff}}| e^{i\Phi_{0,\text{eff}}^{NP}} e^{i\delta^{NP}}. \quad (36)$$

The weak phases change sign in the \bar{B} decay amplitudes. Note that, with this parametrization, $I = 2$ and $I = 0$ NP is described by a single amplitude each, instead of two as was done in the parametrization of reparametrization invariance.

As before, we first assume that both $I = 0$ and $I = 2$ NP are present. In this case, there are 11 unknown parameters: the five magnitudes $|T|$, $|C|$, $|P|$, $|N_{0,\text{eff}}|$ and $|N_{2,\text{eff}}|$, four relative strong phases, and the two NP weak phases $\Phi_{0,\text{eff}}^{NP}$ and $\Phi_{2,\text{eff}}^{NP}$. Since there are only 9 constraints (7 $B \rightarrow \pi\pi$ measurements, along with two theoretical hadronic inputs), there are more theoretical parameters than there are observables, and we cannot perform a fit. We therefore assume that one NP amplitude is dominant.

TABLE VI. Results of a fit assuming only $I = 2$ NP, using the parametrization of Eq. (36). Inputs are the $B \rightarrow \pi\pi$ data (Table I), independent determinations of the CP phases, and the hadronic estimates of Eq. (7). All angles are in degrees. We find χ^2_{\min} d.o.f. = 0.298/1.

| $ T $ $\delta_T - \delta^{NP}$ | $ C $ $\delta_C - \delta^{NP}$ | $ P $ $\delta_P - \delta^{NP}$ | $ N_{2,\text{eff}} $ $\Phi_{2,\text{eff}}^{NP}$ |
|-----------------------------------|-----------------------------------|-----------------------------------|--|
| 24.1 ± 3.8 | 7.5 ± 5.6 | 5.3 ± 1.4 | 10.3 ± 13.2 |
| 114 ± 18 | 115 ± 189 | 33.6 ± 42.8 | 59.0 ± 5.5 |

3. New physics exclusively in $I=2$

For the case in which one has only $I = 2$ NP, there are 8 theoretical parameters and 9 measurements/constraints. We can therefore perform a fit, whose results are shown in Table VI. In this case, we find a single solution, with χ^2_{\min} d.o.f. = 0.298/1. Note that this fit includes the same difficulties with $\cos(\delta_T - \delta_C)$ detailed in Eq. (28). However, since there is 1 degree of freedom in this case, we obtain a good fit.

4. New physics exclusively in $I=0$

For the case with only $I = 0$ new physics, we still cannot perform a fit. As with $I = 2$ NP, there are 8 theoretical parameters and 9 measurements/constraints. However, as discussed earlier, two of the $B \rightarrow \pi\pi$ measurements are redundant and do not probe the NP parameters. That is, for the purpose of a fit, there are effectively only 7 measurements/constraints. We can therefore not measure the NP parameters in this case.

VII. NEW PHYSICS: MORE HADRONIC INPUT

In the previous section, we saw that if we assume that the NP amplitudes are described by a single term each, we can fit for the $I = 2$ NP, but not for $I = 0$. In the latter case, we need an additional theoretical hadronic constraint.

This comes from the strong phases. We have argued that the strong phase of T is also due to self-rescattering, and should therefore be equal to that of NP. We therefore take as the third input

TABLE VII. Results of a fit assuming only $I = 2$ NP, using the parametrization of Eq. (36). Inputs are the $B \rightarrow \pi\pi$ data (Table I), independent determinations of the CP phases, and the hadronic estimates of Eqs. (7) and (37). We find χ^2_{\min} d.o.f. = 0.66/2.

| $ T $ $\delta_T - \delta^{NP}$ | $ C $ $\delta_C - \delta^{NP}$ | $ P $ $\delta_P - \delta^{NP}$ | $ N_{2,\text{eff}} $ $\Phi_{2,\text{eff}}^{NP}$ |
|-----------------------------------|-----------------------------------|-----------------------------------|--|
| 24.8 ± 4.1 | 8.1 ± 5.8 | 5.9 ± 1.3 | 7.0 ± 2.0 |
| -3.7 ± 7.5 | 10.5 ± 23.7 | -131 ± 17 | 58.9 ± 5.7 |

TABLE VIII. Results of a fit assuming only $I = 0$ NP, using the parametrization of Eq. (36). Inputs are the $B \rightarrow \pi\pi$ data (Table I), independent determinations of the CP phases, and the hadronic estimates of Eqs. (7) and (37). We find χ^2_{\min} d.o.f. = 0.84/2.

| $ T $ $\delta_T - \delta^{NP}$ | $ C $ $\delta_C - \delta^{NP}$ | $ P $ $\delta_P - \delta^{NP}$ | $ N_{0,\text{eff}} $ $\Phi_{0,\text{eff}}^{NP}$ |
|-----------------------------------|-----------------------------------|-----------------------------------|--|
| 40.0 ± 18.4 | 16.4 ± 9.3 | 8.0 ± 5.4 | 20.4 ± 17.8 |
| 4.6 ± 9.9 | -118 ± 54.5 | -28.9 ± 28.9 | -86.6 ± 23.1 |
| 40.0 ± 18.4 | 16.4 ± 9.3 | 8.0 ± 5.4 | 20.4 ± 17.8 |
| 4.6 ± 9.9 | -118 ± 54.5 | -28.9 ± 28.9 | -128 ± 18.7 |

$$\delta_T - \delta^{NP} = (0 \pm 11)^\circ, \quad (37)$$

where $I = 0, 2$. With this third input, we can perform a fit for all NP quantities, in particular, the $I = 0$ NP parameters.

In this section we perform fits to both $I = 2$ and $I = 0$ NP. In order to obtain the $I = 0$ NP parameters, we need all three theoretical inputs. For $I = 2$ NP, we need only one, but for simplicity we use all three.

For $I = 2$ NP, the fit yields one solution for $|N_{2,\text{eff}}|$ (Table VII). This solution is rather intriguing, as it yields a good value of χ^2_{\min} d.o.f. and a nonzero value of $|N_{2,\text{eff}}|$ at more than 3σ . However, we must emphasize that this conclusion must be viewed with a great deal of skepticism since this fit has much theoretical input. Indeed, though we did our best to estimate the theoretical quantities, we could easily have incorrect central values and/or errors. Still, the possibility of NP in $B \rightarrow \pi\pi$ decays is rather interesting. As we noted in the discussion of the SM (Secs. II and III), the large value of $|C/T|$ in the SM fit may be pointing to the presence of new physics. Furthermore, the latest measurements of $B \rightarrow \pi K$ seem to indicate the presence of NP in that sector [21]. The two NP solutions may be related.

In the fit for $I = 0$ NP, there are two solutions for $|N_{0,\text{eff}}|$ (Table VIII). In this case, because the number of degrees of freedom is two, both fits are acceptable.

VIII. CONCLUSIONS

If there is new physics (NP) in $B \rightarrow \pi\pi$ decays, it can affect the isospin $I = 0$ or $I = 2$ modes. In this paper, we have discussed several ways of measuring these NP parameters. If no hadronic input is used, one can never detect $I = 0$ NP, and only one piece of the $I = 2$ NP can be observed. Thus, in order to fully detect and measure the NP, it is necessary to add theoretical hadronic input.

The most obvious input consists of theoretical evaluations of the amplitude ratios $|C/T|$ and $|P/T|$, where T and C are the color-allowed and color-suppressed tree amplitudes, respectively, and P is the penguin contribution. (Note that the C and T amplitudes that appear in the $B \rightarrow \pi\pi$ also contain penguin contributions.) Given this input,

there is enough information in principle to measure the $I = 2$ NP parameters, but not those for $I = 0$. In practice, present data yields a fit for $I = 2$ NP which is not great, so that, with this parametrization of new physics, in which the $I = 2$ NP has zero degrees of freedom, this type of NP is disfavored (but not ruled out). For $I = 0$ NP, although one cannot perform a fit to measure its parameters, one can compute limits to detect its presence. The current data suggest that this type of NP might be present, but the errors are still very large.

One can then make a well-motivated assumption that the NP strong phases are all about the same size. This leads to a new parametrization of the NP with more degrees of freedom. In this case, the addition of $I = 2$ NP yields a good fit (although the presence of NP is not favored), but one can still not fit for all the $I = 0$ NP parameters.

Finally, it is then possible to add a third reasonable assumption, that the strong phases of T and the NP are roughly equal. In this case, one can again fit for $I = 2$ NP, obtaining a good fit. More importantly, there are now enough constraints to measure the $I = 0$ NP parameters, and one obtains a good fit in this case as well.

We must stress that there is no compelling evidence for NP in the current $B \rightarrow \pi\pi$ data—the SM can account for it with only a 1.5σ discrepancy. However, should statistically-significant evidence for NP be found in $B \rightarrow \pi\pi$ decays, the various methods discussed in this paper will allow us to distinguish between $I = 2$ and $I = 0$ NP, and measure its parameters.

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